SHAPE OPTIMIZATION OF NOTCHED MECHANICAL MEMBERS TO REDUCE STRESS CONCENTRATION

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ABSTRACT

SHAPE OPTIMIZATION OF NOTCHED MECHANICAL MEMBERS TO REDUCE STRESS CONCENTRATION

Intensive competition in the market place today requires engineers to continually search for better and more economic designs. Current interest especially in structural shape optimization is largely motivated by demands for more competitive designs throughout the industrial sector. Therefore, considerable effort has been devoted to developing efficient techniques for shape optimization.

The aim of this study is to optimize the shape of two-dimensional notched mechanical members to reduce stress concentration. In order to determine the best design, a search algorithm, called Direct Search Simulated Annealing (DSA) technique which seeks the best configurations through randomly generated configurations, is used. A boundary perturbation technique is used to generate random shapes. In this technique, initially, a set of design points is chosen and connected by spline curves, thereby constituting the boundary of the model. In this optimization problem, the objective function to be reduced is the maximum equivalent stress. Besides, connectivity of the geometric model, as a design constraint that should not be violated, is maintained through the optimization process.

In this thesis, a general computer code is used, which was developed using ANSYS Version 8.1 Parametric Design Language for shape optimization. This computer code was applied to three different models in order to check the effectiveness of optimization technique. The results show that this technique can be applied for successfully two dimensional shape optimization problems.

ÖZET

ÇENTİKLİ MEKANİK ELEMANLARIN GERİLME KONSANTRASYONLARINI AZALTMAK İÇİN ŞEKİL OPTİMİZASYONU

Son yıllarda, rekabetçi pazar şartları, mühendisleri sürekli daha iyi ve ekonomik tasarım yapmaya zorlamaktadır. Tüm endüstri kollarında, daha rekabetçi tasarımların oluşturulması için mevcut çalışmalar özellikle yapısal şekil optimizasyonuna yöneltilmektedir. Bu sayede de şekil optimizasyonunda daha etkin yöntemlerin oluşturulmasına olanak sağlanmaktadır.

Bu çalışmanın amacı, gerilme yığılmasını azaltmak üzere iki boyutlu çentikli mekanik elemanların şekillerini eniyilemektir. En iyi tasarımı saptamak amacıyla Doğrudan Arama Tavlama Simülasyonu yöntemi adı verilen ve rasgele oluşturulan konfigürasyonlardan en iyisinin seçimine dayalı bir arama algoritması kullanılmaktadır. Bu teknikte, ilk olarak, belirli sayıda tasarım noktası seçilir ve bu noktalar birleştirilmek suretiyle modelin sınırları tanımlanır. Bu çalışmada azaltılmaya çalışılan hedef fonksiyon, modelin maruz kaldığı azami denk gerilmedir. Ayrıca eniyileme süreci boyunca model bütünlüğünün korunması ihlal edilmemesi gereken durum olarak kabul edilmiştir.

Bu çalışmada, şekil eniyilemesi için ANSYS Versiyon 8.1'de Parametrik Tasarım Dilinde geliştirilen genel bir program kullanılmıştır. Yazılmış olan bu bilgisayar kodu şekil optimizasyonunun verimliliğini kontrol etmek amacıyla 3 farklı modele uygulanmıştır. Elde edilen sonuçlar, iki boyutlu şekil optimizasyonu problemlerine başarılı bir şekilde uygulanabileceğini göstermiştir.

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LIST OF SYMBOLS

A	Array which is subset of N configuration
A_t	Acceptability
A_{ini}	Lateral area of the initial shape
A_0	Lateral area
E	Modulus of elasticity
K_t	Stress concentration factor
L_t	Length of tth Markov chain
N	Number generated configurations
S	Search domain
S_{ref}	Reference stress
S_{max}	Maximum stress
T	Temperature
T_{t+1}	Temperature parameter of the next Markow chain
T_t	Assigned (current) temperature value
T_0	Initial temperature
f_h	Highest cost function
f_l	Lowest cost function
f_x	Cost function for point x
f_y	Cost function for point y
n	Dimension of the problem
n_k	Number of moving key points
r_s	Step size
t	Thickness of the model
α	Cost function constant
α_t	Scale factor
σ_m	Von Mises Stress
σ_{max}	Maximum equivalent stress
σ_x	Normal stress in x direction

σ_y	Normal stress in y direction
τ_{xy}	Shear stress
υ	Poisson ratio

SA	Simulated annealing
DSA	Direct search simulated annealing

1. INTRODUCTION

Intense competition in the market place today requires engineers to continually search for better and more economic designs. Manufacturing becomes the art and science of transforming materials into high quality end products with the most economic way. Engineers do not content themselves with a mere improvement of a few mechanical properties, but they strive for a global optimization of weight, rigidity, strength and cost. Accordingly, structural optimization is a major concern in the design of mechanical systems and structures in the industry (civil engineering, aeronautics and aerospace etc.). Current interest in structural shape optimization is largely motivated by demands better design with regard to cost and performance throughout these industrial sectors. The necessity of optimization arises when minimizing the amount of material, or reducing stresses in critical regions, or new space requirements are imposed. So, especially shape optimization is becoming increasingly important in mechanical engineer design. Shape optimization refers to the optimal design of the boundary of structural components. In other words, the general purpose of shape optimization is to find the best shape (and thus the most efficient and effective use of material) for a structure under various constraints imposed by the requirements of the design.

Finding the optimal shape of mechanical structures containing notches, grooves, holes etc. is the main problem for our present study. Notches are the most critical regions in mechanical members especially if it is subject to fatigue loading. Lowering maximum stress is very effective in increasing fatigue life. Our aim is to find the best shape so that the maximum stress has the lowest possible value. In this study, two-dimensional notched mechanical components are considered, e.g., plate with a shoulder [1, 2], a plate with a hole at its center [3] and a chain link [4] subject to in-plane loads.

The approach that we adopt in this study is to utilize computer aided modeling and predict the behavior of the structure through numerical analysis and optimize the structure by combining the modeling with an optimization algorithm to obtain the desired performance. The conventional design procedure based on a manual trial-and-error approach makes the design process longer and requires extensive human interaction, which

tends to be biased by the designer's intuition. This type of experience-based design approach is not efficient for designing products satisfying complex requirements in a highly competitive world. On the other hand, the modern trend is to use more and more numerical software, which simultaneously analyze and optimize many possible designs, making optimal design an automatic process. Optimization thus provides the engineers with a tool that is essential in finding the best design among countless number of designs [5].

A typical structure may contain many locally optimum shapes with a very complex design space; for this reason determining the globally optimum shape is a difficult problem. A deterministic algorithm, in which a monotonically decreasing value of an objective function is iteratively created, may get stuck into one of the local optima rather than the globally optimum design depending on the starting point. Therefore, its success depends on the choice of initial design. The usual approach is to employ the algorithm many times starting from different configurations with the hope that one of the initial positions be sufficiently close to the globally optimum configuration, and then to choose the lowest value as the globally optimum solution. Another disadvantage is that if the starting point is outside the feasible region, the algorithm may converge to a local optimum within the infeasible domain.

One of the studies based on gradient-based method with relative success was carried out by Song and Baldwin [6], who developed a node-based shape optimization method for planar structures based on the consideration that the critical stress and displacement constraints are generally located along or near the structural boundary. The proposed method puts the maximum weights on selected boundary nodes, referred to as the design points, so that the time-consuming sensitivity analysis is based on the perturbation of only these nodes. The method also allows large shape changes to achieve the optimal shape. The design variables are specified as the moving magnitudes for the prescribed design points that are always located at the structural boundary [6].

An optimization programming technique using gradient-based deterministic search algorithms often fails to find the globally minimum solution. Therefore, many researchers have tried to use some other techniques based on stochastic approaches to find global

minima. Stochastic optimization techniques are quite suitable to find the global optimum points. Among their advantages, they are not sensitive to the starting point, they can search a large solution space, and they can escape local optimum points because they allow occasional uphill moves.

The genetic algorithms are stochastic combinatorial search techniques based on concepts of genetics and Darwinian survival of the fittest. The search points on the design space are expressed as the chromosomes, which are optimized by applying the genetic operators such as the selection, the crossover, the mutation and so on [7]. Using the crossover and mutation operators, the selected chromosomes are then reproduced. The crossover operation consists in taking two selected chromosomes as parents. Then, they are either crossed by using a certain probability value in order to create two new chromosomes (children) or they are directly included into the new population [8].

The optimization scheme can be summarized as follows:

- Select parents.
- Apply genetic operators in order to create the next generation.
- Evaluate objective function and constraints.
- If the optimum has not been reached yet, repeat this process until the optimum is found or the process will be stopped by end criteria [8].

Genetic Algorithms have many advantages over traditional search methods. Among other considerations, they do not need further additional information than objective function values or fitness information. This information is used to determine the success or failure of a design in a particular environment. Moreover, its ability to find out the optimum or quasi-optimum solution gives it a privileged position as a powerful tool in non-conventional optimization problems [8].

Kirkpatrick *et al* first proposed simulated annealing as a powerful stochastic search technique. The method gets its name from the physical process whereby the temperature of a solid is raised to a melting point, where the atoms can move freely and then slowly cooled. The method attempts to model the behavior of the atoms in forming arrangements

in liquid or solid material during annealing. When a material is crystallized from its liquid phase, it must be cooled slowly if it is to assume its highly ordered, lowest-energy, perfect crystalline state. At each temperature level during this annealing process, the material should reach equilibrium. As the temperature decreases, the arrangement of the atoms gets closer and closer to the lower energy state. This process continues until the temperature finally reaches freezing point.

There is an analogy between the physical annealing process and an optimization process. Different configurations of the problem correspond to different arrangements of the atoms. The cost of a configuration corresponds to the energy of the system. Optimal solution corresponds to the lowest energy state. Just in the same manner the atoms find their way to build a perfect crystal structure through random movements, the global optimum is reached through a search within randomly generated configurations.

In the Simulated Annealing algorithm, a random initial point is selected and systematically updated until a stopping criterion is satisfied. Updating is based on an iterative procedure. In each iteration, a random point is generated in the neighborhood of the current configuration. If the new point has a smaller value of cost function compared to that of the current record, the point is accepted. This point replaces the old one. On the other hand, if the new cost function has a larger value, the acceptability of the point is decided according to the probability of Boltzman distribution. The calculation of this probability depends on a temperature parameter, T, which is referred to as temperature, because it plays a similar role in the optimization process as the temperature in the physical annealing process. The temperature is kept constant for a number of trials and then reduced. The rate of reduction should be slow so as not to get trapped at a local minimum point. At initial stages of the algorithm (at high temperatures), the probability of accepting worse designs is higher but later on at low temperatures, this probability becomes smaller and smaller so that in the end the designs having higher cost are almost never accepted [8]. Contrary to genetic algorithm, simulated annealing has a simple control parameter (temperature). Thus the optimization strategy is relatively simple.

In this study, Direct Search Simulated Annealing, which is an improved version of Simulated Annealing, is used. Direct Search Simulated Annealing (DSA) differs from

Simulated Annealing (SA) basically in two aspects. Firstly, DSA retains a set of configurations to be updated during the optimization process rather than just one configuration; secondly it always retains the best of the configurations that was generated up the current iteration.

In DSA, the benefit of using this number of different design configuration is two-fold; firstly it helps keep track of the best solution and secondly some trial points generated based on the existing configurations in the set helps the algorithm recognize the pattern of the landscape of the underlying function.

2. APPROACH TO THE PROBLEM

2.1. Objective

The aim of this study is to obtain optimum shapes for mechanical members containing notches, e.g. fillet, groove, holes subject to static loads. Model connectivity is the main constraint of the optimization problem.

2.2. Formulation of The Problem

The general representation of a shape optimization problem is shown in Figure 2.1. In this study, two-dimensional parts subject to in-plane static loads or axisymmetric parts are considered. The structure is restrained from moving at some points of the boundary. Applied loads and restraints are considered as boundary conditions.

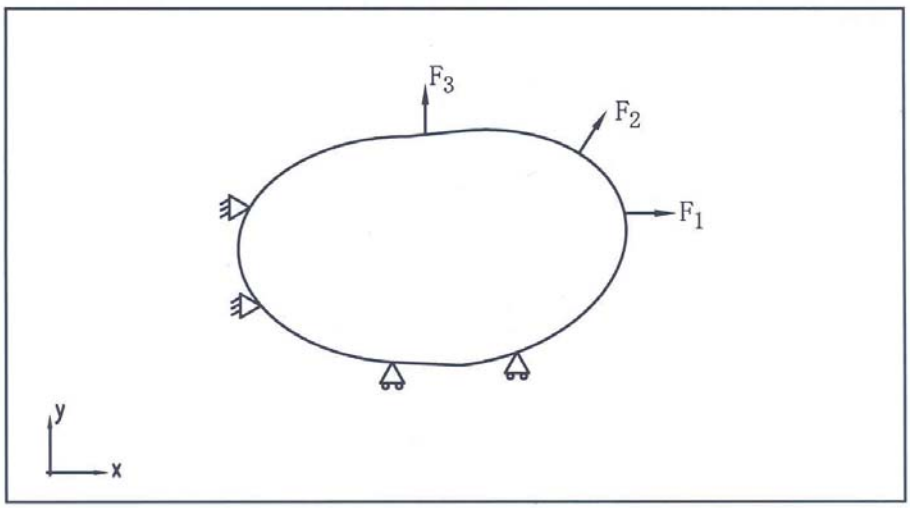

Figure 2.1. The general representation of a shape optimization problem

SA requires random generation of a new configuration (in our case, a new shape) in each iteration. Definition of a configuration should be made based on optimization variables. The algorithm tries to find the optimum values of these variables, which define the best configuration. Shape of a 2D-structure is defined by its thickness and boundary.

Accordingly, its shape can easily be described by spline curves passing through a number of keypoints. Some of these points may be fixed, while others are allowed to move during optimization. Whenever the positions of the moving keypoints are changed in random directions through random distances, a new boundary, thus a new configuration is obtained. Accordingly, the x and y coordinates of the moving keypoints become optimization variables. The design points of the initial configuration are defined by the designer with respect to a Cartesian coordinate system. The initial positions of key points including fixed, moving and restricted are given and the boundary of an initial model is formed by joining these key points by lines or spline curves. In order to prevent the key points from moving to locations far away from the initial design, their movements are restricted be within a closed region, S, to save computational time. The zone in which the moving points are able to move is defined as search domain (S).

The objective in this study is to decrease the maximum stress developed in the structure. However, in order to prevent the algorithm from generating heavy structures, either a constraint should be imposed on weight or it should be included in the cost function. Here, because the structure is two dimensional with a constant thickness and the material is homogeneous, lateral area, A_o, is included in the cost function. Then, the cost function takes the following form:

$$Cost = \frac{A_0}{A_{ini}} + \alpha\left(\frac{\sigma_{\max}}{\sigma_{allow}}\right) \tag{2.1}$$

where A_{ini} is the lateral area of the initial shape, σ_{max} is the maximum equivalent stress, σ_{allow} is the allowable stress, α is a constant and 2 was found to be an appropriate value.

Briefly, the shape optimization problem can be formulated as follows:

Given: Initial configuration, boundary conditions (applied forces and restraints), and material properties
Find: The optimal shape of a two-dimensional structure
Minimize: Maximum equivalent stress together with lateral area
Subject to: The geometric constraint (model connectivity and design domain)

2.3. Methodology

The below written procedure reflects the main steps for shape optimization and Figure 2.2 schematically depicts the steps in the solution procedure for generating the optimal shape design.

In the first step, the initial configuration of the structure (problem) should be defined. For this purpose the initial coordinates of a number of key points are specified, and they are connected by lines, splines and arcs curves to form the boundaries. Some of the key points are assigned as moving key points; so that when their positions are changed a new configuration can be obtained. Then the boundary conditions on the structure such as applied loads and displacements defined. Two-dimensional structure is defined by its boundary and thickness. Model connectivity should also be ensured; this means that the loaded portions of the structure should always remain connected to the fixed portions. ANSYS is used to check model connectivity. The material properties such as modulus of elasticity, Poisson's ratio, and allowable stress must also be known.

In the second step of the solution procedure, positions of moving key points are changed in random directions through random distances to generate a new random configuration.

In the third step, structural analysis of the newly created configuration is carried out. The cost function is calculated. In this step, the maximum stress and area are evaluated.

In the last step, the direct search simulated annealing algorithm is used to decide whether the newly created configuration is acceptable. Lastly, if the new configuration is accepted then it replaces the worst configuration.

After a number of iterations where the temperature parameter is kept constant, initial temperature parameter is decreased. These iteration blocks are called Markow chains. The iterations are continued until stopping criteria are satisfied.

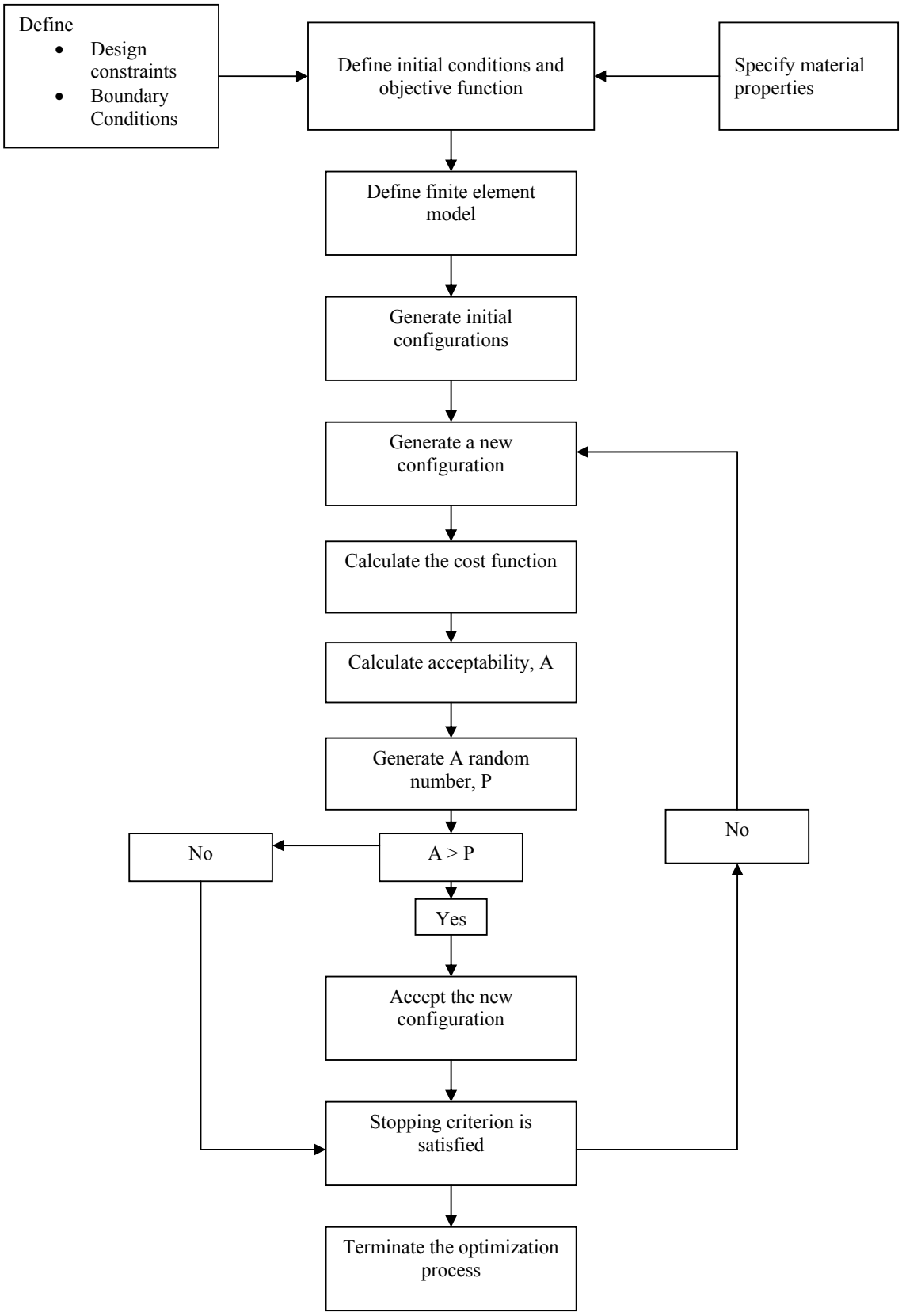

Figure 2.2. The main flow chart of the solution procedure

2.4. Outline of The Thesis

In the first chapter, the introduction of the thesis was given. In this chapter, the importance of the shape optimization, the problem and the solution techniques were explained briefly. In accordance to the solution of the problem, the literature survey was also given.

In the second chapter, the problem was described in more detail. The shape optimization problem was formulated. The main steps of solution procedure were explained and flow chart which reflects this procedure was drawn.

In the third chapter, the solution method, direct search simulated annealing, was described, and how direct search simulated annealing algorithm is applied to general optimization problem was explained. Generating configuration mechanisms, cooling schedule, Markov chain and stooping criterion which are main steps of method are also explained in this chapter. The differences between simulated annealing algorithm and direct search simulated annealing algorithm were given. Besides, the implementation of simulated annealing algorithm to shape optimization problem and model connectivity were explained lastly.

In the fourth chapter, finite element analysis of the structure is explained. Basic steps in finite element method were determined shortly. Material properties of the models and element type/size used during the analysis were given.

In the fifth chapter, results obtained from the analysis of the flat plate with shoulder and chain roller link were given and discussed. The specific material properties belonging to these two models, data used during analysis, finite element models, stress distributions and area/maximum stress temperature graphics were given respectively. During the analysis, a computer program using ANSYS version 8.1 programming language has been developed to solve the optimization problems which were explained. One program module performs the structural analysis to evaluate analysis the maximum stress value and calculates the area of each configuration. Another module carries out the optimization based on the simulated annealing approach.

And in the last chapter, conclusion is drawn. And the improvements, which can lead possible future studies on shape optimization gained from simulated annealing algorithm to the direct search simulated annealing algorithm were discussed.

3. SOLUTION PROCEDURE

Simulated annealing has been developed from the statistical thermodynamics to simulate the behavior of the atomic arrangements in liquid or solid material during the annealing process. Lowering the temperature of the melted material, the material reaches to the lowest energy level (global stable condition). Using this concept, simulated annealing has been successfully applied to large-scale combinatorial optimization problems in various fields [9].

3.1. The Direct Search Simulated Annealing Algorithm

The algorithm that is proposed in this thesis is a simulated annealing algorithm for optimization problems involving continuous variables. This is also a direct search method. The strengths of the new algorithm are;

- It does not require differentiability or any other properties of the function being optimized
- And it is memory-based.

Therefore, the algorithm can be applied to noisy and/or not exactly known functions. Although the algorithm is stochastic in nature, it can memorize the best solution. The difficult global optimization problems were solved in order to show the new simulated annealing algorithm to be reliable, fast, general purpose and efficient.

DSA is robust in terms of locating the global minimum with good accuracy. In DSA an adaptive cooling schedule is proposed and a feature is included, which allows us to retain the best solution during a run. Unlike other SA algorithms, where only one point in the Markov Chain moves in the search space, DSA associates a set of points with each Markov Chain. The benefit of using this set of points is two-fold. Firstly it helps keep track of the best solution and secondly some trial points are generated from the configuration of existing points in the set and this helps the algorithm recognize the pattern of the landscape of the underlying function [10].

The Direct Search Simulated Annealing is adopted with some modifications for optimization algorithm. The solution procedure, which is gained, after all mentioned improvements are explained as follows.

3.2. The Set of Current Configurations

Direct Search Simulated Annealing unlike ordinary Simulated Annealing starts with a set of *N* configurations rather than starting with only one configuration. The initial set of *N* configurations is selected randomly within the search domain, *S*, without any regard to cost and their cost function values are stored. These costs are calculated and stored. A boundary perturbation technique is used to generate the random configurations within this search domain.

DSA avoids gradient calculations by using a subset of *N* points stored in an array *A* [10]. The number of generated configurations depends on the dimension of the problem. The dimension of the problem, *n*, can be calculated by;

$$n = 2 \times n_k \tag{3.1}$$

where n_k is the number of moving key points. This means that for each additional moving key points, one must generate 14 more current configurations.

The x and y coordinates for all points are the variables that generate the initial geometry of the model. Number of configurations can be calculated by;

$$N = 7 \times (n + 1) \tag{3.2}$$

The coordinates of all key points including fixed, restricted and moving must be in the search domain *S*. The coordinates of key points may not exceed the values that determine by the maximum and minimum of both x and y.

3.3. Initial Geometry of The Model

Using key points, which are consisting of fixed, moving and restricted ones, stated the geometry of the model. Determining the both x and y coordinates of the key points can form the initial geometry. These points constitute the geometry by making lines, splines and curves. The different combination of joining these point forms the different configurations that we are looking for the best one whereas starting to form the new configuration rather than this initial one.

3.4. Generation Mechanisms and Restoration of The Best Solution

Starting from a randomly generated initial point $x \in S$ with the cost function f_x and with an assigned value T_t, of the temperature parameter T generates a new point $y \in S$ in the neighborhood of x and the value of the cost function f_y is calculated. The difference between these cost functions is calculated and the new configurations are generated according to the alteration. The computer program, which is developed for optimization can also calculate best, worst cost functions and difference between them. This process is repeated for a large number of iterations for T_t. The sequence of trial points generated in this way is known as the Markov chain. A new Markov chain is then generated (starting from the last accepted point in the previous Markov chain) for a reduced temperature T_{t+1} and the same process continues to decrease values of temperature until the algorithm stops. The whole process therefore consists of two loops:

- The inner loop which generates the trial points in a Markov chain
- The outer loop which reduces the temperature.

The new generation mechanism for DSA is therefore denoted by [10]

$$g_{xy} = \left\{ Random\ Generation\left(\frac{1}{m(S)}\right) \right. \tag{3.3}$$

In random generation, first a configuration is randomly selected from the set of current configurations, and then random movements are given to the moving key points, resulting in a new shape. If any of the moving key points fall outside the search domain *S*, this process is repeated until all of the moving key points fall within *S*. During movement of the keypoints if one segment of the boundary of the boundary crosses another one, or they touch each other, connectivity is lost. In that case, the generation process is repeated.

The DSA algorithm starts with *N* points and their function values stored in *A* and a single starting point of the initial Markov chain; both the *N* points and the starting point are selected at random from *S*. We call f_h the highest and f_l the lowest function values of the points in *A*. Notice that unlike local search a point generated by Eq. (3.4) does not guarantee an improved function value. The acceptability of a new configuration depends on the value of its cost function, f_y; its acceptability, A_t, is calculated by

$$A_t = \begin{cases} 1 & if\ f_y \le f_h \\ \exp\left(-(f_y - f_h)/T\right) & if\ f_y > f_h \end{cases} \tag{3.4}$$

where the trial function value, f_y, is only compared with f_h, the worst point in A. If f_y is accepted then y and f_y replace the worst point and its corresponding function value f_h in *A* and the new f_h and f_l are found in A before the process continues again. (We do not, however, order the whole array but keep the worst and the best points at the top and bottom of *A*, respectively). This process continues until the Markov chain ends. This means that the current point in the Markov chain is always in *A*. In this way, unlike the worst point, the current best point remains in *A* even if it is replaced by a new best. At high temperature it is unlikely that the *N* points will form a dense cluster. On the other hand when *T* is small, it is more likely that points will be accepted with function values less than f_h. In this case the *N* points in *A* would shrink to form a dense cluster.

During the execution of the $t.^{th}$ Markov chain with length L_t, if a point is generated whose function value is less than f_l, the best value in *A*, this ends the current chain. In other words the inner loop stops and a new Markov chain begins. This gives a signal to the procedure of the critical temperature region and call for a slow cooling. If a point cannot be

found whose function value is less than f_l in A, then the complete chain of length L_t is executed. In DSA, therefore, the choice and execution of L_t are not constant throughout the course of the algorithm [10].

3.5. The Cooling Schedule

The simulated annealing process consists of first "melting" the system being optimized at a high "temperature", T, then lowering the temperature slowly until the system "freezes" and no further changes occurs. Here, temperature, T, has no physical meaning; it is just a control parameter. Melting corresponds to the initial stage at which configurations are generated within the solution domain, S, without much regard to the cost.

In any implementation of Simulated Annealing, a cooling schedule must be specified. The temperature parameter, T, is set to an initial value T_0; this is generally relatively high, so that most trials are accepted and there is little chance of the algorithm zooming in on a local minimum in the early stages. A scheme is then required for reducing T and for deciding how many trials are to be attempted at each value of T (the length of the Markov chain). Finally a stopping criterion is required to terminate the algorithm. The choice of a cooling schedule clearly has an important bearing on the performance of a SA algorithm. We propose the following adaptive finite-time cooling schedule for DSA.

3.4.1. Initial Value of The Temperature

The value of the initial temperature parameter, T_0, should be high enough to allow nearly all trials to be accepted. This provides complete "melting" at the initial stages of the optimization process. Otherwise, the region will not be thoroughly searched and thus the algorithm may become trapped in a local minimum. In the physical analogy mentioned earlier, choosing high To corresponds to heating up the solid until all particles are randomly arranged in the liquid phase so that atoms may freely arrange themselves.

3.4.2. Length of Markov Chain

Iterations during which the value of the temperature parameter (T) is kept constant are called Markov Chain (inner loop which generates the trial configurations). The optimal value of this parameter, which has to depend on the problem size, cannot be determined in a rigorous way. Dekkers and Aarts choose the value to be

$$L_t = 10 \times n \tag{3.5}$$

where n is the problem dimension given by Eq. (3.1). Note that this choice leads to a chain length which is constant for a given problem instance n. Therefore, the length of the Markov chain in an adaptive fashion that takes into account f_l and f_h, the lowest and the highest function values in the array A is determined. Therefore, for t.[th] Markov chain is defined;

$$L_t^d = L_t + \left[L_t (1 - e^{(f_l - f_h)}) \right] \tag{3.6}$$

Clearly $L_t^d \in (L_t, 2L_t)$. This strategy allows the possibility of increasing the number of function evaluations at a given temperature if the difference between f_h and f_l increases, but, as mentioned earlier, the whole of the increased Markov chain will not be required if a point is found with function value less than f_l [10].

3.4.3. Decrement of the Control Parameter

After the initial value of the temperature parameter, To, is determined, a decrement rule must be established to find the subsequent values of the temperature parameter. The probability of reaching the global optimum solution depends on how fast the value of the temperature parameter is lowered. If the cooling rate is fast, the optimization process will probably end up with one of the higher-cost local minima. If the cooling rate is slow, the global optimum solution may be reached, but, the optimization process will take excessively long time. Therefore, the choice of the cooling schedule determines the effectiveness of the algorithm.

In DSA, a temperature scale factor (α_{t+1}), is specified to calculate the value of the temperature parameter in the next Markow chain, T_{k+1}.

$$T_{t+1} = \alpha_{t+1}\, Tt, \tag{3.7}$$

where T_k is the value of the current temperature. A value for $\alpha_{t+1} \in [\alpha_{\min}, \alpha_{\max}]$ is calculated using the following equation [10]

$$\alpha_{t+1} = \begin{cases} \alpha_{\max} & \text{if} \quad L_t > L_t' \\ \alpha_t - (\alpha_t - \alpha_{\min})(1 - L_{t-1}'/L_t) & \text{else if} \quad L_t > L_{t-1}' \\ \alpha_{\max} - (\alpha_{\max} - \alpha_t)(L_t/L_{t-1}') & \text{else} \quad L_t \le L_{t-1}' \end{cases} \tag{3.8}$$

where $L_t^{'}$, is the actual number of trials executed in the tth Markow chain. If no configuration is found in the tth Markow chain (inner loop) that is better than the best current configuration $L_t^{'}$, is set to the value of L_t, . If a configuration which is better than all is found, the inner loop is terminated, and $L_t^{'}$, is set to the number of iterations actually executed in this loop. In this study, we chose $\alpha_{\max} = 0.999$ and $\alpha_{\min} = 0.99$ as appropriate values, which provided a compromise between the two effectiveness criteria, i.e. short computational time and high reliability of locating global minimum. Initial value of α is taken as the average of $\alpha_{\max}$ and $\alpha_{\min}$.

3.6. Stopping Criterion

The stopping criterion used is based on two conditions. Firstly, the current temperature has to be small, in other words it should be close to freezing (zero) temperature; i.e. no further improvement may occur. Secondly, the points in array *A* have to form a dense cluster.

Therefore, the following criterion was chosen

$$T_t \le \varepsilon_1 \quad \textit{and} \quad f_h - f_l \le \varepsilon_2, \tag{3.9}$$

ε_1 and ε_2 should be selected enough sufficient for a good estimate of global minimum. For the optimization problems that we considered, it was more than sufficient to take $\varepsilon_1 = \varepsilon_2 = 0.0002$ for a good estimate of the global minimum.

3.7. Model Connectivity

The most important constraints defined for shape optimization problem is the model connectivity. The key points are subjected to two constraints. First of all, the spline curves of the model should never cross each other. The model connectivity should never be violated. The model should remain in one piece. If the model connectivity is violated, then the model will be separated into at least two different parts. When this stage occurs, the new obtained design is not be acceptable. In that case, the parametric language of computer program gives an error and the new configuration will be newly generated using on the previous configuration. Besides, the resulting configuration should be suitable for finite element meshing.

Checking the connectivity of the model using an ANSYS build-in function carried out in a short time. Since ANSYS contains a built-in function, it checks connectivity of the model. There is no necessity to develop any connectivity algorithm.

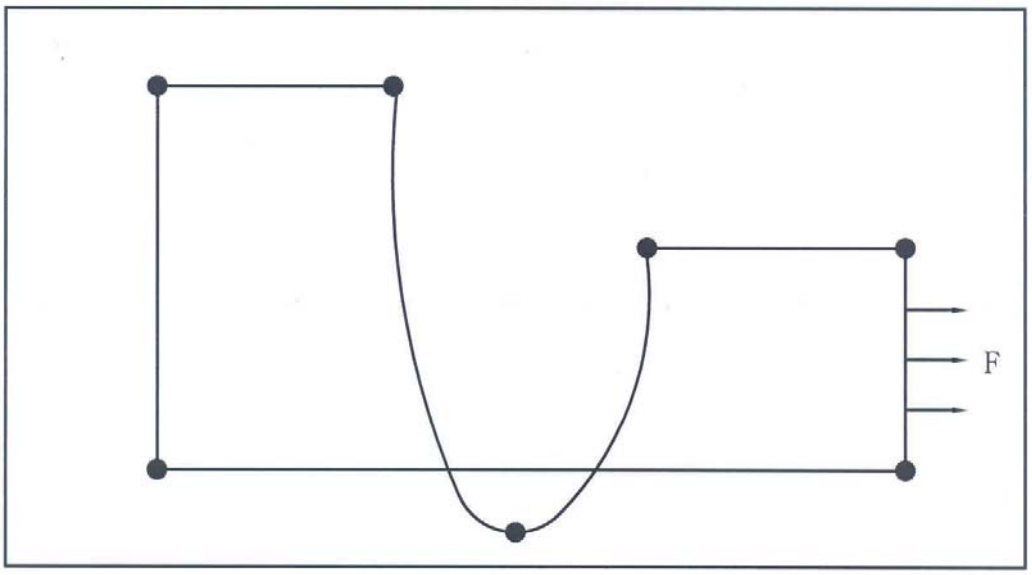

Figure 3.1. Violation of model connectivity by crossing spline segments

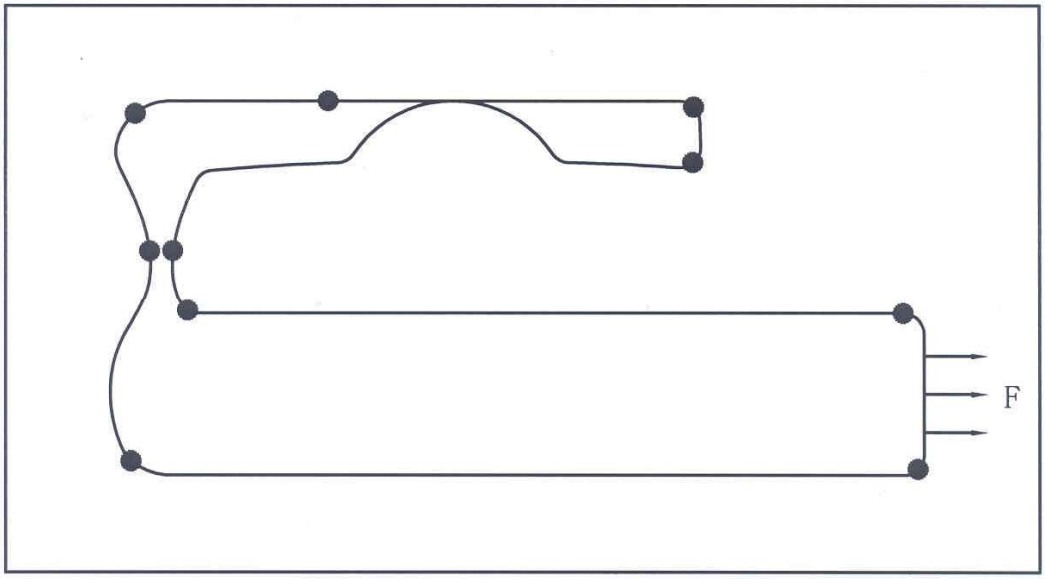

Figure 3.2. Violation of the model connectivity by crossing of design points

3.8. Material Properties & Element Size

During this study, it is assumed that the material used is isotropic, thus the elastic modulus remains the same for every direction, and the Poisson ratio is taken as 0,27.

Two-dimensional structures are analyzed using plane stress 2D-SOLID models. Because of random movements of key points, quite irregular shapes can be generated. All these irregular configurations cannot be properly meshed by using rectangular finite elements (e.g. PLANE 42), so PLANE 2 element is chosen in the finite element analysis of this study.

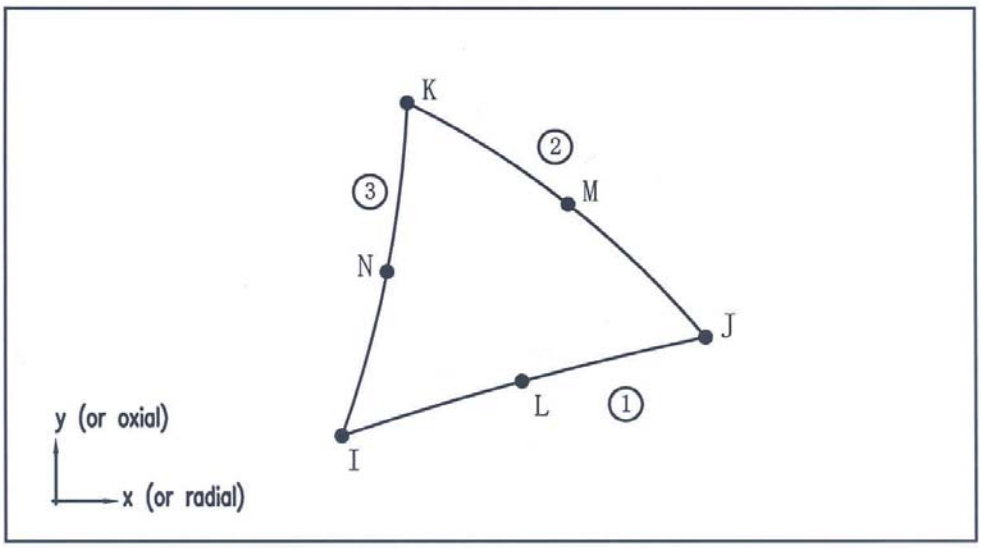

Figure 3.3. Plane2, 2D, 6 node triangular structural solid

PLANE2 element is a 6-node element triangular element compatible with 8-node PLANE82 element. The element has a quadratic displacement behavior and is well suited to model irregular meshes (such as produced from various CAD/CAM systems).

The element is defined by six nodes having two degree of freedom at each node; translations in the nodal x and y directions. It can be used as a plane element (plane stress or plane strain) or as an axisymmetric element. The element also has plasticity, creep, swelling, stress stiffening, large deflection, and large strain capabilities.

4. RESULTS AND DISCUSSIONS

In this chapter, three different numerical examples are presented and discussed to demonstrate the accuracy and effectiveness of the proposed shape optimization method. Parametric design language of ANSYS Version 8.1 was used to implement the optimization procedure of the problem. This software was also used to evaluate the maximum stress and area of the randomly generated two-dimensional models.

The first example is a plate with a shoulder subject to axial loading. This example represents a simple shaft design restrained at both one side end and whole bottom side. Second example is a chain link. And the last example is plate with a hole subject to biaxial in-plane loading. Due to symmetry, for the first problem only one half of the mechanical part, for the second and third problems one quarter of them was modeled and analyzed. During shape optimization process, only outer boundaries were allowed to move.

For the same problem, various improvements in shape were obtained by assigning different number of moving keypoints and imposing different constraints. The most important and remarkable of the generated optimum models will be discussed in the following.

All maximum stress obtained are von Mises stress. It will not be mentioned for each model again.

4.1. Plate with a Shoulder

A plate containing a shoulder (Figure 4.1) is a notched member because of sudden change in cross section. Depending on the fillet shape the maximum stress may be much higher than the nominal stress. The traditional approach is to use a fillet with a circular shape. However, because it is not the optimum shape, high stress concentrations may exist. In order to avoid this, the usual approach is to use a large radius of curvature for the circular fillet; but this may detract from the functionality of the fillet.

By optimizing the shape of a fillet one may considerably extend its fatigue. For this reason, the above mentioned optimization procedure was applied to determine the optimal shape of a shouldered plate. As shown in Figure 4.1, a distributed surface load (pressure) of 300 N/mm^2 is applied to the edge on the right side, and the both left is restrained from moving in the x direction. Because of symmetry only upper portion is analyzed to save computational time and as symmetry condition its bottom is restrained from moving in the y direction. The material properties of the structure are shown on Table 4.1.

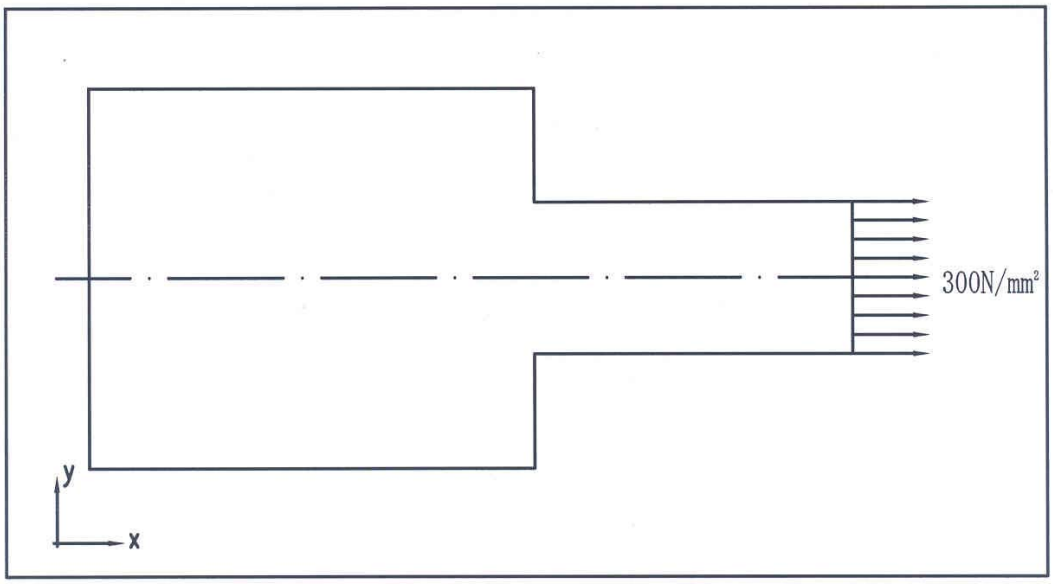

Figure 4.1. Shouldered plate subject to axial loading

Table 4.1. The material properties of the shouldered plate

Material properties	**Description**
E= 2.07 10^5 N/mm^2	Modulus of elasticity
υ= 0.28	Poisson ratio
S_{ref}= 300 N/mm^2	Reference stress
t= 1 mm	Thickness of the model
S= 300 N/mm^2	Applied surface load
9	Number of design points

By changing the number of design points, by fixing some portions of the boundary or letting them move, by imposing restrictions on the movement of the keypoints, various optimum shapes were obtained. Some of them were found to be infeasible with regard to manufacturability and functionality of the part. Here only the details of the most appropriate design will be given. Some of other results will also be discussed.

Figure 4.2 shows the finite element mesh of the initial model where Plane2 (triangular) elements are used. The meshes near the curve defining the shape of the fillet are refined to determine the magnitude of the critical stress correctly.

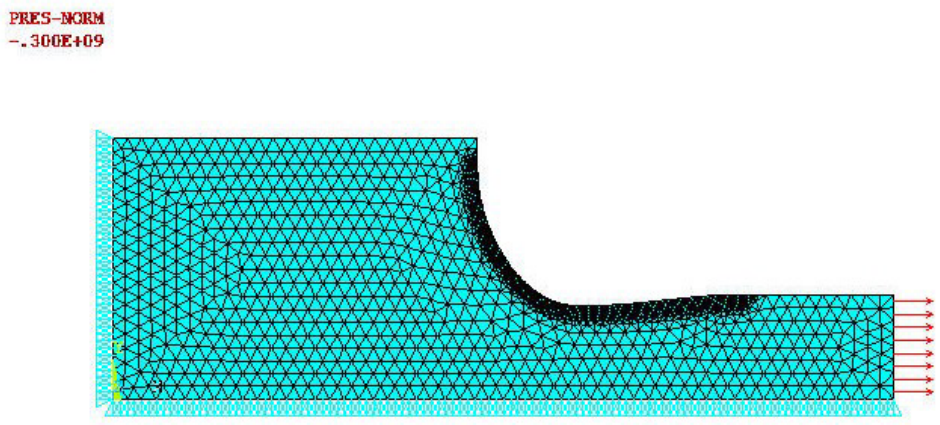

Figure 4.2. The initial configuration of the shouldered plate with its finite element mesh

Figure 4.3 represents the stress distribution in the initial model before applying the shape optimization algorithm. Initial model does not have to be close to the optimum one. The algorithm may start from any initial model and find the globally optimum design.

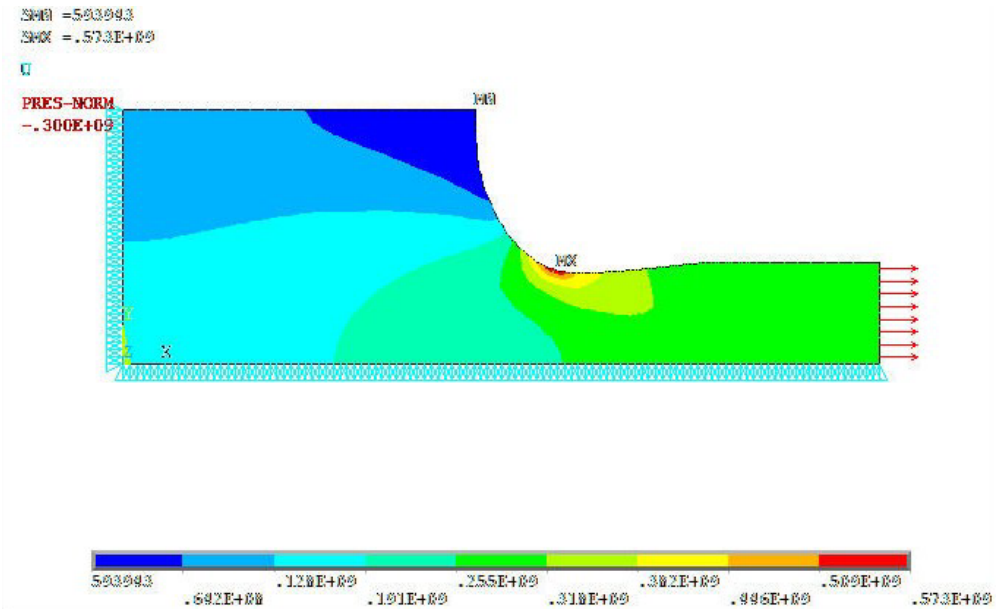

Figure 4.3. Stress distribution in the shouldered plate

The horizontal dimension of the model was chosen to be small to save analysis time. However it is large enough to obtain smooth stress distribution at the sides so that edge effects on the fillet can be avoided. Thus, even though a longer portion of the plate is analyzed, stress state at fillet will not change. Figure 4.4 shows the boundary conditions and the keypoints defining the boundary.

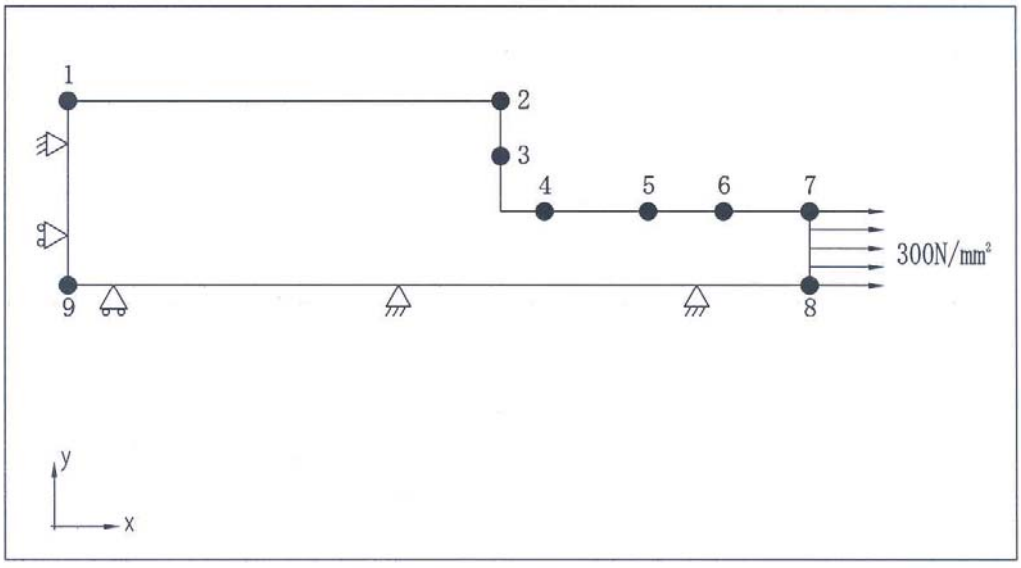

Figure 4.4. The design points and boundary conditions

Figure 4.5 represents the positions of the design points in the initial configuration. Points 4, and 9 are the moving key points. Point 1, 2, 3, 5, 6, 7, and 8 are fixed points. The load is acting on the edge represented by the line defined between points 6 and 7. The coordinates and type of the design points in the initial configuration are given in Table 4.2.

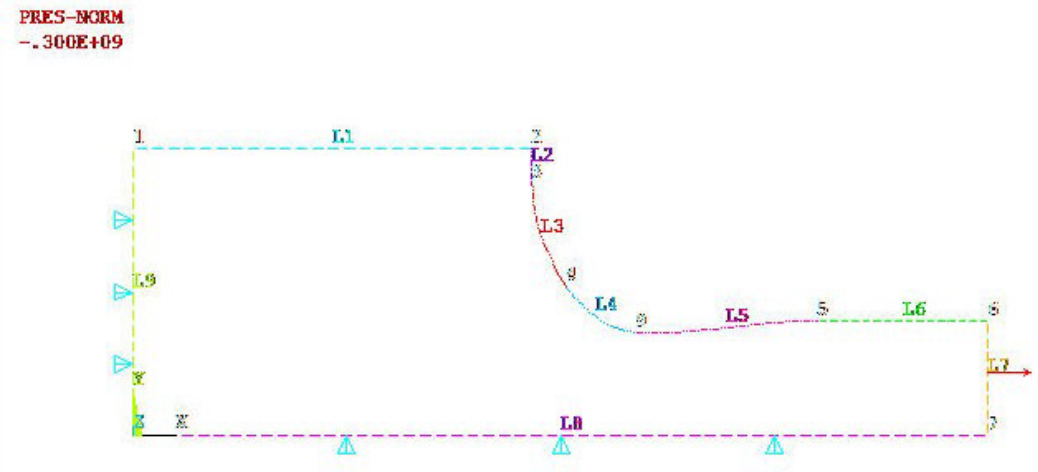

Figure 4.5. The positions of the design points of the initial configuration

Table 4.2. The coordinates and type of the design points in the initial configuration

Number of design points	**Point type**	**The coordinate of the design point**
1	Fixed	(0.0, 0.025)
2	Fixed	(0.035, 0.025)
3	Fixed	(0.035, 0.022)
4	Moving	(0.038, 0.013)
5	Fixed	(0.060, 0.01)
6	Fixed	(0.075, 0.01)
7	Fixed	(0.075, 0.0)
8	Fixed	(0.0, 0.0)
9	Moving	(0.044, 0.009)

The values of the parameters used in shape optimization of the shouldered plate are given in Table 4.3. Number of design points, dimension of the problem and minimum length of Markov chain is calculated according to the formulation given in Chapter 3.

Table 4.3. The values of the parameters used in the shape optimization of shouldered plate

Parameter value	**Description of the data**
Plane 2 (triangular)	Element type
0,0014	Element size
nk=9	Number of design points
n=2	Dimension of the problem (two times number of moving keypoints)
Lt=40	Minimum length of Markov Chain
rs=0,05	Step size
Ti=350	Initial temperature
x_{max}=0,06 x_{min}=0,03 y_{max}=0,022 y_{min}=0,0	Search domain, the area in which the moving points can move
S_{ref}=300 N/mm^2	Reference stress

In order to ensure the accuracy at which the cost was calculated, convergence of the FE solution was frequently checked during the optimization process. If the change in the magnitude of the maximum stress was greater than 1 % when FE analysis of the currently best configuration was carried out using one fourth of the current size of finite elements, then the FE mesh was refined.

Figure 4.6 gives the change in the area of the model with decrement of the temperature during optimization process. Figure 4.7 shows the magnitude of the maximum stress developed in the optimized part with decrement of the temperature.

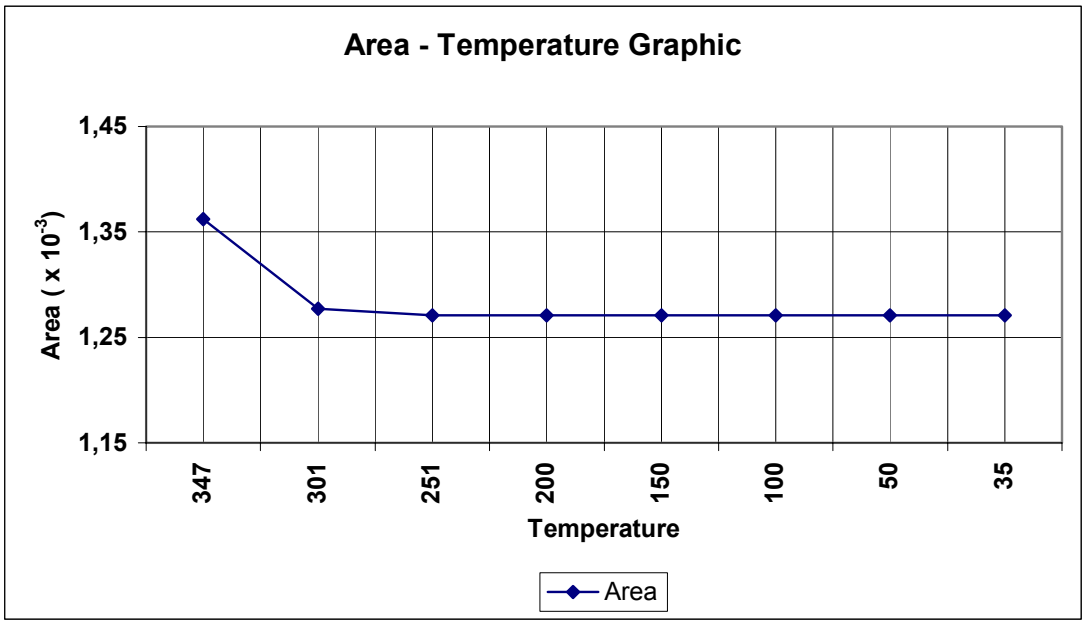

Figure 4.6. Change in the area of the shouldered plate as the temperature parameter decreases during the optimization process

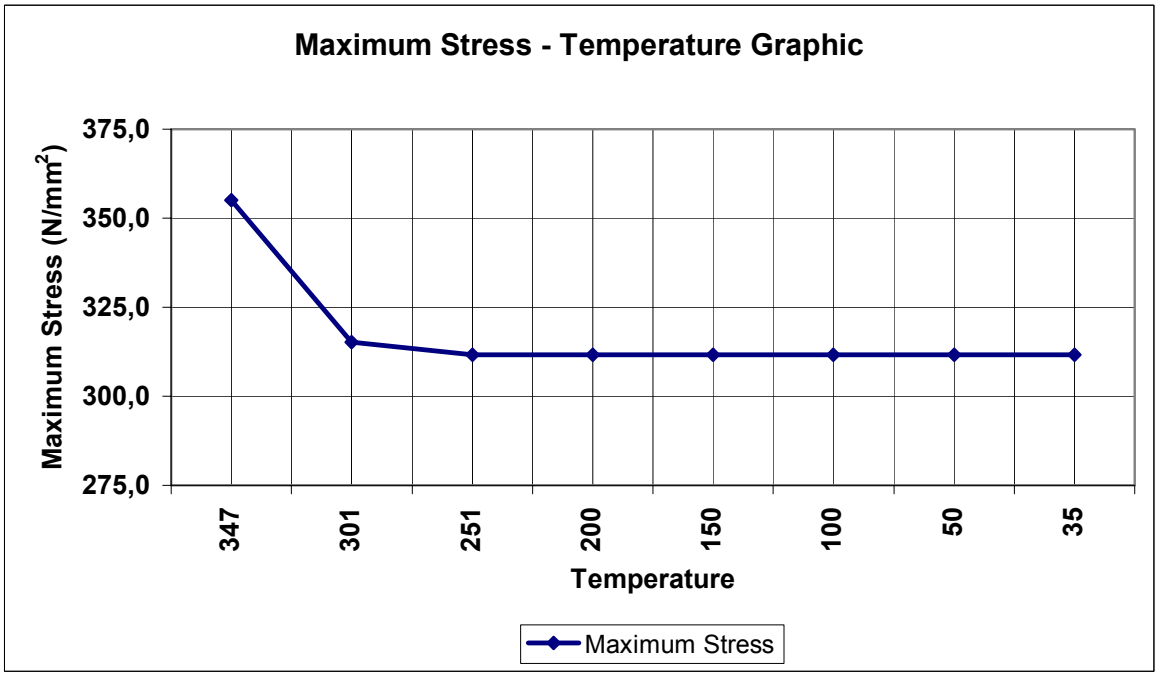

Figure 4.7. Change in the magnitude of the maximum stress developed in the shouldered plate as the temperature parameter decreases during the optimization process

In Figure 4.8, and Figure 4.9, the stress distribution in the plate under specified loading is shown. One is obtained using a mesh with the element size used during the optimization process, and the other using a finer mesh with one fourth of it.

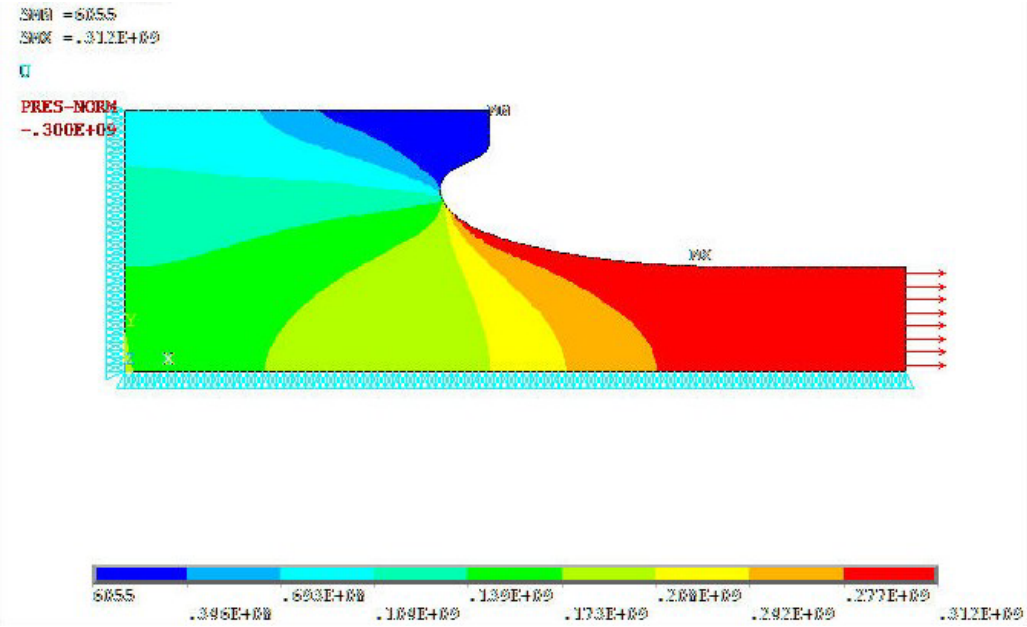

Figure 4.8. Stress distribution developed in the optimal shaped plate with shoulder

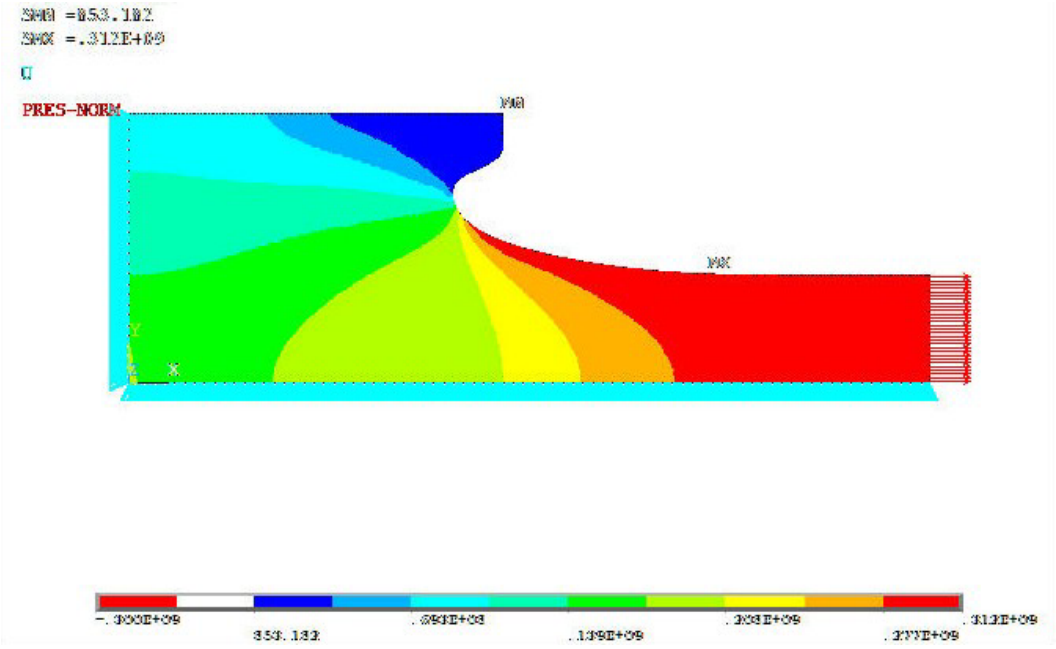

Figure 4.9. Stress distribution obtained with a finer mesh using one fourth of the element size used during optimization process

The stress concentration factor for two-dimensional shaft is given in Figure 4.10. The stress concentration factor (K_t) obtained from our optimal shape is 1.06. The graphic shows us that our K_t factor is reasonable and the optimal shape is appropriate.

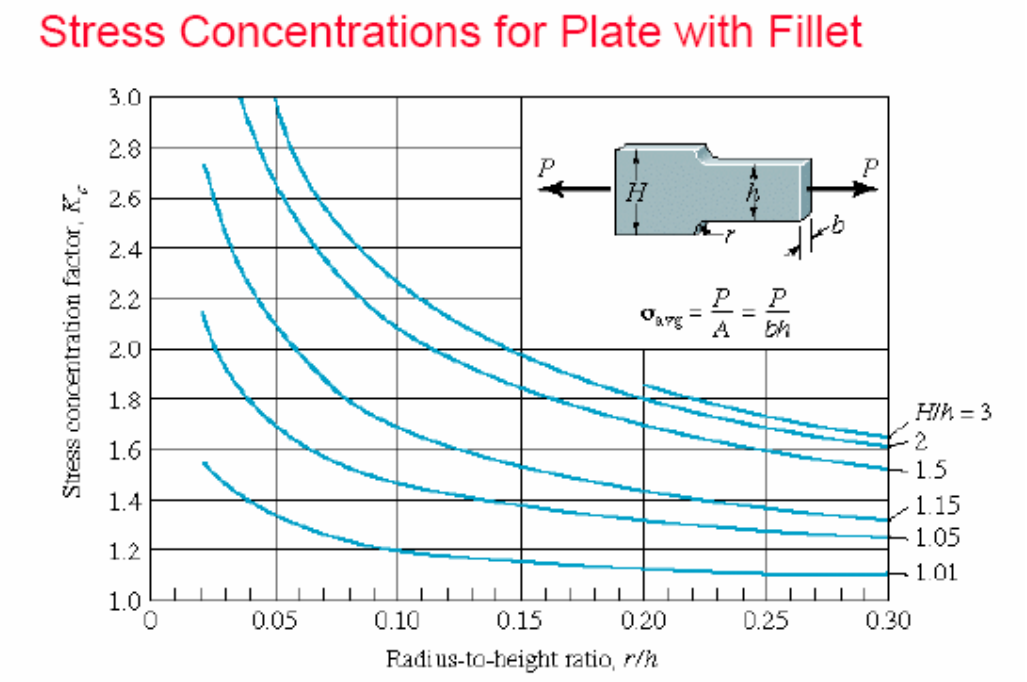

Figure 4.10. Stress concentration factor graphic for two-dimensional plate with shoulder model

On order to model a shouldered shaft the same finite element (Plane2), but with axisymmetric option was used, since Plane2 element can be used as a plane element or as an axisymmetric element. Otherwise, the boundary conditions, the number of keypoints, the sizes of the fixed portions of the model are the same. So the geometric properties, positions of the design points and material properties will not be given again. Only new stress distributions and area/maximum stress temperature graphics are given respectively.

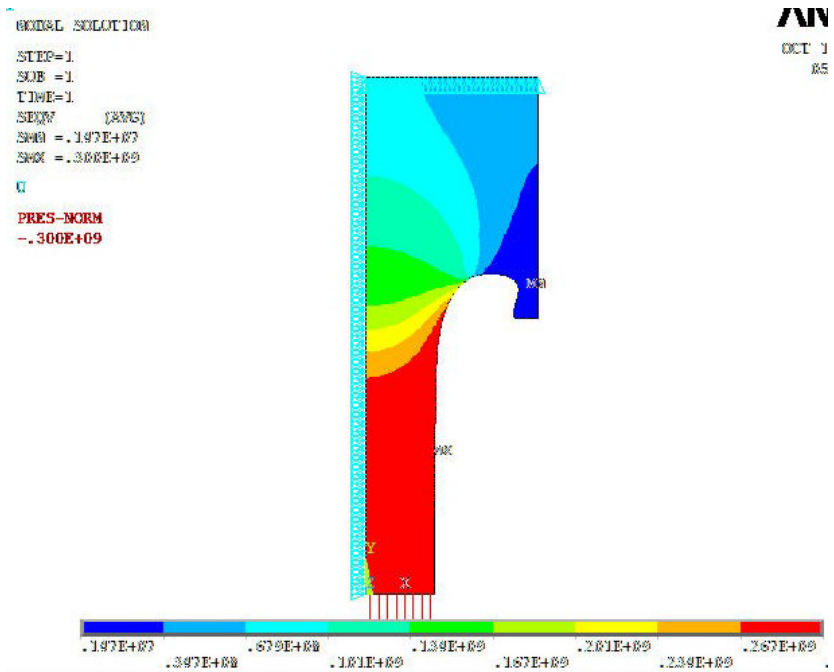

Figure 4.11. Stress distribution developed in the optimal shaped plate with shoulder for axisymmetric model

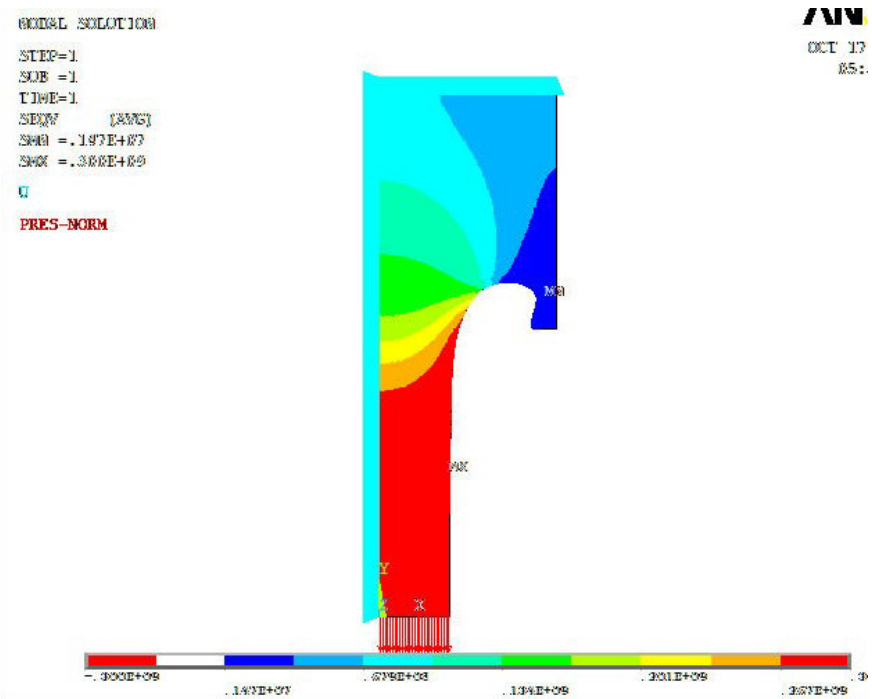

Figure 4.12. Stress distribution obtained with a finer mesh using one fourth of the element size used during optimization process for axisymmetric model

Before obtaining the optimal shape that is feasible, a number of trials were done and different designs were obtained. But they were not acceptable because of inappropriate shapes difficult to produce. Some examples of these shapes are given in the Figures 13-16.

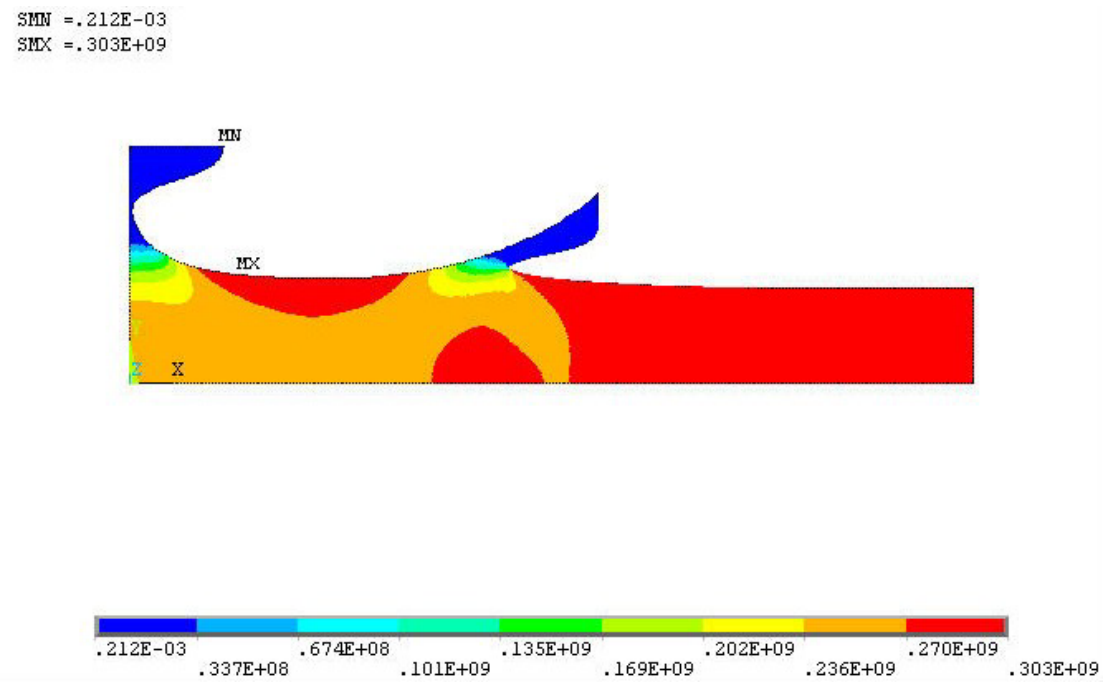

Figure 4.13. Example for inappropriate design

For this trial model, the moving design points are selected for optimizing the two different critical regions. The points are allowed to move in the wide search domain. The algorithm works to provide homogenized stress distribution for whole cross section of the model. Besides, it reduces the stress concentration as it seen on Figure 4.13. Mentioned design points moves near to the left end side of the model which reduces the strength of the model. And also obtained shapes seen on Figure 4.14, 4.15 and 4.16 are not suitable to produce.

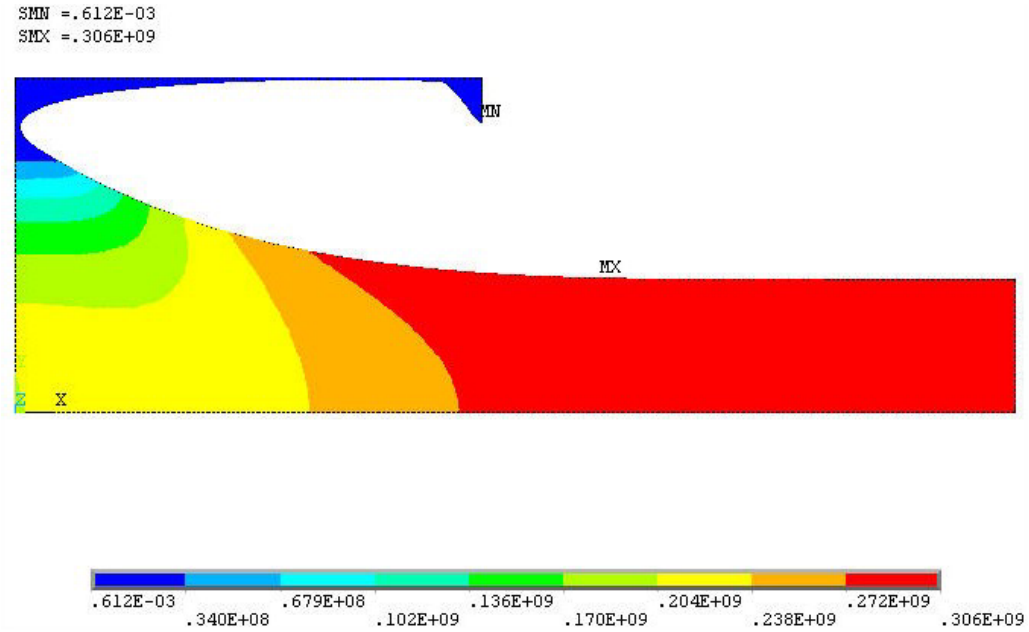

Figure 4.14. Example for inappropriate design

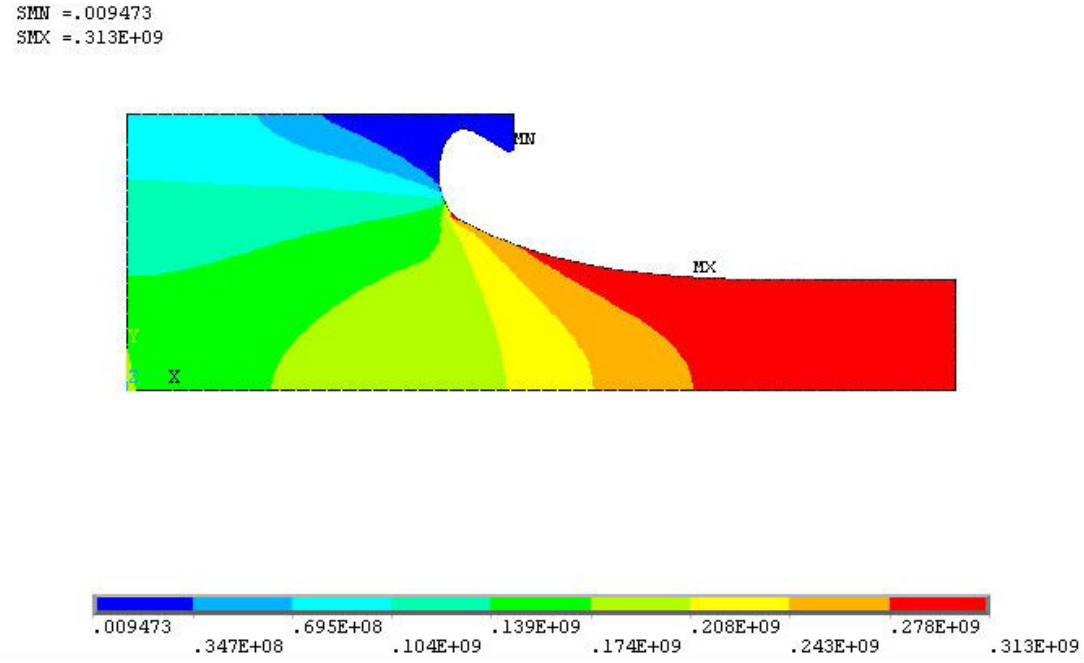

Figure 4.15. Example for inappropriate design

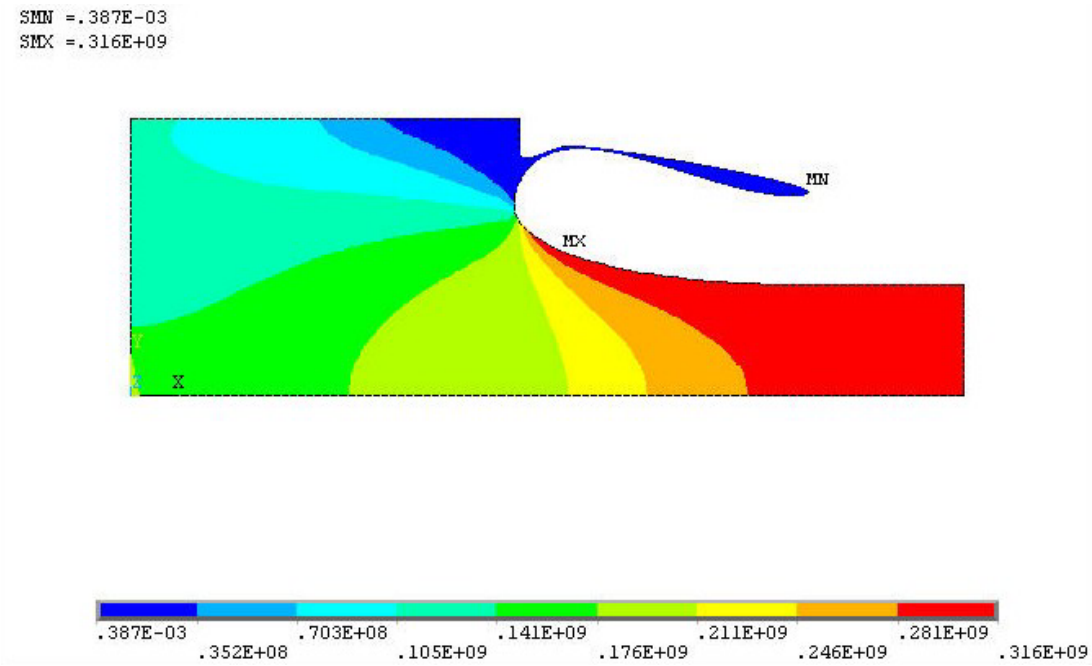

Figure 4.16. Example for inappropriate design

4.2. Chain Roller Link

The second shape optimization example was accomplished for chain roller link. The initial model of the link is displayed as Figure 4.17. As displayed in Figure 4.17, surface distributed load (pressure) of 300 N/mm^2 is to the inner edge on the right side, and both right and bottom edge are restrained from moving in the x direction and moving in the y direction respectively. Because of the symmetry only one quarter portion of the model is analyzed to save computational time. The material properties of the structure are shown in Table 4.4. The optimal shapes that we obtained for this chain link model will be explained accordingly.

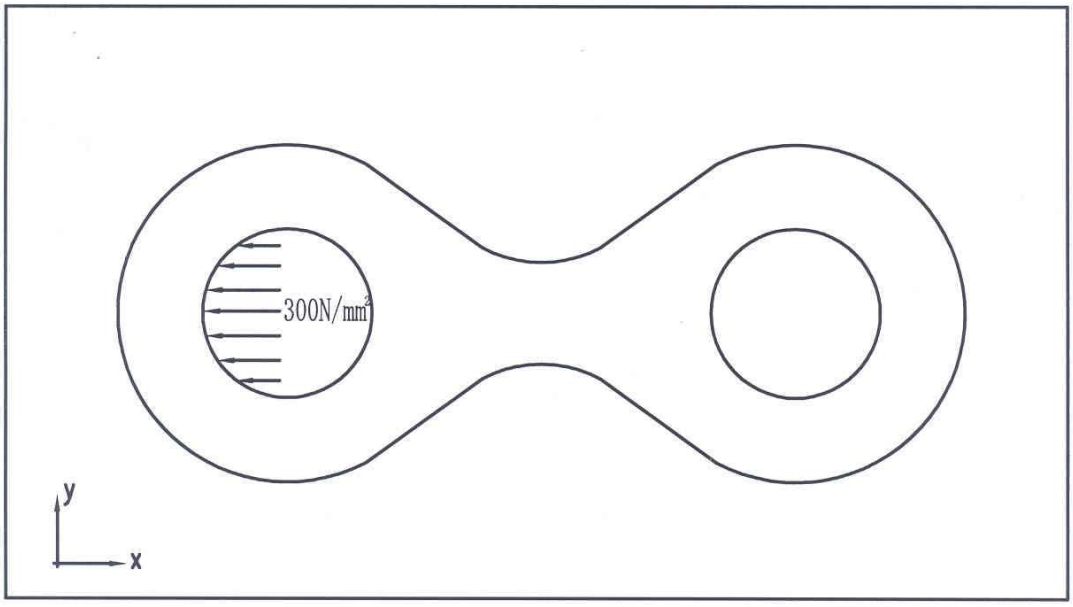

Figure 4.17. Chain roller link subject to axial loading

Table 4.4. The material properties of the chain roller link

Material properties	**Description**
$E= 2.07\ 10^5\ N/mm^2$	Modulus of elasticity
$\upsilon= 0.28$	Poisson ratio
$S_{ref}= 300\ N/mm^2$	Reference stress for evaluating analysis
t= 1 mm	Thickness of the model
$S= 300\ N/mm^2$	Applied surface load
Changes according to the generated model	Number of design points

Figure 4.18 shows the finite element mesh of the initial model. In this model, we used plane 2 (triangular) element same as previous model. Point refinement is applied to get the exact result for the critical region.

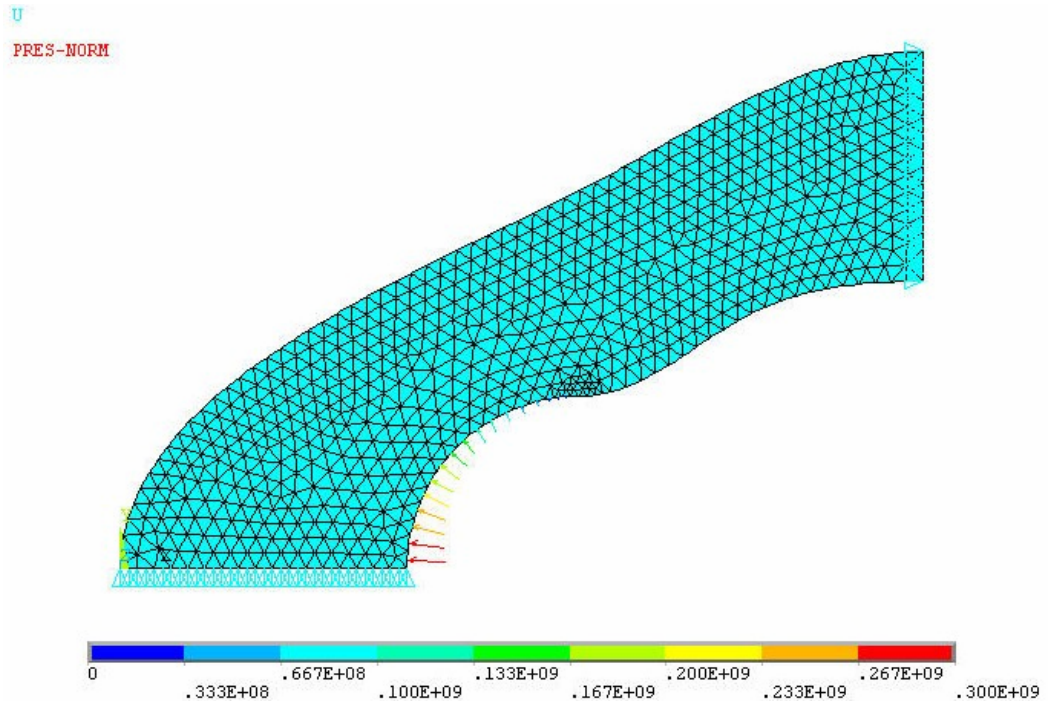

Figure 4.18. The initial configuration of the chain roller link with its finite element mesh

Figure 4.19 represents the stress distribution before applying the optimization algorithm. It is not close the optimum shape because algorithm starts from any initial model and find the global optimum.

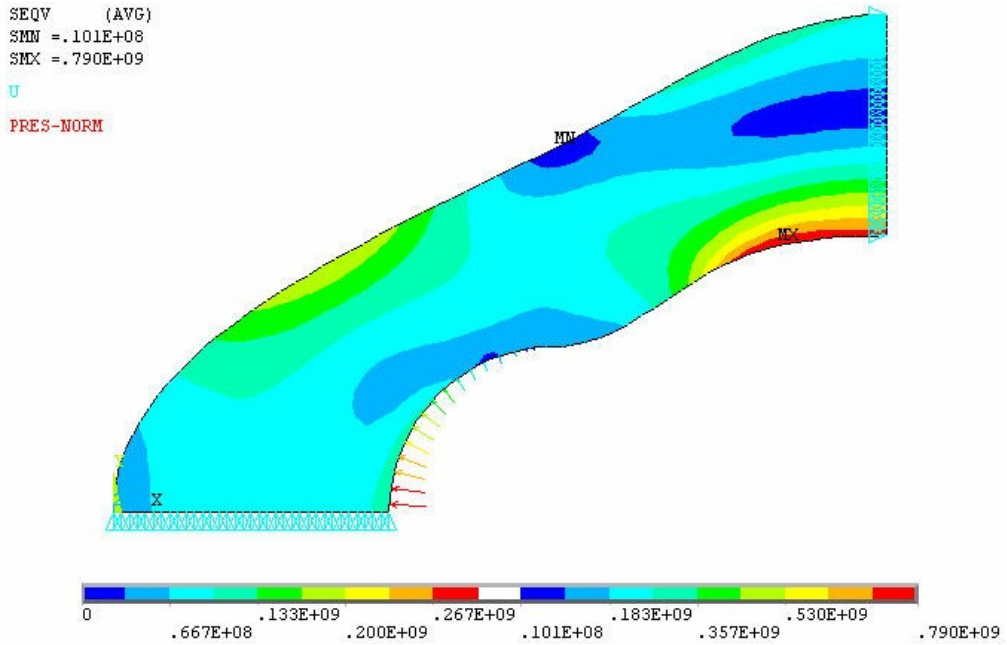

Figure 4.19. Stress distribution of the chain roller link

Because of symmetry, only quarter of the link is needed to evaluate by considering the symmetrical boundary conditions. The dimensions of the model is strived to be kept minimum to save the analysis time. Figure 4.20 shows the optimized shape of the link.

For the finite element modeling, Plane2 Element is used because this type of element is suitable for our configurations in which the boundaries vary randomly. The analysis is performed in ANSYS program. Only point refinement throughout the critical region was applied. This critical region is assigned as Point 3.

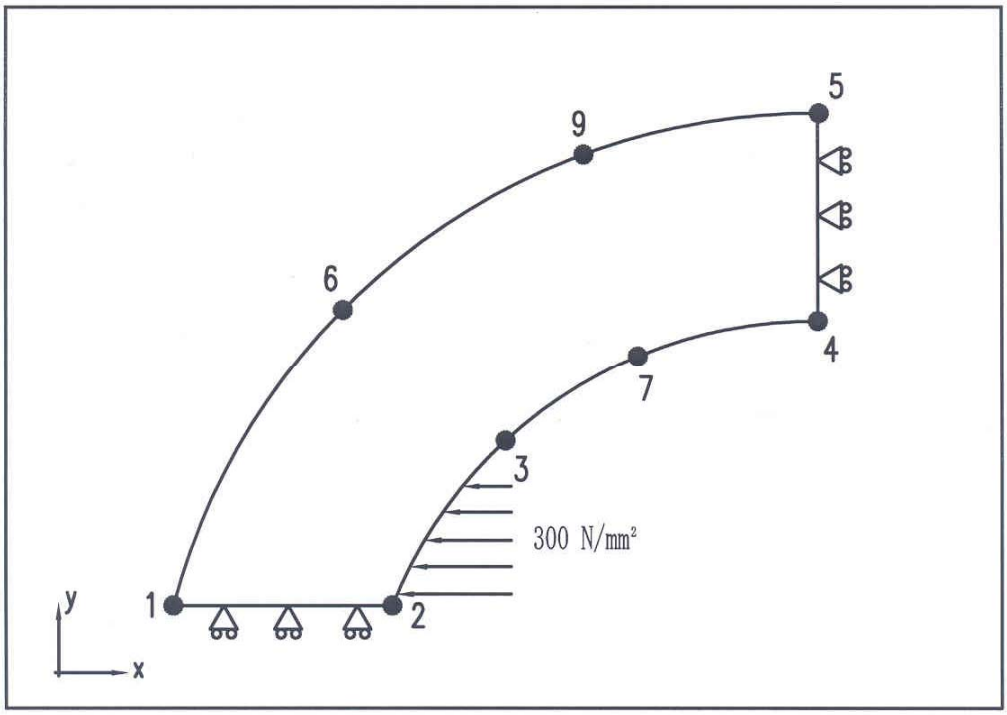

Figure 4.20. The design points and boundary conditions

Figure 4.21 represents the position of the design points of the resulting configuration. Points 7, 6 and 9 are the moving key points. Point 1, 2, 3, 4 and 5 are fixed because the boundary conditions are applied to these points. The acting load is applied to line formed between point 2 and 3. The coordinate values and type of the design points of the resulting configuration are given in Table 4.5.

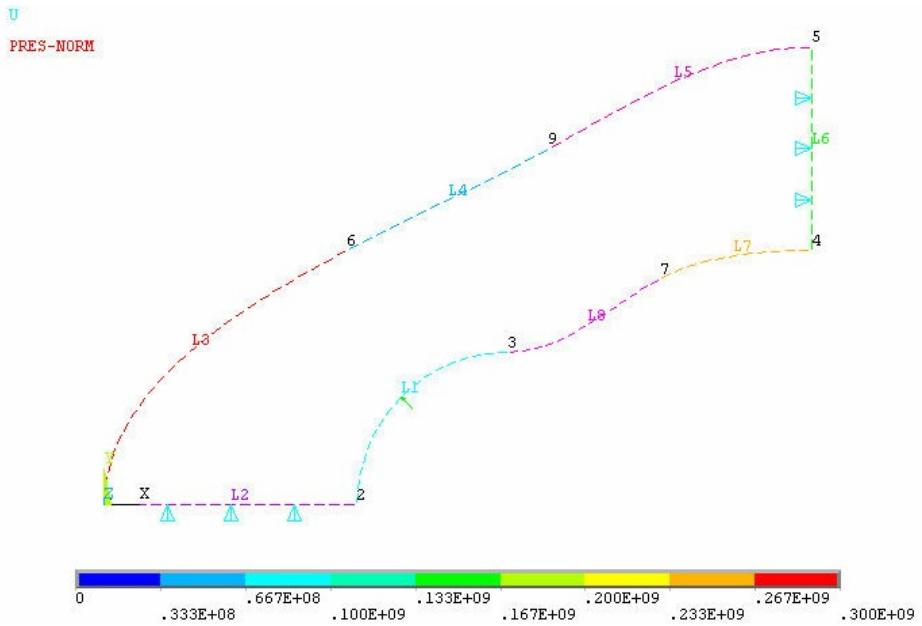

Figure 4.21. The positions of the design points of the resulting configurations

Table 4.5. The coordinates and type of the design points of the initial configuration

Number of design points	**Point type**	**The coordinate of the design point**
1	Restricted	(0.0, 0.0)
2	Restricted	(0.025, 0.0)
3	Fixed	(0.04, 0.015)
4	Restricted	(0.07, 0.025)
5	Restricted	(0.07, 0.045)
6	Moving	(0.024, 0.025)
7	Moving	(0.055, 0.022)
8	Fixed	(0.04, 0.0)
9	Moving	(0.044, 0.035)

The values of parameter used in shape optimization of the chain roller link are given in Table 4.6. Number of design points, dimension of the problem and minimum length of Markov chain are calculated according to the formulation considered in Chapter 3.

Table 4.6. The values of parameters used in the shape optimization of the chain roller link

Parameter values	**Description of the data**
Plane 2 (triangular)	Element type
0,0015	Element size
nk=9	Number of design points
n=10	Dimension of the problem (two times number of moving points)
Lt=100	Minimum length of Markov Chain
Rs=0,05	Step size
Ti=240	Initial temperature
x_{max}=0,07 x_{min}=-0,02 y_{max}=0,045 y_{min}=0,0	Search domain, the area in which the moving points can move
S_{ref}=300 N/mm^2	Reference stress

In Figure 4.22 and Figure 4.23, the stress distribution of the link under specified loading is shown. One is obtained using a mesh with the element size used during the optimization process, and the other using a finer mesh with one fourth of it.

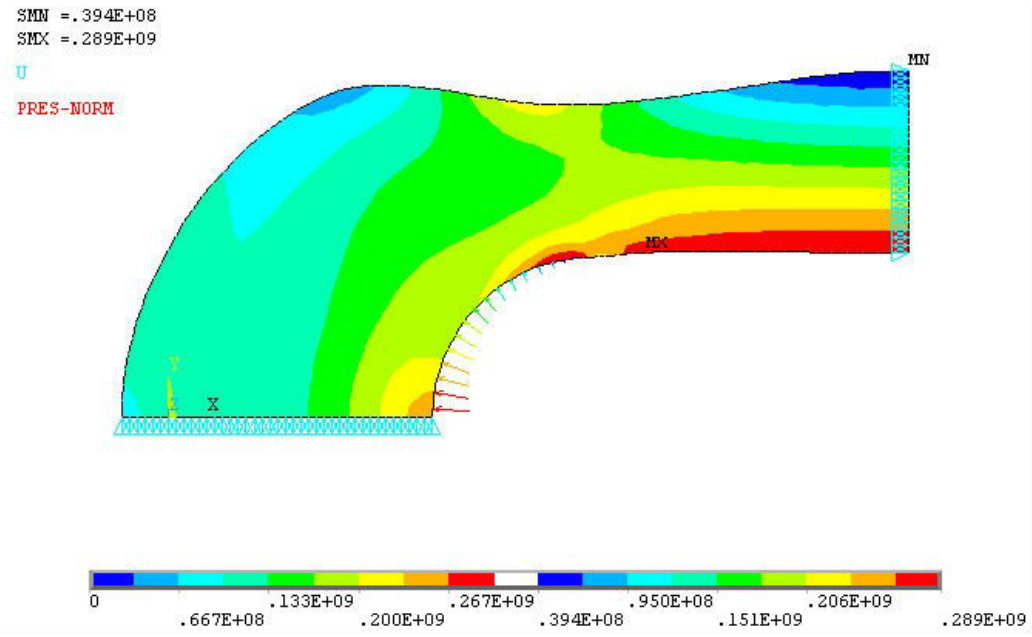

Figure 4.22. Stress distribution developed in the optimal shaped chain roller link

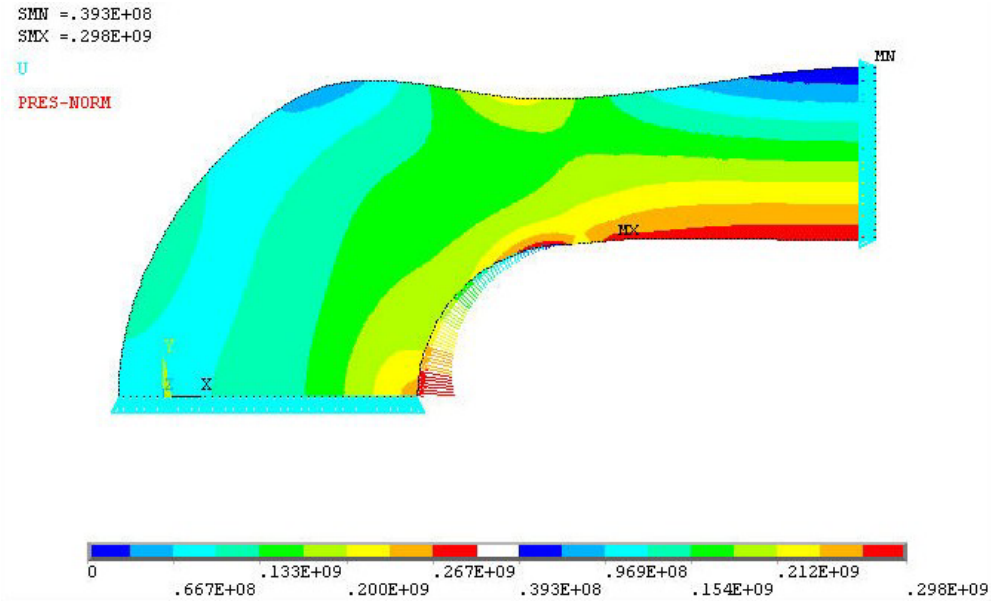

Figure 4.23. Stress distribution obtained with a finer mesh using one fourth of the element size used during optimization process

Figure 4.24 represents the relation of the area and temperature for the chain roller link between 240 (initial temperature) to ~0. The Figure 4.24 gives the change in the area

of the model with decrement of the temperature during optimization process. And Figure 4.25 gives the magnitude of the maximum stress developed in the optimized part with decrement of the temperature.

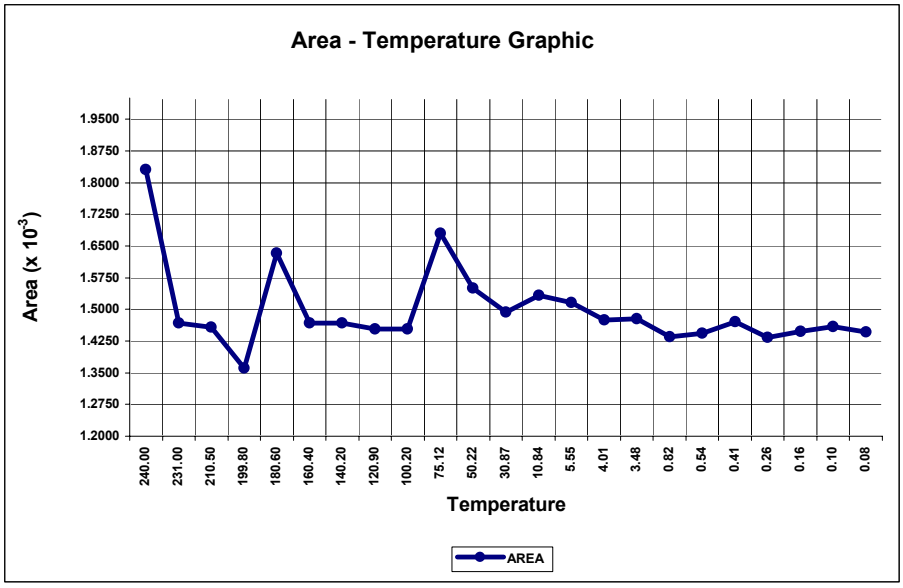

Figure 4.24. Change in the area of the roller chain link as the temperature parameter decreases during the optimization process.

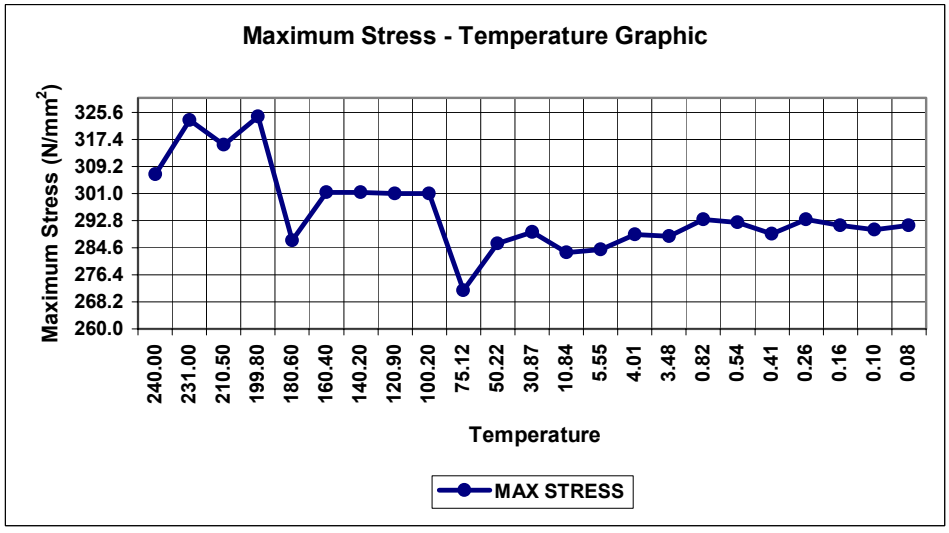

Figure 4.25. Change in the magnitude of the maximum stress developed in the chain roller link as the temperature parameter decreases during the optimization process

4.3. Plate with A Hole

The third model is two-dimensional flat plate with a hole at its center. The objective of this problem is to find out the optimal hole profile to produce the minimum von-Mises stress on the boundary of the hole. The geometry, boundary conditions and loading of the plate are shown in Figure 4.26.

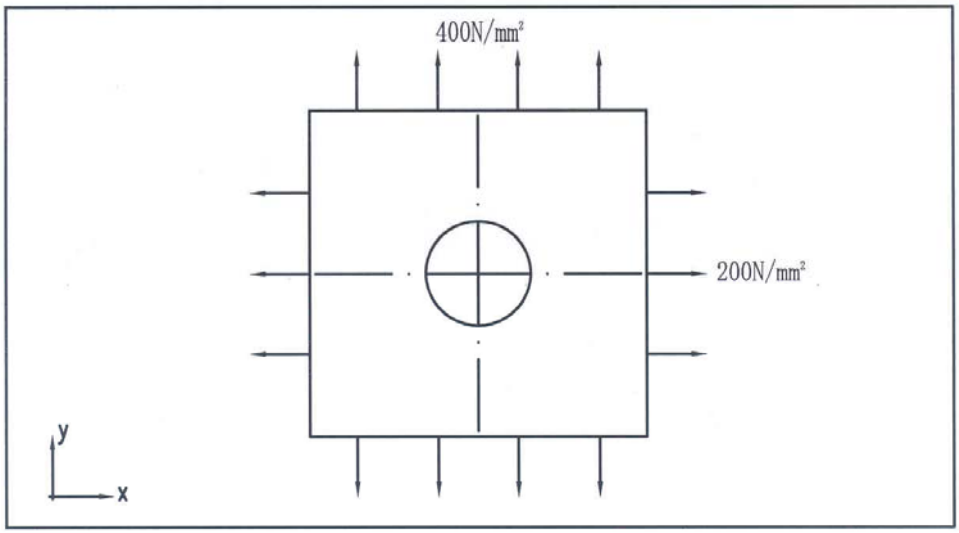

Figure 4.26. Plate with a hole at its center subject to biaxial loading

The material properties of the plate are given in Table.4.7.

Table 4.7. The material properties of the plate with a hole

Material properties	**Description**
E= 2.07 10^5 N/mm^2	Modulus of elasticity
υ= 0.28	Poisson ratio
S_{ref}= 300 N/mm^2	Reference stress
T= 1 mm	Thickness of the model
S= 200 N/mm^2 / S= 400 N/mm^2	Applied surface loads
Changes according to the generated model	Number of design points

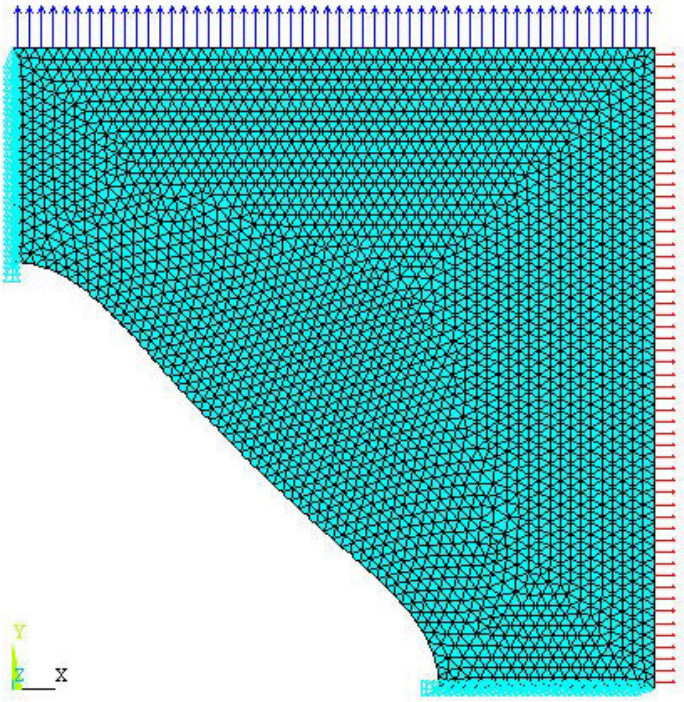

Figure 4.27. The initial configuration of the plate with a hole with its finite element mesh

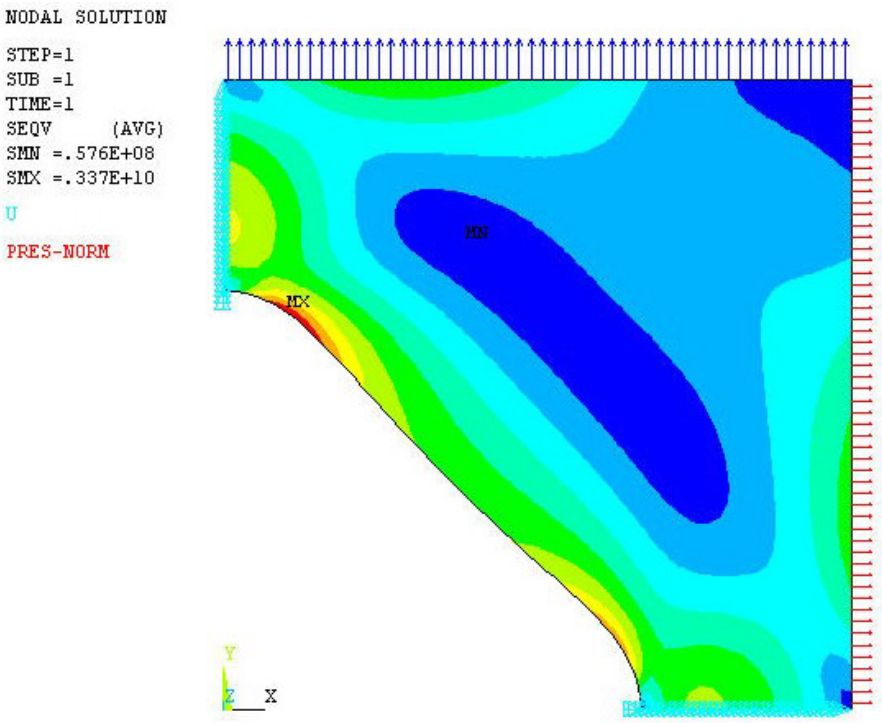

Figure 4.28. Stress distribution in the plate with a hole

Figure 4.27 shows the finite element mesh of one-quarter of the plate with a hole where Plane2 (triangular) elements are used.

Figure 4.28 represents the stress distribution of the model before applying the shape optimization method.

Due to symmetric properties of the model, only one quarter of it was analyzed. Figure 4.29 reflects the optimized shape of the plate with a hole at its center. The bottom and left side of the model is restrained. And different distributed loads are acting on it from upper and right side of the model as it is seen on Figure 4.29.

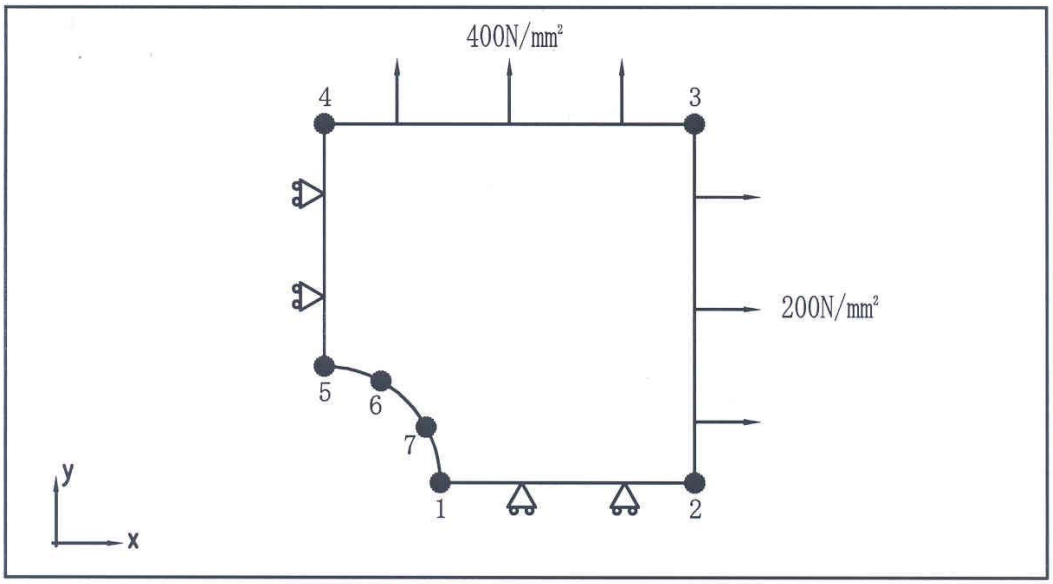

Figure 4.29. Design points and boundary conditions of the plate with a hole

The position of the design points of the resulting configuration is shown in Figure 4.27. Points 1, 2, 3, 4 and 5 are fixed ones and points 6 and 7 are moving points. The distributed load of 200 N/mm^2 is acting on line 2 and other distributed load of 400 N/mm^2 is acting on line 3. The coordinates of the design points are given in Table 4.8.

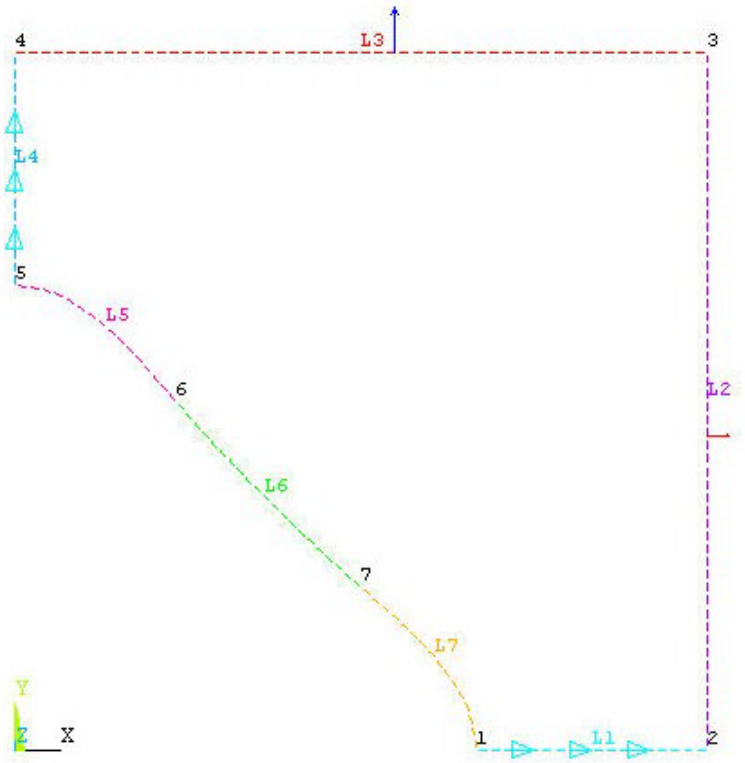

Figure 4.30. The positions of the design points of the initial configuration

Table 4.8. The coordinate values and type of the design points of the resulting configuration

Number of design points	**Point type**	**The coordinate of the design point**
1	Restricted	(0.05, 0.0)
2	Restricted	(0.075, 0.0)
3	Fixed	(0.075, 0.075)
4	Restricted	(0.0, 0.075)
5	Restricted	(0.0, 0.05)
6	Moving	(0.0175, 0.0375)
7	Moving	(0.0375, 0.0175)

The values of the parameter used in shape optimization of plate with a hole at its center are given in Table 4.9.

Table 4.9. The values of the parameter used in the shape optimization of the plate with a hole

Parameter values	Description of the data
Plane 2 (triangular)	Element type
0,0015	Element size
nk=6	Number of design points
n=12	Dimension of the problem
Lt=120	Minimum length of Markov Chain
rs=0,05	Step size
Ti=400	Initial temperature
x_{max}=0,04 x_{min}=-0,01 y_{max}=0,04 y_{min}=0,01	Search domain, the area in which the moving points can move
S_{ref}=300 N/mm^2	Reference stress

The von Mises stress distribution of the model is given in Figure 4.35 and Figure 4.36 from coarse finite element meshing to fine finite element meshing respectively.

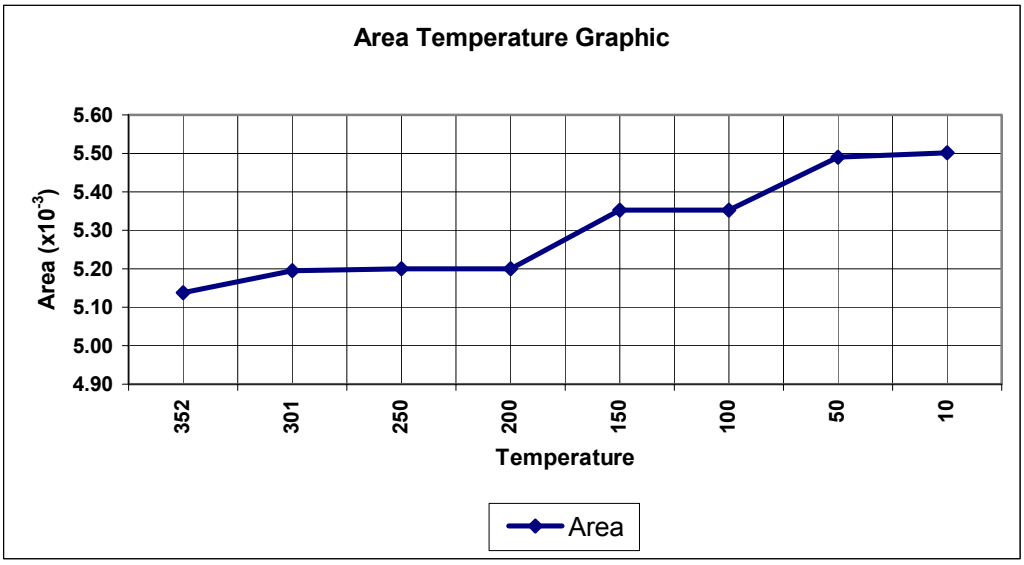

Figure 4.33. Change in the area plate with a hole as the temperature parameter decreases during the optimization process

Figure 4.33 gives the change in the area of the model with decrement of the temperature during optimization process. Figure 4.34 shows the magnitude of the maximum stress developed in the optimized part with the decrement of the temperature.

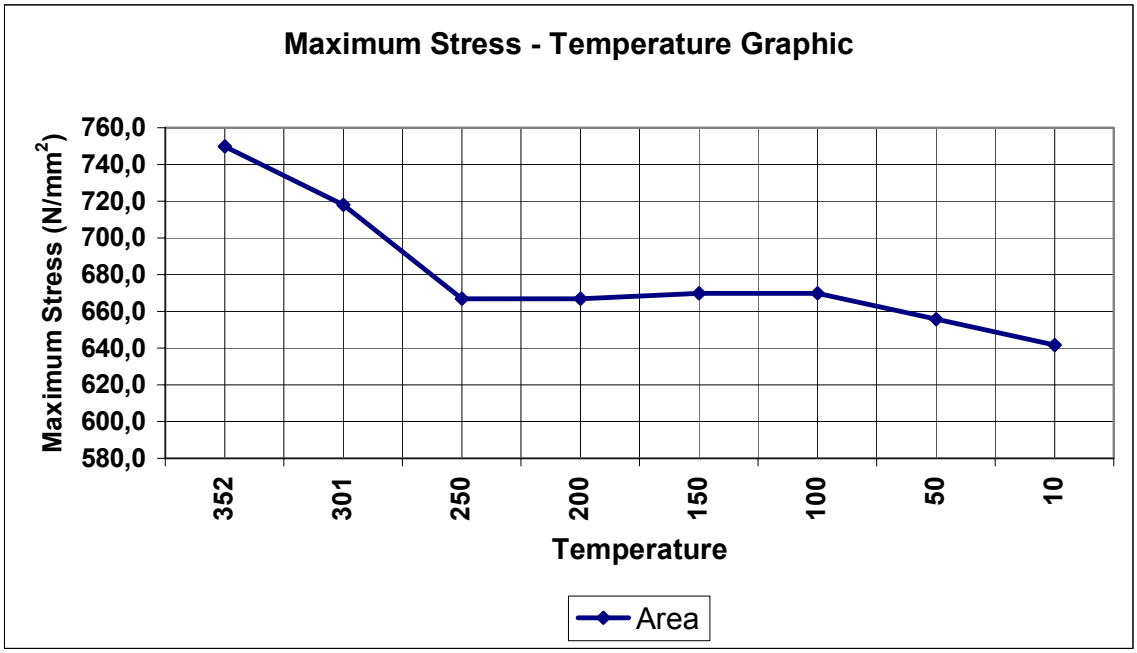

Figure 4.34. Change in the magnitude of the maximum stress developed in the plate with a hole as the temperature parameter decreases during the optimization process

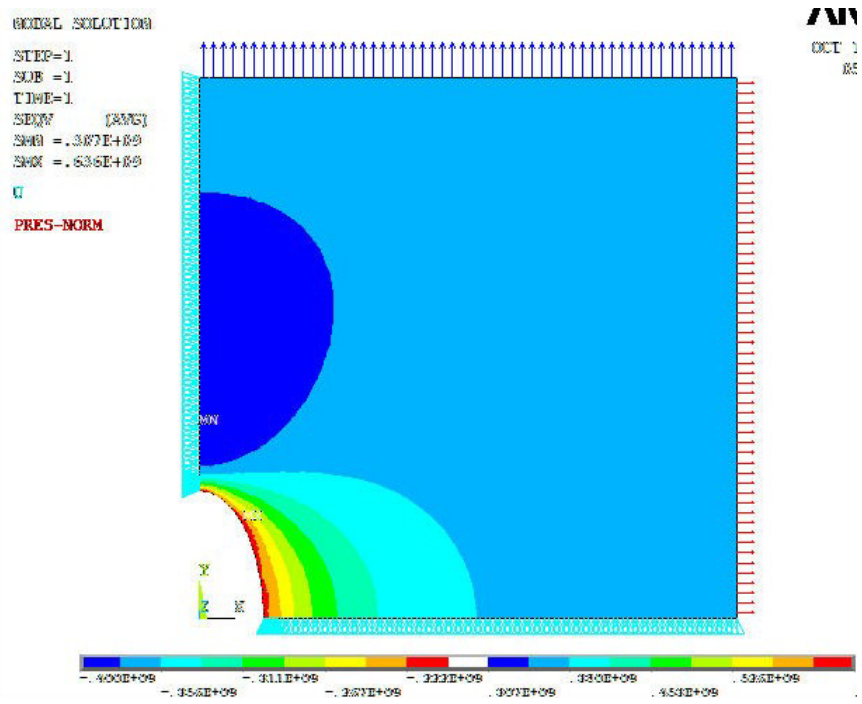

Figure 4.35. Stress distribution developed in the optimal shaped plate with a hole

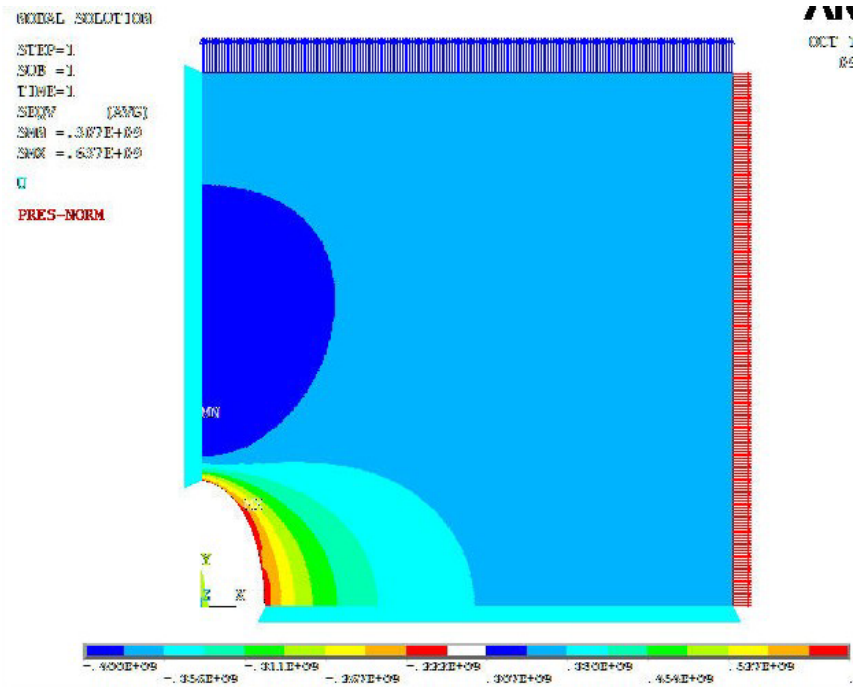

Figure 4.36. Stress distribution obtained with a finer mesh using one fourth of the element size used during optimization process

5. CONCLUSIONS

The most important difficulty that we met during this study is the convergence problem of the maximum stress (S_{max}). As mentioned before, convergence was checked through the optimization process by reevaluating the stress state using much finer mesh It took many times to provide maximum stress convergence. We have started to analyze the models starting from 0,0025 element size to 0,0015 element size. Due to large number of design iterations, nearly more than 2500 iterations for each of the models, it took a lot of time to run the direct search simulated annealing algorithm.

Second important problem, which we met, is finding suitable design criteria. If the initial and boundary conditions are not well defined such as search domain, etc., the results may not be acceptable in terms of manufacturability or functionality of the part. In other words, even though the convergence of maximum stress is ensured and significant reductions in maximum stress and area of the model are obtained, the resulting shape may be unacceptable.

The major advantage of the shape optimization procedure is its flexibility in using many different parameters and finding many different models whose initial conditions such as initial temperature, element size, number and type of design points that means initial geometry differ from each other. The flat plate with shoulder was analyzed using two different finite elements. For the second analysis, Plane2 element has the axisymmetric character. We have chance to see the different analysis results and evaluate.

Stress concentration factor of the shoulder plate model is calculated as 1,06 and it is very reasonable compared with the theoretical values. And the optimal shape obtained form the axisymmetric model of the plate with shoulder has many advantages by means of productivity. The gap formed to the inside of the model gives chance to place a mechanic member such as bearing etc. to the designer. In other words, the obtained shape of the plate with shoulder is usable in order to mount any mechanical member such as gear, bearing etc. to work together without any finishing process during manufacturing of the structure.

By using direct search simulated annealing, we had chance to get the global optimal in a reasonable time without using trail error method. Although the results obtained in this study is optimal where as optimal ones can be obtained by finding suitable cooling schedule and correct initial conditions.

APPENDIX A. FINITE ELEMENT METHOD

The finite element method is a numerical procedure that can be used to obtain solutions to a large class of engineering problems involving stress analysis, heat transfer, electromagnetism, and fluid flow [11].

In general, engineering problems are mathematical models of physical situations. Mathematical models of many engineering problems are differential equations with a set of corresponding boundary and/or initial conditions. The differential equations are derived by applying the fundamentals laws and principles of nature to a system or a control volume [11].

The finite element method is a numerical procedure that can be applied to obtain solutions to a variety of problems in engineering. Steady, transient, linear, or nonlinear problems in stress analysis; heat transfer, fluid flow, and electromagnetism problems may be analyzed with finite element methods. The origin of the modern finite element method may be traced back to the early 1900s when some investigators approximated and modeled elastic continua using discrete equivalent elastic bars. However, Courant (1943) has been credited with being the first person to develop the finite element method. In a paper published in the early 1940s, Courant used piecewise polynomial interpolation over triangular sub-regions to investigate torsion problems.

The next significant step in the utilization of finite element methods was taken by Boeing in the 1950s when Boeing, followed by others, used triangular stress elements to model airplane wings. Yet, it was not until 1960 that Clough made the term finite element popular. During the 1960s, investigators began to apply the finite element method to other areas of engineering, such as heat transfer and seepage flow problems. Zienkiewicz and Cheung (1967) wrote the first book entirely devoted to the finite element method in 1967. In 1971, ANSYS was released for the first time.

ANSYS is a comprehensive general-purpose finite element computer program that contains over 100,000 lines of code. ANSYS is capable of performing static, dynamic, heat

transfer, fluid flow, and electromagnetism analyses. ANSYS has been a leading Finite Element Analysis program for well over 20 years.

In structural optimization problems, one tries to set up some data of the mathematical model that describes the behavior of a structure in order to find a situation in which the structure exhibits a prior given properties. The structural analysis, which we try to evaluate the maximum stresses of the generated different configurations, will be achieved by finite element analysis in shape optimization problem. The structural analyses are carried out using ANSYS finite element software package version 8.1 on an Intel Pc Pentium 4 model.

The general step of finite element analysis for the shape optimization problem is explained in this chapter.

The first step of the solution procedure is to form the objective function of the analysis. In our shape optimization problem, as it is mentioned on the "Approach to the problem" section above, the objective function is reducing the maximum stress for all randomly generated configurations.

Before starting to preprocessing phase of finite element method, the real constants such as thickness, dimensions, and material properties of the model will be determined.

Second step is consisting of preprocessing phase, which is the last step before starting to analyze the model. In this step, we create and discretize the solution domain into finite elements; that is, subdivide the problem into nodes and elements. During this phase we should determine the element size and type. In order to analyze the configuration, first closed curve configuration should be made an area. In other words, the outer boundaries should be appropriate for analyze. ANSYS commands are used to define the element size and type. The algorithm is given at the end of the thesis. Because of existing of moving key points, the shape of he model is not regular so automatic meshing (free) tool is used. It is possible to get more refine results by changing the element size. The point or line refinement are used to get more exact results especially for the critical regions of the model such as the notches or region on which the loads acting.

After meshing the model, the boundary condition, initial conditions are applied. It should be noted that when we restrain the fixed design points, we should also restrain the line between the fixed design points. Otherwise, the stress result will be incorrect.

For the second step, solution phase is started linear or non-linear algebraic equations are solved by ANSYS simultaneously to obtain nodal result, such as stress values of different nodes or temperature values at different nodes [11].

After that, the ANSYS program will solve the model. And then we can evaluate the maximum stress of the configuration.

The von Mises stress has been one of the most frequently used criterions for failure in isotropic materials. For plane stress problems the von Mises stress σ_m is defined as

$$\sigma_m = \sqrt{\left(\sigma_x^2 + \sigma_y^2 - \sigma_x . \sigma_y + 3\tau_{xy}^2\right)}$$

where σ_x and σ_y are normal stress in x and y directions, respectively, and τ_{xy} is the shear stress.

APPENDIX B DESCRIPTION OF THE PROGRAM

The input data which are used during the analysis of the models are as follows:

- The geometric properties (length, thickness, etc.) of the initial geometry (configuration)
- The material properties of the models (modulus of elasticity, poisson ratio, allowable maximum stress, etc.)
- The design variables (the number and the coordinates of design points consisting of fixed, restricted and moving ones)
- The boundary conditions (loads and restraints)
- The initial temperature
- The stopping criterion

The general computer program code is used, which is obtained by using ANSYS parametric design language. The program is developed to test the efficiency of the direct search simulated annealing algorithm. It is possible to apply the program to analyze two-dimensional structures. One module of the program performs the structural analysis for evaluating maximum stress (von Mises Stress) and the other is used to calculate the area of the obtained model.

In the Appendix B section, there are two different computer program code were given. First of them belongs to flat plate with shoulder and the other one is for chain roller link model.

In the program, design points are connecting by spline curves in order to form the geometry of the models. And the new configurations can be created by changing the position moving key point in the search domain.

Sometimes we face with the violation of the model connectivity. Randomly chosen points are forming the new configurations; because of this reason sometimes the line

segments or the boundaries of the configuration cross each other. In this case, the program does not accept this configuration and it creates new one.

We chose Plane2 element due to its advantages Plane2 element is a 6-node element triangular element. The element has a quadratic displacement behavior and is well suited to model irregular meshes. And it is also important to choose the size of element to get the accurate and reliable results.

The boundary conditions such as loads acting on the model should be well defined. It must be specified the line which the loads were applied on and the nodal points which will be restrained. After all these input, program can be run in order to start the analysis. And this process will be repeated for each iteration until end of the solution procedure. Every time maximum stress and are values are obtained and program continues to search the best one by replacing the worst with the best one. In other words, if the cost of the new configuration is better, then the program accepted and replaced it with the worst one. The results are recorded to param.txt and markow.txt files.

The process is controlled by the cooling schedule that consists of determining the initial control parameter such as temperature, decrement rule of temperature and the length of Markov chain and the program runs until the stopping criterion is satisfied. In order to get the optimum shape, optimum cooling schedule should be constructed.

The run time of the program also depends on the boundary of the models. So the model are tried to keep in short dimensions. It takes nearly 6-20 minutes to run the program and take the results for each iteration for both models. The computer using during this thesis is Pentium ® 4 CPU 3.00 GHz, 1.00 GB ram model.

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