SCREENING LOSS-AVERSE BUYERS THROUGH BEST-PRICE PROVISIONS

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Screening Loss-Averse Buyers Through Best-Price Provisions

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ABSTRACT

Screening Loss-Averse Buyers Through Best-Price Provisions

We consider a monopolist who is facing loss-averse and loss-neutral buyers over two periods, with an uncertainty about the type of buyers. Monopolist announces its price today, while the price tomorrow can be high or low, which is exogenous. Monopolist aims to screen loss-averse buyers using most-favored-customer (MFC) clause by offering two contracts, one with MFC, and the other without MFC, with different prices, such that loss-averse buyers self-select the former, while loss-neutral ones choose the latter. First, we identify when the incentive problem is an issue, so that for a parameter subspace, the first-best prices are incentive compatible while for the rest of the parameter space they are not. Then, we characterize the second-best prices for patient buyers and show that the price attached to the contract with an MFC clause is higher than the price without an MFC clause. This resembles those pricing strategies for airline tickets and hotel stays, where usually a flexible (free cancellation or free rescheduling) version is offered at a higher price.

ÖZET

Kayıptan Kaçınma Tercihleri Olan Alıcıları En İyi Fiyat Provizyonları ile Ayrıştırma

Bu tezde iki dönem boyunca kayıptan kaçınma tercihi olan ve kayıptan etkilenmeyen iki tür alıcı ile karşılaşan bir tekeli ele aldık. Bu tekel, hangi alıcıların kayıptan kaçınan ve hangilerinin kayıptan etkilenmeyen olduklarını bilmiyor. Tekel, bugünün fiyatını belirlerken yarının fiyatı düşük veya yüksek olabilir. Tekel, en çok kayırılan tüketici (MFC) koşulunu kullanarak kayıptan kaçınma tercihleri olan alıcıları ayrıştırmayı amaçlamaktadır. Bunun için farklı fiyatları olan iki sözleşme sunarak (birinde MFC olan, diğerinde MFC olmayan) kayıptan kaçınma tercihleri olan alıcıların MFC olan sözleşmeyi ve kayıptan etkilenmeyen alıcıların de MFC olmayan sözleşmeyi tercih etmelerini istemektedir. İlk olarak, teşvik probleminin ne zaman bir sorun olduğunu belirledik ve belli parametreler için birinci en iyi fiyatların teşvik uyumlu olduğunu ancak geri kalan parametreler için uyumlu olmadığını bulduk. Ardından, sabırlı alıcılar için ikinci en iyi fiyatları karakterize ettik ve MFC olan sözleşmeye bağlı fiyatın, MFC olmayan sözleşme fiyatından daha yüksek olduğunu gösterdik. Bulduğumuz bu sonuç, genellikle uçak biletleri ve otel konaklamaları için sunulan esnek (ücretsiz iptal veya ücretsiz yeniden planlama) versiyonun daha yüksek bir fiyatla sunulduğu gibi fiyatlandırma stratejilerini andırır.

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TABLE OF CONTENTS

CHAPTER 1: INTRODUCTION 1
CHAPTER 2: LITERATURE 4
CHAPTER 3: MODEL 6
3.1 Loss-averse buyers 6
3.2 Monopolist's problem 8
CHAPTER 4: COMPLETE INFORMATION 10
4.1 Loss-neutral buyers 10
4.2 Loss-Averse Buyers 11
CHAPTER 5: INCOMPLETE INFORMATION 13
5.1 Incentive Compatibility Constraints 14
CHAPTER 6: CONCLUSION 20
APPENDIX A: 21
REFERENCES 68

CHAPTER 1

INTRODUCTION

The prices can change over time in numerous markets due to demand shocks. In the case of a potential demand shock, the price in the future can increase or decrease. We can see such price fluctuations in real-life, for instance, in the airline industry, where prices can rise or fall depending on the travel demand. In the case of such uncertain future prices, buyers decide whether to buy today or tomorrow, according to their expectations about tomorrow's price. If a buyer expects a lower price tomorrow, she can choose to wait and buy tomorrow. Moreover, if the actual price buyers pay and their expected price differ, then they will experience some losses or gains, which makes loss aversion relevant; where they value losses more than same sized gains. Within such an environment, for the monopolist the *most-favored-customer* (MFC) clause, through which the seller promises to the buyers to pay back the price difference if there is a lower price in the future, becomes also relevant. MFC clauses are very common in real-life. Airplane tickets and hotel stays are usually offered with a nonrefundable low price and with a higher price which has a free cancellation policy. This free cancellation policy can be seen as an MFC contract because in the case of a potential drop in the price, the buyer can cancel the purchase, get the full refund and buy at the lower price.

We consider an adverse selection problem of a monopolist, who faces two types of buyers, *loss-neutral* and *loss-averse*, and does not know the type of buyers. The monopolist also does not know the buyers' valuations. In a two-period model where the monopolist announces its price today and the price tomorrow can be high or low, we analyze the set of optimal contracts that the monopolist uses to screen the loss-averse buyers via an MFC clause. First, we find the first-best prices in the case of complete information and show that it is never optimal for the monopolist to adopt an MFC clause if the buyers are loss-neutral. For the loss-averse buyers, we show that the monopolist adopts an MFC clause if the buyers are sufficiently patient. Then, we analyze the incentive problems and show that the first-best prices are not incentive

compatible for sufficiently loss-averse buyers. Thus, there is an incentive problem if the buyers are sufficiently loss-averse. Moreover, we characterize the second-best prices for perfectly patient buyers and show that the price in the presence of an MFC clause is higher than that in the absence of an MFC clause. We also show that the first-best prices are not incentive compatible for loss-averse buyers if they are sufficiently patient.

In this context, the seller posts two prices today, one attached to the MFC clause and the other to the one without the MFC clause. After observing these prices, a loss-averse buyer today forms a reference point. If the actual price she paid is higher than her reference price, there will be a loss, and a gain in the opposite case. If she chooses to buy today, then the loss and gain are realized after tomorrow's price is observed. If she chooses to buy today and observes that tomorrow's price is higher than today's price, she will experience a loss. In both cases, losses compared to the same-sized gains affect the buyers more. Then, a buyer today chooses to buy today or tomorrow by considering the potential losses and gains.

An MFC clause both serves as an option to fix the effective price for the buyer who buys today against a potential decrease in price tomorrow and as an insurance mechanism against a potential loss from a price drop. Due to this insurance channel, the monopolist is able to charge a higher price for the contract with an MFC clause. Also, due to the option channel, the monopolist achieves a higher profit from using an MFC clause if the loss-averse buyers are sufficiently patient in the case of complete information. In other words, we show that as loss-averse buyers become more patient, in the case of complete information, the monopolist's profit when using an MFC clause increases, while the profit without an MFC clause decreases.

The plan of the thesis is as follows. In Chapter 2 we provide a brief review of the literature on MFC clauses and loss aversion preferences. Chapter 3 describes our model, where we specify how we model loss aversion in Section 3.1 and we state the monopolist' problem in Section 3.2. Chapter 4 provides the analysis for the optimal prices if the monopolist knows the types of buyers. Chapter 5 provides the analysis

for the incomplete information case where we analyze the incentive compatibility constraints in Section 5.1. Chapter 6 concludes. All proofs are given in the Appendix.

CHAPTER 2

LITERATURE

The MFC clause is a commonly used pricing policy for the seller, especially when the seller cannot commit credibly today to a future price. A durable good monopolist lacks this commitment power because buyers may predict that the prices will be lower if the demand decreases over time. This may lead to a situation where they postpone their purchasing decision until the price falls to the marginal cost. This causes the seller to earn only competitive profits (Coase, 1972). Using an MFC clause, a durable good monopolist can generate some commitment power and earn profits, typically through rental agreements (Butz, 1990). A monopolist can choose to use an MFC clause to increase the price for advance sales if the buyers' valuations are private information (Png, 1991). When there are new identical buyers in each period, by using an MFC clause, the seller can achieve a higher profit (Neilson & Winter, 1994). Also, the possibility of collusion among sellers in a duopolistic competition can be affected by MFC clauses. In a noncooperative equilibrium, at least one seller offers an MFC clause, facilitating the tacit collusion (Cooper, 1986). On the other hand, without a restrictive assumption about firms' demand, there is no equilibrium in which both sellers offer an MFC clause (Neilson & Winter, 1993).

Loss aversion is a form of reference-dependent preference (Kahneman & Tversky, 1991; Kőszegi & Rabin, 2006), and its implications have been widely studied in many economic contexts, following the prospect theory (Kahneman & Tversky, 1979). In a circular city type of product differentiation model, loss aversion in the match-value dimension has anti-competitive effects while it has pro-competitive effects in the price dimension (Karle & Peitz, 2014). If there is an advance purchase setting, loss aversion has anti-competitive effects on prices (Karle & Möller, 2020). Even when firms have different cost distributions, a focal price equilibrium can exist with loss-averse buyers (Heidhues & Koszegi, 2008). A monopolist may choose to introduce risk when facing loss-averse buyers by offering low and variable sale prices and a high regular price. Sales prices make a buyer

realize that she will purchase with positive probability, which in turn leads her to purchase at the high regular price to avoid loss (Heidhues & Koszegi, 2014). A monopolist can charge different prices for two substitute goods where the one with higher price is bought by loss-averse buyers to avoid any possible losses (Rosato, 2016). When a seller offers a menu of bundles to loss-averse buyers where bundles consist of quality and payment of the product, the selller chooses not to screen high willingness-to-pay and low willingness-to-pay buyers if the latter type of buyers are sufficiently more likely, and the degree of loss aversion is in an intermediate range (Hahn, Kim, Kim, & Lee, 2018).

CHAPTER 3

MODEL

A monopolist produces and sells a good over two periods, $t = 0, 1$. Each buyer buys at most one unit of the good. There is a unit measure of buyers in the first period. Each buyer is loss-averse with probability $\beta \in (0, 1)$. Valuations of the buyers at period 0 are distributed with uniformly on [0,1]. In the next period price can be p^H with probability α and can be p^L with probability 1-α, where $p^L < p^H$ and $p^L < \frac{1}{2}$, $1 < p^H$. Each buyer discounts future utilities with a discount factor $0 < \delta \leq 1$. Loss-averse buyers weigh the losses with $\lambda > 1$ and gains with 1. Probabilities α and β, discount factor δ and the loss aversion parameter λ are common knowledge. Monopolist does not know which buyers are loss-averse or not. Valuations are private information.

3.1 Loss-averse buyers

The utility of a loss-averse buyer has two parts: intrinsic utility and loss/gain utility. The first one is the net utility from consuming the good. If she buys the good, it is equal to her valuation minus the price she paid, and if she does not buy the good, then it is 0. The loss/gain utility comes from the difference between the reference point utility and actual intrinsic utility. In this model, loss/gain utility comes from the difference between the actual price she paid and the expected price.

A buyer at period 0 may choose to buy at period 0, or wait for period 1. The reference points of a buyer when she chooses to wait for tomorrow and when she chooses to buy today is different. Therefore, the loss/gain utility of a buyer depends on the decision made.

Expected utility of a buyer at period 0, who chooses to wait: When a buyer with valuation v at period 0 chooses to wait for period 1, her utility for each possible state $i = H, L$ is given by

$$u_1^i(v) = \max\{v - p^i, 0\} + \mathbb{I}\big[\max\{v - p^i, 0\} - R(v)\big]$$

where

$$\mathbb{I}[m] = \begin{cases} m & \text{if} \quad m \geq 0 \\ \lambda m & \text{if} \quad m < 0 \end{cases}$$

where $\lambda > 1$ and *reference point* $R(v)$ is defined as

$$R(v) = \alpha \max\{v - p^H, 0\} + (1 - \alpha) \max\{v - p^L, 0\}$$

With probability α, price in period 1 will be p^H and in this case, buyer's intrinsic utility is $\max\{v - p^H, 0\}$. Similarly, with probability $1 - \alpha$, price in period 1 will be p^L and buyer's intrinsic utility is $\max\{v - p^L, 0\}$. The loss/gain utility is given by the difference between $\max\{v - p^i, 0\}$ and $R(v)$. If this difference negative, then buyer experiences a loss, and if it is positive then there is a gain utility.
The expected utility of a loss-averse of a buyer who waits for period 1 is given by

$$\mathbb{E}U_1(v) = \alpha u_1^H(v) + (1 - \alpha) u_1^L(v)$$

Expected utility of a buyer at period 0, who chooses to buy at period 0: When a buyer with valuation v at period 0 chooses to buy today, her utility is given by by

$$u_0(v) = v - P + \delta\big(\alpha \mathbb{I}\big[v - P - R_H\big] + (1 - \alpha)\mathbb{I}\big[v - P - R_L\big]\big)$$

where $R_i = \max\{v - p^i, 0\}$, $i = H, L$, and $\mathbb{I}[\cdot]$ is as defined above.
When a buyer decides to buy at period 0, her reference point is different for each possible state. For the loss/gain utility, she compares her intrinsic utility, $v - P$, with what she would have receive if she had waited for period 1. With probability α, she would have received $\max\{v - p^H, 0\}$ and with probability $1 - \alpha$, she would have received $\max\{v - p^L, 0\}$. If the difference between her intrinsic utility and state specific reference point is negative, then she experiences a loss and if it is positive she experiences a gain. The loss/gain utility is realized at period 1, so it enters the utility function as discounted with δ.

3.2 Monopolist's problem

Monopolist offers two contracts to screen buyers: (P_A, MFC) and $(P_N, NoMFC)$ where $P_A > P_N$ and $p^H > P_A > p^L$. So, the monopolist maximizes the expected profit such that loss-averse buyers will choose the contract with high price and MFC, while loss-neutral buyers will choose the one without MFC and with the low price. The monopolist's problem is given by

$$max_{P_A,P_N}\ \mathbb{E}\pi_0(P_A,P_N) + \mathbb{E}\pi_1(P_A,P_N) \qquad \text{subject to}$$

$$\begin{aligned} &u_0^{notLA}(P_N, noMFC) > u_0^{notLA}(P_A, MFC) \\ &u_0^{notLA}(P_N, noMFC) > \delta\mathbb{E}U_1(v) \qquad \text{for } v > \hat{v}_N \end{aligned}$$

and

$$\begin{aligned} &\delta\mathbb{E}U_1(v) > u_0^{notLA}(P_A, MFC) \\ &\delta\mathbb{E}U_1(v) > u_0^{notLA}(P_N, noMFC) \qquad \text{for } v < \hat{v}_N \end{aligned}$$

$$\begin{aligned} &u_0^{LA}(P_A, MFC) > u_0^{LA}(P_N, noMFC) \\ &u_0^{LA}(P_A, MFC) > \delta\mathbb{E}U_1(v) \qquad \text{for } v > \hat{v}_A \end{aligned}$$

and

$$\begin{aligned} &\delta\mathbb{E}U_1(v) > u_0^{LA}(P_N, noMFC) \\ &\delta\mathbb{E}U_1(v) > u_0^{LA}(P_A, MFC) \qquad \text{for } v < \hat{v}_A \end{aligned}$$

where $\mathbb{E}\pi_0(P_A, P_N)$ is the expected profit at period 0, and $\mathbb{E}\pi_1(P_A, P_N)$ is the expected profit at period 1, where

$$\mathbb{E}\pi_0 \equiv \beta\pi_0^{LA}(P_A, P_N) + (1-\beta)\pi_0^{notLA}(P_A, P_N)$$

$$\begin{aligned}\mathbb{E}\pi_1 &\equiv \beta[\alpha\pi_1^{H,LA}(P_A, P_N) + (1-\alpha)\pi_1^{L,LA}(P_A, P_N))] \\ &+ (1-\beta)[\alpha\pi_1^{H,notLA}(P_A, P_N) + (1-\alpha)\pi_1^{L,notLA}(P_A, P_N)]\end{aligned}$$

and $\pi_0^j(P_A, P_N)$ is the profit that the monopolist gets at period 0 from selling to type $j = LA, notLA$ buyers, and $\pi_1^{i,j}(P_A, P_N)$ is the realized profit in state $i = H, L$. The utility $u_0^j(P, noMFC)$ denotes the utility of a type j buyer if she buys today at $(P, noMFC)$ and the utility $u_0^j(P, MFC)$ denotes the utility of a type j buyer if she buys today at (P, MFC) where $j = LA, notLA$.

In Chapter 4, we will first solve the monopolist's problem as if the monopolist knew which buyers are loss-averse and which ones are not. In this case, we will not have the above constraints in the maximization problem. Then, in Chapter 5, we will solve the problem when the monopolist does not know which buyers are loss-averse and which ones are not. We first find two thresholds to solve the monopolist's problem: $\hat{v}_A$ and $\hat{v}_N$. A loss-averse buyer at period 0 with valuation v buys at period 0 at the price attached to the contract with the MFC clause if and only if $v \geq \hat{v}_A$ and a loss-neutral buyer at period 0 with valuation v at period 0 at the price attached to the contract without the MFC clause if and only if $v \geq \hat{v}_N$. Thus, the residual demand left to the period 1 is $\beta\hat{v}_A + (1-\beta)\hat{v}_N$. At period 0, the monopolist announces P_A and P_N and these induces $\hat{v}_A$ and $\hat{v}_N$. So, we form the monopolist's problem in terms of $\hat{v}_A$ and $\hat{v}_N$. Note that a buyer who chooses to wait for period 1 buys if $v > p^i$ for $i = H, L$.

CHAPTER 4

COMPLETE INFORMATION

We first consider the complete information case, where the monopolist knows which buyer is loss-neutral and which buyer is loss-averse. We characterize the optimal prices, with and without the MFC clause, for both loss-neutral and loss-averse buyers, separately.

4.1 Loss-neutral buyers

A loss-neutral buyer has no loss/gain utility component and a loss-neutral buyer with valuation v will get $u_0(v) = v - P_N$ if she buys at period 0. If she waits for period 1, then she will get an expected utility $\mathbb{E}U_1(v)$ which is given by

$$\mathbb{E}U_1(v) = \begin{cases} v - \alpha p^H - (1-\alpha)p^L & \text{for } v \geq p^H > p^L \\ (1-\alpha)(v - p^L) & \text{for } p^H > v > p^L \\ 0 & \text{for } p^H > p^L \geq v \end{cases}$$

When there is an MFC clause, the monopolist pays the price difference to the buyer at period 1 if the price that the buyer paid at period 0 is higher than the price at period 1.Thus, if the monopolist offers a contract with $(P_{N,MFC}, MFC)$ to loss-neutral buyers then the utility of a loss-neutral buyer who chooses to buy at period 0 will be given by

$$u_0(v) = \begin{cases} v - P_{N,MFC} + \delta[\alpha(P_{N,MFC} - p^H) + (1-\alpha)(P_{N,MFC} - p^L)] & \text{for } P_{N,MFC} \geq p^H > p^L \\ v - P_{N,MFC} + \delta(1-\alpha)(P_{N,MFC} - p^L) & \text{for } p^H \geq P_{N,MFC} > p^L \\ v - P_{N,MFC} & \text{for } p^H > p^L \geq P_{N,MFC} \end{cases}$$

Lemma 1 *For any α, δ, p^L, it is never optimal for monopolist to adopt the MFC clause.*

Proof. See the Appendix. ■

Although the MFC clause serves as *option* (to fix the effective price), since the low price tomorrow is low enough $p^L < \frac{1}{2}$, the MFC clause makes the amount

to be paid back tomorrow large, which in turn hurts the monopolist. To show this, we first find the optimal prices in the absence of the MFC clause and in the presence of the MFC clause. Then we compare the monopolist's expected profit under these two cases. In Lemma 1, we show that if the buyers are loss-neutral then the monopolist does not adopt the MFC clause. In Altinok and Yilmaz (2023), they show that the monopolist adopts the MFC clause if and only if the buyers are sufficiently patient. The difference is that, in that paper, the next period's prices are endogenous, while we assume that the next period's prices are exogenous.

4.2 Loss-Averse Buyers

In the absence of an MFC clause, utility of a loss-averse buyer with valuation v is given in 3.1. In the presence of an MFC clause, utility of a loss-averse buyer with valuation v if she buys today is given by

$$u_0(v) = v - P_{A,MFC} + \delta\Big((1-\alpha)(P_{A,MFC} - p^L) + \alpha\mathbb{I}\big[v - P_{A,MFC} - R_H\big] + (1-\alpha)\mathbb{I}\big[v - p^L - R_L\big]\Big)$$

where $R_i = \max\{v - p^i, 0\}$, $i = H, L$, and $\mathbb{I}[\cdot]$ is as defined above.

Since we assume that $p^H > P_{A,MFC} > p^L$, the MFC clause is effective only in the state $i = L$ in the next period. So the monopolist pays the price difference to the buyer with probability $1 - \alpha$. Thus, for a buyer who chooses to buy at period 0, the valuation net of effective payment after the prices in next period are realized is $v - P_{A,MFC}$ with probability α, while it is $v - p^L$ with probability $1 - \alpha$. Hence, for loss/gain utility, she compares $v - P_{A,MFC}$ with her reference point in state $i = H$ with probability α, and compares $v - p^L$ with her reference point in state $i = L$ with probability $1 - \alpha$.

Now, we will consider when it is optimal for the monopolist to adopt the MFC clause when there is loss aversion by comparing the expected profit of the monopolist with the MFC clause and without the MFC clause. In Lemma 2, we show that the monopolist adopts the MFC clause for loss-averse buyers if buyers are sufficiently patient.

Lemma 2 *For any α, p^L, and λ there exists $\bar{\delta} < 1$ such that $\mathbb{E}\pi^{LA}_{MFC} > \mathbb{E}\pi^{LA}_{noMFC}$ if and only if $\delta > \bar{\delta}$.*

Proof. See the Appendix. ■

Lemma 2 says that when there is loss aversion the MFC clause is optimal for the monopolist if the buyers are sufficiently patient. The MFC clause serves an *option* (to fix the effective price). Since under the MFC clause, the seller can charge higher price, adopting the MFC clause is good for the seller. As the buyers are more patient, this potential refund becomes more valuable and therefore, the seller can charge higher price under the MFC clause as the patience increases. Thus, as the buyers are more patient, the benefit of adopting the MFC clause increases.

CHAPTER 5

INCOMPLETE INFORMATION

So far, we show that the MFC clause is never optimal for the monopolist if the buyers are loss-neutral, and it is optimal if the buyers are loss-averse and sufficiently patient. That is, if the monopolist knew the type of each buyer, then he would offer the contract $(P_N^{FB}, noMFC)$ and the contract $(P_{A,MFC}^{FB}, MFC)$ to the loss-neutral buyers and to the loss-averse buyers, respectively. We find two thresholds, $\hat{v}_{A,MFC}^{FB}$ and $\hat{v}_N^{FB}$. Suppose the valuation of a loss-averse buyer, v, is above $\hat{v}_{A,MFC}^{FB}$ threshold. In that case, the buyer will choose to buy today at $(P_{A,MFC}^{FB}, MFC)$ over waiting for tomorrow, and if the valuation is below $\hat{v}_{A,MFC}^{FB}$ threshold, then the buyer will choose to wait for tomorrow. Similarly, for a loss-neutral buyer, if the valuation is above $\hat{v}_N^{FB}$ threshold, then she will choose to buy today at $(P_N^{FB}, noMFC)$ over waiting for tomorrow. If the valuation is below $\hat{v}_N^{FB}$ threshold, then the buyer will choose to wait for tomorrow.

In this chapter, we consider the case where the monopolist does not know which buyer is loss-averse and which buyer is loss-neutral. The monopolist tries to screen the loss-averse buyers by using an MFC clause. There are two possible screening methods. The seller offers two contracts: one with an MFC clause and one without an MFC clause. The seller can construct the contracts such that the loss-neutral buyers will choose the contract with the MFC clause, and the loss-averse buyers will choose the contract without the MFC clause. In the other screening method, the seller can ensure that the loss-neutral buyers will prefer the contract without the MFC clause and the loss-averse buyers will prefer the contract with the MFC clause. We show that the first option is not possible and to screen the buyers, the monopolist offers two contracts, one without the MFC clause, $(P_N, noMFC)$, and the other with the MFC clause, (P_A, MFC), and wants the loss-neutral buyers to choose $(P_N, noMFC)$, while the loss-averse buyers to choose (P_A, MFC).

In this screening problem, we first analyze the incentive compatibility constraints, which will provide us with certain thresholds. These thresholds will

make sure that the contract targeted at a specific type provides her with a payoff at least as large as that from waiting. Since the expected payoff from waiting is at least zero, the individual rationality constraints will already be satisfied.

5.1 Incentive Compatibility Constraints

A buyer has three options: she can buy the good at a high price with an MFC contract or at a low price without an MFC contract, or she can wait for tomorrow. We can construct the incentive compatibility constraints in different ways. In one way we can find the threshold by considering the condition of being indifferent between buying today at the low price without the MFC contract and buying today at the high price with the MFC contract. However, this will not be possible. Another alternative is to consider the condition of being indifferent between buying today and waiting for tomorrow. In this second case, we have two possibilities. The seller can screen the buyers by offering two contracts,(P_N, MFC) and $(P_A, noMFC)$, so that the loss-neutral buyers will choose (P_N, MFC), while the loss-averse buyers will choose $(P_A, noMFC)$ where $P_N > P_A$. We will show that there are no optimal contracts for the seller in this screening method. The other possibility is to screen the buyers by offering them the contracts (P_A, MFC) and $(P_N, noMFC)$ so that the loss-averse buyers will prefer (P_A, MFC) and loss-neutral buyers will prefer the other contract.

In the first screening method, we first find the thresholds which make sure that the contract targeted at a specific type provides her with a payoff at least as large as that from waiting. There is a threshold, $\hat{\hat{v}}_N$, such that the loss-neutral buyers at period 0 with valuation $v > \hat{\hat{v}}_N$ choose to buy the good today at (P_N, MFC) over waiting for period 1. For loss-averse buyers, there is a threshold, $\hat{\hat{v}}_A$, such that buyers at period 0 with valuation $v > \hat{\hat{v}}_A$ choose to buy the good today at $(P_A, noMFC)$ over waiting for period 1. We find these thresholds, $\hat{\hat{v}}_N$ and $\hat{\hat{v}}_A$, by solving the condition of being indifferent between buying today at (P_N, MFC) and waiting for period 1 and buying today at $(P_A, noMFC)$ and waiting for period 1, respectively. Also, we need loss-neutral buyers with valuation above $\hat{\hat{v}}_N$ to choose (P_N, MFC)

over $(P_A, noMFC)$ and buyers with valuation below $\hat{\hat{v}}_N$ to choose waiting for tomorrow over the other options. Similaly, loss-averse buyers with valuation above $\hat{\hat{v}}_A$ should choose $(P_A, noMFC)$ over (P_N, MFC) and buyers with valuation below $\hat{\hat{v}}_A$ should choose waiting over the other options. In Lemma 3 we show that this type of screening is not possible.

Lemma 3 *The incentive compatibilty constraint of loss-neutral buyers is given by*

$$(ICLN) \qquad -P_N(1-\delta+\delta\alpha)+P_A-\delta(1-\alpha)p^L \geq 0 \tag{5.1}$$

The incentive compatibility constraints for loss-averse buyers with valuation v where $v > \hat{\hat{v}}_A$ or $\hat{\hat{v}}_A > v > P_N$ is given by

$$(ICLA_1) \qquad -P_A(1+\delta\alpha+\delta(1-\alpha)\lambda)+P_N(1-\delta+2\delta\alpha)+\delta(1-\alpha)p^L(\lambda+1) \geq 0 \tag{5.2}$$

The incentive compatibility constraints for loss-averse buyers with valuation v where $v < P_N$ is given by

$$(ICLA_2) \qquad -v(1+\delta\alpha\lambda-\delta(1-\alpha)(1+\alpha-\alpha\lambda)+P_N(1-\delta+\delta\alpha+\delta\alpha\lambda)+\delta(1-\alpha)p^L\alpha(\lambda-1) > 0 \tag{5.3}$$

The inequality 5.2 does not hold for any parameter values.

Proof. See the Appendix. ■

In the proof of Lemma 3, we show that the incentive compatibility constraint of loss-averse buyers with a valuation above the threshold and loss-averse buyers with a valuation between the threshold and the price attached to the MFC contract does not hold for any parameter values. That is, the loss-averse buyers with a valuation above the threshold will have an incentive to deviate from the contract without the MFC clause to the contract with the MFC clause, and the loss-averse buyers with a valuation between the threshold and the price attached to the MFC contract will have an incentive to deviate from waiting for tomorrow to the contract with the MFC clause. Therefore, Lemma 3 says there are no optimal contracts such

that loss-averse buyers will choose the contract without the MFC clause, and loss-neutral buyers will choose the contract with the MFC clause. Even if the incentive compatibility constraint of loss-neutral buyers given by the equation 5.1 and the incentive compatibility constraint of loss-averse buyers with a valuation below the price attached to the MFC contract given by the equation 5.3 hold together, there will be no screening. In such a case, we will have the following scenario: loss-neutral buyers with valuation $v > \hat{\hat{v}}_N$ will choose the contract with the MFC clause and loss-neutral buyers with valuation $v < \hat{\hat{v}}_N$ will choose to wait for tomorrow. Also, loss-averse buyers with valuation $v > \hat{\hat{v}}_A$ or $\hat{\hat{v}}_A > v > P_N$ will choose the contract with the MFC clause and the loss-averse buyers with valuation $v < P_N = \hat{\hat{v}}_N$ will choose to wait. Thus, regardless of the type of buyers, everyone will choose the contract with the MFC clause if their valuation is above the $\hat{\hat{v}}_N$ threshold and will choose to wait for tomorrow otherwise.

The only possibility for the seller to screen the buyers is to offer them the contracts, (P_A, MFC) and $(P_N, noMFC)$, where loss-averse buyers will prefer (P_A, MFC) and loss-neutral buyers will prefer $(P_N, noMFC)$. In this case, there is a threshold,$\hat{v}_N$, such that loss-neutral buyers at period 0 with valuation $v > \hat{v}_N$ prefers to buy the good today at $(P_N, noMFC)$ over waiting for period 1. Similarly, for loss-averse buyers, there is a threshold ,$\hat{v}_A$, such that buyers at period 0 with valuation $v > \hat{v}_A$ choose to buy the good today at (P_A, MFC) instead of waiting for period 1. These thresholds,$\hat{v}_N$ and $\hat{v}_A$, arise from the condition of being indifferent between buying today at $(P_N, noMFC)$ and waiting for period 1 and buying today at (P_A, MFC) and waiting for period 1, respectively. We also need that loss-neutral buyers with valuation above $\hat{v}_N$ to choose $(P_N, noMFC)$ over (P_A, MFC) and buyers with valuation below $\hat{v}_N$ to choose waiting for period 1 instead of buying it at either of the options. Similarly, we need loss-averse buyers with valuations above $\hat{v}_A$ to choose (P_A, MFC) over $(P_N, noMFC)$ and those with valuation below $\hat{v}_A$ to choose to wait for period 1 instead of buying it at either of the options. Lemma 4 characterizes these incentive compatibility constraints.

Lemma 4 *The incentive compatibility constraint of loss-neutral buyers is given by*

$$(ICLN) \qquad -P_N + P_A(1-\delta+\delta\alpha) + \delta(1-\alpha)p^L \geq 0 \tag{5.4}$$

The incentive compatibility constraints for loss-averse buyers with valuation v where $P_A > v > \hat{v}_A$ or $\hat{v}_A > v$ is given by

$$(ICLA_1) \qquad -\frac{1+\delta\alpha-\delta(1-\alpha)(1+\alpha-\alpha\lambda)}{1+\delta\alpha\lambda-\delta(1-\alpha)(1+\alpha-\alpha\lambda)}(1-\delta+\delta\alpha+\delta\alpha\lambda)P_A$$
$$+P_N(1+\delta\alpha+\delta(1-\alpha)\lambda)+\delta(1-\alpha)p^L\left(-1-\lambda+\frac{\delta\alpha^2(\lambda-1)^2}{1+\delta\alpha\lambda-\delta(1-\alpha)(1+\alpha-\alpha\lambda)}\right)\geq 0 \tag{5.5}$$

The incentive compatibility constraints for loss-averse buyers with valuation v where $v > P_A$ is given by

$$(ICLA_2) \qquad P_A(-1+\delta-2\delta\alpha)+\delta(1-\alpha)p^L(-1-\lambda)+P_N(1+\delta\alpha+\delta\lambda(1-\alpha)) \geq 0 \tag{5.6}$$

Moreover, the inequality 5.5 implies the inequality 5.6.

Proof. See the Appendix. ■

Lemma 5 *If a loss-averse buyer with valuation $\hat{v}_A$ chooses to buy today at (P_A, MFC) over buying today at $(P_N, noMFC)$, then another loss-averse buyer with valuation v where $v > \hat{v}_A$ chooses to buy today at (P_A, MFC). Also, if a loss-averse buyer with valuation $\hat{v}_A$ chooses to wait for tomorrow over buying today at $(P_N, noMFC)$, then another loss-averse buyer with valuation v' where $v' < \hat{v}_A$ chooses to wait for tomorrow.*

Proof. See the Appendix. ■

Lemma 4 shows that there are two incentive compatibility constraints that we need to consider when we solve the monopolist's problem and they are given by the inequalities 5.4 and 5.5.

Before we proceed with the optimal second-best prices, we first analyze the incentive compatibility constraints at the first-best prices we found above. This will

allow us to see when the incentive problem arises and when it does not. We first show that for the loss-neutral buyers, there is no incentive problem at the first-best prices (Lemma 6). Then, for loss-averse we characterize when there is an incentive problem at the first-best prices and not (Propositions 1 and 2).

Lemma 6 *The incentive compatibility of loss-neutral buyers (ICLN), calculated at the first-best prices P_A, P_N, always holds with strict inequality for all parameter values.*

Proof. See the Appendix. ■

Lemma 6 shows that when these inequalities are calculated at the first-best prices, given in Lemma 7 and Lemma 10, loss-neutral buyers have no incentive to deviate from $(P_N, noMFC)$ to (P_A, MFC) if they are offered the first-best prices.

Proposition 1 *For $\alpha = \frac{1}{2}$, there exists $\bar{\delta}$ such that the incentive compatibility of loss-averse buyers ($ICLA_1$), calculated at the first-best prices does not hold for $\delta > \bar{\delta}$, and holds for $\delta \leq \bar{\delta}$.*

Proof. See the Appendix. ■

Proposition 1 says that if buyers are sufficiently patient then they will have an incentive to deviate from (P_A, MFC) to $(P_N, noMFC)$, where P_A and P_N are the first-best prices. As the buyers are more patient, the first-best price increases when there is an MFC clause, and the first-best price decreases when there is no MFC clause. This decrease in the first-best price when there is no MFC clause attracts the buyers, so they want to switch to the contract with no MFC clause as they are more patient.

Proposition 2 *For $\alpha = \frac{1}{2}$, if $p^L > \frac{3(1-\delta)}{4-3\delta}$ where $\delta < 1$, there exists $\bar{\lambda}$ such that the incentive compatibility of loss-averse buyers ($ICLA_1$), calculated at the first-best prices does not hold for $\lambda > \bar{\lambda}$ and holds for $\lambda \leq \bar{\lambda}$. Moreover, if $p^L < \frac{1-\delta}{2-\delta}$ or $\tilde{p}^L < p^L < \frac{3(1-\delta)}{4-3\delta}$, then it holds with inequality for all values of λ.*

Proof. See the Appendix. ■

Proposition 2 says that if buyers are sufficiently loss-averse then they have an incentive to deviate from (P_A, MFC) to $(P_N, noMFC)$ if P_A and P_N are the first-best prices. As the buyer, who is buying today, becomes more loss-averse, the possible loss from a price drop increases. Also, if the low price in the next period is not too low, then the potential refund from the MFC clause becomes less attractive. Therefore, in both of the cases, the MFC clause is not sufficient so that she chooses to buy at the lower price with no MFC clause.

For $\delta = 1$, we have $\bar{\lambda} = 1$, thus first-best prices are not incentive compatible. The following proposition characterizes the optimal second-best prices for perfectly patient buyers, $\delta = 1$.

Proposition 3 *For $\delta = 1$, the optimal prices are given by*

$$P_A^{SB} = \frac{(\alpha + 2\lambda - \alpha\lambda)(1 - \beta + \beta\alpha)}{2\alpha((1-\beta)(\alpha + 2\lambda - \alpha\lambda) + \beta(\lambda + 1))} - \frac{(1-\alpha)p^L\beta(\lambda - 1)}{2((1-\beta)(\alpha + 2\lambda - \alpha\lambda) + \beta(\lambda + 1))}$$

$$P_N^{SB} = \frac{(\alpha + 2\lambda - \alpha\lambda)(1 - \beta + \beta\alpha)}{2((1-\beta)(\alpha + 2\lambda - \alpha\lambda) + \beta(\lambda + 1))} + \frac{(1-\alpha)p^L}{2}\left[2 - \frac{\beta\alpha(\lambda - 1)}{(1-\beta)(\alpha + 2\lambda - \alpha\lambda) + \beta(1 + \lambda)}\right]$$

Proof. See the Appendix. ■

Proposition 3 says that when $\delta = 1$, the ICC of the loss-averse buyers at the first-best prices is negative for all λ and p^L. This means that loss-averse buyers have an incentive to deviate from (P_A, MFC) to $(P_N, noMFC)$ if these prices are the first-best prices given in Lemma 7 and Lemma 10.

Proposition 4 *For $\delta = 1$, the second-best price in the contract with the MFC clause is higher than the second-best price in the contract without the MFC clause given in Proposition 3.*

Proof. See the Appendix. ■

Proposition 4 is a key result resembling our motivating examples, where a more flexible option is made available via a higher price while a less flexible option is present at a lower price.

CHAPTER 6

CONCLUSION

We studied the adverse selection problem of a monopolist facing loss-averse and loss-neutral buyers with uncertainty about the type of buyers. We first found the optimal prices attached to the contract with an MFC clause and the contract without one if the monopolist knew the type of each buyer. We showed that an MFC clause is never optimal for the monopolist if the buyers are loss-neutral, while it is optimal if the buyers are loss-averse and sufficiently patient. We considered the incentive problem and showed that loss-averse buyers will deviate from the contract with the MFC clause to the contract without one if they are sufficiently patient and the prices attached to the contracts are the first-best. We also showed that at these first-best prices, for a given patience level, sufficiently loss-averse buyers will deviate from the contract with the MFC clause to the contract without the MFC clause. We found the second-best prices for perfectly patient buyers.

An alternative modelling would be to have no asymmetric information regarding the valuation of the buyers, but introduce the types in terms of the loss aversion parameter that is different for each buyer. Each buyer has the same valuation v but different λ distributed over some range $[1, \bar{\lambda}]$. While this modelling may sound more natural and simpler, we may not be able to see the effect of degree of loss aversion on the optimal contracts.

Also, we have the loss-aversion formulation in terms of comparing net payoffs, while it is usually done in the valuation and price dimension separately. However, since there is no match-value dimension in our model (only a possible difference between buying and not buying), we believe our results would not change qualitatively.

APPENDIX A

Before proving Lemma 1, we will find optimal prices in the absence of MFC and in the presence of MFC. Lemma 7 characterizes the optimal residual demand and price when the monopolist offers the contract $(P_N, noMFC)$ to loss-neutral buyers.

Lemma 7 *In the absence of MFC, when buyers are loss-neutral, the optimal price and the residual demand are characterized by*

$$\begin{aligned} P_N^{FB} &= \frac{1-\delta+\delta\alpha+(1-\alpha)(1+\delta)p^L}{2} \\ \hat{v}_N^{FB} &= \frac{1}{2}+\frac{p^L(1-\alpha)(1-\delta)}{2(1-\delta+\delta\alpha)} \end{aligned}$$

where $p^H > \hat{v}_N^{FB} > P_N^{FB} > p^L$.

PROOF of LEMMA 7. There is a threshold $\hat{v}_N$, such that a buyer at period 0 with valuation v purchases if and only if $v \geq \hat{v}_N$. Threshold $\hat{v}_N$ solves $u_0(\hat{v}_N) = \delta \mathbb{E}U_1(\hat{v}_N)$. Buyers will purchase the good today if and only if $u_0(\hat{v}_N) = \hat{v}_N - P_N > 0$, that is, $\hat{v}_N > P_N$. There are three cases:

Case (i): $v \geq p^H > p^L$: This case is not possible because it contradicts with the model's assumption that $p^H > 1$.

Case (ii): $p^H > \hat{v}_N > p^L$: In this case, $\mathbb{E}U_1(\hat{v}_N) = (1-\alpha)(\hat{v}_N - p^L)$. Hence, $u_0(\hat{v}_N) = \delta \mathbb{E}U_1(\hat{v}_N)$ implies $\hat{v}_N - P_N = \delta(1-\alpha)(\hat{v}_N - p^L)$. Thus,

$$\hat{v}_N = \frac{P_N - (1-\alpha)\delta p^L}{1-\delta(1-\alpha)}$$

. Then using this expression we can write P_N as a function of $\hat{v}_N$

$$P_N = \hat{v}_N(1-\delta+\delta\alpha) + (1-\alpha)\delta p^L$$

The overall profit of the monopolist in this case is

$$\pi = (1 - \hat{v}_N)P_N + (1 - \alpha)p^L(\hat{v}_N - p^L)$$

By maximizing this profit with respect to $\hat{v}_N$, we get the first order condition with respect to $\hat{v}_N$ as follows

$$-P_N + (1 - \hat{v}_N)\frac{\partial P_N}{\partial \hat{v}_N} + (1 - \alpha)p^L = 0$$

Then using P_N expression, we find $\frac{\partial P_N}{\partial \hat{v}_N} = 1 - \delta + \delta\alpha$. Thus, the FOC becomes

$$\hat{v}_N(1 - \delta + \delta\alpha) + (1 - \alpha)\delta p^L = (1 - \hat{v}_N)(1 - \delta + \delta\alpha) + (1 - \alpha)p^L$$

which implies

$$\hat{v}_N = \frac{1}{2} + \frac{(1 - \alpha)(1 - \delta)p^L}{2(1 - \delta + \delta\alpha)}$$

Then P_N is given by

$$P_N = \frac{1 - \delta + \delta\alpha + (1 - \alpha)p^L)(1 + \delta)}{2}$$

Case (iii): $p^H > p^L \geq \hat{v}_N$: We have $\mathbb{E}U_1(\hat{v}_N) = 0$. Thus, $u_0(\hat{v}_N) = \delta\mathbb{E}U_1(\hat{v}_N)$ implies $\hat{v}_N - P_N = 0$. Hence, we have $\hat{v}_N = P_N$.

The overall profit of the monopolist is given by

$$\pi = (1 - \hat{v}_N)P_N$$

By maximizing this profit with respect to $\hat{v}_N$, we get the first order condition with respect to $\hat{v}_N$ as follows

$$-P_N + (1 - \hat{v}_N)\frac{\partial P_N}{\partial \hat{v}_N} = 0$$

Then using P_N expression, we find $\frac{\partial P_N}{\partial \hat{v}_N} = 1$.Thus, the FOC becomes

$$\hat{v}_N = (1 - \hat{v}_N)$$

which implies $\hat{v}_N = P_N = \frac{1}{2}$. However, in this case we would have $p^L \geq \hat{v}_N = \frac{1}{2}$ which contradicts the model's assumption that $p^L < \frac{1}{2}$.

Thus, the only possibility is *Case (ii)*. For this case we need to check $p^H > \hat{v}_N > p^L$ and $\hat{v}_N > P_N$ hold for any parameter values. Note that $P_N = \hat{v}_N(1 - \delta + \delta\alpha) + (1 - \alpha)\delta p^L$. For $\hat{v}_N > P_N$ to hold we need

$$\begin{aligned} \hat{v} &> \hat{v}_N(1 - \delta + \delta\alpha) + (1 - \alpha)\delta p^L \\ \delta(1 - \alpha)\hat{v} &> (1 - \alpha)p^L \\ \delta\hat{v} &> p^L \end{aligned}$$

Since $\hat{v} > \delta\hat{v}$, to show $\hat{v} > P_N$ and $\hat{v} > p^L$, it suffices to show that $\delta\hat{v} > p^L$. To see this,plug $\hat{v}$ into the inequality.

$$\delta\Big(\frac{1}{2} + \frac{p^L(1 - \alpha)(1 - \delta)}{2(1 - \delta + \delta\alpha)}\Big) > p^L$$

This holds if and only if

$$\delta\Big(\frac{1 - \delta + \delta\alpha + p^L(-1 + \delta - \alpha - \delta\alpha)}{2(1 - \delta + \delta\alpha)}\Big) > 0$$

The denominator in the above expression is positive. Therefore the inequality holds if the numerator is positive. To see this, we use that $p^L < \frac{1}{2}$.

$$\begin{aligned} 1 - \delta + \delta\alpha + p^L(-1 + \delta - \alpha - \delta\alpha) &> 1 - \delta + \delta\alpha + \frac{(-1 + \delta - \alpha - \delta\alpha)}{2} \\ &> \frac{(1 - \delta)(1 - \alpha)}{2} \\ &> 0 \end{aligned}$$

Also we need $\hat{v} < p^H$.

$$\begin{aligned}
\hat{v} - p^H &= \frac{1}{2} + \frac{p^L(1-\alpha)(1-\delta)}{2(1-\delta+\delta\alpha)} - p^H \\
&< \frac{1}{2} + \frac{(1-\alpha)(1-\delta)}{4(1-\delta+\delta\alpha)} - 1 \\
&= -\frac{1}{2} + \frac{(1-\alpha)(1-\delta)}{4(1-\delta+\delta\alpha)} \\
&= \frac{-1+\delta-\alpha-\delta\alpha}{4(1-\delta+\delta\alpha)} \\
&< 0
\end{aligned}$$

In the first inequality, we use that $p^L < \frac{1}{2}$ and $p^H > 1$. For the last inequality, the denominator is positive and the numerator is negative.Therefore we end up with *Case (ii)* which gives us the optimal price and residual demand given in the Lemma. ■

Lemma 8 characterizes the optimal residual demands and prices when the monopolist offers the contract $(P_{N,MFC}, MFC)$ to loss-neutral buyers .

Lemma 8 *In the presence of MFC, when buyers are loss-neutral, the optimal price and the residual demand are characterized by*

$$P^{FB}_{N,MFC} = \hat{v}^{FB}_{N,MFC} = \frac{1}{2}$$

where $p^H \geq \hat{v}^{FB}_{N,MFC} = P^{FB}_{N,MFC} > p^L$.

Proof. See the Appendix. ■

PROOF of LEMMA 8. There is a threshold $\hat{v}_{N,MFC}$, such that a buyer at period 0 with valuation v purchases if and only if $v \geq \hat{v}_{N,MFC}$. We need to solve $u_0(\hat{v}_{N,MFC}) = \delta \mathbb{E}U_1(\hat{v}_{N,MFC})$ to find threshold v.

The expected utility of a buyer who chooses to wait for period 1 is given by

$$\mathbb{E}U_1(v) = \alpha \max\{v - p^H, 0\} + (1-\alpha)\max\{v - p^L, 0\}$$

If a buyer with valuation v chooses to buy today then her utility is given by

$$u_0(v) \;=\; v - P_{N,MFC} + \delta(\alpha \max\{P_{N,MFC} - p^H, 0\} + (1-\alpha)\max\{P_{N,MFC} - p^L, 0\})$$

There are three cases that we need to consider.

Case (i): $v \geq p^H > p^L$: This case is not possible because it contradicts with the model's assumption that $p^H > 1$.

Case (ii): $p^H \geq v \geq p^L$: In this case $\mathbb{E}U_1(v) = (1-\alpha)(v - p^L)$.

Subcase (ii.1): $P_{N,MFC} \geq p^H > p^L$: The utility of a buyer with valuation v who chooses to buy today, in the presence of MFC, is given by

$$u_0(v) \;=\; v - P_{N,MFC} + \delta[\alpha(P_{N,MFC} - p^H) + (1-\alpha)(P_{N,MFC} - p^L)]$$

By solving $u_0(\hat{v}_{N,MFC}) = \mathbb{E}U_1(\hat{v}_{N,MFC})$, we find $\hat{v}_{N,MFC}$ as

$$\hat{v}_{N,MFC} \;=\; \frac{(1-\delta)P_{N,MFC} + \delta\alpha p^H}{1 - \delta + \delta\alpha}$$

But this case is not possible. To show this, we check $\hat{v}_{N,MFC} - p^H$.

$$\begin{aligned}
\hat{v}_{N,MFC} - p^H &= \frac{(1-\delta)P_{N,MFC} + \delta\alpha p^H}{1 - \delta + \delta\alpha} - p^H \\
&= \frac{(1-\delta)P_{N,MFC} - p^H(1-\delta)}{1 - \delta + \delta\alpha} \\
&= \frac{(1-\delta)(P_{N,MFC} - p^H)}{1 - \delta + \delta\alpha} \\
&\geq 0
\end{aligned}$$

The inequality comes from $P_{N,MFC} \geq p^H$. Thus, we have $\hat{v}_{N,MFC} \geq p^H$ which contradicts with *Case ii*.

Subcase (ii.2): $p^H \geq P_{N,MFC} > p^L$: The utility of a buyer with valuation v who chooses to buy today, in the presence of MFC, is given by

$$u_0(v) \;=\; v - P_{N,MFC} + \delta(1-\alpha)(P_{N,MFC} - p^L)$$

By solving $u_0(\hat{v}_{N,MFC}) = \mathbb{E}U_1(\hat{v}_{N,MFC})$, we find that $\hat{v}_{N,MFC} = P_{N,MFC}$. Then the overall profit of the monopolist is given by

$$\begin{aligned}\pi \;&=\; (1-\hat{v}_{N,MFC})P_{N,MFC} + (1-\alpha)p^L(\hat{v}_{N,MFC} - p^L)\\ &\quad -(1-\alpha)(1-\hat{v}_{N,MFC})(P_{N,MFC} - p^L)\\ &=\; (1-\hat{v}_{N,MFC})\alpha P_{N,MFC} + (1-\alpha)p^L(1-p^L)\end{aligned}$$

Maximizing this profit with respect to $\hat{v}_{N,MFC}$, we get the first order condition as follows

$$-\alpha P_{N,MFC} + \alpha(1-\hat{v})\frac{\partial P_{N,MFC}}{\partial \hat{v}} = 0$$

By plugging $P_{N,MFC} = \hat{v}$ into the FOC, we get $\hat{v} = P_{N,MFC} = \frac{1}{2}$

Subcase (ii.3): $p^H > p^L \geq P_{N,MFC}$: The utility of a buyer with valuation v who chooses to buy today, in the presence of MFC, is given by $u_0(v) = v - P_{N,MFC}$.

$$\begin{aligned}u_0(v) \;&=\; v - P_{N,MFC} \geq v - p^L\\ &>\; (1-\alpha)(v-p^L)\\ &>\; \delta(1-\alpha)(v-p^L)\\ &=\; \delta\mathbb{E}U_1(v)\end{aligned}$$

The first inequality comes from $p^L \geq P_{N,MFC}$, the second and third inequalities are due to $\delta \in (0,1)$ and $\alpha \in (0,1)$. Thus, for all v we have $u_0(v) > \delta\mathbb{E}U_1(v)$, which implies $\hat{v}_{N,MFC} = 0$. Then since $\hat{v}_{N,MFC} \geq p^L \geq P_{N,MFC}$, we have $P_{N,MFC} \leq 0$, which cannot be optimal.

Case (iii): $p^H > p^L \geq v$: In this case, the expected utility of a buyer who chooses to wait for period 1 is given by $\mathbb{E}U_1(v) = 0$.

Subcase (iii.1): $P_{N,MFC} \geq p^H > p^L$: The utility of a buyer with valuation v who chooses to buy today, in the presence of MFC, is given by

$$\begin{aligned} u_0(v) &= v - P_{N,MFC} + \delta[\alpha(P_{N,MFC} - p^H) + (1-\alpha)(P_{N,MFC} - p^L)] \\ &< v - \delta p^L - (1-\delta)P_{N,MFC} \\ &< v - p^L \\ &\leq 0 \end{aligned}$$

The first inequality comes from $p^H > p^L$, and the second inequality comes from $P_{N,MFC} > p^L$. The last inequality is due to $v \leq p^L$. Thus, for all v we have $u_0(v) < \delta\mathbb{E}U_1$, which implies $\hat{v}_{N,MFC} = 1$. Then since $p^L \geq \hat{v}_{N,MFC}$, we have $p^L \geq 1$, which is a contradiction to the model's assumption $p^L < \frac{1}{2}$.

Subcase (iii.2): $p^H \geq P_{N,MFC} > p^L$: The utility of a buyer with valuation v who chooses to buy today, in the presence of MFC, is given by

$$\begin{aligned} u_0(v) &= v - P_{N,MFC} + \delta(1-\alpha)(P_{N,MFC} - p^L) \\ &= v - P_{N,MFC}(1 - \delta + \delta\alpha) - \delta(1-\alpha)p^L \\ &< v - p^L \\ &\leq 0 \end{aligned}$$

The first inequality comes from $P_{N,MFC} > p^L$, and the last inequality comes from $v \leq p^L$. Thus, for all v we have $u_0(v) < \delta\mathbb{E}U_1$, which implies $\hat{v}_{N,MFC} = 1$. Then since $p^L \geq \hat{v}_{N,MFC}$, $p^L \geq 1$, which is a contradiction to the model's assumption $p^L < \frac{1}{2}$.

Subcase (iii.3): $p^H > p^L \geq P_{N,MFC}$: The utility of a buyer with valuation v who chooses to buy today, in the presence of MFC, is given by $u_0(v) = v - P_{N,MFC}$.

There are two possibilities: either $v \geq P_{N,MFC}$ or $v < P_{N,MFC}$. If $v \geq P_{N,MFC}$, then $u_0(v) \geq \delta \mathbb{E}U_1$, for all v. This implies $\hat{v}_{N,MFC} = 0$ which implies $P_{N,MFC} = 0$ and this cannot be optimal. If $v < P_{N,MFC}$, then $u_0(v) < \delta \mathbb{E}U_1$, for all v. This implies $\hat{v}_{N,MFC} = 1$. Then since $p^L \geq \hat{v}_{N,MFC}$, we have $p^L \geq 1$, which is a contradiction to the model's assumption $p^L < \frac{1}{2}$.

Thus, in the presence of MFC, the optimal price and residual demands are given by $\hat{v}_{N,MFC} = P_{N,MFC} = \frac{1}{2}$ where $p^H \geq \hat{v}_{N,MFC} = P_{N,MFC} > p^L$. ■

PROOF of LEMMA 1. Let $\mathbb{E}\pi^{noLA}_{noMFC}$ denote the expected profit of the monopolist when there is no loss aversion and in the absence of MFC. From Lemma 7 profit of the monopolist is given by

$$\begin{aligned}
\mathbb{E}\pi^{noLA}_{noMFC} &= (1-\hat{v}_N)P_N + (1-\alpha)p^L(\hat{v}_N - p^L) \\
&= \frac{1-\delta+\delta\alpha}{4} + \frac{p^L(1-\alpha)(1+\delta)}{2} + \frac{P_1^{L^2}(1-\alpha)^2(1-\delta)^2}{4(1-\delta+\delta\alpha)} - (1-\alpha)P_1^{L^2}
\end{aligned}$$

Let $\mathbb{E}\pi^{noLA}_{MFC}$ denote the expected profit of the monopolist when there is no loss aversion and in the presence of MFC.From the Lemma 8 profit of the monopolist is given by

$$\begin{aligned}
\mathbb{E}\pi^{noLA}_{MFC} &= (1-\hat{v}_{N,MFC})P_{N,MFC} + (1-\alpha)p^L(\hat{v}_{N,MFC} - p^L) \\
&\quad - (1-\alpha)(1-\hat{v}_{N,MFC})(P_{N,MFC} - p^L) \\
&= (1-\hat{v}_{N,MFC})\alpha P_{N,MFC} + (1-\alpha)p^L(1-p^L) \\
&= \frac{1}{4} - (1-\alpha)\left(\frac{1}{2} - p^L\right)^2
\end{aligned}$$

Now look at the difference between $\mathbb{E}\pi^{noLA}_{noMFC}$ and $\mathbb{E}\pi^{noLA}_{MFC}$. The difference $\mathbb{E}\pi^{noLA}_{noMFC} - \mathbb{E}\pi^{noLA}_{MFC}$ is given by

$$\begin{aligned}
&= \frac{-\delta(1-\alpha)}{4} + \frac{1-\alpha}{4} + \frac{(\delta-1)p^L(1-\alpha)}{2} + \frac{(p^L)^2(1-\alpha)^2(1-\delta)^2}{4(1-\delta+\delta\alpha)} \\
&= \frac{(1-2p^L)(1-\alpha)(1-\delta)}{4} + \frac{(p^L)^2(1-\alpha)^2(1-\delta)^2}{4(1-\delta+\delta\alpha)} \\
&> 0
\end{aligned}$$

The last inequality follows from $p^L < \frac{1}{2}$. Thus, we have $\mathbb{E}\pi^{noLA}_{noMFC} > \mathbb{E}\pi^{noLA}_{MFC}$. ■

Before proving Lemma 2, we will find optimal prices in the absence of MFC and in the presence of MFC. Lemma 9 characterizes the optimal residual demands and price when the monopolist offers the contract $(P_A, noMFC)$ to loss-averse buyers.

Lemma 9 *In the absence of MFC, when buyers are loss-averse, the optimal residual demand and price are characterized by*

$$P_A^{FB} = \frac{1+\delta\alpha-\delta(1-\alpha)(1+\alpha-\alpha\lambda)+p^L(1-\alpha)(1+\delta+2\delta\alpha+2\delta\lambda-2\delta\alpha\lambda)}{2(1+\delta\alpha+\delta\lambda-\delta\alpha\lambda)}$$

$$\hat{v}_A^{FB} = \frac{1}{2}+\frac{p^L(1-\alpha)(1-\delta)}{2(1+\delta\alpha-\delta(1-\alpha)(1+\alpha-\alpha\lambda))}$$

where $p^H > \hat{v}_A^{FB} > P_A^{FB} > p^L$.

PROOF of LEMMA 9. There is a threshold $\hat{v}_A$ such that buyers with valuation above that threshold buys today and buyers with valuation below that threshold waits for tomorrow. We need to solve $u_0(\hat{v}_A) = \delta\mathbb{E}U_1(\hat{v}_A)$ to find $\hat{v}_A$. The expected utility of a buyer who chooses to wait for tomorrow is given by

$$\mathbb{E}U_1(v) = \begin{cases} v-\alpha p^H-(1-\alpha)p^L+\alpha(1-\alpha)(1-\lambda)(p^H-p^L) & \text{for } v \geq p^H > p^L \\ (v-p^L)(1-\alpha)(1+\alpha-\alpha\lambda) & \text{for } p^H \geq v \geq p^L \\ 0 & \text{for } p^H > p^L \geq v \end{cases}$$

If a buyer with valuation v chooses to buy today then her utility is given by

$$\begin{aligned} u_0(v) &= v-P_A+\delta\big(\alpha\mathbb{I}\big[v-P_A-\max\{v-p^H,0\}\big] \\ &\quad +(1-\alpha)\mathbb{I}\big[v-P_A-\max\{v-p^L,0\}\big]\big) \end{aligned}$$

There are three cases and their subcases that we need to consider.

Case (i): $v \geq p^H > p^L$: This case is not possible because it contradicts with the model's assumption that $P^H > 1$.

Case (ii): $p^H \geq v \geq p^L$: The profit maximizing $\hat{v}$ satisfies the same FOC as in Lemma 7 under *Case (ii)* and it is given by

$$-P_A + (1-\hat{v}_A)\frac{\partial P_A}{\partial \hat{v}_A} + (1-\alpha)p^L = 0$$

Assume $v \geq P_A$.

Subcase (ii.1): $P_A \geq p^H > p^L$: We assume $v \geq P_A$ which implies $v \geq p^H$ and this contradicts with *Case ii*.

Subcase (ii.2): $p^H \geq P_A > p^L$: The utility of a buyer who chooses to buy today is given by

$$u_0(v) = v - P_A + \delta[\alpha(v-P_A) + (1-\alpha)\lambda(p^L - P_A)]$$

. By solving $u_0(\hat{v}_A) = \delta \mathbb{E}U_1(\hat{v}_A)$, we find $\hat{v}_A$ as

$$\hat{v}_A = \frac{P_A(1+\delta\alpha+\delta\lambda-\delta\alpha\lambda) - \delta(1-\alpha)(1+\alpha+\lambda-\alpha\lambda)p^L}{1+\delta\alpha-\delta(1-\alpha)(1+\alpha-\alpha\lambda)}$$

Using $\hat{v}_A$ expression, we can write P_A as a function of $\hat{v}_A$ as follows

$$P_A = \frac{\hat{v}_A(1+\delta\alpha-\delta(1-\alpha)(1+\alpha-\alpha\lambda)) + \delta(1-\alpha)(1+\alpha+\lambda-\alpha\lambda)p^L}{1+\delta\alpha+\delta\lambda-\delta\alpha\lambda}$$

and we get

$$\frac{\partial P_A}{\partial \hat{v}_A} = \frac{(1+\delta\alpha-\delta(1-\alpha)(1+\alpha-\alpha\lambda))}{1+\delta\alpha+\delta\lambda-\delta\alpha\lambda}$$

Plugging these above equations in the FOC, we find

$$P_A = \frac{1+\delta\alpha-\delta(1-\alpha)(1+\alpha-\alpha\lambda) + p^L(1-\alpha)(1+\delta+2\delta\alpha+2\delta\lambda-2\delta\alpha\lambda)}{2(1+\delta\alpha+\delta\lambda-\delta\alpha\lambda)}$$

$$\hat{v}_A = \frac{1}{2} + \frac{p^L(1-\alpha)(1-\delta)}{2(1+\delta\alpha-\delta(1-\alpha)(1+\alpha-\alpha\lambda))}$$

Subcase (ii.3): $p^H > p^L \geq P_A$: The utility of a buyer who chooses to buy today is given by

$$u_0(v) = v - P_A + \delta[\alpha(v - P_A) + (1-\alpha)(p^L - P_A)]$$

. By solving $u_0(\hat{v}_A) = \delta \mathbb{E}U_1(\hat{v}_A)$, we find $\hat{v}_A$ as

$$\hat{v}_A = \frac{P_A(1+\delta) - \delta(1-\alpha)(2-\alpha(\lambda-1))p^L}{1+\delta\alpha - \delta(1-\alpha)(1+\alpha-\alpha\lambda)}$$

But this case is not possible. To show this, we check $\hat{v}_A - p^L$:

$$\begin{aligned}
\hat{v}_A - p^L &= \frac{P_A(1+\delta) - \delta(1-\alpha)(2-\alpha(\lambda-1))p^L - p^L(1+\delta\alpha - \delta(1-\alpha)(1+\alpha-\alpha\lambda))}{1+\delta\alpha - \delta(1-\alpha)(1+\alpha-\alpha\lambda)} \\
&= \frac{P_A(1+\delta) - p^L(1+\delta\alpha) + \delta(1-\alpha)p^L(-2+\alpha\lambda-\alpha+1+\alpha-\alpha\lambda)}{1+\delta\alpha - \delta(1-\alpha)(1+\alpha-\alpha\lambda)} \\
&= \frac{P_A(1+\delta) - p^L(1+\delta\alpha+\delta(1-\alpha)}{1+\delta\alpha - \delta(1-\alpha)(1+\alpha-\alpha\lambda)} \\
&= \frac{(1+\delta)(P_A - p^L}{1+\delta\alpha - \delta(1-\alpha)(1+\alpha-\alpha\lambda)} \\
&\leq 0
\end{aligned}$$

The inequality comes from $P_A \leq p^L$. Thus, we have $\hat{v}_A < p^L$ which contradicts the assumption in this case $\hat{v}_A \geq p^L$.[1]

Case (iii): $p^H > p^L \geq v$: Assume $v \geq P_A$. Thus, the only possibility is $p^H > p^L \geq v \geq P_A$. In this case, the utility of a buyer with valuation v who chooses to buy today is given by

$$u_0(v) = (v - P_A)(1+\delta)$$

. By solving $u_0(\hat{v}_A) = \delta \mathbb{E}U_1(\hat{v}_A)$, we get $\hat{v}_A = P_A$.

[1]This case is possible only when $P_A = \hat{v} = p^L$. But, then $u_0(\hat{v}_A) = \delta \mathbb{E}U_1(\hat{v}_A)$ holds for any $\hat{v}_A$. Then the first order condition for this Case (i) is greater than 0, which means that we can increase the profit further and hence this $\hat{v}_A$ is not the profit maximizing one.

The profit maximizing $\hat{v}_A$ satisfies the same FOC as in Lemma 7 under *Case (iii)* and it is given by

$$-P_A + (1 - \hat{v}_A)\frac{\partial P_A}{\partial \hat{v}_A} = 0$$

Then using $\hat{v}_A = P_A$ in the FOC, we get $\hat{v}_A = P_A = \frac{1}{2}$. But this case is not possible because it contradicts the model's assumption that $p^L < \frac{1}{2}$.

Thus, the only possibility is *Subcase (ii).2*. For this case, we need to check $p^H \geq \hat{v}_A \geq P_A > p^L$ holds for any parameter values.

Note that $P_A = \frac{\hat{v}_A(1+\delta\alpha-\delta(1-\alpha)(1+\alpha-\alpha\lambda))+\delta(1-\alpha)(1+\alpha+\lambda-\alpha\lambda)p^L}{1+\delta\alpha+\delta\lambda-\delta\alpha\lambda}$. For $\hat{v}_A \geq P_A$ to hold, we need

$$\begin{aligned}
&\hat{v}_A \geq \frac{\hat{v}_A(1+\delta\alpha-\delta(1-\alpha)(1+\alpha-\alpha\lambda))+\delta(1-\alpha)(1+\alpha+\lambda-\alpha\lambda)p^L}{1+\delta\alpha+\delta\lambda-\delta\alpha\lambda} \\
&\delta(1-\alpha)(1+\alpha+\lambda-\alpha\lambda)\hat{v}_A \geq \delta(1-\alpha)(1+\alpha+\lambda-\alpha\lambda)p^L \\
&\hat{v}_A \geq p^L
\end{aligned}$$

Also we need $P_A > p^L$. For this to hold, we need

$$\begin{aligned}
&\frac{\hat{v}_A(1+\delta\alpha-\delta(1-\alpha)(1+\alpha-\alpha\lambda))+\delta(1-\alpha)(1+\alpha+\lambda-\alpha\lambda)p^L}{1+\delta\alpha+\delta\lambda-\delta\alpha\lambda} > p^L \\
&\hat{v}_A(1+\delta\alpha-\delta(1-\alpha)(1+\alpha-\alpha\lambda)) > p^L(1+\delta\alpha-\delta(1-\alpha)(1+\alpha-\alpha\lambda)) \\
&\hat{v}_A > p^L
\end{aligned}$$

Thus, it suffices to show that $\hat{v}_A > p^L$. To see this, plug $\hat{v}_A$ into the inequality.

$$\frac{1}{2} + \frac{p^L(1-\alpha)(1-\delta)}{2(1+\delta\alpha-\delta(1-\alpha)(1+\alpha-\alpha\lambda))} > p^L$$

This holds if and only if

$$\frac{1+\delta\alpha-\delta(1-\alpha)(1+\alpha-\alpha\lambda)+p^L(2\delta(1-\alpha)(1+\alpha-\alpha\lambda)-(1+\alpha)(1+\delta))}{2(1+\delta\alpha-\delta(1-\alpha)(1+\alpha-\alpha\lambda))} > 0$$

The denominator is positive. To see this, we arrange the denominator and get $1-\delta+\delta\lambda(1+\alpha+\lambda-\alpha\lambda)$ which is positive. Therefore the inequality holds if the numerator is positive. To see this, we use that $p^L < \frac{1}{2}$.

$$\begin{aligned}
& 1+\delta\alpha-\delta(1-\alpha)(1+\alpha-\alpha\lambda)+p^L(2\delta(1-\alpha)(1+\alpha-\alpha\lambda)-(1+\alpha)(1+\delta)) \\
> \; & 1+\delta\alpha-\delta(1-\alpha)(1+\alpha-\alpha\lambda)+\delta(1-\alpha)(1+\alpha-\alpha\lambda)-\frac{(1+\alpha)(1+\delta)}{2} \\
= \; & 1+\delta\alpha-\frac{(1+\alpha)(1+\delta)}{2} \\
= \; & \frac{(1-\alpha)(1-\delta)}{2} \\
> \; & 0
\end{aligned}$$

Also we need $\hat{v}_A < p^H$.

$$\begin{aligned}
\hat{v}_A - p^H &= \frac{1}{2}+\frac{p^L(1-\alpha)(1-\delta)}{2(1+\delta\alpha-\delta(1-\alpha)(1+\alpha-\alpha\lambda))}-p^H \\
&< \frac{1}{2}+\frac{(1-\alpha)(1-\delta)}{4(1+\delta\alpha-\delta(1-\alpha)(1+\alpha-\alpha\lambda))}-1 \\
&= -\frac{1}{2}+\frac{(1-\alpha)(1-\delta)}{4(1+\delta\alpha-\delta(1-\alpha)(1+\alpha-\alpha\lambda))} \\
&= \frac{-2(1+\delta\alpha-\delta(1-\alpha)(1+\alpha-\alpha\lambda))+(1-\alpha)(1-\delta)}{4(1+\delta\alpha-\delta(1-\alpha)(1+\alpha-\alpha\lambda))} \\
&= \frac{(1-\alpha)(-1+\delta+2\delta\alpha(1-\lambda))}{4(1+\delta\alpha-\delta(1-\alpha)(1+\alpha-\alpha\lambda))} \\
&< 0
\end{aligned}$$

In the first inequality, we use that $p^L < \frac{1}{2}$ and $p^H > 1$. For the last inequality, the denominator is positive and the numerator is negative.Therefore we end up with *Subcase (ii).2* which gives us the optimal price and residual demand given in the Lemma. ■

Lemma 10 characterizes the optimal residual demands and price when the monopolist offers the contract $(P_{A,MFC}, MFC)$ to loss-averse buyers.

Lemma 10 *In the presence of MFC, when buyers are loss-averse, the optimal residual demand and price are characterized by*

$$\begin{aligned} P^{FB}_{A,MFC} &= \frac{1+\delta\alpha\lambda-\delta(1-\alpha)(1+\alpha-\alpha\lambda)-\delta(1-\alpha)\alpha(\lambda-1)p^L}{2(1-\delta(1-\alpha)+\delta\alpha\lambda)} \\ \hat{v}^{FB}_{A,MFC} &= \frac{1}{2}+\frac{\delta(1-\alpha)\alpha(\lambda-1)p^L}{2(1+\delta\alpha\lambda-\delta(1-\alpha)(1+\alpha-\alpha\lambda)} \end{aligned}$$

where $p^H > P^{FB}_{A,MFC} > \hat{v}^{FB}_{A,MFC} > p^L$.

PROOF of LEMMA 10. There is a threshold $\hat{v}_{A,MFC}$ such that buyers with valuation above that threshold buys today and buyers with valuation below that threshold waits for tomorrow. We need to solve $u_0(\hat{v}_A) = \delta\mathbb{E}U_1(\hat{v}_A)$ to find $\hat{v}_A$. The expected utility of a buyer who chooses to wait for tomorrow is the same as in the proof of Lemma 9.

If a buyer with valuation chooses to buy today then her utility is given by

$$\begin{aligned} u_0(v) = v - P_{A,MFC} + \delta\Big((1-\alpha)(P_{A,MFC}-p^L) + \alpha\mathbb{I}\big[v - P_{A,MFC} - \max\{v-p^H,0\}\big] \\ + (1-\alpha)\mathbb{I}\big[v-p^L-\max\{v-p^L,0\}\big]\Big) \end{aligned}$$

There are four cases that we need to consider.

Case (i): $v \geq p^H > P_{A,MFC} > p^L$: This case is not possible because it contradicts with the model's assumption that $p^H > 1$.

Case (ii): $p^H \geq v > P_{A,MFC} > p^L$: In this case, the utility of a buyer who chooses to buy today is given by

$$\begin{aligned} u_0(v) &= v - P_{A,MFC} + \delta[(1-\alpha)(P_{A,MFC}-p^L) + \alpha(v-P_{A,MFC}) \\ &\quad + (1-\alpha)(v-p^L-(v-p^L))] \\ &= v - P_{A,MFC} + \delta[(1-\alpha)(P_{A,MFC}-p^L) + \alpha(v-P_{A,MFC})] \end{aligned}$$

By solving $u_0(\hat{v}_{A,MFC}) = \mathbb{E}U_1(\hat{v}_{A,MFC})$, we find $\hat{v}_{A,MFC}$ as

$$\hat{v}_{A,MFC} = \frac{P_{A,MFC}(1-\delta+2\delta\alpha)+\delta\alpha(1-\alpha)(\lambda-1)p^L}{1+\delta\alpha-\delta(1-\alpha)(1+\alpha-\alpha\lambda)}$$

But this case is not possible. To see this,we check $\hat{v}_{A,MFC} - P_{A,MFC}$. It is given by

$$\begin{aligned}
&= \frac{P_{A,MFC}(1-\delta+2\delta\alpha)+\delta\alpha(1-\alpha)(\lambda-1)p^L}{1+\delta\alpha-\delta(1-\alpha)(1+\alpha-\alpha\lambda)} - P_{A,MFC} \\
&= \frac{P_{A,MFC}(-\delta+\delta\alpha+\delta(1-\alpha)(1+\alpha-\alpha\lambda))+\delta\alpha(1-\alpha)(\lambda-1)p^L}{1+\delta\alpha-\delta(1-\alpha)(1+\alpha-\alpha\lambda)} \\
&= \frac{\delta\alpha(1-\alpha)(\lambda-1)(-P_{A,MFC}+p^L)}{1+\delta\alpha-\delta(1-\alpha)(1+\alpha-\alpha\lambda)} \\
&< 0
\end{aligned}$$

The inequality comes from $P_{A,MFC} > p^L$. Thus, we have $\hat{v}_{A,MFC} < P_{A,MFC}$ which contradicts the assumption in this case.

Case (iii): $p^H > P_{A,MFC} > v > p^L$: In this case, the utility of a buyer who chooses to buy today is given by

$$\begin{aligned}
u_0(v) &= v - P_{A,MFC} + \delta[(1-\alpha)(P_{A,MFC}-p^L) + \alpha\lambda(v-_{A,MFC}) \\
&+ (1-\alpha)(v-p^L-(v-p^L))] \\
&= v - P_{A,MFC} + \delta[(1-\alpha)(P_{A,MFC}-p^L) + \alpha\lambda(v-P_{A,MFC})]
\end{aligned}$$

By solving $u_0(\hat{v}_{A,MFC}) = \mathbb{E}U_1(\hat{v}_{A,MFC})$, we find $\hat{v}_{A,MFC}$ as

$$\hat{v}_{A,MFC} = \frac{P_{A,MFC}(1-\delta(1-\alpha)+\delta\alpha\lambda)+\delta\alpha(1-\alpha)(\lambda-1)p^L}{1+\delta\alpha\lambda-\delta(1-\alpha)(1+\alpha-\alpha\lambda)}$$

Using $\hat{v}_{A,MFC}$ expression we can write $P_{A,MFC}$ as a function of $\hat{v}_{A,MFC}$ as follows

$$P_{A,MFC} = \frac{\hat{v}_{A,MFC}(1+\delta\alpha\lambda-\delta(1-\alpha)(1+\alpha-\alpha\lambda))-\delta\alpha(1-\alpha)(\lambda-1)p^L}{1-\delta(1-\alpha)+\delta\alpha\lambda}$$

and we get

$$\frac{\partial P_{A,MFC}}{\partial \hat{v}_{A,MFC}} = \frac{1+\delta\alpha\lambda-\delta(1-\alpha)(1+\alpha-\alpha\lambda)}{1-\delta(1-\alpha)+\delta\alpha\lambda}$$

The profit maximizing $\hat{v}_{A,MFC}$ satisfies the same FOC as in Lemma 8 under *Subcase (ii.2)* and it is given by

$$-\alpha P_{A,MFC} + \alpha(1-\hat{v}_{A,MFC})\frac{\partial P_{A,MFC}}{\partial \hat{v}_{A,MFC}} = 0$$

Plugging equations for $\hat{v}_{A,MFC}$, $P_{A,MFC}$, and $\frac{\partial P_{A,MFC}}{\partial \hat{v}_{A,MFC}}$ in the FOC, we find

$$\begin{aligned} P_{A,MFC} &= \frac{1+\delta\alpha\lambda-\delta(1-\alpha)(1+\alpha-\alpha\lambda)-\delta\alpha(1-\alpha)(\lambda-1)p^L}{2(1-\delta(1-\alpha)+\delta\alpha\lambda)} \\ \hat{v}_{A,MFC} &= \frac{1}{2}+\frac{\delta\alpha(1-\alpha)(\lambda-1)p^L}{2(1+\delta\alpha\lambda-\delta(1-\alpha)(1+\alpha-\alpha\lambda))} \end{aligned}$$

Case (iv): $p^H > P_{A,MFC} > p^L \geq v$: In this case, the utility of a buyer who chooses to buy today is given by

$$u_0(v) = v - P_{A,MFC} + \delta[(1-\alpha)(P_{A,MFC}-p^L) + \alpha\lambda(v-P_{A,MFC}) + (1-\alpha)\lambda(v-p^L)]$$

By solving $u_0(\hat{v}_{A,MFC}) = \mathbb{E}U_1(\hat{v}_{A,MFC})$, we find $\hat{v}_{A,MFC}$ as

$$\hat{v}_{A,MFC} = \frac{P_{A,MFC}(1-\delta(1-\alpha-\alpha\lambda))+\delta(1-\alpha)(\lambda+1)p^L}{1+\delta\lambda}$$

But this case is not possible. To see this we check, $\hat{v}_{A,MFC} - p^L$:

$$\begin{aligned} \hat{v}_{A,MFC} - p^L &= \frac{P_{A,MFC}(1-\delta(1-\alpha-\alpha\lambda))+\delta(1-\alpha)(\lambda+1)p^L - p^L(1+\delta\lambda)}{1+\delta\lambda} \\ &= \frac{(1-\delta(1-\alpha-\alpha\lambda))(P_{A,MFC}-p^L)}{1+\delta\lambda} \\ &> 0 \end{aligned}$$

The inequality comes from $P_{A,MFC} > p^L$. Thus, we have $\hat{v}_{A,MFC} > p^L$ which contradicts with the assumption in this case.

Thus the only possibility is *Case (iii)*. For this case, we need to check $p^H > P_{A,MFC} > \hat{v}_{A,MFC} > p^L$ holds for any parameter values.Note that $P_{A,MFC} = \frac{\hat{v}_{A,MFC}(1+\delta\alpha\lambda-\delta(1-\alpha)(1+\alpha-\alpha\lambda))-\delta\alpha(1-\alpha)(\lambda-1)p^L}{1-\delta(1-\alpha)+\delta\alpha\lambda}$. For $P_{A,MFC} > \hat{v}_{A,MFC}$ to hold, we need

$$\frac{\hat{v}_{A,MFC}(1+\delta\alpha\lambda-\delta(1-\alpha)(1+\alpha-\alpha\lambda))-\delta\alpha(1-\alpha)(\lambda-1)p^L}{1-\delta(1-\alpha)+\delta\alpha\lambda} > \hat{v}_{A,MFC}$$

$$\delta\alpha(1-\alpha)(\lambda-1)\hat{v}_{A,MFC} > \delta\alpha(1-\alpha)(\lambda-1)p^L$$

$$\hat{v}_{A,MFC} > p^L$$

Thus, it suffices to show that $\hat{v}_{A,MFC} > p^L$. To see this, plug $\hat{v}_{A,MFC}$ into the inequality.

$$\frac{1}{2} + \frac{\delta\alpha(1-\alpha)(\lambda-1)p^L}{2(1+\delta\alpha\lambda-\delta(1-\alpha)(1+\alpha-\alpha\lambda))} > p^L$$

This holds if and only if

$$\frac{1+\delta\alpha\lambda-\delta(1-\alpha)(1+\alpha-\alpha\lambda)+p^L(-2-2\delta\alpha\lambda+\delta(1-\alpha)(2+\alpha-\alpha\lambda))}{2(1+\delta\alpha\lambda-\delta(1-\alpha)(1+\alpha-\alpha\lambda))} > 0$$

Since $1+\delta\alpha\lambda-\delta(1-\alpha)(1+\alpha-\alpha\lambda) > 1+\delta\alpha-\delta(1-\alpha)(1+\alpha-\alpha\lambda)$ and the second expression is positive, the denominator is positive. Therefore, the inequality holds if the numerator is positive. To see this, use $p^L < \frac{1}{2}$.

$$\begin{aligned}
& 1+\delta\alpha\lambda-\delta(1-\alpha)(1+\alpha-\alpha\lambda)+p^L(-2-2\delta\alpha\lambda+\delta(1-\alpha)(2+\alpha-\alpha\lambda)) \\
> \; & 1+\delta\alpha\lambda-\delta(1-\alpha)(1+\alpha-\alpha\lambda)-1-\delta\alpha\lambda+\frac{\delta(1-\alpha)(2+\alpha-\alpha\lambda)}{2} \\
= \; & \frac{\delta\alpha(1-\alpha)(\lambda-1)}{2} \\
> \; & 0
\end{aligned}$$

Also we need $\hat{v}_{A,MFC} < p^H$.

$$\begin{aligned}
\hat{v}_{A,MFC} - p^H &= \frac{1}{2} + \frac{\delta\alpha(1-\alpha)(\lambda-1)p^L}{2(1+\delta\alpha\lambda-\delta(1-\alpha)(1+\alpha-\alpha\lambda))} - p^H \\
&< \frac{1}{2} + \frac{\delta\alpha(1-\alpha)(\lambda-1)}{4(1+\delta\alpha\lambda-\delta(1-\alpha)(1+\alpha-\alpha\lambda))} - 1 \\
&= -\frac{1}{2} + \frac{\delta\alpha(1-\alpha)(\lambda-1)}{4(1+\delta\alpha\lambda-\delta(1-\alpha)(1+\alpha-\alpha\lambda))} \\
&= \frac{-2(1+\delta\alpha\lambda-\delta(1-\alpha)(1+\alpha-\alpha\lambda)) + \delta\alpha(1-\alpha)(\lambda-1)}{4(1+\delta\alpha\lambda-\delta(1-\alpha)(1+\alpha-\alpha\lambda))} \\
&= \frac{-2-2\delta\alpha\lambda+\delta(1-\alpha)(2+\alpha-\alpha\lambda)}{4(1+\delta\alpha\lambda-\delta(1-\alpha)(1+\alpha-\alpha\lambda))}
\end{aligned}$$

The first inequality comes from $p^L < \frac{1}{2}$ and $p^H > 1$. In the last line, the denominator is positive. To see that the numerator is negative, arrange it to get $-2(1-\delta) - \delta\alpha(1+\alpha+2\lambda) + \delta\alpha\lambda(\alpha-1) < 0$. Thus, we have $\hat{v}_{A,MFC} < p^H$. Therefore we end up with *Case (iii)* which gives us the optimal price and residual demand given in the Lemma.

■

PROOF of LEMMA 2. Let $\mathbb{E}\pi^{LA}_{MFC}$ denote the expected profit of the monopolist when there is loss aversion and in the presence of MFC. From Lemma 10, we have

$$\begin{aligned}
\mathbb{E}\pi^{LA}_{MFC} &= (1-\hat{v}_{A,MFC})P_{A,MFC} + (1-\alpha)p^L(\hat{v}_{A,MFC} - p^L) \\
&\quad - (1-\alpha)(1-\hat{v}_{A,MFC})(P_{A,MFC} - p^L) \\
&= (1-\hat{v}_{A,MFC})\alpha P_{A,MFC} + (1-\alpha)p^L(1-p^L) \\
&= \frac{\alpha}{4(1-\delta+\delta\alpha+\delta\alpha\lambda)}\Bigg[1+\delta\alpha\lambda-\delta(1-\alpha)(1+\alpha-\alpha\lambda) - 2\delta\alpha(1-\alpha)(\lambda-1)p^L \\
&\quad + \frac{(\delta\alpha(1-\alpha)(\lambda-1)p^L)^2}{1+\delta\alpha\lambda-\delta(1-\alpha)(1+\alpha-\alpha\lambda)}\Bigg] + (1-\alpha)p^L(1-p^L)
\end{aligned}$$

Claim 1 *$\mathbb{E}\pi^{LA}_{MFC}(\delta)$ is increasing in δ for any λ, α and p^L.*

Proof. By taking the derivative of $\mathbb{E}\pi^{LA}_{MFC}$ with respect to δ, we get

$$\begin{aligned}
\frac{\partial \mathbb{E}\pi^{LA}_{MFC}}{\partial \delta} &= -\frac{(-1+\alpha)\alpha^2(-1+\lambda)}{4(-1+\delta-\alpha^2\delta+(-2+\alpha)\alpha\delta\lambda)^2(1-\delta+\delta\alpha+\delta\alpha\lambda)^2} \\
&\times \underbrace{\Big(1-2P_1^L+\delta(-1+2P_1^L+\alpha(2\lambda+\alpha(-1+\lambda)(-1+P_1^L)-P_1^L-3\lambda P_1^L)\Big)}_{A(\delta)} \\
&\times \underbrace{\Big(1+\delta(-1+\alpha(2\lambda+\alpha(-1+\lambda)(-1+P_1^L)+P_1^L-\lambda P_1^L))\Big)}_{B(\delta)}
\end{aligned}$$

The first term in the above expression is positive. Thus, $\frac{\partial \mathbb{E}\pi^{LA}_{MFC}}{\partial \delta} > 0$ if and only if $A(\delta)B(\delta) > 0$.

Derivative of $A(\delta)$ with respect to delta is independent of δ:

$$\frac{\partial A(\delta)}{\partial \delta} = -1+2p^L+\alpha(2\lambda+\alpha(\lambda-1)(p^L-1)-p^L-3\lambda p^L)$$

At $\delta = 0$, $A(\delta) = 1-2p^L > 0$ because we assume $p^L < \frac{1}{2}$.

At $\delta = 1$, $A(\delta) > 0$:

$$\begin{aligned}
A(1) &= 1-2p^L-1+2p^L+\alpha(2\lambda-\alpha(\lambda-1)+\alpha(\lambda-1)p^L-p^L-3\lambda p^L) \\
&= \alpha(2\lambda-\alpha\lambda+\alpha+p^L(\alpha\lambda-\alpha-1-3\lambda)) \\
&> \alpha\Big(2\lambda-\alpha\lambda+\alpha+\frac{\alpha\lambda-\alpha-1-3\lambda}{2}\Big) \\
&= \frac{\alpha(\lambda-\alpha\lambda+\alpha-1)}{2} \\
&= \frac{\alpha(1-\alpha)(\lambda-1)}{2} \\
&> 0
\end{aligned}$$

The first inequality comes from $p^L < \frac{1}{2}$. Thus, we have $A(\delta) > 0$ at $\delta = 0$ and $\delta = 1$. Also the derivative of $A(\delta)$ with respect to δ does not depend on δ. Therefore, $A(\delta) > 0$ always.

Now,we consider $B(\delta)$:

$$\begin{aligned} B(\delta) &= 1+\delta(-1+\alpha(2\lambda+\alpha(-1+\lambda)(-1+P_1^L)+P_1^L-\lambda P_1^L)) \\ &= 1+\delta(-1+\alpha(2\lambda-\alpha\lambda+\alpha+p^L(1-\alpha)(1-\lambda))) \\ &> 1+\delta\Big(-1+\alpha(2\lambda-\alpha\lambda+\alpha+\frac{(1-\alpha)(1-\lambda))}{2}\Big) \\ &= 1+\delta\Big(-1+\alpha\frac{2\lambda+\alpha+1+\lambda(1-\alpha)}{2}\Big) \\ &> 0 \end{aligned}$$

The first inequality comes from $p^L < \frac{1}{2}$. Thus, we have shown that $\frac{\partial \mathbb{E}\pi_{MFC}^{LA}}{\partial \delta} > 0$ and this finishes the proof of Claim 1. □

Similarly, let $\mathbb{E}\pi_{noMFC}^{LA}$ denote the expected profit of the monopolist when there is loss aversion and in the absence of MFC. From Lemma 9, we have

$$\begin{aligned} \mathbb{E}\pi_{noMFC}^{LA} &= (1-\hat{v}_A)P_A+(1-\alpha)p^L(\hat{v}_A-p^L) \\ &= \frac{1}{4(1+\delta\alpha+\delta\lambda-\delta\alpha\lambda)}\Bigg[1+\delta\alpha-\delta(1-\alpha)(1+\alpha-\alpha\lambda) \\ &\quad +(1-\alpha)(1+\delta+2\delta\alpha+2\delta\lambda-2\delta\alpha\lambda)p^L-p^L(1-\alpha)(1-\delta) \\ &\quad +\frac{(1-\alpha)^2P_1^{L^2}(1-\delta)(1+\delta+2\delta\alpha+2\delta\lambda-2\delta\alpha\lambda)}{1+\delta\alpha-\delta(1-\alpha)(1+\alpha-\alpha\lambda)}\Bigg] \\ &\quad +\frac{(1-\alpha)p^L}{2}+\frac{(1-\alpha)^2P_1^{L^2}(1-\delta)}{2(1+\delta\alpha-\delta(1-\alpha)(1+\alpha-\alpha\lambda))}-(1-\alpha)P_1^{L^2} \end{aligned}$$

Claim 2 *$\mathbb{E}\pi_{noMFC}^{LA}(\delta)$ is decreasing in δ for any λ, α and p^L.*

Proof. By taking derivative of $\mathbb{E}\pi_{noMFC}^{LA}(\delta)$ with respect to δ, we get

$$\begin{aligned} \frac{\partial \mathbb{E}\pi_{noMFC}^{LA}}{\partial \delta} &= \frac{(-1+\alpha)(-1+\alpha(-1+\lambda)-\lambda)}{4(1+\delta\alpha+\delta\lambda-\delta\alpha\lambda)^2(1+\delta(-1++\alpha(1+\alpha+\lambda-\alpha\lambda))^2} \\ &\times \underbrace{\Big(-1+\delta+p^L-\alpha p^L-\delta p^L+\alpha\delta(-1+\alpha(-1+\lambda)-\lambda+p^L)\Big)}_{C(\delta)} \\ &\times \underbrace{\Big(1-(1+\alpha)p^L+\delta(-1+p^L+\alpha(1+\lambda-p^L-2\lambda p^L+\alpha(\lambda-1)(-1+2p^L)))\Big)}_{D(\delta)} \end{aligned}$$

First,we look at the first term in the above expression. This term's denominator is a square, so it is positive. The numerator is also positive because $(-1+\alpha)(-1+\alpha(-1+\lambda)-\lambda) = (-1+\alpha)(-1-\alpha+\lambda(\alpha-1))$ and both terms in the parentheses are negative. Thus $\frac{\partial \mathbb{E}\pi^{LA}_{noMFC}}{\partial \delta} < 0$ if and only if $C(\delta)D(\delta) < 0$.

$$\begin{aligned} C(\delta) &= -1+\delta+p^L-\alpha p^L-\delta p^L+\alpha\delta(-1+\alpha(-1+\lambda)-\lambda+p^L) \\ &= (1-p^L)(\delta-1)-\alpha p^L+\alpha\delta(-1+p^L-\alpha+\lambda(\alpha-1)) \\ &< 0 \end{aligned}$$

Since $p^L < \frac{1}{2}$ and $\delta \in (0,1)$, $(1-p^L)(\delta-1)$ is negative. Also $\alpha \in (0,1)$ and hence $(-1+p^L-\alpha+\lambda(\alpha-1))$ is negative.
To see $D(\delta) > 0$, observe that

$$\begin{aligned} D(\delta) &= 1-(1+\alpha)p^L+\delta(-1+p^L+\alpha(1+\lambda-p^L-2\lambda p^L+\alpha(\lambda-1)(-1+2p^L))) \\ &= 1-(1+\alpha)p^L+\delta(-1+p^L+\alpha(1+\lambda+\alpha-\alpha\lambda+p^L(-1-2\alpha+2\lambda(\alpha-1)))) \\ &> 1-(1+\alpha)p^L+\delta(-1+p^L+\alpha(1+\lambda+\alpha-\alpha\lambda-\frac{1}{2}-\alpha+\lambda(\alpha-1))) \\ &= 1-(1+\alpha)p^L+\delta(-1+p^L+\frac{\alpha}{2}) \\ &= 1-\delta+\frac{\delta\alpha}{2}+p^L(-1-\alpha+\delta) \\ &> 1-\delta+\frac{\delta\alpha}{2}-\frac{1-\alpha+\delta}{2} \\ &= \frac{(1-\delta)(1-\alpha)}{2} \\ &> 0 \end{aligned}$$

The first and second inequality comes from $p^L < \frac{1}{2}$ and the last one comes from $\delta, \alpha \in (0,1)$. Thus, we have shown that $\frac{\partial \mathbb{E}\pi^{LA}_{noMFC}}{\partial \delta} < 0$ and this finishes the proof of Claim 2. □

Now, we show that

$$\mathbb{E}\pi^{LA}_{MFC}(\delta=0) < \mathbb{E}\pi^{LA}_{noMFC}(\delta=0)$$

and

$$\mathbb{E}\pi^{LA}_{MFC}(\delta = 1) < \mathbb{E}\pi^{LA}_{noMFC}(\delta = 1)$$

. With Claims 1 and 2, this implies that there exists $\bar{\delta} < 1$ such that $\mathbb{E}\pi^{LA}_{MFC} > \mathbb{E}\pi^{LA}_{noMFC}$ for $\delta > \bar{\delta}$.The expected profits at $\delta = 0$ are given by

$$\begin{aligned}
\mathbb{E}\pi^{LA}_{MFC}(\delta = 0) &= \frac{\alpha}{4} + (1-\alpha)p^L - (1-\alpha)P_1^{L^2} \\
\mathbb{E}\pi^{LA}_{noMFC}(\delta = 0) &= \frac{1}{4} + \frac{(1-\alpha)^2 P_1^{L^2}}{4} + \frac{(1-\alpha)p^L}{2} - (1-\alpha)P_1^{L^2}
\end{aligned}$$

Therefore, $\mathbb{E}\pi^{LA}_{MFC}(\delta = 0) < \mathbb{E}\pi^{LA}_{noMFC}(\delta = 0)$ if and only if

$$\frac{\alpha}{4} + (1-\alpha)p^L < \frac{1}{4} + \frac{(1-\alpha)^2 P_1^{L^2}}{4} + \frac{(1-\alpha)p^L}{2}$$

which holds if and only if

$$\frac{(1-\alpha)(-1+2p^L) - P_1^{L^2}(1-\alpha)^2}{4} < 0$$

. This inequality holds because $p^L < \frac{1}{2}$. Thus, at $\delta = 0$, $\mathbb{E}\pi^{LA}_{MFC}(\delta = 0) < \mathbb{E}\pi^{LA}_{noMFC}(\delta = 0)$ for any λ, p^L.

The expected profits at $\delta = 1$ are given by

$$\begin{aligned}
\mathbb{E}\pi^{LA}_{MFC}(\delta = 1) = \frac{1}{4(1+\lambda)}\Big[1 + \alpha\lambda - (1-\alpha)(1+\alpha-\alpha\lambda) - 2\alpha(1-\alpha)(\lambda-1)p^L \\
+ \frac{\alpha^2(1-\alpha)^2(\lambda-1)^2 P_1^{L^2}}{1+\alpha\lambda-(1-\alpha)(1+\alpha-\alpha\lambda)}\Big] + (1-\alpha)p^L - (1-\alpha)P_1^{L^2} \\
\mathbb{E}\pi^{LA}_{noMFC}(\delta = 1) = \frac{1}{4(1+\alpha+\lambda-\alpha\lambda)}\Big[1 + \alpha - (1-\alpha)(1+\alpha-\alpha\lambda) \\
+ 2p^L(1-\alpha)(1+\alpha+\lambda-\alpha\lambda)\Big] + \frac{(1-\alpha)p^L}{2} - (1-\alpha)P_1^{L^2}
\end{aligned}$$

To show that $\mathbb{E}\pi^{LA}_{MFC}(\delta = 1) > \mathbb{E}\pi^{LA}_{noMFC}(\delta = 1)$, we look at the difference $\mathbb{E}\pi^{LA}_{MFC}(\delta = 1) - \mathbb{E}\pi^{LA}_{noMFC}(\delta = 1)$:

$$\begin{aligned}
&= \frac{\alpha^2 - \alpha^2\lambda + 2\alpha\lambda}{4(1+\lambda)} - \frac{p^L(\alpha^2 - \alpha + \alpha\lambda - \alpha^2\lambda)}{2(1+\lambda)} - \frac{\alpha}{4} + \frac{\alpha^2(1-\alpha)^2(\lambda-1)^2 P_1^{L^2}}{4(\alpha^2 - \alpha^2\lambda + 2\alpha\lambda)} \\
&> \frac{\alpha^2 - \alpha^2\lambda + 2\alpha\lambda}{4(1+\lambda)} - \frac{\alpha^2 - \alpha + \alpha\lambda - \alpha^2\lambda}{4(1+\lambda)} - \frac{\alpha}{4} + \frac{\alpha^2(1-\alpha)^2(\lambda-1)^2 P_1^{L^2}}{4(\alpha^2 - \alpha^2\lambda + 2\alpha\lambda)} \\
&= \frac{\alpha^2(1-\alpha)^2(\lambda-1)^2 P_1^{L^2}}{4(\alpha^2 - \alpha^2\lambda + 2\alpha\lambda)} \\
&> 0
\end{aligned}$$

The first inequality comes from p^L<$\frac{1}{2}$. Thus, at δ=1, $\mathbb{E}\pi^{LA}_{MFC}(\delta = 1)$ > $\mathbb{E}\pi^{LA}_{noMFC}(\delta = 1)$ for any λ, p^L.This finishes the proof of Lemma 2. ■

PROOF of LEMMA 3. Let $u_0^j(P, noMFC)$ denote the utility of a type j buyer if she buys today at $(P, noMFC)$ and let $u_0^j(P, MFC)$ denote the utility of a type j buyer if she buys today at (P, MFC) where $j = LA, notLA$. For loss-neutral buyers we need

$$u_0^{notLA}(P_N, MFC) > u_0^{notLA}(P_A, noMFC) \tag{A.1}$$

$$u_0^{notLA}(P_N, MFC) > \delta\mathbb{E}U_1(v) \qquad \text{for } v > \hat{\hat{v}}_N \tag{A.2}$$

and

$$\delta\mathbb{E}U_1(v) > u_0^{notLA}(P_A, noMFC) \tag{A.3}$$

$$\delta\mathbb{E}U_1(v) > u_0^{notLA}(P_N, MFC) \qquad \text{for } v < \hat{\hat{v}}_N \tag{A.4}$$

We find $\hat{\hat{v}}_N$ by solving $u_0^{notLA}(P_N, MFC) = \delta\mathbb{E}U_1(v)$. Hence, from the definition of $\hat{\hat{v}}_N$, if $v > \hat{\hat{v}}_N$ then the buyer will choose to buy today at (P_N, MFC) instead of waiting for tomorrow and if $v < \hat{\hat{v}}_N$ then the buyer will choose to wait for tomorrow. Thus, A.2 and A.4 is satisfied. First we find $\hat{\hat{v}}_N$. We analyze this under the proof of Lemma 8 and we find $\hat{\hat{v}}_N = P_N$ where $p^H \geq P_N > p^L$. Note that $u_0^{notLA}(P_N, MFC) = v - P_N + \delta(1-\alpha)(P_N - p^L)$ and

$u_0^{notLA}(P_A, noMFC) = v - P_A$. The inequality A.1 holds if and only if

$$\begin{aligned} u_0^{notLA}(P_N, MFC) - u_0^{notLA}(P_A, noMFC) &= v - P_N + \delta(1-\alpha)(P_N - p^L) - (v - P_A) \\ &= P_A - P_N(1 - \delta + \delta\alpha) - \delta(1-\alpha)P^L \geq 0 \end{aligned}$$

Note that this condition is independent of v. Hence, if this condition holds, then we have $u_0^{notLA}(P_N, MFC) \geq u_0^{notLA}(P_A, noMFC)$ for all v. For $v < \hat{\hat{v}}_N$, we have $\delta\mathbb{E}_1(v) > u_0^{notLA}(P_N, MFC)$. Thus, combining these two inequalities, for $v < \hat{\hat{v}}_N$ we have $\delta\mathbb{E}_1(v) > u_0^{notLA}(P_A, noMFC)$ which gives us A.3. Thus, the incentive compatibility constraint for loss-neutral buyers is

$$P_A - P_N(1 - \delta + \delta\alpha) - \delta(1-\alpha)p^L \geq 0$$

For loss-averse buyers we need

$$u_0^{LA}(P_A, noMFC) > u_0^{LA}(P_N, MFC) \tag{A.5}$$

$$u_0^{LA}(P_A, noMFC) > \delta\mathbb{E}U_1(v) \qquad \text{for } v > \hat{\hat{v}}_A \tag{A.6}$$

and

$$\delta\mathbb{E}U_1(v) > u_0^{LA}(P_N, MFC) \tag{A.7}$$

$$\delta\mathbb{E}U_1(v) > u_0^{LA}(P_A, noMFC) \qquad \text{for } v < \hat{\hat{v}}_A \tag{A.8}$$

We find $\hat{\hat{v}}_A$ by solving $u_0^{notLA}(P_A, noMFC) = \delta\mathbb{E}U_1(v)$. Hence, from the definition of $\hat{\hat{v}}_A$, if $v > \hat{\hat{v}}_A$ then the buyer will choose to buy today at $(P_A, noMFC)$ instead of waiting for tomorrow and if $v < \hat{\hat{v}}_A$ then the buyer will choose to wait for tomorrow. Thus, A.6 and A.8 is satisfied. First we find $\hat{\hat{v}}_A$. We analyze this under the proof of Lemma 9. We have two possible cases:

(a): $\hat{\hat{v}}_A = \frac{P_A(1+\delta\alpha+\delta(1-\alpha)\lambda) - \delta(1-\alpha)(1+\alpha+\lambda-\alpha\lambda)p^L}{1+\delta\alpha-\delta(1-\alpha)(1+\alpha-\alpha\lambda)}$ where $p^H \geq \hat{\hat{v}}_A \geq P_A > p^L$

(b): $\hat{\hat{v}}_A = P_A$ where $p^H > p^L \geq P_A$

However, *(b)* is not possible because the incentive compatibility constraint for loss-neutral buyers does not hold.

$$P_A - P_N(1-\delta+\delta\alpha) - \delta(1-\alpha)p^L \;\; < \;\; \delta(1-\alpha)(P_A - p^L)$$

This inequality comes from $P_A < P_N$. Under *(b)*, we have $P_A \leq p^L$ which makes the above expression nonpositive. Thus, loss-neutral buyers have an incentive to deviate from (P_N, MFC) to $(P_A, noMFC)$ under this case. Now, under *(b)* we check A.5 and A.7.

Case (i): $p^H > v > \hat{\hat{v}}_A \geq P_A > p^L$: First, we show $\hat{\hat{v}}_N < \hat{\hat{v}}_A$. Consider the difference $\hat{\hat{v}}_N - \hat{\hat{v}}_A$:

$$\begin{aligned}
&= P_N - \left(\frac{P_A(1+\delta\alpha+\delta(1-\alpha)\lambda) - \delta(1-\alpha)(1+\alpha+\lambda-\alpha\lambda)p^L}{1+\delta\alpha-\delta(1-\alpha)(1+\alpha-\alpha\lambda)}\right) \\
&= \frac{P_N(1+\delta\alpha-\delta(1-\alpha)(1+\alpha-\alpha\lambda)) - P_A(1+\delta\alpha+\delta(1-\alpha)\lambda) + \delta(1-\alpha)p^L(1+\alpha+\lambda-\alpha\lambda)}{1+\delta\alpha-\delta(1-\alpha)(1+\alpha-\alpha\lambda)} \\
&\leq \frac{P_N(1+\delta\alpha-\delta(1-\alpha)(1+\alpha-\alpha\lambda)) - (1+\delta\alpha+\delta(1-\alpha)\lambda)[P_N(1-\delta+\delta\alpha)+\delta(1-\alpha)p^L]}{1+\delta\alpha-\delta(1-\alpha)(1+\alpha-\alpha\lambda)} \\
&\quad + \frac{\delta(1-\alpha)p^L(1+\alpha+\lambda-\alpha\lambda)}{1+\delta\alpha-\delta(1-\alpha)(1+\alpha-\alpha\lambda)} \\
&= \frac{\delta(1-\alpha)(\alpha+\lambda-\alpha\lambda)(1-\delta)(p^L-P_N)}{1+\delta\alpha-\delta(1-\alpha)(1+\alpha-\alpha\lambda)} \\
&< 0
\end{aligned}$$

From the incentive compatibility constraint for loss-neutral buyers we have $P_A \geq P_N(1-\delta+\delta\alpha) + \delta(1-\alpha)p^L$. We use this to get the first inequality in the above expressions.

In this case, utilities of a loss-averse buyer with valuation v from buying today at both of the options are given by

$$\begin{aligned}
u_0^{LA}(P_A, noMFC) &= v - P_A + \delta\alpha(v-P_A) + \delta(1-\alpha)\lambda(p^L - P_A) \\
u_0^{LA}(P_N, MFC) &= v - P_N + \delta(1-\alpha)(P_N - p^L) + \delta\alpha(v-P_N)
\end{aligned}$$

Now, consider the difference $u_0^{LA}(P_A, noMFC) - u_0^{LA}(P_N, MFC)$:

$$\begin{aligned} &= v - P_A + \delta\alpha(v - P_A) + \delta(1-\alpha)\lambda(p^L - P_A) \\ &\quad -(v - P_N + \delta(1-\alpha)(P_N - p^L) + \delta\alpha(v - P_N)) \\ &= -P_A(1 + \delta\alpha + \delta(1-\alpha)\lambda) + P_N(1 - \delta + 2\delta\alpha) + \delta(1-\alpha)p^L(\lambda + 1) \quad \text{(A.9)} \end{aligned}$$

For $u_0^{LA}(P_A, noMFC) - u_0^{LA}(P_N, MFC) > 0$ to hold, we need A.9$\geq$0.

Case (ii): $p^H > \hat{\hat{v}}_A > v > P_N > P_A > p^L$: In this case, utilities of a loss-averse buyer with valuation v from buying today at (P_N, MFC) and from waiting for tomorrow are given by

$$\begin{aligned} \mathbb{E}(v) &= (v - p^L)(1-\alpha)(1 + \alpha - \alpha\lambda) \\ u_0^{LA}(P_N, MFC) &= v - P_N + \delta(1-\alpha)(P_N - p^L) + \delta\alpha(v - P_N) \end{aligned}$$

Now, consider the difference $\delta\mathbb{E}(v) - u_0^{LA}(P_N, MFC)$:

$$\begin{aligned} &= \delta(v - p^L)(1-\alpha)(1 + \alpha - \alpha\lambda) - (v - P_N + \delta(1-\alpha)(P_N - p^L) + \delta\alpha(v - P_N)) \\ &= -v(1 + \delta\alpha - \delta(1-\alpha)(1 + \alpha - \alpha\lambda)) + P_N(1 - \delta + 2\delta\alpha) + \delta(1-\alpha)p^L\alpha(\lambda - 1) \\ &> -\hat{\hat{v}}_A(1 + \delta\alpha - \delta(1-\alpha)(1 + \alpha - \alpha\lambda)) + P_N(1 - \delta + 2\delta\alpha) + \delta(1-\alpha)p^L\alpha(\lambda - 1) \end{aligned}$$

The inequality comes from $\hat{\hat{v}}_A > v$. Plug $\hat{\hat{v}}_A$ into the last expression to get

$$= -P_A(1 + \delta\alpha + \delta(1-\alpha)\lambda) + P_N(1 - \delta + 2\delta\alpha) + \delta(1-\alpha)p^L(\lambda + 1)$$

This is the same as A.9. We can also write the difference $\delta\mathbb{E}(v) - u_0^{LA}(P_N, MFC)$ as follows:

$$\begin{aligned}
&= -v(1+\delta\alpha-\delta(1-\alpha)(1+\alpha-\alpha\lambda)) + P_N(1-\delta+2\delta\alpha) + \delta(1-\alpha)p^L\alpha(\lambda-1) \\
&< -P_N(1+\delta\alpha-\delta(1-\alpha)(1+\alpha-\alpha\lambda)) + P_N(1-\delta+2\delta\alpha) + \delta(1-\alpha)p^L\alpha(\lambda-1) \\
&= P_N(-\delta+\delta\alpha+\delta(1-\alpha)(1+\alpha-\alpha\lambda)) + \delta(1-\alpha)p^L\alpha(\lambda-1) \\
&= \delta(1-\alpha)\alpha(\lambda-1)(p^L-P_N) \\
&< 0
\end{aligned}$$

The first inequality comes from $v > P_N$ and the last inequality comes from $P_N > p^L$. Thus, we show that $\delta\mathbb{E}(v) - u_0^{LA}(P_N, MFC) < 0$. We also know that $\delta\mathbb{E}(v) - u_0^{LA}(P_N, MFC) > -P_A(1+\delta\alpha+\delta(1-\alpha)\lambda) + P_N(1-\delta+2\delta\alpha) + \delta(1-\alpha)p^L(\lambda+1)$. So, by putting these two inequalities together we get $-P_A(1+\delta\alpha+\delta(1-\alpha)\lambda) + P_N(1-\delta+2\delta\alpha) + \delta(1-\alpha)p^L(\lambda+1) < 0$. Hence, the equation A.9 does not hold.

Case (iii): $p^H > \hat{\hat{v}}_A > P_N > v > P_A > p^L$: In this case, utilities of a loss-averse buyer with valuation v from buying today at (P_N, MFC) and from waiting for tomorrow are given by

$$\begin{aligned}
\mathbb{E}(v) &= (v-p^L)(1-\alpha)(1+\alpha-\alpha\lambda) \\
u_0^{LA}(P_N, MFC) &= v - P_N + \delta(1-\alpha)(P_N-p^L) + \delta\alpha\lambda(v-P_N)
\end{aligned}$$

Now, consider the difference $\delta\mathbb{E}(v) - u_0^{LA}(P_N, MFC)$. This difference is positive if and only if the following holds:

$$\begin{aligned}
&= \delta(v-p^L)(1-\alpha)(1+\alpha-\alpha\lambda) - (v - P_N + \delta(1-\alpha)(P_N-p^L) \\
&+ \delta\alpha\lambda(v-P_N)) \\
&= -v(1+\delta\alpha\lambda-\delta(1-\alpha)(1+\alpha-\alpha\lambda)) + P_N(1-\delta(1-\alpha)+\delta\alpha\lambda) \\
&+ \delta(1-\alpha)p^L\alpha(\lambda-1) \\
&> 0
\end{aligned}$$

■

PROOF of LEMMA 4. Let $u_0^j(P, noMFC)$ denote the utility of a type j buyer if she buys today at $(P, noMFC)$ and let $u_0^j(P, MFC)$ denote the utility of a type j buyer if she buys today at (P, MFC) where $j = LA, notLA$. For loss-neutral buyers we need

$$u_0^{notLA}(P_N, noMFC) > u_0^{notLA}(P_A, MFC) \tag{A.10}$$

$$u_0^{notLA}(P_N, noMFC) > \delta \mathbb{E}U_1(v) \qquad \text{for } v > \hat{v}_N \tag{A.11}$$

and

$$\delta \mathbb{E}U_1(v) > u_0^{notLA}(P_A, MFC) \tag{A.12}$$

$$\delta \mathbb{E}U_1(v) > u_0^{notLA}(P_N, noMFC) \qquad \text{for } v < \hat{v}_N \tag{A.13}$$

From the definition of $\hat{v}_N$, if $v > \hat{v}_N$ then the buyer will choose to buy today at $(P_N, noMFC)$ instead of waiting for tomorrow and if $v < \hat{v}_N$ then the buyer will choose to wait for tomorrow. Thus, A.11 and A.13 is satisfied. Note that $u_0^{notLA}(P_N, noMFC)$=v-P_N and $u_0^{notLA}(P_A, MFC) = v - P_A + \delta(1-\alpha)[P_A - p^L]$. The inequality A.10 holds if and only if

$$\begin{aligned} u_0^{notLA}(P_N, noMFC) - u_0^{notLA}(P_A, MFC) &= v - P_N - (v - P_A + \delta(1-\alpha)[P_A - p^L]) \\ &= -P_N + P_A(1 - \delta + \delta\alpha) + \delta(1-\alpha)p^L \\ &\geq 0 \end{aligned}$$

Notice that this above condition is independent of the buyer's valuation v. We also need A.12. For $v < \hat{v}_N$, we have $\delta \mathbb{E}U_1(v) > u_0^{notLA}(P_N, noMFC)$. If $-P_N + P_A(1 - \delta + \delta\alpha) + \delta(1-\alpha)p^L \geq 0$, then we have $u_0^{notLA}(P_N, noMFC) \geq u_0^{notLA}(P_A, MFC)$ for all v.Then we have $\delta \mathbb{E}U_1(v) > u_0^{notLA}(P_A, MFC)$ which gives us A.12. Thus, the

incentive compatibility constraint for loss-neutral buyer is

$$-P_N + P_A(1-\delta+\delta\alpha) + \delta(1-\alpha)p^L \geq 0$$

For loss-averse buyers we need

$$u_0^{LA}(P_A, MFC) > u_0^{LA}(P_N, noMFC) \tag{A.14}$$

$$u_0^{LA}(P_A, MFC) > \delta\mathbb{E}U_1(v) \qquad \text{for } v > \hat{v}_A \tag{A.15}$$

and

$$\delta\mathbb{E}U_1(v) > u_0^{LA}(P_N, noMFC) \tag{A.16}$$

$$\delta\mathbb{E}U_1(v) > u_0^{LA}(P_A, MFC) \qquad \text{for } v < \hat{v}_A \tag{A.17}$$

From the definition of $\hat{v}_A$, if $v > \hat{v}_A$ then the buyer will choose to buy today at (P_A, MFC) instead of waiting for tomorrow and if $v < \hat{v}_A$ then the buyer will choose to wait for tomorrow. Thus, A.15 and A.17 is satisfied. We find $\hat{v}_A$ by solving $u_0^{LA}(P_A, MFC) = \delta\mathbb{E}U_1(v)$. We analyze this under the proof of Lemma 10 and we find

$$\hat{v}_A = \frac{P_A(1-\delta(1-\alpha)+\delta\alpha\lambda) + \delta\alpha(1-\alpha)(\lambda-1)p^L}{1+\delta\alpha\lambda-\delta(1-\alpha)(1+\alpha-\alpha\lambda)}$$

where $p^H > P_A > \hat{v}_A > p^L$. Now, we check A.14 and A.16.

Case (i): $p^H > P_A > v > \hat{v}_A > p^L$: In this case, utilities of a loss-averse buyer with valuation v from buying today at both of the options are given by [2]

$$\begin{aligned} u_0^{LA}(P_A, MFC) &= v - P_A + \delta[(1-\alpha)(P_A - p^L) + \alpha\lambda(v - P_A)] \\ u_0^{LA}(P_N, noMFC) &= v - P_N + \delta[\alpha(v - P_N) + (1-\alpha)\lambda(p^L - P_N)] \end{aligned}$$

[2] These utilities are given in the proof of Lemma 9 and 10.

Now, consider the difference $u_0^{LA}(P_A, MFC) - u_0^{LA}(P_N, noMFC)$:

$$\begin{aligned} &= v\delta\alpha(\lambda-1) + P_A(-1+\delta-\delta\alpha-\delta\alpha\lambda) + p^L\delta(1-\alpha)(-1-\lambda) \\ &+ P_N(1+\delta+\delta\lambda-\delta\alpha\lambda) \\ &> \hat{v}_A\delta\alpha(\lambda-1) + P_A(-1+\delta-\delta\alpha-\delta\alpha\lambda) + p^L\delta(1-\alpha)(-1-\lambda) \\ &+ P_N(1+\delta+\delta\lambda-\delta\alpha\lambda) \end{aligned}$$

The inequality comes from $v > \hat{v}_A$. Plug $\hat{v}_A$ into the last expression to get

$$-\frac{1+\delta\alpha-\delta(1-\alpha)(1+\alpha-\alpha\lambda)}{1+\delta\alpha\lambda-\delta(1-\alpha)(1+\alpha-\alpha\lambda)}(1-\delta+\delta\alpha+\delta\alpha\lambda)P_A + P_N(1+\delta\alpha+\delta(1-\alpha)\lambda) \\ +\delta(1-\alpha)p^L\left(-1-\lambda+\frac{\delta\alpha^2(\lambda-1)^2}{1+\delta\alpha\lambda-\delta(1-\alpha)(1+\alpha-\alpha\lambda)}\right) \tag{A.18}$$

For $u_0^{LA}(P_A, MFC) - u_0^{LA}(P_N, noMFC) > 0$ to hold, we need $A.18 \geq 0$.

Case (ii): $p^H > v > P_A > \hat{v}_A > p^L$ and $P_N > p^L$: In this case, utilities of a loss-averse buyer with valuation v from buying today at both of the options are given by

$$\begin{aligned} u_0^{LA}(P_A, MFC) &= v - P_A + \delta[(1-\alpha)(P_A - p^L) + \alpha(v - P_A)] \\ u_0^{LA}(P_N, noMFC) &= v - P_N + \delta[\alpha(v - P_N) + (1-\alpha)\lambda(p^L - P_N)] \end{aligned}$$

Now, consider the difference $u_0^{LA}(P_A, MFC) - u_0^{LA}(P_N, noMFC)$:

$$P_A(-1+\delta-2\delta\alpha) + p^L\delta(1-\alpha)(-1-\lambda) + P_N(1+\delta\alpha+\delta\lambda-\delta\alpha\lambda) \tag{A.19}$$

For $u_0^{LA}(P_A, MFC) - u_0^{LA}(P_N, noMFC) > 0$ to hold, we need $A.19 \geq 0$.

Case (iii): $v > p^H > P_A > \hat{v}_A > p^L$ and $P_N > p^L$: This case is not possible since we assume $p^H > 1$ and $0 \leq v \leq 1$.

Case (iv): $p^H > P_A > \hat{v}_A > v > P_N > p^L$: In this case, utilities of a loss-averse buyer with valuation v from buying today at $(P_N, noMFC)$ and from waiting for tomorrow are given by

$$\begin{aligned}
\mathbb{E}U_1(v) &= (v - p^L)(1-\alpha)(1+\alpha-\alpha\lambda) \\
u_0^{LA}(P_N, noMFC) &= v - P_N + \delta[\alpha(v-P_N) + (1-\alpha)\lambda(p^L - P_N)]
\end{aligned}$$

Now, consider the difference $\delta\mathbb{E}U_1(v) - u_0^{LA}(P_N, noMFC)$:

$$\begin{aligned}
&= v(\delta(1-\alpha)(1+\alpha-\alpha\lambda) - 1 - \delta\alpha) + p^L\delta(1-\alpha)(-1-\alpha-\lambda+\alpha\lambda) \\
&\quad + P_N(1+\delta\alpha+\delta\lambda-\delta\alpha\lambda) \\
&> \hat{v}_A(\delta(1-\alpha)(1+\alpha-\alpha\lambda) - 1 - \delta\alpha) + p^L\delta(1-\alpha)(-1-\alpha-\lambda+\alpha\lambda) \\
&\quad + P_N(1+\delta\alpha+\delta\lambda-\delta\alpha\lambda)
\end{aligned}$$

The inequality comes from $v < \hat{v}_A$. Plug $\hat{v}_A$ into the last expression to get

$$\begin{aligned}
&-\frac{1+\delta\alpha-\delta(1-\alpha)(1+\alpha-\alpha\lambda)}{1+\delta\alpha\lambda-\delta(1-\alpha)(1+\alpha-\alpha\lambda)}(1-\delta+\delta\alpha+\delta\alpha\lambda)P_A + P_N(1+\delta\alpha+\delta(1-\alpha)\lambda) \\
&+\delta(1-\alpha)p^L\left(-1-\lambda+\frac{\delta\alpha^2(\lambda-1)^2}{1+\delta\alpha\lambda-\delta(1-\alpha)(1+\alpha-\alpha\lambda)}\right)
\end{aligned}$$

This is the same as A.18. For the second part of the Lemma, consider the difference between expressions in A.18 and A.19. Then $ICLA_1 - ICLA_2$ is given by the following.

$$\begin{aligned}
&= -\frac{1+\delta\alpha-\delta(1-\alpha)(1+\alpha-\alpha\lambda)}{1+\delta\alpha\lambda-\delta(1-\alpha)(1+\alpha-\alpha\lambda)}(1-\delta+\delta\alpha+\delta\alpha\lambda)P_A - P_A(-1+\delta-2\delta\alpha) \\
&\quad + \delta(1-\alpha)p^L\left(-1-\lambda+\frac{\delta\alpha^2(\lambda-1)^2}{1+\delta\alpha\lambda-\delta(1-\alpha)(1+\alpha-\alpha\lambda)}\right) - p^L\delta(1-\alpha)(-1-\lambda) \\
&= \left(\frac{\delta\alpha^2(\lambda-1)^2}{1+\delta\alpha\lambda-\delta(1-\alpha)(1+\alpha-\alpha\lambda)}\right)(p^L - P_A) \\
&< 0
\end{aligned}$$

The inequality comes from $P_A > p^L$. Therefore, ICLA$_1$<ICLA$_2$ which implies that if ICLA$_1 \geq 0$ then ICLA$_2 > 0$. Thus, ICLA$_2$ becomes redundant. ■

PROOF of LEMMA 5. Suppose that a loss-averse buyer with valuation $\hat{v}_A$ buys today at (P_A, MFC). So we have $u_0^{LA}(P_A, MFC) > u_0^{LA}(P_N, noMFC)$.

$$\begin{aligned} u_0^{LA}(P_A, MFC) &= \hat{v}_A - P_A + \delta(1-\alpha)(P_A - p^L) + \delta\alpha\lambda(\hat{v}_A - P_A) \\ u_0^{LA}(P_N, noMFC) &= \hat{v}_A - P_N + \delta\alpha(\hat{v}_A - P_N) + \delta(1-\alpha)\lambda(p^L - P_N) \end{aligned}$$

Thus, $u_0^{LA}(P_A, MFC) > u_0^{LA}(P_N, noMFC)$ implies

$$\hat{v}_A - P_A + \delta(1-\alpha)(P_A - p^L) + \delta\alpha\lambda(\hat{v}_A - P_A) > \hat{v}_A - P_N + \delta\alpha(\hat{v}_A - P_N) + \delta(1-\alpha)\lambda(p^L - P_N)$$

$$\hat{v}_A\delta\alpha(\lambda-1) + P_A(-1+\delta-\delta\alpha-\delta\alpha\lambda) - \delta(1-\alpha)p^L(1+\lambda) + P_N(1+\delta\alpha+\delta\lambda-\delta\alpha\lambda) > 0 \tag{A.20}$$

Now, we consider a loss-averse buyer with valuation $v > \hat{v}_A$. There are two possibilities: (i) $v < P_A$ and (ii) $v > P_A$

Case (i): $v < P_A$: In this case, $u_0^{LA}(P_A, MFC) = v - P_A + \delta(1-\alpha)(P_A - p^L) + \delta\alpha\lambda(v - P_A)$ and $u_0^{LA}(P_N, noMFC) = v - P_N + \delta\alpha(v - p^L) + \delta(1-\alpha)\lambda(p^L - P_N)$. Then, this buyer chooses to buy today at the contract (P_A, MFC) instead of buying today at the contract $(P_N, noMFC)$ if and only if $u_0^{LA}(P_A, MFC) > u_0^{LA}(P_N, noMFC)$. Thus, we consider the difference $u_0^{LA}(P_A, MFC) - u_0^{LA}(P_N, noMFC)$.

$$\begin{aligned} &= v - P_A + \delta(1-\alpha)(P_A - p^L) + \delta\alpha\lambda(v - P_A) - [v - P_N + \delta\alpha(v - p^L) \\ &\quad + \delta(1-\alpha)\lambda(p^L - P_N)] \\ &= v\delta\alpha(\lambda-1) + P_A(-1+\delta-\delta\alpha-\delta\alpha\lambda) - \delta(1-\alpha)p^L(1+\lambda) \\ &\quad + P_N(1+\delta\alpha+\delta\lambda-\delta\alpha\lambda) \\ &> \hat{v}_A\delta\alpha(\lambda-1) + P_A(-1+\delta-\delta\alpha-\delta\alpha\lambda) - \delta(1-\alpha)p^L(1+\lambda) \\ &\quad + P_N(1+\delta\alpha+\delta\lambda-\delta\alpha\lambda) \\ &> 0 \end{aligned}$$

The first inequality comes from $v > \hat{v}_A$ and the last inequality comes from the inequality A.20. Thus, we show that a loss-averse buyer with valuation v where $P_A > v > \hat{v}_A$ also buys today at (P_A, MFC).

Case (ii): $v > P_A$: In this case, $u_0^{LA}(P_A, MFC) = v - P_A + \delta(1-\alpha)(P_A - p^L) + \delta\alpha(v - P_A)$ and $u_0^{LA}(P_N, noMFC) = v - P_N + \delta\alpha(v - p^L) + \delta(1-\alpha)\lambda(p^L - P_N)$. Then, this buyer chooses to buy today at the contract (P_A, MFC) instead of buying today at the contract $(P_N, noMFC)$ if and only if $u_0^{LA}(P_A, MFC) > u_0^{LA}(P_N, noMFC)$. Thus, we consider the difference $u_0^{LA}(P_A, MFC) - u_0^{LA}(P_N, noMFC)$.

$$
\begin{aligned}
&= v - P_A + \delta(1-\alpha)(P_A - p^L) + \delta\alpha(v - P_A) - [v - P_N + \delta\alpha(v - p^L) \\
&\quad + \delta(1-\alpha)\lambda(p^L - P_N)] \\
&= P_A(-1 + \delta - 2\delta\alpha) + \delta(1-\alpha)p^L(-\lambda - 1) \\
&\quad + P_N(1 + \delta\alpha + \delta\lambda - \delta\alpha\lambda)
\end{aligned}
$$

This last expression is the $ICLA_2$ in Lemma 4. Also note that the expression in A.20 is the $ICLA_1$ in Lemma 4. From Lemma 4, we know that if $ICLA_1$ holds then $ICLA_2$ holds as well. Thus, we show that a loss-averse buyer with valuation v where $v > P_A$ also buys today at (P_A, MFC).

Suppose that a loss-averse buyer with valuation $\hat{v}_A$ chooses to wait for tomorrow. So we have $\delta\mathbb{E}U_1(\hat{v}_A) > u_0^{LA}(P_N, noMFC)$. The expected utility from waiting for tomorrow is $\mathbb{E}U_1(\hat{v}_A) = (1-\alpha)(\hat{v}_A - p^L)(1 + \alpha - \alpha\lambda)$. Thus, $\delta\mathbb{E}U_1(\hat{v}_A) > u_0^{LA}(P_N, noMFC)$ implies

$$
\begin{aligned}
&\delta(1-\alpha)(\hat{v}_A - p^L)(1 + \alpha - \alpha\lambda) > \hat{v}_A - P_N + \delta\alpha(\hat{v}_A - p^L) + \delta(1-\alpha)\lambda(p^L - P_N) \\
&\hat{v}_A(-1 - \delta\alpha + \delta(1-\alpha)(1 + \alpha - \alpha\lambda)) + \delta(1-\alpha)p^L(-1 - \alpha - \lambda + \alpha\lambda) \\
&\quad + P_N(1 + \delta\alpha + \delta\lambda - \delta\alpha\lambda) > 0 \qquad \text{(A.21)}
\end{aligned}
$$

Now, consider a loss-averse buyer with valuation $v < \hat{v}_A$. Then the expected utility from waiting is given by $\mathbb{E}U_1(v) = (1-\alpha)(v - p^L)(1 + \alpha - \alpha\lambda)$ and the utility from buying today at $(P_N, noMFC)$ is given by $u_0^{LA}(P_N, noMFC) = v - P_N + \delta\alpha(v -$

$p^L) + \delta(1-\alpha)\lambda(p^L - P_N)$. Then, this buyer chooses to wait for tomorrow instead of buying today at $(P_N, noMFC)$ if and only if $\delta\mathbb{E}U_1(v) > u_0^{LA}(P_N, noMFC)$. Thus, we consider the difference $\delta\mathbb{E}U_1(v) > u_0^{LA}(P_N, noMFC)$.

$$
\begin{aligned}
&= \delta(1-\alpha)(v-p^L)(1+\alpha-\alpha\lambda) - [v - P_N + \delta\alpha(v-p^L) + \delta(1-\alpha)\lambda(p^L - P_N)] \\
&= v(-1-\delta\alpha+\delta(1-\alpha)(1+\alpha-\alpha\lambda)) + \delta(1-\alpha)p^L(-1-\alpha-\lambda+\alpha\lambda) \\
&\quad + P_N(1+\delta\alpha+\delta\lambda-\delta\alpha\lambda) \\
&> \hat{v}_A(-1-\delta\alpha+\delta(1-\alpha)(1+\alpha-\alpha\lambda)) + \delta(1-\alpha)p^L(-1-\alpha-\lambda+\alpha\lambda) \\
&\quad + P_N(1+\delta\alpha+\delta\lambda-\delta\alpha\lambda) \\
&> 0
\end{aligned}
$$

The first inequality comes from $v < \hat{v}_A$ and the last inequality comes from A.21. Thus, we show that a loss-averse buyer with valuation $v < \hat{v}_A$ also chooses to wait for tomorrow. ■

PROOF of LEMMA 6. From Lemma 4 we know that the ICC of loss-neutral is given by

$$-P_N + P_A(1-\delta+\delta\alpha) + \delta(1-\alpha)p^L \geq 0$$

To see that this holds at the first-best prices, plug P_N and P_A given in Lemma 7 and Lemma 10,respectively.

$$
\begin{aligned}
&p^L(1+\delta\alpha+\delta\lambda-\delta\alpha\lambda) + \frac{-1+\delta-\delta\alpha+(\alpha-1)(\delta+1)p^L}{2} \\
&\quad + \frac{(1-\delta+\delta\alpha)}{2(1-\delta+\delta\alpha+\delta\alpha\lambda)}(1+\delta(-1+\alpha(2\lambda-\alpha(\lambda-1)+p^L(\lambda-1)(\alpha-1)))) \\
&> \frac{p^L}{2}(1-\delta+\alpha+3\delta\alpha+2\delta\lambda(1-\alpha)) + \frac{-1+\delta-\delta\alpha}{2} \\
&\quad - \frac{1-\delta+\delta\alpha}{2(1-\delta+\delta\alpha+\delta\alpha\lambda)}(1-\delta+\delta\alpha+\delta\alpha\lambda) \\
&= \frac{p^L}{2}(1-\delta+\alpha+3\delta\alpha+2\delta\lambda(1-\alpha)) \\
&> 0
\end{aligned}
$$

The first inequality comes from $p^L < 1$. This finishes the proof of Lemma 6. ■

PROOF of PROPOSITION 1. First, we need to calculate the ICC of loss-averse buyers given by 5.5 at the prices P_N and P_A given in Lemma 7 and Lemma 10, respectively. So, it is given by

$$(1-\alpha)\delta\left(-1-\lambda-\frac{\alpha^2\delta(-1+\lambda)^2}{-1+\delta+\alpha^2\delta(-1+\lambda)-2\alpha\delta\lambda}\right)p^L$$
$$+\frac{1}{2}(-1+\alpha\delta(-1+\lambda)-\delta\lambda)(-1+\delta-\alpha\delta+(-1+\alpha)(1+\delta)p^L)$$
$$-\frac{(-1-(-1+\alpha+\alpha^2)\delta+(-1+\alpha)\alpha\delta\lambda)(1+\delta(-1+\alpha(2\lambda+\alpha(-1+\lambda)(-1+p^L)+p^L-\lambda p^L)))}{2(-1+\delta-\alpha^2\delta+(-2+\alpha)\alpha\delta\lambda)} \tag{A.22}$$

Then at $\alpha=\frac{1}{2}$, this incentive compatibility constraint of loss-averse buyers is given by

$$\frac{1}{8}\left(-(-1+\delta)\delta(1+\lambda)+\frac{(8+\delta(-3(-4+\delta)\delta+\delta(-4+3\delta)\lambda^2-2(7+\lambda)))p^L}{4+3\delta(-1+\lambda)}\right) \tag{A.23}$$

We calculate A.23 at $\delta=0$ and at $\delta=1$. At $\delta=0$, it is positive.

$$\frac{1}{4}p^L \;>\; 0$$

Also at $\delta=1$, it is negative.

$$-\frac{(-1+\lambda)(3+\lambda)p^L}{8+24\lambda} \;<\; 0$$

By taking the derivative of the expression in A.23 with respect to δ, we get

$$-\frac{(-1+2\delta)(4+3\delta(-1+\lambda))^2(1+\lambda)}{8(4+3\delta(-1+\lambda))^2}-\frac{32(1+\lambda)p^L}{8(4+3\delta(-1+\lambda))^2}$$
$$+\frac{2\delta(48-16\lambda^2+9\delta^2(-1+\lambda)^2(1+\lambda)-6\delta(-1+\lambda)(-6+(-3+\lambda)\lambda))p^L}{8(4+3\delta(-1+\lambda))^2}$$

This derivative at $\delta=0$ is positive:

$$-\frac{1}{8}(\lambda+1)(-1+2p^L) \;>\; 0$$

This derivative at $\delta = 1$ is negative. Note that denominator is always positive. So we need to check that at $\delta = 1$ the numerator is negative:

$$\begin{aligned} -(1+\lambda)(1+3\lambda)^2 + 2(\lambda-1)^2(5+3\lambda)p^L &< -(1+\lambda)(1+3\lambda)^2 + \frac{1}{2}(\lambda-1)^2(5+3\lambda) \\ &= -6\lambda^3 - 16\lambda^3 - 14\lambda + 4 \\ &= -2(3\lambda^3 + 8\lambda^2 + 7\lambda - 2) \\ &< 0 \end{aligned}$$

The first inequality comes from $p^L < \frac{1}{2}$.

We show that ICC of loss-averse buyers is positive and it increases when $\delta = 0$ because its derivative is positive at $\delta = 0$. Also it is negative and it decreases when $\delta = 1$ because its derivative is negative at $\delta = 1$. To show the existence of a delta threshold, it suffices to show that the derivative is 0 only at one value of δ. Let $g(\delta)$ denote the negative of the numerator of the derivative expression.

$$g'(\delta) = -2(4 + 3\delta(\lambda - 1))f(\delta)$$

where $f(\delta) = -7 + \lambda\left(-4 + \lambda(3 - 4p^L)\right) + 9\delta(-1+\lambda^2)(-1+p^L) + 12p^L$. Notice that $g'(\delta)$ and $f(\delta)$ have the opposite signs.

Note that $f'(\delta) = 9(-1+\lambda^2)(-1+p^L) < 0$ since $p^L < \frac{1}{2}$. Thus, f is decreasing in δ. So it takes its lowest value when $\delta = 1$, and largest value when $\delta = 0$.

$$\begin{aligned} f(1) &= 2 + 3p^L + \lambda\left(-4 + \lambda(-6 + 5p^L)\right) \\ &< 2 - 4\lambda - 6\lambda^2 + \frac{3}{2} + \frac{5\lambda^2}{2} \\ &= \frac{7}{2}(-1+\lambda^2) - 4\lambda \\ &< 0 \end{aligned}$$

$$f(0) = -7 + \lambda\left(-4 + \lambda(3 - 4p^L)\right) + 12p^L$$

So, at $\delta = 1$, f is negative for all values of λ and p^L but at $\delta = 0$ it can be positive or negative depending on the parameter values.

If $f(0) < 0$, then since f is decreasing with δ and $f(1) < 0$ we have $f(\delta) < 0$ for all δ. Then $g'(\delta) > 0$ for all δ which implies that g increases with δ. We know that the numerator is positive at $\delta = 0$ and negative when $\delta = 1$. Since $g(\delta)$ is the negative of the numerator, we have $g(0) < 0$ and $g(1) > 0$. So there exists $\tilde{\delta}$ such that $g(\tilde{\delta}) = 0$.

If $f(0) > 0$, then there exists $\hat{\delta}$ such that $f(\delta) > 0$ for $\delta < \hat{\delta}$ and $f(\delta) < 0$ for $\delta > \hat{\delta}$. This implies that $g'(\delta) < 0$ for $\delta < \hat{\delta}$ and $g'(\delta) > 0$ for $\delta > \hat{\delta}$. Thus, g increases with δ at first and then decreases. Together with $g(0) < 0$ and $g(1) > 0$, this implies that there exists $\tilde{\delta}$ such that $g(\tilde{\delta}) = 0$. In each cases, we show that the numerator is 0 only at one value of δ. Since the denominator is always positive, this implies that the derivative is 0 only at one value of δ and this finishes the proof of Proposition 1. ■

PROOF of PROPOSITION 2. At $\alpha = \frac{1}{2}$, the incentive compatibility constraint of loss-averse buyers is given in equation A.23. At $\lambda = 1$, it is positive.

$$\frac{(1-\delta)}{4}(\delta + (1-\delta)p^L) \quad > \quad 0$$

Now, consider the limit of A.23 as $\lambda \to \infty$. It can be negative or positive infinity depending on the sign of $3\delta^2(1-\delta)+\delta^2 p^L(-4+3\delta)$. If $3\delta^2(1-\delta)+\delta^2 p^L(-4+3\delta) > 0$, then the limit is positive infinity. Thus, the limit is positive if $p^L < \frac{3(1-\delta)}{4-3\delta}$.

By taking the derivative of the expression in A.23 with respect to λ, we get

$$\frac{1}{8}\delta\left(1-\delta+\frac{-32+\delta\left(48+3\delta\left(-4(-3+\lambda)+3\delta(-1+\lambda)\right)(-1+\lambda)-32\lambda\right)p^L}{(4+3\delta(-1+\lambda))^2}\right) \quad \text{(A.24)}$$

At $\lambda = 1$, this derivative is given by

$$\frac{1}{8}\delta(1+\delta(-1+p^L)-2p^L)$$

Thus, the derivative at $\lambda = 1$ is positive if and only if $1+\delta(-1+p^L)-2p^L > 0$ which holds if and only if $p^L < \frac{1-\delta}{2-\delta}$. Now, consider the derivative as $\lambda \to \infty$. The limit is

given by

$$\frac{9\delta(1-\delta)+3\delta p^L(-4+3\delta)}{7}$$

This limit is positive if $9\delta(1-\delta)+3\delta p^L(-4+3\delta) > 0$ which holds if $p^L < \frac{3(1-\delta)}{4-3\delta}$. Thus, if $p^L < \frac{3(1-\delta)}{4-3\delta}$, then both the limits of equations A.23 and A.24 are positive. Now, we consider the second derivative of equation A.23 with respect to λ. It is given by

$$\frac{8\delta^2 p^L}{(4+3\delta(-1+\lambda))^3} \quad > \quad 0$$

Thus, the second derivative is positive. This implies equation A.24 is increasing in λ.

Case (i): $p^L < \frac{1-\delta}{2-\delta}$: In this case, A.24 at $\lambda = 1$ is positive. We also know that A.24 is increasing with λ. Therefore, the first derivative of ICC with respect to λ is positive which implies that ICC is increasing with λ. This together with ICC being positive at $\lambda = 1$ implies that ICC is positive.

Case (ii) $\frac{1-\delta}{2-\delta} < p^L < \frac{3(1-\delta)}{4-3\delta}$: In this case, A.24 at $\lambda = 1$ is negative and the limit of A.24 is positive. Since A.24 is increasing with λ, there is $\tilde{\lambda}$ such that it is zero at $\tilde{\lambda}$. That is, at $\tilde{\lambda}$ the first derivative of ICC is zero which means that ICC takes its minimum value at $\tilde{\lambda}$. We know that ICC is positive at $\lambda = 1$ and the limit of ICC is positive. Thus, it suffices to show that the minimum value of ICC is positive.
By setting A.24 to zero we get

$$1-\delta \quad = \quad -\left(\frac{-32+\delta\left(48+3\delta\left(-4(-3+\tilde{\lambda})+3\delta(-1+\tilde{\lambda})\right)(-1+\tilde{\lambda})-32\tilde{\lambda}\right)p^L}{(4+3\delta(-1+\tilde{\lambda}))^2}\right) \quad \text{(A.25)}$$

Plug A.25 into A.23 to get the minimum value of ICC.

$$\frac{p^L(16-\delta(-24+\delta(21+6\delta(-1+\tilde{\lambda})-5\tilde{\lambda}))(-1+\tilde{\lambda}))}{4(4+3\delta(-1+\tilde{\lambda}))^2} \qquad \text{(A.26)}$$

Minimum value of ICC is decreasing with δ. To see this, take derivative of A.26 with respect to δ:

$$-\frac{\delta(-1+\tilde{\lambda})(9\delta(4+\delta(-1+\tilde{\lambda}))(-1+\tilde{\lambda})+16(3+\tilde{\lambda}))p^L}{2(4+3\delta(-1+\tilde{\lambda}))^3} \quad < \quad 0$$

Thus, the minimum value of ICC takes its lowest value when $\delta = \frac{3-4p^L}{3(1-p^L)}$. Since the denominator is always positive, we only consider the numerator. At $\delta = \frac{3-4p^L}{3(1-p^L)}$, it is given by

$$h(p^L) \quad = \quad 16 - \frac{(-1+\tilde{\lambda})(-3+4p^L)(-27+5(9-4p^L)p^L+\tilde{\lambda}(-1+3p^L)(-3+4p^L))}{9(-1+p^L)^3}$$

For p^L, $h(0) = 7 - \tilde{\lambda}(\tilde{\lambda}-10)$ and it can be positive or negative depending on the value of $\tilde{\lambda}$. For $p^L = \frac{1}{2}$, $h(\frac{1}{2}) = 16 + \frac{4}{9}(\lambda-1)(19+\lambda) > 0$. We look at the first, second, and third derivatives of h with respect to p^L:

$$\begin{aligned}
h'(p^L) &= \frac{2(-1+\tilde{\lambda})(-3p^L+\tilde{\lambda}(-3+4p^L)(-4+7p^L))}{9(-1+p^L)^4} \\
h''(p^L) &= -\frac{2(-1+\tilde{\lambda})(-3-9p^L+\tilde{\lambda}(11+p^L(-55+56p^L)))}{9(-1+p^L)^5} \\
h'''(p^L) &= \frac{8(-1+\tilde{\lambda})(-2+p^L(-3+\tilde{\lambda}(-9+14p^L)))}{3(-1+p^L)^6}
\end{aligned}$$

$h'''(p^L)$ is negative. The denominator is positive, so it is negative if and only if the numerator is negative. To see this, note that

$$\begin{aligned}
(-1+\tilde{\lambda})(-2+p^L(-3+\tilde{\lambda}(-9+14p^L))) &< (-1+\tilde{\lambda})(-2+p^L(-3-2\tilde{\lambda})) \\
&< 0
\end{aligned}$$

Thus, $h''(p^L)$ is decreasing with p^L. Hence, it takes its largest value at $p^L = 0$ which is positive and it takes its lowest value at $p^L = \frac{1}{2}$ which is negative. To see this, note

that

$$\begin{aligned} h''(0) &= \frac{2}{9}(\tilde{\lambda}-1)(-3+11\tilde{\lambda}) > 0 \\ h''(1) = &= \frac{32}{9}(\tilde{\lambda}-1)(-15-5\tilde{\lambda}) < 0 \end{aligned}$$

Therefore, there exists $\hat{P}_1^L$ such that $h''(\hat{P}_1^L) = 0$. Then, we have $h''(p^L) > 0$ for $p^L < \hat{P}_1^L$ and $h''(p^L) < 0$ for $p^L > \hat{P}_1^L$.

Note that $h'(0)$ is positive:

$$h'(0) = \frac{8}{3}\tilde{\lambda}(\tilde{\lambda}-1) > 0$$

For $\tilde{\lambda} > 3$, $h'(\frac{1}{2}) > 0$ and for $\tilde{\lambda} < 3$, $h'(\frac{1}{2}) < 0$:

$$h'(\frac{1}{2}) = \frac{8}{9}(\tilde{\lambda}-1)(-3+\tilde{\lambda})$$

If $p^L < \hat{P}_1^L$, then h' is increasing with p^L. We also know that at $p^L = 0$, h' is positive. This implies that h' is positive for all $p^L < \hat{P}_1^L$. Thus, h is increasing with p^L for $p^L < \hat{P}_1^L$. If $p^L > \hat{P}_1^L$, then h' is decreasing with p^L. For $\tilde{\lambda} > 3$, we have $h'(\frac{1}{2}) > 0$. We also have $h'(0) > 0$. Thus, h' is positive for $p^L > \hat{P}_1^L$. With these two arguments, we show that if $\tilde{\lambda} > 3$, h' is positive for all p^L. Hence, if $\tilde{\lambda} > 3$, h is increasing with p^L. If $h(0) > 0$, then $h > 0$ for all p^L which implies that minimum value of ICC is positive. If $h(0) > 0$, then there exists $\tilde{P}_1^L$ such that $h(\tilde{P}_1^L) = 0$ because $h(1) < 0$ and h is increasing with p^L. Thus, the minimum value of ICC is positive for all $p^L > \tilde{P}_1^L$.

If $p^L > \hat{P}_1^L$, then h' is decreasing with p^L. For $\tilde{\lambda} < 3$, we have $h'(\frac{1}{2}) < 0$. We also have $h'(0) > 0$. Thus, there exists $\bar{P}_1^L$ such that $h'(\bar{P}_1^L) = 0$. Then we have $h'(p^L) > 0$ for $p^L < \bar{P}_1^L$ and $h'(p^L) < 0$ for $p^L > \bar{P}_1^L$. Suppose $p^L < \bar{P}_1^L$. Then h is increasing with p^L. This together with h being positive at 0 for $\tilde{\lambda} < 3$ implies that $h > 0$ for all $p^L < \bar{P}_1^L$. That is, minimum value of ICC is positive for all $p^L < \bar{P}_1^L$.

Suppose $p^L > \bar{P}_1^L$. Then h is decreasing with p^L. This together with h being positive at 0 and at 1 implies that $h > 0$ for all $p^L > \bar{P}_1^L$. That is, minimum value of ICC is positive for all $p^L > \bar{P}_1^L$.

Thus, if $\tilde{P}_1^L < p^L < \frac{3(1-\delta)}{4-3\delta}$, then the minimum value of ICC is positive in all cases.

Case (iii) $p^L > \frac{3(1-\delta)}{4-3\delta}$: In this case, A.24 at $\lambda = 1$ is negative and the limit of A.24 is negative. We also know that A.24 is increasing with λ. Therefore, the first derivative of ICC with respect to λ is negative which implies that ICC is decreasing with λ. This together with ICC being positive at $\lambda = 1$ and limit of ICC being negative implies that there exists $\bar{\lambda}$ such that ICC takes the value 0. For $\lambda > \bar{\lambda}$, ICC is negative and for $\lambda \leq \bar{\lambda}$, ICC is positive. This finishes the proof of Proposition 2. ■

PROOF of PROPOSITION 3. When $\delta = 1$, ICC of loss-averse buyers at first-best prices given by equation A.22 is negative for all parameter values. To see this, plug $\delta = 1$ in A.22.

$$\frac{(-1+\alpha)\alpha(-1+\alpha(-1+\lambda)-\lambda)(-1+\lambda)p^L}{2\alpha(-1+\lambda)-4\lambda} \tag{A.27}$$

By rearranging the denominator we get $2(-\alpha+\lambda(\alpha-2)) < 0$. Thus, equation A.27 is negative if and only if the numerator is positive. Note that $(-1+\alpha)\alpha(-1+\lambda)p^L$ is negative. By rearranging the remaining term in the numerator we get $-1-\alpha+\lambda(\alpha-1) < 0$. Thus, numerator is positive. Therefore, there is an incentive problem when $\delta = 1$

Before solving the problem, note that we find residual demands $\hat{v}_A$ and $\hat{v}_N$ by solving $u_0(v) = \delta \mathbb{E}U_1(v)$ for loss-averse buyers and loss-neutral buyers where $u_0(v)$ for loss-averse is the utility from buying today at (P_A, MFC) and for loss-neutral buyers is the utility from buying today at $(P_N, noMFC)$. These thresholds,$\hat{v}_N$ and $\hat{v}_A$, are given in the proof of Lemma 7 and Lemma 10. By plugging δ=1 in these

expressions, we get

$$\begin{aligned}\hat{v}_N &= \frac{P_N - (1-\alpha)p^L}{\alpha} \\ \hat{v}_A &= \frac{P_A(1+\lambda) + (1-\alpha)(\lambda-1)p^L}{\alpha + 2\lambda - \alpha\lambda}\end{aligned}$$

where $p^H > \hat{v}_N > P_N > p^L$ and $p^H > P_A > \hat{v}_A > p^L$.

From Lemma 4, we have two incentive compatibility constraints given by *(ICLN)* and $(ICLA_1)$ where the first one is for loss-neutral buyers and the second one is for the loss-averse buyers. When $\delta = 1$, they are given by

$$-P_N + \alpha P_A + (1-\alpha)p^L \geq 0 \tag{A.28}$$

$$\frac{(-1-\alpha-\lambda+\alpha\lambda)(P_A\alpha(\lambda+1) + P_N(-2\lambda-\alpha+\alpha\lambda) + 2p^L\lambda(1-\alpha))}{\alpha+2\lambda-\alpha\lambda} \geq 0 \tag{A.29}$$

At period 0, monopolist's expected profit is given by

$$\mathbb{E}\pi_0 = \beta(1-\hat{v}_A)P_A + (1-\beta)(1-\hat{v}_N)P_N$$

At period 1, monopolist's expected profit is given by

$$\begin{aligned}\mathbb{E}\pi_1 = \beta[(1-\alpha)p^L(\hat{v}_A - p^L) - (1-\alpha)(1-\hat{v}_A)(P_A - p^L)] \\ + (1-\beta)[(1-\alpha)p^L(\hat{v}_N - p^L)]\end{aligned}$$

Using these two expected profit expressions we write the overall profit as

$$\begin{aligned}\mathbb{E}\pi &= \mathbb{E}\pi_0 + \mathbb{E}\pi_1 \\ &= \beta[\alpha(1-\hat{v}_A)P_A + (1-\alpha)p^L(1-p^L)] + (1-\beta)[(1-\hat{v}_N)P_N \\ &\quad +(1-\alpha)(\hat{v}_N - p^L)p^L]\end{aligned} \tag{A.30}$$

Now, we solve the monopolist's problem when $\delta = 1$ subject to the constraints A.28 and A.29.

$$\begin{aligned}\mathcal{L} = {} & \beta[\alpha(1-\hat{v}_A)P_A + (1-\alpha)p^L(1-p^L)] + (1-\beta)[(1-\hat{v}_N)P_N + (1-\alpha)(\hat{v}_N - p^L)p^L] \\ & + \mu_1(-P_N + \alpha P_A + (1-\alpha)p^L) \\ & + \mu_2\left(\frac{(-1-\alpha-\lambda+\alpha\lambda)(P_A\alpha(\lambda+1) + P_N(-2\lambda-\alpha+\alpha\lambda) + 2p^L\lambda(1-\alpha))}{\alpha+2\lambda-\alpha\lambda}\right)\end{aligned}$$

We have the following Kuhn-Tucker conditions:

(i): $\frac{\partial \mathcal{L}}{\partial P_A} = 0$ and $\frac{\partial \mathcal{L}}{\partial P_N} = 0$

(ii): $\mu_i \geq 0$ for $i = 1, 2$

(iii): A.28 and A.29 hold

(iv):

$$\mu_1(-P_N + \alpha P_A + (1-\alpha)p^L) = 0 \tag{A.31}$$

$$\mu_2\left(\frac{(-1-\alpha-\lambda+\alpha\lambda)(P_A\alpha(\lambda+1) + P_N(-2\lambda-\alpha+\alpha\lambda) + 2p^L\lambda(1-\alpha))}{\alpha+2\lambda-\alpha\lambda}\right) = 0 \tag{A.32}$$

By taking the derivative of $\mathcal{L}$ with respect to P_A, we get

$$\begin{aligned}\frac{\partial \mathcal{L}}{\partial P_A} &= \beta\left[-\alpha\frac{\partial \hat{v}_A}{\partial P_A}P_A + \alpha(1-\hat{v}_A)\right] + \mu_1\alpha + \mu_2\left(\frac{(-1-\alpha-\lambda+\alpha\lambda)\alpha(\lambda+1)}{\alpha+2\lambda-\alpha\lambda}\right) \\ &= \beta\alpha - \frac{\beta\alpha(1-\alpha)(\lambda-1)p^L}{\alpha+2\lambda-\alpha\lambda} - \frac{2\beta\alpha(1+\lambda)P_A}{\alpha+2\lambda-\alpha\lambda} + \mu_1\alpha \\ &\quad + \mu_2\left(\frac{(-1-\alpha-\lambda+\alpha\lambda)\alpha(\lambda+1)}{\alpha+2\lambda-\alpha\lambda}\right) \qquad \text{(A.33)} \\ &= 0\end{aligned}$$

To get equation A.33, we use $\hat{v}_A$ expression and $\frac{\partial \hat{v}_A}{\partial P_A} = \frac{1+\lambda}{\alpha+2\lambda-\alpha\lambda}$.

By taking the derivative of $\mathcal{L}$ with respect to P_N, we get

$$\begin{aligned}\frac{\partial \mathcal{L}}{\partial P_N} &= (1-\beta)\left[-\frac{\partial \hat{v}_N}{\partial P_N}P_N + (1-\hat{v}_N) + (1-\alpha)\frac{\partial \hat{v}_N}{\partial P_N}p^L\right] - \mu_1 - \mu_2(-1-\alpha-\lambda+\alpha\lambda) \\ &= \frac{1-\beta}{\alpha} + \frac{2(1-\beta)(1-\alpha)p^L}{\alpha} - \frac{2(1-\beta)P_N}{\alpha} - \mu_1 - \mu_2(-1-\alpha-\lambda+\alpha\lambda) \qquad \text{(A.34)} \\ &= 0\end{aligned}$$

To get equation A.34, we use $\hat{v}_N$ expression and $\frac{\partial \hat{v}_N}{\partial P_N} = \frac{P_N}{\alpha}$. Now, we consider 4 cases.

Case (i): Suppose $\mu_1 = 0$ and $\mu_2 = 0$. Then P_A and P_N are the first-best prices. However, at these prices we know that if $\delta = 1$, ICC of loss-averse buyers is negative. That is, A.29 does not hold. Thus, this case is not possible.

Case (ii): Suppose $\mu_1 > 0$ and $\mu_2 = 0$. Then by A.31, ICC of loss-neutral buyers binds. We rewrite the first order conditions using $\mu_2 = 0$ and we get

$$\begin{aligned}
\frac{\partial \mathcal{L}}{\partial P_A} &= \beta\alpha - \frac{\beta\alpha(1-\alpha)(\lambda-1)p^L}{\alpha+2\lambda-\alpha\lambda} - \frac{2\beta\alpha(1+\lambda)P_A}{\alpha+2\lambda-\alpha\lambda} + \mu_1\alpha = 0 \\
\frac{\partial \mathcal{L}}{\partial P_N} &= \frac{1-\beta}{\alpha} + \frac{2(1-\beta)(1-\alpha)p^L}{\alpha} - \frac{2(1-\beta)P_N}{\alpha} - \mu_1 = 0
\end{aligned}$$

Solving these two equations simultaneously, we find P_A and P_N as functions of μ_1.

$$\begin{aligned}
P_A &= \frac{\alpha+2\lambda-\alpha\lambda}{2(1+\lambda)} - \frac{(1-\alpha)(\lambda-1)p^L}{2(1+\lambda)} + \frac{\mu_1(\alpha+2\lambda-\alpha\lambda)}{2\beta(1+\lambda)} \\
P_N &= \frac{1}{2} + (1-\alpha)p^L - \frac{\mu_1\alpha}{2(1-\beta)}
\end{aligned}$$

Since ICC of loss-neutral buyers binds we have $-P_N + \alpha P_A + (1-\alpha)p^L = 0$. By plugging P_A and P_N expressions into this equation, we get

$$\mu_1 = \frac{\beta(1-\beta)\left[1+\lambda-\alpha(\alpha+2\lambda-\alpha\lambda)+\alpha(1-\alpha)(\lambda-1)p^L\right]}{\alpha((1-\beta)(\alpha+2\lambda-\alpha\lambda)+\beta(1+\lambda))}$$

We need to check $\mu_1 > 0$. Note that the fraction term in the above expression is positive. Thus, $\mu_1 > 0$ if and only if

$$1+\lambda-\alpha(\alpha+2\lambda-\alpha\lambda)+\alpha(1-\alpha)(\lambda-1)p^L > 0$$

To see that this inequality holds, consider $1+\lambda-\alpha(\alpha+2\lambda-\alpha\lambda)$. By rearranging this expression, we get $1-\alpha^2+\lambda(1-\alpha)^2$ which is positive. Thus, the above inequality holds which implies that $\mu_1 > 0$. Now, by plugging μ_1 expression into P_A and P_N

equations, we get

$$\begin{aligned} P_A &= \frac{(\alpha+2\lambda-\alpha\lambda)(1-\beta+\beta\alpha)}{2\alpha((1-\beta)(\alpha+2\lambda-\alpha\lambda)+\beta(1+\lambda))} - \frac{\beta(1-\alpha)(\lambda-1)p^L}{2((1-\beta)(\alpha+2\lambda-\alpha\lambda)+\beta(1+\lambda))} \\ P_N &= \frac{(\alpha+2\lambda-\alpha\lambda)(1-\beta+\beta\alpha)}{2((1-\beta)(\alpha+2\lambda-\alpha\lambda)+\beta(1+\lambda))} - \frac{\beta\alpha(1-\alpha)(\lambda-1)p^L}{2((1-\beta)(\alpha+2\lambda-\alpha\lambda)+\beta(1+\lambda))} \\ &\quad +(1-\alpha)p^L \end{aligned}$$

Case (iii): Suppose $\mu_1 = 0$ and $\mu_2 > 0$. Then by A.32, ICC of loss-averse buyers binds. We rewrite the first order conditions using $\mu_1 = 0$ and we get

$$\begin{aligned} \frac{\partial \mathcal{L}}{\partial P_A} &= \beta\alpha - \frac{\beta\alpha(1-\alpha)(\lambda-1)p^L}{\alpha+2\lambda-\alpha\lambda} - \frac{2\beta\alpha(1+\lambda)P_A}{\alpha+2\lambda-\alpha\lambda} \\ &\quad +\mu_2\left(\frac{(-1-\alpha-\lambda+\alpha\lambda)\alpha(\lambda+1)}{\alpha+2\lambda-\alpha\lambda}\right) = 0 \\ \frac{\partial \mathcal{L}}{\partial P_N} &= \frac{1-\beta}{\alpha} + \frac{2(1-\beta)(1-\alpha)p^L}{\alpha} - \frac{2(1-\beta)P_N}{\alpha} \\ &\quad -\mu_2(-1-\alpha-\lambda+\alpha\lambda) = 0 \end{aligned}$$

Solving these two equations simultaneously, we find P_A and P_N as functions of μ_2.

$$\begin{aligned} P_A &= \frac{\alpha+2\lambda-\alpha\lambda}{2(1+\lambda)} - \frac{(1-\alpha)(\lambda-1)p^L}{2(1+\lambda)} + \frac{\mu_2(-1-\alpha-\lambda+\alpha\lambda)}{2\beta} \\ P_N &= \frac{1}{2} + (1-\alpha)p^L - \frac{\mu_2\alpha(-1-\alpha-\lambda+\alpha\lambda)}{2(1-\beta)} \end{aligned}$$

Since ICC of loss-averse buyers bind we have

$$\frac{(-1-\alpha-\lambda+\alpha\lambda)(P_A\alpha(\lambda+1)+P_N(-2\lambda-\alpha+\alpha\lambda)+2p^L\lambda(1-\alpha))}{\alpha+2\lambda-\alpha\lambda} = 0$$

By plugging P_A and P_N expressions into this equation, we get

$$\mu_2 = \frac{\beta(1-\beta)[(\alpha+2\lambda-\alpha\lambda)(\alpha-1)+\alpha(1-\alpha)(\lambda-1)p^L]}{\alpha(1+\alpha+\lambda(1-\alpha))(1+\lambda+\beta(1-\alpha)(\lambda-1))}$$

We need to check $\mu_2 > 0$. Note that the denominator is positive. Thus, $\mu_2 > 0$ if and only if the numerator is positive.

$$\begin{aligned} & \beta(1-\beta)[(\alpha+2\lambda-\alpha\lambda)(\alpha-1)+\alpha(1-\alpha)(\lambda-1)p^L] \\ < \; & \beta(1-\beta)\left[(\alpha+2\lambda-\alpha\lambda)(\alpha-1)+\frac{\alpha(1-\alpha)(\lambda-1)}{2}\right] \\ = \; & \frac{\beta(1-\beta)(1-\alpha)}{2}[-3\alpha-4\lambda+3\alpha\lambda)] \\ < \; & 0 \end{aligned}$$

The first inequality comes from $p^L < \frac{1}{2}$. To see the last inequality, rearrange $-3\alpha - 4\lambda + 3\alpha\lambda$ to get $-3\alpha - \lambda + 3\lambda(\alpha - 1)$ which is negative. Therefore, we show that the numerator is negative which implies that $\mu_2 < 0$. However, this is a contradiction to this case assumption. Hence, this case is not possible.

Case (iv): Suppose $\mu_1 > 0$ and $\mu_2 > 0$. Then by A.31 and A.32, both ICC bind. That is, we have

$$\begin{aligned} & -P_N + \alpha P_A + (1-\alpha)p^L = 0 \\ & \frac{(-1-\alpha-\lambda+\alpha\lambda)(P_A\alpha(\lambda+1)+P_N(-2\lambda-\alpha+\alpha\lambda)+2p^L\lambda(1-\alpha))}{\alpha+2\lambda-\alpha\lambda} = 0 \end{aligned}$$

By solving, these two equations simultaneously, we find $P_A = P_N = p^L$ which is a contradiction to our assumption that $P_A > P_N > p^L$.

Thus, the only possibility is *Case (ii)*. We need to check $p^H > \hat{v}_N > P_N > p^L$ and $p^H > P_A > \hat{v}_A > p^L$.

$\hat{v}_N > P_N$ holds if and only if $\frac{(1-\alpha)(P_N - p^L)}{\alpha} > 0$. This holds if and only if $P_N > p^L$. To see that $P_N > p^L$ holds, we use $p^L < \frac{1}{2}$. The difference $P_N - p^L$ is given by

$$\begin{aligned}
&= \frac{(\alpha + 2\lambda - \alpha\lambda)(1 - \beta + \beta\alpha)}{2((1-\beta)(\alpha + 2\lambda - \alpha\lambda) + \beta(1+\lambda))} - \frac{\beta\alpha(1-\alpha)(\lambda - 1)p^L}{2((1-\beta)(\alpha + 2\lambda - \alpha\lambda) + \beta(1+\lambda))} \\
&+ (1-\alpha)p^L - p^L \\
&= \frac{(\alpha + 2\lambda - \alpha\lambda)(1 - \beta + \beta\alpha) + \alpha\beta(\lambda - 1)(1-\alpha)p^L - 2\alpha p^L(\alpha + 2\lambda - \alpha\lambda)}{2((1-\beta)(\alpha + 2\lambda - \alpha\lambda) + \beta(1+\lambda))} \\
&> \frac{(\alpha + 2\lambda - \alpha\lambda)(1 - \beta + \beta\alpha) + \alpha\beta(\lambda - 1)(1-\alpha)p^L - \alpha p^L(\alpha + 2\lambda - \alpha\lambda)}{2((1-\beta)(\alpha + 2\lambda - \alpha\lambda) + \beta(1+\lambda))} \\
&= \frac{(\alpha + 2\lambda - \alpha\lambda)(1 - \beta)(1-\alpha) + \alpha\beta(\lambda - 1)(1-\alpha)p^L}{2((1-\beta)(\alpha + 2\lambda - \alpha\lambda) + \beta(1+\lambda))} \\
&> 0
\end{aligned}$$

Therefore, we show that $\hat{v}_N > P_N > p^L$.

We check $P_A > P_N$. To see this, we use $p^L < \frac{1}{2}$. We consider the difference $P_A - P_N$:

$$\begin{aligned}
&= \frac{(1-\alpha)(\alpha + 2\lambda - \alpha\lambda)(1 - \beta + \beta\alpha)}{2\alpha((1-\beta)(\alpha + 2\lambda - \alpha\lambda) + \beta(1+\lambda))} - \frac{\beta(1-\alpha)^2(\lambda - 1)p^L}{2((1-\beta)(\alpha + 2\lambda - \alpha\lambda) + \beta(1+\lambda))} \\
&\quad -(1-\alpha)p^L \\
&> \frac{(1-\alpha)(\alpha + 2\lambda - \alpha\lambda)(1 - \beta + \beta\alpha)}{2\alpha((1-\beta)(\alpha + 2\lambda - \alpha\lambda) + \beta(1+\lambda))} - \frac{\beta(1-\alpha)^2(\lambda - 1)}{4((1-\beta)(\alpha + 2\lambda - \alpha\lambda) + \beta(1+\lambda))} \\
&\quad -\frac{(1-\alpha)}{2} \\
&= \frac{(1-\alpha)^2}{4\alpha((1-\beta)(\alpha + 2\lambda - \alpha\lambda) + \beta(1+\lambda))}[2(1-\beta)(\alpha + 2\lambda - \alpha\lambda) + \beta\alpha(\lambda - 1)] \\
&> 0
\end{aligned}$$

$P_A > p^L$ holds since we also have $P_N > p^L$. Note that $P_A > \hat{v}_A$ holds if and only if $\frac{(1-\alpha)(\lambda-1)(P_A - p^L)}{\alpha + 2\lambda - \alpha\lambda} > 0$ and $\hat{v}_A > p^L$ holds if and only if $\frac{(1+\lambda)(P_A - p^L)}{\alpha + 2\lambda - \alpha\lambda} > 0$. These two hold since we have $P_A > p^L$. Therefore we show that $P_A > \hat{v}_A > p^L$. ■

PROOF of PROPOSITION 4. This is done in the proof of Proposition 3. ■

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