REDESIGNING THE HIGH SCHOOL PLACEMENT MECHANISM IN TURKEY

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DECLARATION OF ORIGINALITY

I, Emine Taşçı, certify that

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ABSTRACT

Redesigning the High School Placement Mechanism in Turkey

In this thesis, we study the placement mechanism used for matching eight grade students and high schools in Turkey. We observe that the mechanism does not satisfy the most desirable properties such as individual rationality, strategy-proofness, respecting improvement, non-wastefulness, and fairness. In order for the system to have the desired properties, students must be able to present their original preferences. However, in the current system, this is not possible due to preference restrictions. Our approach is to improve the students' welfare by redesigning the mechanism.

ÖZET

Türkiye'deki Lise Yerleştirme Mekanizmasının Yeniden Tasarlanması

Bu tezde, Türkiye'de sekizinci sınıf öğrencilerini ve liseleri eşleştirmek için kullanılan yerleştirme mekanizmasını inceliyoruz. Mekanizmanın, bireysel rasyonellik, stratejilere dayanıklılık, gelişmeleri ödüllendirme, israfsızlık, ve adelet gibi en çok istenen özellikleri karşılamadığını gözlemliyoruz. Sistemin istenen özelliklere sahip olması için, öğrencilerin orijinal tercihlerini sunabilmeleri gerekmektedir. Ancak mevcut sistemde, tercih listesindeki kısıtlamalar nedeniyle bu mümkün değildir. Yaklaşımımız mevcut mekanizmayı yeniden tasarlayarak öğrencilerin refahını arttırmaktır.

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CHAPTER 1

INTRODUCTION AND RELATED LITERATURE

In Turkey, the average score differences in the university entrance exam are outstanding according to graduated high school type (YKS 2018 Report). As competition increases for a high-quality university, the demand for high-quality high schools is more and more. Since the demand is more than their capacities, it is impossible to assign each student to her top choice. As a result, the problem of how to match high schools and students becomes very essential.

The matching theory concept that solves such problems is introduced by Gale and Shapley (1962). The market includes two sides that need to be matched with each other and monetary transfers between two sides are not available. Each of the sides has preferences over the other and a central planner collects preferences of two sides. In real life, these sides could be firms-workers, men-women, doctors-hospitals, students-schools.

We can refer to the matching problem of schools and students as school admission problem. School admissions problem can be introduced under three different models: the college admissions problem, the student placement problem, and the school choice problem.

College admissions problem is a many-to-one matching problem where agents on one of the sides can be matched with more than one in the other. Gale and Shapley Deferred Acceptance Algorithm (Gale and Shapley, 1962) is the suggested mechanism in their paper to the college admissions problem and always produces a stable matching. Moreover, Gale and Shapley Student Proposal Deferred Acceptance

Algorithm gives the most efficient outcome for students among stable matchings (Gale and Shapley, 1962).

If the proposing side is the students, the DA works as follows: At the first step, each student proposes to her best acceptable college. Each college rejects any unacceptable proposals and, if received proposals are more than its quota, "holds" the most preferred students, otherwise, it temporarily holds all of them. If no proposals are rejected, then each college is matched with the students (if any) whose proposal it is "holding" and the procedure terminates. At step k, any student who was rejected at step k-1 makes a new proposal to her most preferred acceptable college which hasn't yet rejected her. Each college "holds" its most preferred acceptable offers from this step and the ones it is holding from the previous step according to its quota and rejects the rest if there are more than its quota. Otherwise, it "holds" all of the acceptable offers from this step and the ones from the previous step. It rejects all unacceptable offers from this step. If no proposals are rejected, each student is matched with the college (if any) whose proposal she is "holding" and the procedure terminates.

Although college admissions can be decentralized, some countries such as Turkey use a centralized model for college admissions. In these countries, colleges are not strategic agents but students are. In student placement problem described by Balinski and Sönmez (1999), school seats are public goods and priority orderings of schools over students are based on their exam scores.

However, priorities that order students for each school do not always base on exam scores. The problem where priorities can be exogenously determined, for example by geography and demographics, is called the school choice problem (Abdulkadiroğlu and Sönmez, 2003).

In general, as in Turkey, countries have different school types. More specifically, one side of the two-sided matching market is partitioned into groups. In Manjunath and Turhan (2016)'s model, the process of matching students to different groups of

schools are independent from each other. This implies that some students choose between two school offers, while some students are left unmatched. To solve this problem, Manjunath and Turhan (2016) propose iteratively rematching students to schools. Although it is costly to rematch, they show that even rematching a few times has significant gains.

Apart from Manjunath and Turhan (2016)'s problem, the assignments to different groups of schools are done sequentially in our problem. Our problem originates from the fact that students' submitted preferences may be potentially different from original preferences because of restrictions on preferences conducted by Ministry of Education (MEB). It is similar to the problem of the allocation of cadets to military branches in the US (Sönmez and Switzer, 2013). The suggested mechanism in their paper, Cadet-Optimal Stable Mechanism (COSM) is simply the extension of the agent-optimal stable mechanism (Gale and Shapley, 1962).

Until the year 2018, the placement of the eighth-grade students to the high schools was made by the Transition from Elementary Education to Secondary Education System (TEOG). Students are assigned to schools according to their scores in the high school entrance exams and their scores in the secondary school (sixth, seventh and eighth grades). The first high school entrance exam took place at the end of the first semester of eighth grade and the second one was at the end of the second semester. Students were able to submit preference lists up to twenty-five schools.

Since 2018, TEOG has been replaced by a new placement system that aims to remove the examination over time. Therefore, in the current system, there are three types of schools. These are schools whose places are allocated by the centralized exam (i.e. "exam schools"), local schools and boarding schools. Moreover, the local schools are consists of Anatolian High Schools, Vocational and Technical Anatolian High Schools, Anatolian Imam Hatip High Schools.

Another purpose of the new system is to enable students to go to schools which are in walk-zone. Therefore, regions have been created according to students' residence addresses. The Registration Area includes schools within walking distance. The Neighbor Recording Area contains schools which are a little further away from walking distance. Lastly, Other Registration Area contains more remote schools.

Firstly, students are required to present a preference list for local schools in order to make a preference list for exam and boarding schools. Students will be able to choose up to five schools in the local placement by selecting the first three schools from the Registration Area. A maximum of three schools can be selected from the same school type (Anatolian High School, Vocational and Technical Anatolian High School, and Anatolian Imam Hatip High School). After completing their preference list for the schools that have taken place with the local placement, students will be able to present preference lists for exam and boarding schools. Each preference lists can include at most five schools. Hence, a student's preference list can include at most fifteen schools.

The students are assigned to exam schools based on their scores in the centralized exam. For local schools and boarding schools, priority orderings are exogenously determined. Accordingly, our problem is a hybrid between student placement problem and school choice; since there are some schools with endogenous priorities over students and some with exogenous.

In the case of local schools, priorities depend on students' residence addresses, secondary school district, preference priority, school achievement scores, attendance-absenteeism and age criteria. If the students prefer a school from the "Registration Area" according to their residence address, they will be in an advantageous position than the students "Neighbor Registration Area" who prefer the same school. The students in the "Neighbor Registration Area" will also be advantageous than the students in the "Other Registration Area". In the case of

boarding schools, priorities depend on students' residence addresses such that the student whose address is farthest from school is most privileged.

In the first step of the algorithm, the serial dictatorship mechanism is used to assign students to exam schools. The mechanism used in the second and third step must be fair and non-wasteful. Another concern is that students may manipulate the mechanism by misreporting their preferences. We investigate the ongoing problems even if the individual mechanisms used in three steps are non-manipulable. More specifically, throughout the paper we consider Student Proposal Deferred Acceptance Algorithm (Gale and Shapley, 1962) as the mechanism used in the later steps; as Alcalde and Barberà (1994) suggest that it is the unique fair, non-wasteful and strategy-proof mechanism.

We proceed as follows: first, we describe the current system as a mathematical model; then we investigate the properties of the system. Finally, we describe how to redesign the system.

CHAPTER 2

THE MODEL

Let S and I be sets of schools and students, respectively. The set of schools, S, is partitioned into three subsets S^1, S^2, S^3. These are respectively exam schools, local schools and boarding schools. Moreover, the set of local schools, S^2, is also partitioned into three subsets S_1^2, S_2^2, S_3^2. These are Anatolian High Schools, Vocational and Technical Anatolian High Schools, Anatolian Imam Hatip High Schools, respectively. For each $s \in S$, q_s is the capacity of school s.

For each student $i \in I$, P_i is a strict preference relation over $S \cup \{s_0\}$, where s_0 denotes being unassigned and $q_{s_0} =\mid I \mid$. Let $\mathcal{P}$ denote the set of all such preference relations. Let R_i denote the at-least-as-good-as relation associated with the preference relation P_i for all $i \in I$. That means, for all $s^{'}$, $s^{''} \in S \cup \{s_0\}$ we have $s^{'} R_i s^{''}$ if and only if $s^{'} P_i s^{''}$ or $s^{'} = s^{''}$. If a student prefers school s to remain unassigned, then this school is acceptable for the student.

For each $k \in \{1, 2, 3\}$, $\mathcal{P}^k$ is a subset of preference relations over $S^k \cup \{s_0\}$. This is because the preference list to be submitted for local schools is subject to three special rules: Firstly, any P^2 can include a maximum of three schools from S_1^2, S_2^2, S_3^2. Secondly, the first three school on the prefence list P^2 must be from the "Registration Area". Thirdly, any P^2 must include at least one school.

The artificial preference list P_i^* is constructed from P_i as follows:

i. For any acceptable school $s \in S^i$ and $s^{'} \in S^j$, $s\ P_i^*\ s^{'}$ if $i < j$

ii. For any acceptable school s, $s^{'} \in S^k$ for $k \in \{1, 2, 3\}$, $s\ P_i^*\ s^{'}$ if $s\ P_i^k\ s^{'}$

iii. If s^* is an unacceptable school such that s^* is the top choice among local schools according to preference list P_i^2, then $s^* \; P_i^* \; s_0 \; P_i^* \; s$ for any $s \in S^2$.

Let f^i denote the test score of student i for each $i \in I$. Assume that there are no ties in the test scores.

For each $s \in S$, $\succ_s$ is strict priority order over I. For each $s \in S^1$, it is defined such that $i \succ_s j$ if and only if $f^i > f^j$ for all $i, j \in I$, i.e, student i has a higher test score than student j. For local schools and boarding schools, priority orderings are exogenously determined.

The matching market is denoted by the tuple $(I, S, q, \{\succ_s\}_{s \in S} \{P_i\}_{i \in I})$.

A *matching* μ is an allocation of students to schools such that no student is matched with more than one school and no school is matched with more students than its capacity. Formally, it is a function $\mu : I \to S \cup \{s_0\}$ such that $\mid \mu^{-1}(s) \mid \leq q_s$, for all $s \in S$. If $\mu(i) = s_0$, this means that student i is not assigned to any school. Let $\mathcal{M}$ be the set of all possible matchings.

A *mechanism* is a function which assigns a matching outcome for any matching market. A *direct mechanism*, ϕ, is a mechanism which uses preferences as strategy space. For any preference profile P, let $\phi[P] \in \mathcal{M}$ denote the matching outcome of ϕ for preference profile P.

Definition: A matching μ is *individually rational* if $\mu(i) R_i s_0$ for all $i \in I$. A mechanism is individually rational if it chooses an individually rational matching for any matching market.

A matching is individually rational if no student is assigned to a school that is worse than being unassigned to a school from the set S.

Definition: A matching μ is *non-wasteful* if $sP_i\mu(i)$ implies $|\mu^{-1}(s)| = q_s$ for all $i \in I$ and for all $s \in S$. A mechanism is non-wasteful if it chooses a non-wasteful matching for any matching market.

A matching μ is non-wasteful if, whenever a student prefers a school s to her assignment, the school s has no empty slot under μ.

Definition: A matching μ *Pareto dominates* a matching μ' if $\mu(i)R_i\mu'(i)$ for all $i \in I$ and $\mu(j)P_j\mu'(j)$ for some $j \in I$. A matching μ is *Pareto efficient* if it is not Pareto dominated by any other matching. A mechanism is Pareto efficient if it chooses a Pareto efficient matching for any matching market.

A matching μ is Pareto dominates a matching μ' if no student prefers μ' over μ and there is at least one student that prefers μ over μ'.

Definition: A matching μ is *fair* if for all $i, j \in I$ and for all $s \in S$ such that $j \in \mu^{-1}(s)$, $sP_i\mu(i)$ implies $j \succ_s i$. A mechanism is fair if it chooses a fair matching for any matching market.

A matching μ is fair if the priorities are respected.

Definition: Let $\succ_s$ be the strict priority order for school s. Let $\phi[\succ_s](i)$ be the match of student i given the priorities. Let $\succ'_s$ be another strict priority order for school s. We say that student i's priority improved for school s, if the followings holds:

i. For any $j, k \in I \setminus \{i\}$ if $j \succ_s k$, then $j \succ'_s k$

ii. For some $j \in I \setminus \{i\}$, $j \succ_s i$ and $i \succ'_s j$

Suppose that an arbitrary student i's priority improved for the school s and it stayed the same for other schools. A mechanism *respects improvements* if for all $i \in I$, $\phi[P \mid \succ_s'](i)\ R_i\ \phi[P \mid \succ_s](i)$.

A mechanism respects improvements if a student never receives a worse assignment as a result of an increase in her priority.

Definition: Let P_i be the preference list of student i and P_{-i} be the preference lists of the rest of the students. Let $\phi[P_i, P_{-i}](i)$ be the match of student i given the preference lists P_i and P_{-i}. A mechanism is *strategy-proof* if for all $i \in I$, for any announced preference profile P_i', $\phi[P_i, P_{-i}](i)\ R_i\ \phi[P_i', P_{-i}](i)$.

A mechanism is strategy-proof if no student can ever benefit by misrepresenting her preferences.

The current mechanism:

The students who take the exam are ranked according to their test scores and the serial dictatorship mechanism is used to assign students to the exam schools. If the students are placed in an exam school, then the local school preference list and the boarding school preference list will not be taken into consideration. The rest of the students is matched by using the student-proposing deferred acceptance algorithm according to their preference lists P^2's. If the students are placed in a local school, then the boarding school preference list will not be taken into consideration.The remaining students are matched by using the student-proposing deferred acceptance algorithm according to their preference lists P^3's. Formally:

Step 1.1 : The student with the highest test score is assigned to her top choice in her preference list P^1.

Step 1.k : The student with the next highest score is assigned to her top choice in her preference list P^1 among the remaining slots.

Assign s_0 to all students who are not assigned to a school.

In step 2 only the students who are not matched at step 1 and their preference list P^2 are considered.

Step 2.1: Each student "proposes" to her first choice. Each school seats its proposers one at a time in their priority order. Any remaining proposers are rejected.

Step 2.k: Each student who was rejected in the previous step proposes to her next choice if one remains. Each school considers the students whom it has been holding together with its new proposers and tentatively assigns its seats to these students one at a time in priority order. Any remaining proposers are rejected.

Step 2 terminates after a step where no rejections are made by matching each school to the students (if any) whose proposals it is "holding."

Assign s_0 to all students who are not assigned a school.

In step 3 only the students who are not matched at step 1 and step 2 and their preference list P^3 are considered.

Step 3.1: Each student "proposes" to her first choice. Each school seats its proposers one at a time in their priority order. Any remaining proposers are rejected.

Step 3.k: Each student who was rejected in the previous step proposes to her next choice if one remains. Each school considers the students whom it has been holding together with its new proposers and tentatively assigns its seats to these students one at a time in priority order. Any remaining proposers are rejected.

Step 3 terminates after a step where no rejections are made by matching each school to the students (if any) whose proposals it is "holding."

Assign s_0 to all students who are not assigned to a school.

CHAPTER 3

PROPERTIES OF THE CURRENT SYSTEM

Theorem: The current mechanism is equal to the student proposal deferred accenptance algorithm according to the artificial preferences.

Proof: Since the mechanism has three steps, we will call it 3S. We need to prove these claims:

1)The students matched at step 1 of 3S will match with the same school in the SPDA.

2) The students matched at step 2 of 3S will match with the same school in the SPDA.

3) The students matched at step 3 of 3S will match with the same school in the SPDA.

Proof of claim 1: Let I^1, I^2, I^3 be the sets of students matched at step 1, 2 and 3 of 3S, respectively. Suppose $A = \{i \in I^1 : \mu^{SPDA}(i) \neq \mu^{3S}(i)\}$ is a nonempty set and i^* has the highest test score among the students in A. Let $\mu^{SPDA}(i^*)=s'$ and $\mu^{3S}(i^*)=s^*$.

Case 1: $\mu^{SPDA}(i^*)\ P_{i^*}\ \mu^{3S}(i^*)$

This means i^* could not find a seat in the school s' when 3S is in the process because s' was full. In the first part of the mechanism serial dictatorship is used and that tells us that the seats are taken by the students with higher scores. Since i^* has the highest score among the students in A, $\mu^{SPDA}(j)=\mu^{3S}(j)=s'$ for any j $\in (\mu^{3S})^{-1}(s')$. The students who fill the school s' under 3S fill it under SPDA and thus there is no seat for i^*. So, $\mu^{SPDA}(i^*)$ cannot be s'.

Case 2: $\mu^{3S}(i^*)\ P_{i^*}\ \mu^{SPDA}(i^*)$

Then, since SPDA is non-wasteful s^* must be full. If there is any $j \in (\mu^{SPDA})^{-1}(s^*)$ such that $f^{i^*} > f^j$, then (i^*, s^*) is a blocking pair. This is because $i^* \succ_{s^*} j$ and $s^*\ P_{i^*}\ s'$. This contradicts with the stability of SPDA. Thus, any $j \in (\mu^{SPDA})^{-1}(s^*)$, $f^j > f^{i^*}$. Since i^* has the highest test score among the students in A, $\mu^{SPDA}(j)=\mu^{3S}(j)=s^*$. So, s^* is full and i^* cannot be matched with s^* in the 3S.

Proof of claim 2: Suppose there exists $i \in I^2$ such that $\mu^{SPDA}(i) \neq \mu^{3S}(i)$.

Case 1: If there is a student who becomes worse-off in the SPDA, let i^* be the first student who gets a reflection from her assignment under 3S in the process of SPDA. Let $\mu^{SPDA}(i^*) = s'$ and $\mu^{3S}(i^*) = s^*$. At step k, s^* rejects i^*. Let $i' \in (\mu_k^{SPDA})^{-1}(s^*)$. So, $i' \succ_{s^*} i^*$. Since i^* is the first student who get a reflection from her assignment under 3S in the process of SPDA, $s^*\ R_{i'}\ \mu^{3S}(i')$.

If all students who matched with s^* at step k match with s^* in the 3S, then s^* has enough quota for all $i' \in (\mu_k^{SPDA})^{-1}(s^*)$ and i^*. Thus, s^* does not reject i^* at step k.

If there is some $i' \in (\mu_k^{SPDA})^{-1}(s^*)$ such that $s^*\ P_{i'}\ \mu^{3S}(i')$, this means $\mu^{3S}(i') \notin S^1$ and in the second part of the mechanism i' proposes to s^* and s^* rejects i' at some step m. If i^* proposes to s^* at some step $l \geq m$, s^* rejects i^* because it rejects i' and $i' \succ_{s^*} i^*$. If i^* proposes to s^* before i' does, s^* rejects i^* when it gets a proposal from i'.

Case 2: If there is a student who becomes better-off in the SPDA, let i^* be the first student who gets a reflection from her assignment under SPDA in the process of 3S. Let $\mu^{SPDA}(i^*) = s'$ and $\mu^{3S}(i^*) = s^*$. At step k of 3S, s' rejects i^*. Let $i' \in (\mu_k^{3S})^{-1}(s')$. So, $i' \succ_{s'} i^*$. Since i^* is the first student who gets a reflection from her assignment under SPDA in the process of 3S, $s'\ R_{i'}\ \mu^{SPDA}(i')$.

If all students who matched with s' at step k match with s' in the SPDA, then s' has enough quota for all $i' \in (\mu_k^{3S})^{-1}(s')$ and i^*. Thus, s' does not reject i^* at step k of 3S.

If there is some $i' \in (\mu_k)^{-1}(s')$ such that $s' \; P_{i'} \; \mu^{SPDA}(i')$, then ($i'$,$s'$) is a blocking pair.

Proof of claim 3: By proof of claim 2, we can say that the students matched at step 3 of 3S will match with the same school in the SPDA.

We proved that everyone matches with the same school in the SPDA.

Proposition 1: The current mechanism, 3S, is not individually rational according to the original preferences.

Example 1: Let I=$\{i_1, i_2, i_3\}$ and S=$\{s_1, s_2, s_3\}$ such that S^1=$\{s_1\}$, S^2=$\{s_2\}$, S^3=$\{s_3\}$. Let q_{s_1}=q_{s_2}=q_{s_3}=1. Let the ranking of the test scores and the priorities be such that $f^1 > f^2 > f^3$ and $i_1 \succ_{s_2} i_3 \succ_{s_2} i_2$ and $i_2 \succ_{s_3} i_3 \succ_{s_3} i_1$. Let preferences be as follows:

$s_1 \; P_{i_1} \; s_2 \; P_{i_1} \; s_3 \; P_{i_1} \; s_0$

$s_1 \; P_{i_2} \; s_3 \; P_{i_2} \; s_2 \; P_{i_2} \; s_0$

$s_1 \; P_{i_3} \; s_0 \; P_{i_3} \; s_2 \; P_{i_3} \; s_3$

At the end of the step 1 the outcome is: $\mu = \begin{pmatrix} i_1 & i_2 & i_3 \\ s_1 & s_0 & s_0 \end{pmatrix}$

At the end of the step 2 the outcome is: $\mu = \begin{pmatrix} i_1 & i_2 & i_3 \\ s_1 & s_0 & s_2 \end{pmatrix}$

At the end of the step 3 the final outcome is: $\mu = \begin{pmatrix} i_1 & i_2 & i_3 \\ s_1 & s_3 & s_2 \end{pmatrix}$

This example shows that the current mechanism is individually irrational.

Proposition 2: The direct mechanism that is equivalent to the current mechanism 3S is not strategy-proof according to the original preferences.

Example 2: Let I=$\{i_1, i_2, i_3\}$ and S=$\{s_1, s_2, s_3\}$ such that S^1=$\{s_1, s_2\}$, S^2=$\{s_3\}$. Let q_{s_1}=q_{s_2}=q_{s_3}=1. Let the ranking of test scores and the priorities be such that $f^2 > f^1 > f^3$ and $i_1 \succ_{s_3} i_2 \succ_{s_3} i_3$. Let preferences be as follows:

$s_1 \, P_{i_1} \, s_3 \, P_{i_1} \, s_2 \, P_{i_1} \, s_0$

$s_1 \, P_{i_2} \, s_2 \, P_{i_2} \, s_3 \, P_{i_2} \, s_0$

$s_2 \, P_{i_3} \, s_1 \, P_{i_3} \, s_3 \, P_{i_3} \, s_0$

At the end of the step 1 the outcome is: $\mu = \begin{pmatrix} i_1 & i_2 & i_3 \\ s_2 & s_1 & s_0 \end{pmatrix}$

At the end of the step 2 the final outcome is: $\mu = \begin{pmatrix} i_1 & i_2 & i_3 \\ s_2 & s_1 & s_3 \end{pmatrix}$

Assume student i_1 announced her preferences as $s_1 \, P_{i_1} \, s_3 \, P_{i_1} \, s_0 \, P_{i_1} \, s_2$.

According to the new preference lists at the end of the step 1 the outcome is:

$\mu = \begin{pmatrix} i_1 & i_2 & i_3 \\ s_0 & s_1 & s_2 \end{pmatrix}$

According to the new preference lists at the end of the step 2 the final outcome is:

$$\mu = \begin{pmatrix} i_1 & i_2 & i_3 \\ \\ s_3 & s_1 & s_2 \end{pmatrix}$$

This example shows that the direct mechanism that is equivalent to the current mechanism is not strategy-proof.

Proposition 3: The current mechanism, 3S, does not respect improvements according to the original preferences.

Example 3: Let I=$\{i_1, i_2, i_3\}$ and S=$\{s_1, s_2, s_3\}$ such that S^1=$\{s_1, s_2\}$, S^2=$\{s_3\}$. Let q_{s_1}=q_{s_2}=q_{s_3}=1. Let the ranking of test scores and the priorities be such that $f^1 > f^3 > f^2$ and $i_1 \succ_{s_3} i_2 \succ_{s_3} i_3$. Let preferences be as follows:

$s_1 \, P_{i_1} \, s_2 \, P_{i_1} \, s_3 \, P_{i_1} \, s_0$

$s_1 \, P_{i_2} \, s_3 \, P_{i_2} \, s_2 \, P_{i_2} \, s_0$

$s_2 \, P_{i_3} \, s_1 \, P_{i_3} \, s_3 \, P_{i_3} \, s_0$

At the end of the step 1 the outcome is: $\mu = \begin{pmatrix} i_1 & i_2 & i_3 \\ \\ s_1 & s_0 & s_2 \end{pmatrix}$

At the end of the step 2 the final outcome is: $\mu = \begin{pmatrix} i_1 & i_2 & i_3 \\ \\ s_1 & s_3 & s_2 \end{pmatrix}$

Assume that student i_2 increased her test score such that $f'^2 \geq f^2$ and $f^1 > f'^2 > f^3$.

According to the new test scores at the end of the step 1 the outcome is:

$$\mu = \begin{pmatrix} i_1 & i_2 & i_3 \\ \\ s_1 & s_2 & s_0 \end{pmatrix}$$

According to the new test scores at the end of the step 2 the final outcome is:

$$\mu = \begin{pmatrix} i_1 & i_2 & i_3 \\ s_1 & s_2 & s_3 \end{pmatrix}$$

This example shows that the current mechanism does not respect improvements.

Proposition 4: The current mechanism, 3S, is wasteful according to the original preferences.

Example 4: Let I={i_1, i_2, i_3} and S={s_1, s_2, s_3} such that S^1={s_1, s_2}, S^2={s_3}. Let q_{s_1}=q_{s_3}=1 and q_{s_2}=2. Let the ranking of test scores and the priorities be such that $f^2 > f^1 > f^3$ and $i_1 \succ_{s_3} i_2 \succ_{s_3} i_3$. Let preferences be as follows:

$s_1 \; P_{i_1} \; s_3 \; P_{i_1} \; s_2 \; P_{i_1} \; s_0$

$s_1 \; P_{i_2} \; s_2 \; P_{i_2} \; s_3 \; P_{i_2} \; s_0$

$s_2 \; P_{i_3} \; s_3 \; P_{i_3} \; s_1 \; P_{i_3} \; s_0$

At the end of the step 1 the outcome is: $\mu = \begin{pmatrix} i_1 & i_2 & i_3 \\ s_2 & s_1 & s_2 \end{pmatrix}$

Since $s_3 \; P_{i_1} \; s_2$ and $|\mu^{-1}(s_3)| < q_{s_3}$, this example shows that the current mechanism is wasteful.

Proposition 5: The current mechanism, 3S, is not fair according to the original preferences.

Example 5: Let I={i_1, i_2, i_3, i_4} and S={s_1, s_2, s_3, s_4} such that S^1={s_1, s_2}, S^2={s_3, s_4}. Let q_{s_1}=q_{s_2}=q_{s_3}=q_{s_4}=1. Let the ranking of test scores and the priorities be such that $f^4 > f^1 > f^2 > f^3$, $i_2 \succ_{s_3} i_3 \succ_{s_3} i_1 \succ_{s_3} i_4$ and $i_2 \succ_{s_4} i_3 \succ_{s_4} i_1 \succ_{s_4} i_4$. Let preferences be as follows:

$s_1\ P_{i_1}\ s_3\ P_{i_1}\ s_2\ P_{i_1}\ s_4\ P_{i_1}\ s_0$

$s_2\ P_{i_2}\ s_1\ P_{i_2}\ s_3\ P_{i_2}\ s_4\ P_{i_2}\ s_0$

$s_1\ P_{i_3}\ s_3\ P_{i_3}\ s_2\ P_{i_3}\ s_4\ P_{i_3}\ s_0$

$s_1\ P_{i_4}\ s_4\ P_{i_4}\ s_2\ P_{i_4}\ s_3\ P_{i_4}\ s_0$

Suppose that i_1's reported preferences are such that: $s_1\ P^*_{i_1}\ s_3\ P^*_{i_1}\ s_4\ P^*_{i_1}\ s_0\ P^*_{i_1}\ s_2$.

At the end of the step 1 the outcome is: $\mu = \begin{pmatrix} i_1 & i_2 & i_3 & i_4 \\ s_0 & s_2 & s_0 & s_1 \end{pmatrix}$

At the end of the step 2 the outcome is: $\mu = \begin{pmatrix} i_1 & i_2 & i_3 & i_4 \\ s_0 & s_2 & s_3 & s_1 \end{pmatrix}$

At the end of the step 3 the final outcome is: $\mu = \begin{pmatrix} i_1 & i_2 & i_3 & i_4 \\ s_4 & s_2 & s_3 & s_1 \end{pmatrix}$

Since $i_2 \in \mu^{-1}(s_2)$, $s_2\ P_{i_1}\ s_4$ and $f^1 > f^2$, this example shows that the current mechanism is not fair.

The analysis above shows that the restrictions on the students' preference lists are the main reason for the failure of the current mechanism. We know that the outcome of the student proposing deferred acceptance mechanism is at least as good as any other stable matching for all students (Gale and Shapley, 1962). Furthermore, the student proposing deferred acceptance mechanism fulfills individual rationality, fairness, non-wastefulness, strategy proofness, and respecting improvements (Balinski and Sönmez, 1999). On the other hand, we showed that 3S mechanism does not fulfill these properties at all. This is because the 3S mechanism uses artificial preferences instead of the original preferences. Hence, we can easily solve these problems by

applying the student proposal acceptance mechanism to original preferences. This means that for the mechanism to have desirable properties students should not be forced to change their original preferences according to some rules. Since the student proposing deferred acceptance mechanism assigns the best matching among all stable matchings to students, we can reach the best system for the students.

CHAPTER 4

CONCLUSION

In this thesis, we studied high school placement system in Turkey. Firstly, we have shown that the most desirable properties are lost even though they may be fulfilled under SPDA or SD as a result of the sequentiality of the current mechanism. We proved that 3S mechanism is equivalent to SPDA when artficial preferences are used. Instead of eliciting the preferences of students over different type schools separately, we propose to elicit students' preferences over all schools and to use SPDA. Redesigning the system in this way improves students' welfare compared to their welfare in the current design of the system.

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