TEACHING FUNCTIONS TO 9TH GRADE STUDENTS USING REALISTIC MATHEMATICS EDUCATION APPROACH: AN ACTION RESEARCH

by

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ABSTRACT

TEACHING FUNCTIONS TO 9TH GRADE STUDENTS USING REALISTIC MATHEMATICS EDUCATION APPROACH: AN ACTION RESEARCH

This action research study sought to improve teaching and learning of functions and to develop positive perceptions of mathematics and mathematics learning. For these purposes, instructional sequences based on *Realistic Mathematics Education (RME) principles of teaching functions for understanding* were developed. Twenty-four 9th grade students attended RME based instruction which took four weeks. In order to investigate the effectiveness of the instruction, students' knowledge of functions was assessed by Function Knowledge Test before and after the instruction. In addition, students' conceptions of mathematics and perceptions of mathematics learning were also investigated before and after the instruction. Changes in the students' conceptions of mathematics and perceptions of mathematics learning were evaluated through the completion of a pre and post open ended questionnaire. Furthermore, students' experiences about learning functions based on RME approach were examined and evaluated through a post open ended questionnaire and students' journals.

The results of the study showed that the function instruction based on *RME principles of teaching functions for understanding* has positive impacts on development of students' knowledge of functions. In addition, it was found that the instruction may help some participants to improve their conceptions of mathematics or to ensure participants with *cohesive* conceptions of mathematics to hold the same conception of mathematics after the instruction. Also, it was found that there were significant changes in the results of some perceptions, but not all perceptions of mathematics learning. It is deduced that RME approach is promising. On the contrary, findings related to perceptions of learning functions revealed that students overemphasize RME approach in their learning of functions.

ÖZET

9.SINIF ÖĞRENCİLERİNE GERÇEKÇİ MATEMATİK EĞİTİMİ İLE FONKSİYON ÖĞRETİMİ: BİR EYLEM ARAŞTIRMASI

Bu eylem araştırmasının amacı, fonksiyonların öğrenimini ve öğretimini geliştirmektir. Ayrıca, bu çalışmada matematik ve matematik öğrenme ile ilgili algıların pozitif yönde geliştirilmesi de amaçlanmıştır. Bu amaçlar doğrultusunda, *Gerçekçi Matematik Eğitimine dayanan fonksiyon öğretimi prensiplerine* göre dersler dizisi geliştirilmiştir. 24 tane 9.sınıf öğrencisi, dört hafta süren öğretime katılmıştır. Geliştirilen öğretim uygulamasının etkililiğini araştırmak için, öğretimden önce ve sonra Fonksiyon Bilgi Testi ile öğrencilerin fonksiyon bilgisi değerlendirilmiştir. Ayrıca, öğretimden önce ve sonra öğrencilerin matematik anlayışları ve matematik öğrenme ile ilgili algıları incelenmiştir. Öğrencilerin matematik anlayışlarının ve matematik öğretimi hakkındaki algılarının değişimi ön ve son anket ile değerlendirilmiştir. Bununla beraber, öğrencilerin Gerçekçi Matematik Öğretimine dayalı fonksiyon öğretimi hakkındaki görüşleri, son anket ve öğrencilerin tuttukları günlükler aracılığıyla incelenip değerlendirilmiştir.

Elde edilen sonuçlara göre *Gerçekçi Matematik Eğitimine dayanan fonksiyon öğretimi prensiplerine* göre geliştirilen dersler dizisinin, öğrencilerin fonksiyon bilgisinin gelişmesine olumlu etkileri olduğu sonucuna varılmıştır. Elde edilen bulgulara göre, geliştirilen dersler dizisinin bazı öğrencilerin matematik anlayışlarının olumlu yönde gelişmesine ya da olumlu yönde matematik anlayışına sahip öğrencilerin aynı anlayışta kalmasına katkı sağlayabileceği sonucuna varılmıştır. Ayrıca, öğrencilerin tüm algılarında olmamakla birlikte bazı algılarında anlamlı değişimler gözlenmiştir. Bu noktadan hareketle Gerçekçi Matematik Eğitimin umut verici bir yaklaşım olduğu sonucuna varılabilir. Matematik öğrenme algıları ile ilgili bulguların aksine öğrencilerin fonksiyon öğretimi hakkındaki algıları incelendiğinde, öğrencilerin fonksiyon öğrenim sürecinde Gerçekçi Matematik Eğitimi yaklaşımı üzerinde önemle durdukları bulunmuştur.

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LIST OF SYMBOLS

Asymp Sig	Asymptotic Significance
df	Degree of Freedom
N	Number
Std	Standard

LIST OF ACRONYMS/ABBREVIATIONS

MEB	Milli Eğitim Bakanlığı / Ministry of National Education
NCTM	National Council of Teachers of Mathematics
RME	Realistic Mathematics Education

1. INTRODUCTION

According to National Council of Teachers of Mathematics, "the concept of function is an important unifying idea in mathematics." (NCTM, 1989, p.154). The function concept appears throughout elementary and secondary school mathematics from grade one to grade twelve. However, 9th grade mathematics is where functions are officially introduced to students (Dreyfus and Eisenberg, 1982). This is same for Turkish secondary school mathematics (MEB, 2013).

I have been teaching for five years and I teach at least one 9th grade class every academic year. The goal of teaching functions is the development of conceptual understanding of functions and procedural fluency in functions. Even though understanding the concept of function and its related concepts are central to learning secondary school mathematics, I have experienced that students have difficulties in understanding the concept of function and may hold misconceptions about it. According to Dreyfus and Eisenberg (1982), "the function concept is a complex one." (p.361). First, it is not a single concept; because there are many sub-concepts related to it. In addition, it plays a unifying role in linking diverse mathematics domains such as geometry, algebra, and statistics. Function also has many definitions and there are many ways to express and describe it. Tables, graphs, algebraic symbols, words, and problem situations are common ways of describing or representing functions (Dreyfus and Eisenberg, 1982). All of above contribute to the complexity of the concept; which leads difficulties that students face while they learn functions.

Ponte (1992) argued that students face difficulties in understanding functions; because students are expected to deal with the more abstract entities in school mathematics. Students should construct and analyze tables, calculate numerical values and develop a quantitative relationship between variables. However, if they deal with concrete numbers or real life situations, students may better develop important aspects of functions. According to NCTM (1989), "the study of functions should begin with a sampling of those that exist in the students' world." (p.154). Use of real life context or realistic situations in

teaching mathematics is encouraged in both 2013 and 2017 Turkish Mathematics Curriculums for Grade 9-12. It is emphasized that learning is meaningful when it is connected to students' lives. Also, by providing students with meaningful learning experiences, they may develop positive attitudes toward mathematics learning.

Realistic Mathematics Education (RME) is a teaching and learning theory for mathematics which has progressed significantly since 1971. According to RME, mathematics must be connected to reality. In this sense, 'realistic' situations have important place in the learning process. Realistic situations involve not only real-world situations but also problem situations which students can imagine (Van den Heuvel-Panhuizen and Drijvers, 2014). Considering the goal of teaching functions, instructional sequences based on Realistic Mathematics Education (RME) approach were developed. In this study, the effects of RME approach to teaching functions on 9^{th} grade students' knowledge of functions, their conceptions of mathematics, perceptions of mathematics learning and learning functions were investigated.

2. REVIEW OF LITERATURE

This chapter involves a brief review of the literature on studies related to the teaching and learning of functions. It includes analysis of definitions of function, students' difficulties in learning and understanding concepts related to the function concept, teaching approaches of the function concept, Realistic Mathematics Education and teaching functions through Realistic Mathematics Education.

2.1. The Function Concept

The function concept is one of the most important concepts in mathematics; since it has played a central role in the development of mathematics (Ponte, 1992; NCTM, 2000). In addition, the function concept is essential in related areas of sciences and every field of applied mathematics (Ronau *et al.*, 2014). In the Curriculum and Evaluation Standards for School Mathematics proposed by NCTM (1989), it is stated that "one of the central themes of mathematics is the study of patterns and functions." (p.98); because the function concept is essential to unify algebra, geometry and trigonometry (Selden and Selden, 1992; Dreyfus and Eisenberg, 1982; NCTM, 2000). Moreover, the function concept "provides a means for thinking quantitatively about real world phenomena and a context for studying relationships and change." (Cooney *et al.*, 2010,p.7).

2.1.1. Historical Development of the Function Concept

The definition of function has undergone an evolution within centuries due to developments in mathematics (Malik, 1980); thus its definition was in a state of flux (Kleiner, 1989). Even though the function concept in implicit form dates back to about 2000 B.C, it emerged explicitly at the end of 17^{th} century (Kjeldsen and Lützen, 2015). A function was considered as a curve described by a motion in the 17^{th} century (Even, 1993). Then, it was changed as an analytic expression (Ponte, 1992). In 1829, Dirichlet proposed

a new definition of function; since he aimed to separate the function concept from its analytical representation. According to Dirichlet (1826), a function is an arbitrary correspondence between two variables such that there is one and only one value of dependent variable for each value of independent variable (as cited in Ponte, 1992). From the concept of correspondence, mathematicians moved to the concept of relation between variables (Ponte, 1992). In 1939, Bourbaki introduced the ordered pair definition of a function. According to the Bourbaki approach, a function is a certain subset of the Cartesian product A×B (as cited in Kleiner, 1989). In the literature, there are terms for this definition; but they have been used interchangeably; the modern definition (Kjeldsen and Lützen, 2015; Even, 1993), the set theoretic definition (Malik, 1980), the formal definition (Clement, 2001) and the formal set definition (Markovits *et al.*, 1986). For consistency, the term 'formal set definition' will be used for the rest of the thesis.

According to Markovits, Eylon and Bruckheimer (1986), "…these definitions were a reflection of the state of the function concept then currently in use in higher mathematics." (p.18). Considering the historical development of the function concept, changes in the definitions of function have been influenced by the developments in mathematics which have occurred throughout centuries. In other words, it is not possible to think separately of the development of the function concept from developments in higher mathematics. On the other hand, the function concept leads to further developments in mathematics, applied mathematics and other related fields. Even (1993) indicated that teachers should be familiar with the historical development of functions; because in order to have a meaningful understanding of the function concept, teachers should explain why functions are defined in these ways.

2.1.2. Definitions of Function

As explained in the above section, function can be defined in many ways. Some examples of informal and formal definitions of function stated in textbooks or articles are as follows:

- "A function f from A to B is defined as any subset of the *Cartesian product* of A and B, such that for every a∈A there is exactly one b∈B such that (a, b)∈f."(Even, 1993,p.95, italics added).
- "A function is a *relation* in which each element of the domain *corresponds* to one and only one element of the range." (Ratti and McWaters, 2012, p.211, italics added).
- "A function is a rule that takes certain numbers as inputs and assigns to each a definite number. The set of all *input* numbers is called the domain of the function and the set of resulting *output* numbers is called the range of the function." (Hughes-Hallet *et al.*,2012, p.2, italics added).
- "A function is f on a set D into a set S is a *rule* that assigns a unique element $f(x)$ in S to each element x in D." (Adams and Essex, 2012, p.24, italics added).
- "A function, sometimes called a *mapping*, is a relation in which no two different *ordered pairs* have the same x- coordinate or first component." (Haese *et al.*, 2012, p.55, italics added).

Freundenthal (1983) stated that the formal definition of function has two essential features; *arbitrariness* and *univalence* (as cited in Even, 1993). As Even (1993) pointed out that the arbitrariness of a function means that regularity is not required for functions. Any graphs or expressions can be used to define functions. Also, functions can be defined on any sets of objects that are not necessarily sets of numbers. However, arbitrary nature of the function is implicit in its definition. On the other hand, the univalence feature is explicit in the definition. It is stated that there is only one element in the range for each element in the domain; which is the univalence feature of functions. Considering the development of the function concept, the univalence requirement was added to the definition of a function later; since according to Freudenthal (1983), it was difficult to work with differentials of orders higher than one and multivalued symbols. Thus, mathematicians wanted to facilitate it by adding the univalence requirement to the definition (as cited in Even, 1993).

Malik (1980) argued that "the definition of function as an expression or formula representing a relation between variables is for calculus or a pre-calculus course; is a rule

of correspondence between reals for analysis; and a set theoretic definition with domain and range is required in the study of topology." (p.492). Malik (1980) and Markovits *et al.* (1986) claimed that for school mathematics, the formal set definition is more abstract than the earlier definitions which emphasize relations between variables. Furthermore, in the sciences and applied mathematics, the definition of function as a relation between variables is mostly used. Therefore, Malik (1980) advocated that the introduction of the set definition of a function can be postponed after secondary school level. On the other hand, understanding of the formal set definition of a function is essential to build additional mathematical concepts. In fact, the formal set definition is essential for students who will study analysis and topology at the university. Moreover, the earlier definitions of functions include numerical functions; but not non-numerical functions and transformations. The formal set definition is necessary to study non-numerical functions and transformations (Markovits *et al.*, 1986).

It is deduced that different definitions of function emphasize different characteristics of functions. Both formal and informal definitions complete each other. Considering Turkish Mathematics Curriculum for Grade 9 (MEB, 2013), both formal and informal definitions of function are emphasized in the curriculum. It is stated that students should consider functions as a relation between dependent and independent variables. Besides that, students also should define a function as a relation in which each element of the domain corresponds to one and only one element of the range (MEB, 2013, p.7).

2.1.3. Understanding of Functions

As Jones (2006) asserted "true understanding of functions runs deeper than just the ability to regurgitate definitions" (p.7), it is important to specify the understanding of the function concept. Based on Sfard's study (1991), Sajka (2003) noted that there are two essentially different ways in which the function can be understood. Functions can be considered structurally — as objects and operationally — as processes (as cited in Sajka, 2003). In the operational conception, functions are conceived as something that transforms inputs to outputs. On the other hand, in the structural (object) conception of function,

functions are conceived "as a possibly infinite set of ordered pairs (x,y) in which each x-coordinate is paired with only one y-coordinate." (Jones, 2006, p.8). According to Sfard (1991), these two ways of understanding of the function concept are complementary and parts of a coherent unity (as cited in Sajka, 2003). Based on historical examples, Sfard (1991) suggested that the operational aspect of function concept preceded the structural one. The structural understanding of function is important; but it is too abstract to acquire for high school students. Therefore, Sfard (1991) asserted that students should first acquire function as operations, and then they will develop the structural (object) conception of function. Sfard's discussion of the function concept is supported by other researchers (Breidenbach *et al.*, 1992; Carlson *et al.*, 2010). According to them, students acquire at least a process conception of function in order to develop strong understanding of functions.

Markovits, Eylon and Bruckheimer (1986) explained that a learner has a general understanding of the function concept if he or she is able to use the concept in different context within mathematics and in different fields other than mathematics. They elaborated the stages that students pass through when they learn about functions. First, students learn that there are three sub-concepts within the formal definition of a function; namely domain, range and rule of correspondence. Second, students learn that functions can be represented in many ways such as algebraic, graphical, verbal and tabular representations. According to Doorman, Drijvers, Gravemeijer, Boon and Reed (2012), each of the representations highlights different characteristics of functions; but it may suppress some aspects of functions. In addition, some representations of functions can be more useful than others in certain contexts (Cooney *et al.* (2010). Cooney and his colleagues' discussion of different aspects of representations can also be seen in Markovits and her colleagues' last stage; translation between representations. They argued that students need to learn how to translate a given function from one representation to another representation.

Considering different conceptions of the function concept and stages of learning functions, it is important to specify which big ideas involve understanding of the function concept. According to Cooney and his colleagues (2010), there are several big ideas that are important in the teaching and learning of functions; but they focus on five of these ideas; "*the function concept, covariation & rate of change, families of functions,*

combining and transforming functions, and *multiple representations of functions."* (Cooney *et al.*, 2010, pp.7-8, italics added). Within the scope of this thesis study, the focus is on *the function concept, covariation & rate of change* and *multiple representations of functions;* since according to Turkish Mathematics Curriculum for Grade 9, learning objectives are pre-determined. In the curriculum, learning objectives regarding functions are stated as follows:

- O1.Students will be able to explain the function concept.
- O2. Students will be able to represent functions as graphs.
- O3.Students will be able to sketch the graphs of $f(x) = x^n$ $(n \in Z)$.
- O4. Students will able to explain one to one and onto functions. (MEB, 2013, pp.7-8).

9^{th} grade mathematics is where the function concept is officially introduced to students. However, students already develop knowledge of functions at middle schools without knowing its relation to the function concept. According to Turkish Mathematics Curriculum for Grade 6-8 (MEB, 2013, 2017), students are required to

- write the rule of given arithmetic sequence.
- write simple algebraic expressions to model word problems.
- decide whether the relationship between two quantities is proportional by investigating the graph of and the table of the relationship.
- translate the relationship between two quantities into the equation or table.
- explain linear relationship between dependent and independent variables.
- graph a linear equation.
- represent real life situations including linear relationship in various ways which include algebraic means, graphs and tables.
- explain the concept of slope by using models.
- explain the relationship between slope, linear equation and its graph.

Combining learning objectives concerning the function concept for grade 6-8 and for grade 9, it can be concluded that they are arranged in a way that reaching learning objectives for grade 6-8 supports a continuum of learning functions at high schools.

While this thesis study had been conducted, Turkish Mathematics Curriculum for Grade 9-12 was revised and updated Mathematics curriculum was approved on September, 2017. The 2017 grade 9-12 Mathematics are based on the newly revised 2017 Mathematics Curriculum. Even though changes in the recent curriculum do not affect the framework of the study, it is important to compare 2013 Mathematics Curriculum and recent Mathematics Curriculum in terms of the unit of function. In the recent curriculum, learning objectives regarding functions are stated as follows:

- O1. Students will be able to solve questions related to functions.
- O2. Students will be able to sketch the graphs of functions.
- O3. Students will be able to interpret the graphs of functions.
- O4. Students will be able to sketch graphs for real life situations modeled by linear functions.

When learning objectives of 2013 and 2017 Mathematics curriculums are compared, it seems that all four objectives are revised. However, considering explanations given after each objective in the recent curriculum, it can be realized that there are few changes in the scope of learning objectives. The most significant difference between two curriculums is the introduction time for functions. In 2013 curriculum, functions are introduced at grade 9. However, according to 2017 curriculum introduction to functions was postponed to grade 10. Terms and concepts introduced in the function unit are exactly same except for the horizontal line test. In the recent curriculum, horizontal line test is not specified. In the 2013 curriculum, students are guided to link the concept of slope and rate of change whereas according to recent curriculum, rate of change will be introduced to students at the grade 11. To sum up, it can be concluded that the scope of learning objectives in both curriculums is almost same.

According to national curriculum, teachers are guided to focus on the balanced development of both procedural and conceptual knowledge. In this respect, students are

expected to link among procedural and conceptual knowledge under the guidance of teachers (MEB, 2013). As Star (2007) pointed out, conceptual and procedural knowledge are mutually interdependent and both of them are important in students' learning. According to Haapasalo and Kadijevich (2000), *procedural knowledge* is defined as "dynamic and successful utilization of particular rules, algorithms or procedures within relevant representation form(s). This usually requires not only the knowledge of the objects being utilized, but also the knowledge of format and syntax for the representational system(s) expressing them." (p.141, italics added). On the other hand, *conceptual knowledge* is defined as "knowledge of and a skillful "drive" along particular networks, the elements of which can be concepts, rules (algorithms, procedures, etc.), and even problems (a solved problem may introduce a new concept or rule) given in various representation forms." (Haapasalo and Kadijevich, 2000, p.141, italics added). In the national curriculum, knowledge of mathematics that students need to know is specified through explanations given below each objective. Following is a collection of the combination of both procedural and conceptual knowledge of functions; as stated in the curriculum (MEB, 2013, pp.7-8).

- Students will explain that a function is a relation between dependent and independent variables.
- Students will explain that a function is a relation in which each element of the domain corresponds to one and only one element of the range.
- Students will investigate graphical and tabular representations of functions.
- Students will determine the domain and range of a function by looking at a graph of the function.
- Students will determine the image of an element in the domain or the inverse image of an element in the range by looking at a graph of the function.
- Students will determine whether a graph represents a function by using the vertical line test.
- Students will find the set of images corresponding to elements in the subset of domain of a function that is given graphically.
- Students will make applications on the graphs of linear functions.
- Students will sketch the graph of piecewise functions.

- Students will find the set of images corresponding to elements in the subset of range of a function that is given graphically.
- Students will sketch the graphs of $f(x) = x^n$ for $n = 1,2,3,-1$.
- Students will explain that if a horizontal line intersects a function's graph more than once, then the function is not one-to-one.

Considering *big ideas* on developing essential understanding of functions (Cooney *et al.*, 2010) and learning objectives regarding functions and explanations given after the objectives in the curriculum (MEB, 2013), I assume that learners understand the function concept, if they are able to

- F.1 explain and use the basic characteristics of the function concept which are univalence and arbitrariness.
- F.2 explain that a function is a relation in which each element of the domain corresponds to one and only one element of the range.
- F.3 explain that functions can show relationships between varying quantities or variables.
- F.4 describe a rate of change as the covariation between two variables.
- F.5 explain linear functions which are characterized by a constant rate of change.
- F.6 represent functions in various ways which include algebraic means, graphs, word descriptions and tables.
- F.7 transfer from one representation of a function to another.
- F.8 link between different representations of functions.
- F.9 explain one to one and onto functions.

2.2. Students' Misconceptions and Difficulties in Learning Functions

The function concept is not a simple one (Ronda, 2009; Tall and Bakar, 1992). Dreyfus and Eisenberg (1982) explained the reasons why the function concept is complex. First, the function concept involves a variety of sub-concepts. It is also used to unify subjects which seems unrelated to each other. It unifies algebra, trigonometry and

geometry together (Dreyfus and Eisenberg, 1982). Furthermore, the same function may be represented in many ways. Each representation is tied to each other. However, students have difficulties in connecting different representations of the same function (Dreyfus and Eisenberg, 1982; Selden and Selden, 1992). All of above are some of major contributors to difficulties which students face in learning functions. In addition, the function concept has a dual nature discussed in section 2.1.3; which leads another difficulty in understanding the function concept.

Research on students' understanding of functions revealed that students have a variety of misconceptions about and difficulties in learning the function concept (for example Cooney et. al, 2010; Tall and Bakar, 1992; Even, 1993; Markovits *et al.*, 1986; Vinner, 1983). Vinner (1983) investigated 10^{th} and 11^{th} grade students' learning of functions. 65 students in grade 10 and 81 students in grade 11 participated in his study. Based on the analysis of participants' answers, he revealed that students think that a function should be given by one single rule. Even (1993) found that even prospective secondary school teachers hold the same misconception. In her study, seven of ten participants expressed that all functions can be represented by using a formula. According to Cooney *et al.* (2010), this may stem from the frequent use of linear and quadratic functions in mathematics classroom. Hence, students often have a narrow view of the function concept (Cooney *et. al*, 2010).

Vinner (1983) noted that students may consider that a function should be one-to-one. According to Vinner (1983), students assume that "if for every x in the domain there is only one y in the range then the contrary should also be true." (p.303). However, Cooney *et al.* (2010) argued that many to one correspondence or irregular functions can be very useful in representing real world situations.

Students' difficulties with constant functions were found by many researchers (for example Vinner, 1983; Tall and Bakar, 1992; Markovits *et al.*, 1986). Students tend to think that a constant function is not a function. In the study of Tall and Bakar (1992), 53% of participants think that y=4 is not a function; because they think that the algebraic expression does not involve the variable x. Students also have difficulties with piecewise functions. Students do not consider a piecewise function as a function. According to

Markovits *et al.* (1986), students did not realize that different rules of correspondence can be applied to different parts of the domain.

Vinner (1983) also found that "functions (which are not algebraic) exist only if mathematicians officially recognize them (by giving them a name or denoting them by specific symbols)." (p. 302). In this sense, students may believe that circles or ellipses are functions. Tall and Bakar (1992) encountered the same misconception in their study. They found that three quarters of participants who started a university mathematics course considered that a circle is a function.

According to Tall and Bakar (1992), students frequently encounter regular shaped graphs such as x^2 and sinx in their mathematics classes. This may lead to a misconception related to the graph of a function. Students consider that a graph of functions should be "nice" and "smooth" (Even, 1993). In the study of Vinner (1983), many participants claimed that the graph in the given question is not a graph of a function; because it seems irregular.

Univalence feature of functions refers that there is only one element in the range for each element in the domain. Students are expected to use univalence feature of functions to determine whether given relations are functions or not. However, students prefer to use the vertical line test directly to determine whether a given graph is a function without considering features of functions. Clement (2001) noted that students tend to use vertical line test as a rule regardless of the fact that they understand it. In her study, 60 % of the precalculus students applied the vertical line test directly even though it does not work for the given question. Ronau *et al.* (2014) emphasized that vertical line test is useful for a limited set of functions.

Functions can be represented in several ways such as algebraic, graphical, verbal and tabular representations. Each of the representations highlights different characteristics of functions (Doorman *et al.*, 2012); but different representations of functions complete each other (Gagatsis and Shiakalli, 2004). Even though they convey the same information, students do not realize that equation, table and graph provide the same information in different forms. In fact, students consider different representations as separate entities

(Lambertus, 2007). Studies (for example Hitt, 1998; Gagatsis and Shiakalli, 2004) indicated that students have difficulties in handling the different representations of functions and in translating between different representations. However, in order to develop understanding of the function concept, students should recognize coherently different representations of functions and translate between different representations. According to Gagatsis and Shiakalli, (2004), students tend to prefer algebraic representation rather than the graphical one. This may stem from the fact that teachers tend to use algebraic expressions and undermine the use of graphs in their mathematics classroom. In the study, Gagatsis and Shiakalli (2004) found that students are successful in solving questions algebraically; but percentages of success are lower when students are presented with graphical representations.

2.3. Teaching the Concept of Function

The function concept is one of the central concepts in mathematics (Ponte, 1992). However, it is one of the least understood concepts (Eisenberg, 1991). Most teachers use conventional approach to teach functions. They often start teaching with stating definitions and solving examples. Later, they allow students to do a few applications. In this approach, students are not engaged in knowledge construction (Chimhande, 2013). There are many approaches to teach functions rather than conventional approach. This section will follow by a brief review on different teaching approaches used by teachers while teaching the function concept.

Tall and Bakar (1992) explained that the function concept can be introduced initially by giving the examples and non-examples of functions. However, they argued that it is not possible for students to construct the function concept if they do not experience examples of the function concept.

Pattern approach for teaching the function concept is suggested by Kwari (2007) and Van de Valle (2004) (as cited in Chimhande, 2013). In this approach, the focus is on studying change, relationships, rules and patterns. Students are guided to observe changes,

identify relationships between variables, obtain a rule and make predictions by using the rule (as cited in Chimhande, 2013). Even though this approach provides students with the visual representations of the function concept (Kwari, cited in Chimhande, 2013), due to this approach students may conclude that functions are sequences (Sierpinska, 1992).

Selden and Selden (1992) noted that some teachers use the function machine metaphor while introducing the function concept. In this approach, teachers introduce functions as a machine which takes an input and produces an output (Jones, 2006). Selden and Selden (1992) discussed that this approach may helpful for students to develop a process conception of the function. Moreover, it may be useful in helping students to develop covariation and correspondence relationship. However, they (1992) emphasized that if students do not know how machines process numbers, this approach might be ineffective. In addition, use of the function machine metaphor does not provide the complete understanding of the concept (Jones, 2006).

Carlson, Jacobs, Coe, Larsen and Hsu (2002) supported covariational approach to teach functions. They defined covariational reasoning as "the cognitive activities involved in coordinating two varying quantities while attending to the ways in which they change in relation to each other." (p. 354). In this approach, students are guided to focus on the rate of change in variables. Correspondence between variables is less emphasized in covariational approach. Carlson *et al.* (2002) emphasized that in this way, students can develop better understanding of change in order to construct functional relationship.

As discussed above, functions have many definitions and representations. Moreover, functions have a dual nature. Considering all of them, it is understandable why students face many difficulties in understanding the function concept and why teachers also have difficulties in teaching the function concept. In order to help students have a better understanding of the function concept, it seems essential to use a variety of teaching approaches when introducing the function concept. In this sense, it is necessary to investigate which approaches to teaching functions are illustrated in the Turkish Mathematics Curriculum for Grade 9. According to 2013 Mathematics curriculum for secondary school mathematics,

- Teachers should introduce only numerical functions to students. Teachers should also introduce functions qualitatively to students. In this sense, teachers can give realistic or real life situations modeled by not only one to one but many to one functions to students.
- Teachers should encourage students to define and view functions as a covariation between dependent and independent variables and as a correspondence between values of two quantities.
- Teachers can explain functions as a machine which takes an input and produces an output. Within this scope, students should be guided to find the values of $f(1)$, $f(2)$, $f(a)$, $f(2x)$, $f(x+1)$ according to rule or table of a function f(x).
- Teachers should explain identity, constant, linear and equivalent functions within the context of examples. Teachers should put emphasis on the relationship between rate of change and the slope of a line.

The requirements from MEB for teaching functions are indeed in parallel with the above principles of teaching functions. In line with literature on understanding of functions, students' misconceptions and difficulties in learning functions, and Turkish Mathematics Curriculum for Grade 9, the following ideas emerged as principles of teaching functions. I used these principles in developing the instructional sequences for this thesis study.

- I.1. Functions should be introduced qualitatively.
- I.2. Functions should be introduced non-algebraically.
- I.3. Realistic or real life situations modeled by one to one and many to one functions should be used as examples of functions.
- I.4. Functions as a covariation between two quantities and as a correspondence between values of two quantities should be emphasized.
- I.5. Characteristic of the function concept; univalence and arbitrariness should be highlighted.
- I.6. The idea of a function machine can be used to help students view functions from the operational conception.
- I.7. Any particular shaped graphs should be used as examples of functions.

- I.8.Students should be guided to pay attention on domain and range of the function.
- I.9. Multiple examples of different representations including a table, an input/output function machine, a graph, and a word problem describing a functional relationship should be given.

As mentioned before, Turkish Mathematic curriculum for grade 9-12 was revised. Their differences between curriculums were discussed in terms of what students need to know about functions. These two curriculums also have differences in terms of teaching approaches to functions. For both curriculums, students are expected to explain the function concept. However, introduction of functions seems different. In the 2013 curriculum, teachers are expected to encourage students to define and view functions as a *covariation* between dependent and independent variables, and as a *correspondence* between values of two quantities. In the 2017 curriculum, teachers are expected to emphasize that function is a special kind of *relation.* This difference may stem from that the topic of relation is included in the recent curriculum whereas it is not included in 2013 curriculum for grade 9-12. Also, in the 2013 curriculum, teachers are guided to introduce functions qualitatively to students by giving realistic or real life situations modeled by one to one and many to one functions. On the other hand, this kind of suggestion which involves pedagogical approaches to teaching functions does not take place in the recent curriculum. It is important to underline that this does not mean that the recent curriculum does not encourage teachers to use pedagogical approaches to teaching functions specified in the 2013 curriculum. It can be inferred that pedagogical approaches to teaching functions are not specified in the recent curriculum. In addition to all, MEB emphasizes use of real life context or realistic situations for teaching functions in both 2013 and 2017 Mathematics curriculums for grade 9-12. The literature on teaching mathematics through real life contexts or realistic situations will be discussed below.

2.4. Teaching Functions through Realistic Mathematics Education

Realistic Mathematics Education (RME) is a teaching and learning theory in mathematics education originally developed in the Netherlands and based on Freudenthal's

views on mathematics. According to Freudenthal (1977), “mathematics must be must be connected to reality, stay close to children and should be relevant to society.”(as cited in Van den Heuvel-Panhuizen, 2003, p.9). Hence, the use of realistic contexts is one of basic concepts of RME. However, the words ‘realistic’ that is used to describe RME can lead to misunderstanding. The meaning of the word ‘realistic’ is not limited to the connection with the real life. It also refers to problem situations that are real in students' mind (Van den Heuvel-Panhuizen, 2003; Zulkardi, 1999). In other words, it is not necessary to give real world situations to students. If puzzles, fairy tales and even formal mathematics are real in the students’ minds, they can also provide suitable contexts (Van den Heuvel-Panhuizen, 2003).

Freundenthal (1968) stressed that mathematics is not a subject to be transmitted or a body of knowledge; but it is a human activity. He (1968) stated that “What humans have to learn is not mathematics as a closed system, but rather as an activity, the process of mathematizing reality and if possible even that of mathematizing mathematics.” (p. 7). He added that mathematics is best learned by doing and mathematizing. “His primary focus on was on mathematizing reality in the common sense meaning of the world out there.” (Van den Heuvel-Panhuizen, 2003, p.11). In RME, students should not receive ready-made mathematics; but they should develop and apply concepts and tools related to mathematics in problem situations that are meaningful to them (Van den Heuvel-Panhuizen, 2003, 2014).

Based on work of Treffers (1987), Freundenthal (1991) defined horizontal mathematization to go from the world of life to the world of symbols; and vertical mathematization to move within the world of symbols. However, he discussed that it is not possible to consider these two types of mathematization separately. He added that they are closely related and have equal importance in doing mathematics (as cited in Van den Heuvel-Panhuizen, 2003). Zulkardi (1999) discussed further horizontal and vertical mathematization. Horizontal mathematization involves “identifying or describing the specific mathematics in a general context, schematizing, formulating and visualizing a problem in different ways, discovering relations, discovering regularities, recognizing isomorphic aspect in different problems, transferring a real world problem to a mathematical problem, and transferring a real world problem to a known mathematical

problem" (Zulkardi, 1999, p.4). On the other hand, vertical mathematization involves "representing a relation in a formula, proving regularities, refining and adjusting models, using different models, combining and integrating models, formulating a mathematical model, and generalizing."(Zulkardi, 1999, p.4).

2.4.1. Teaching Principles of Realistic Mathematics Education

All these discussions on mathematization also influence perspectives on learning mathematics. In that sense, these discussions are reflected in understanding of Realistic Mathematics Education approach. Van den Heuvel-Panhuizen and Drijvers (2014) distinguished six core principles of RME for teaching mathematics. The *activity principle* means that students actively contribute to their learning process. In addition, as has been said before, mathematics is learned best by doing. The *reality principle* involves two meanings. First, it is important to consider students' ability to apply mathematics in solving problem situations. Second, it underlines that teaching should start with problem situations that make sense to students rather than introduction of definitions or abstractions to be applied later. In other words, students should develop informal context-related solution strategies while they solve problems in rich context in the first step of their learning process. The *level principle* means that students pass various level of understanding while they learn mathematics. Mathematizing takes place from developing informal context-related solutions to create some level of schematization, and then to have insight on the relation between concepts and strategies. Models are important to connect informal context-related mathematics and the more formal mathematics. The *intertwinement principle* emphasizes that mathematical content domains such as number, geometry, measurement and data are heavily integrated and should not taught as isolated curriculum chapters. The *interactivity principle* refers to the idea of learning mathematics as a social activity. In RME, whole class discussion and group work are favored. The *guidance principle* is based on Freundenthal's view on "guided re-invention" of mathematics. In RME, teachers play a proactive role in students' learning (Van den Heuvel-Panhuizen and Drijvers, 2014). In a similar sense, Gravemeijer (1994) discussed

the role of the RME teacher in the classroom is a facilitator, an organizer, a guide, and an evaluator (as cited in Zulkardi, 1999). Teachers in RME should

- "Give the students a contextual problem that relate to the topic as the starting point.
- During interaction activity, give the students a clue, for instance, by drawing a table on the board, guide the students individually or in a small group in case they need help;
- Stimulate the students to compare their solutions in a class discussion. The discussion refers to the interpretation of the situation sketched in the contextual problem and also focuses on the adequacy and the efficiency of various solution procedures.
- Let the students find their own solution. It means the students are free to make discoveries at their own level, to build on their own experiential knowledge, and perform shortcuts at their own pace.
- Give another problem in the same context." (as cited in Zulkardi, 1999, p.11).

2.4.2. Developing Understanding of Functions through Realistic Mathematics Education

According to the synthesis of studies reviewed in literature, characteristics and principles of RME and big ideas in understanding functions, the following *RME principles of teaching functions for understanding* emerged to guide instructional sequences for this thesis study:

- Contextual problems or problem situations which students can imagine should be the starting point in teaching the function concept. Contextual problems or problem situations which can be real in students' mind should provide opportunities to introduce functions *qualitatively* and *non-algebraically* in the first step. Contextual problems or realistic problem situations should be modeled by one to one and many to one functions. Moreover, contextual problems or problem situations should

involve other mathematical content domains: number, geometry, measurement and data if appropriate.

- Functions can be *represented* in various ways which include algebraic means, graphs, word descriptions and tables. Hence, multiple examples of symbolic, tabular, graphical and verbal representations of functions within problem situations should be given to students.
- According to the *level principle* of RME, students should pass various levels of understanding while they learn mathematics. In the case of the function concept, students should develop operational conception of the function first, and then develop a sense of structural understanding of function concept. In this sense, functions as a covariation between two variables and as a correspondence between values of two quantities should be emphasized in order to guide students to develop operational conception of the function. To achieve this, teachers can use function machine metaphor in order to help students view functions from the operational conception.
- The teacher should arrange tasks for students in a way that they interact and collaborate with each other, and take an active part in the discussions. In this way, students actively contribute to their learning process. In addition, the teacher should encourage students to develop informal context-related solution strategies while they solve problems in rich context in the first step of their learning process.

2.4.3. Research Studies on Realistic Mathematics Education

RME originally has been developed in the Netherlands; however many countries such as England, Germany, Denmark, Spain, Portugal, South Africa, Brazil, USA, Japan, and Malaysia has been adopted RME(de Lange, 1996 as cited in Zulkardi, 1999). The body of research on RME from different countries has reported its positive results. Results related to RME will be discussed through the following studies.

Fauzan, Slettenhaar and Plomp (2002) conducted a research project in order to investigate to what extent RME addresses problems in geometry instruction at Indonesian

primary schools. They designed 10 lessons about the topic of area and perimeter. Students at two primary schools attended these RME-based lessons. Students' activities and reactions were recorded through observations, logbook and interviews (Fauzan *et al.*, 2002). Findings of their research showed that students were used to traditional way of teaching and faced troubles in working as a group. At first, students demonstrated less reasoning and understanding of concepts. However, Fauzan *et al.* (2002) recorded that students' learning behavior tends to change in a positive way. They (2002) concluded that "RME is a potential approach for teaching and learning mathematics." (p.2).

Webb, Kooij and Geist (2011) designed a unit in order to promote understanding of logarithms using the instructional design theory of RME. U.S. community college students enrolled in College Algebra course attended two-week pilot study. Results of the study showed that "students could acquire a profound understanding of the connection between exponential growth and logarithms." (Webb *et al.*, 2011, p.51). Also, it is noted that graduated students tended to perceive logarithms as a set of rules which have to be memorized in order to pass the test. However, students in this study demonstrated appreciation for learning and understanding mathematics at the end-of-unit evaluation survey.

Rasmussen and King (2000) searched how to adapt the instructional design perspective of RME to the learning and teaching of differential equations. For this purpose, they developed learning activities for differential equations based on RME. Twelve students attended classroom teaching investigation conducted by the researchers and three students' learning was analyzed. Considering analysis of students' learning, Rasmussen and King (2000) pointed out that RME instructional design is promising in promoting conceptual reasoning about the rate in their case. Same suggestion is supported by the study of Kwon (2002). Kwon conducted a classroom teaching experiment in an introductory course for differential equations at Ewha Womans University. 43 first-year undergraduate students attended teaching experiment. Through an RME design, students experienced reinventing conventional representations out of mathematizing experiences. Kwon (2002) argued that an RME design can be developed for not only primary school mathematics but also for undergraduate mathematics.

Zulkardi (2002) indicated that the low achievements of students in mathematics and their poor attitude toward mathematics are main problems of mathematics education in Indonesia. On the contrary, Indian government aimed to improve the quality of mathematics education. Hence, a project was initiated by introducing realistic mathematics education (RME). Zulkardi (2002) reported this study about the development of a *learning environment* based on realistic mathematics education (RME). In this study, RME was introduced to the student teachers and these student teachers conducted teaching practices based on RME. In other words, student teachers were both learning and teaching RME in the schools. Throughout the study, student teachers were assisted by providing them with resources and tools for communication and collaboration. Findings of the study showed that student teachers developed more positive beliefs or attitudes toward mathematics and demonstrated good performance on teaching in RME classroom after they followed the *learning environment.* Besides, students' experiences about RME based instruction were also investigated. The results showed that students perceived RME materials as interesting; since RME materials included use of daily applications which are real to them. Also, students indicated that they liked group discussion. According to Zulkardi (2002), these students could learn by freely discussing, communicating and justifying the adequacy and the efficiency of their solutions.

Besides international studies on RME, there are national studies on implementation of RME approach in schools ranging from primary to high school (Akyüz, 2010; Gelibolu, 2018; Ersoy, 2013; Özdemir, 2015; Çakır, 2006). Majority of these studies focus on effectiveness of RME based instruction on students' achievement for a specific mathematics topic such as integral (Akyüz, 2010), logic (Gelibolu, 2008), set (Özdemir, 2015), algebra and area (Çakır, 2011), fraction (Ayvalı, 2013). Effectiveness of RME approachwas investigated through experimental studies which were conducted as a pre-test-post-test design with a control group. For example, Akyüz (2010) investigated the effect of teaching the topic of integral based on RME approach on 12^{th} grade students' achievement. The results of achievement test showed that even though students in experimental and control groups performed similarly in pre-test, students taught by RME approach performed better in the post-test compared to those in control group. Similar results were also found in the studies of Gelibolu (2008), Ersoy (2013), Özdemir (2015), Çakır, (2011). Considering the results of these studies, it can be summarized that RME

based instruction may lead to higher achievement on a specific mathematics topic than the traditional method.

In addition to studies investigating effects of RME based instruction on achievement; there are also some studies which also focused on students' views about implementing RME approach in classrooms (Cansız, 2016; Özdemir, 2015; Ersoy, 2013). For example, Özdemir (2015) investigated students' views about learning sets based on RME approach by using semi-structured interviews. Students indicated that RME approach facilitates learning, makes learning permanent, and uses real life examples. From the findings, Özdemir (2015) discussed that students had positive views about RME approach. Cansız (2016) also examined students' views about RME approach by using interviews. Participants in the study answered which teaching method(s) is appropriate to use in mathematics classroom. Most participants in the study believed that they were active through discussion guided by teacher during the RME based instruction. 28 out of 33 participants indicated that RME based instruction facilitates their learning**.** Cansız (2016) further analyzed views of participants who indicated RME based instruction is appropriate to use in mathematics classroom. Among 28 participants, 12of them indicated that RME based instruction makes connections to real life. Also, 10 of them indicated that RME based instruction makes learning permanent and 9 participants indicated that they show more interest in mathematics by learning based on RME approach.

In the study of Ersoy (2013), students attended RME based instruction for the topic of statistics – probability. After instruction, students completed open-ended questionnaires about their views on RME based instruction. Results of the study showed that 58.3% of students indicated learning through group work is one of the positive sides of RME approach. 44.4% of them indicated that they improved their success by learning based on RME approach. Also, 30.5% of them indicated that they learned better by learning based on RME approach. Ersoy (2013) concluded that students' perceptions towards RME approach were positive. He further discussed that RME approach was promising in forming positive attitudes toward mathematics. This argument is supported by the findings of studies focusing on effects of RME based instruction on attitudes toward mathematics. Çakır (2011) investigated the effect of RME based instruction on 6^{th} grade students' achievement on the topic of algebra and area, and attitudes toward mathematics. In the

experimental study conducted as pre-test-post-test design with a control group, experimental group attended RME based instruction whereas control group received traditional instruction. Results showed that there was a significant improvement in attitudes toward mathematics in the experimental group. Effects of RME on improvement in attitudes toward mathematics or mathematics learning were found in the studies of Fauzan *et al.*(2002), Zulkardi (2002), Üzel and Uyangör (2006), Cansız (2016), Çakır (2011) and Ersoy (2013). It can be concluded that RME based instruction is seen as a promising approach for improving positive attitudes toward mathematics or mathematics learning.

As Dickinson and Hough (2012) reported, it was found that students who had experienced RME were much more positive about mathematics or mathematics learning compared to those taught by more traditional instruction. In this respect, it is important to investigate beliefs about or conceptions of mathematics held by students who do not experience RME. It is also important to note that as Grady (2013) explained, there is no consensus about the definition of conceptions and beliefs in the literature. Grady (2013) further pointed out that some researchers prefers one of the terms or use these terms interchangeably. For consistency, the distinction between belief and conception made by Ponte (1994) was used in this study. According to Ponte, "beliefs are the incontrovertible personal 'truths' held by everyone, deriving from experience or from fantasy, having a strong affective and evaluative component (Pajares, 1992). Conceptions are the underlying organizing frames of concepts, having essentially a cognitive nature. Both beliefs and conceptions are part of knowledge." (Ponte, 1994, p.5). Assessment of students' mathematical beliefs or conceptions of mathematics is important; since according to Spangler (1992), there is a relationship between beliefs and learning. As Spangler (1992) stated, this relationship is cyclic. In other words, students' learning experiences influence their beliefs about mathematics learning and their beliefs also influence their approach to the new learning experiences. Same argument is also supported for the case of conceptions. As Gravoso, Pasa and Mori (2002) argued, students' learning outcomes are influenced by their prior learning experiences and their conceptions of learning. To sum up, it can be concluded that it is important to assess students' conceptions of mathematics and mathematics learning.

Walker (1999) pointed out that development of conceptions held by students takes long periods of time. He also added that school experiences and classroom expectations primarily shape these conceptions. Combining works of Ball (1988), Schoenfeld (1985) and Stodoldsky (1985), Lampert (1990) summarized the conceptions of mathematics as follows:

> Commonly mathematics is associated with certainty: knowing it, with being able to get the right answer, quickly. These cultural assumptions are shaped by school experience, in which doing mathematics means following the rules laid down by the teacher; knowing mathematics means remembering and applying the correct rule when the teacher asks a question; and mathematical truth is determined when the answer is ratified by the teacher. Beliefs about how to do mathematics and what it means to know it in school are acquired through years of watching, listening, and practicing, (p. 32).

Crawford, Gordon, Nicholas and Prosser (1994) conducted phenomenographic study to investigate first year university mathematics students' conceptions of mathematics. Students were asked to answer "Think about the maths you've done so far. What do you think mathematics is?". From analysis of students' written statements, they identified conceptions of mathematics as *fragmented* and *cohesive* conceptions of mathematics (as cited in Crawford et al, 1998). Their categorization of conceptions of mathematics is summarized in the following table.

Table 2.1.Crawford *et al.*'s categorization of conceptions of mathematics (1998, pp.88)

Crawford *et al.*'s Categorization of Conceptions of Mathematics	
Fragmented conceptions	"A. Maths is numbers, rules and formulae. B. Maths is numbers, rules and formulae which can be applied to solve problems."
Cohesive conceptions	"C. Maths is a complex logical system and way of thinking. D. Maths is a complex logical system which can be used to solve complex problems. E. Maths is a complex logical system which can be used to solve complex problems and provides insights used for understanding the world."

Similar conceptions of mathematics or mathematics learning can be found in the Turkish studies related to mathematical beliefs. Aksu, Demir and Sümer (2002) investigated primary school students' beliefs about mathematics.563 students from two primary schools were administered the "Beliefs about Mathematics Survey (BMS)". Findings of the study revealed that students believed that they had to use the methods taught by the teacher correctly and quickly in order to solve problems. It was also found that these students tended to make efforts for remembering the methods taught by the teacher rather than attempting to reason their way through the problem. Beliefs that elementary school students hold about mathematics were also investigated in the study of Toluk Uçar, Pişkin, Akkaş and Taşçı (2010). Nineteen 6^{th}, 7^{th}, and 8^{th}grade students attended the study. Analysis of data collected through interviews revealed that students perceived mathematics as consisting of calculation, numbers and operations. Participants believed that problem solving means solving questions on a test. Further, they indicated that being successful in mathematics requires speed and correctness in calculation. Half of the participants also stated that success in mathematics is equivalent to being smart.

In the light of the above studies, I developed instructional sequences based on RME approach. To evaluate the effects of RME approach to teaching functions, my 9^{th} grade students' knowledge of functions were investigated before and after the instruction. Also, considering the literature on beliefs and conceptions, it seems essential to consider my students' views about what mathematics is or what it means to learn mathematics. Hence, students' conceptions of mathematics and perceptions of mathematics learning were also investigated before and after the instruction of function. In addition to all, my students attended RME based instruction for four weeks; which was a new experience for them. Thus, their experiences about learning functions based on RME approach were also examined in this study.

3. SIGNIFICANCE OF STUDY

The function concept is central to mathematics and is related to almost every field of mathematics. In addition, functions have an important place in the secondary mathematics curriculum (NCTM, 2000). Even though the function concept appears throughout elementary and secondary school mathematics from grade one to grade twelve, 9th grade mathematics is where functions are officially introduced to students (MEB, 2013).

Eisenberg (1991) argued that the function concept is "…one of the most difficult concepts to master in the learning of school mathematics" (p.140). As it is parallel to the findings of research on teaching and learning functions, I have experienced that students have many difficulties in learning the function concept and concepts related to it. In addition, they may hold misconceptions regarding functions; which prevents them developing conceptual understanding and procedural fluency of functions. On the other hand, I want my 9th grade students to develop conceptual understanding of functions and procedural fluency in this subject. According to Kalchman and Koedinger (2005), an instructional plan is necessary to build and secure conceptual understanding and procedural fluency. For this purpose, instructional sequences based on Realistic Mathematics Education (RME) approach were prepared after a very intense period of literature review. In this study, the effects of RME approach to teaching functions on 9th grade students' knowledge of functions were investigated. In addition to this, literature regarding students' beliefs showed that beliefs that students hold influence their mathematics learning and their learning experiences also influence their beliefs about mathematics (Spangler, 1992). In this regard, teachers are encouraged to consider their students' beliefs about mathematics or mathematics learning. Hence, in this study students' conceptions of mathematics and perceptions of mathematics learning were also investigated. It is expected that the results which were obtained from this study will not only improve my teaching practices, but also inform other mathematics teachers about teaching the function concept through RME, and may have implications for mathematics education field.

4. STATEMENT OF THE PROBLEM

The present study aims to investigate the effects of RME based instruction on 9th grade students 'knowledge of functions. Also, students' conceptions of mathematics and perceptions of mathematics learning were investigated. Students attended RME based instruction of functions. In this regard, students' experiences about learning functions based on RME approach were also examined in this study.

4.1. Variables

The variables of this study are conceptual knowledge, procedural knowledge, knowledge of functions, and conceptions. The definitions of these variables are as follows:

- Mathematical knowledge consists of procedural and conceptual knowledge. In this study, *knowledge of functions* consists of procedural and conceptual knowledge of functions.
- *Procedural knowledge* is defined as "dynamic and successful utilization of particular rules, algorithms or procedures within relevant representation form(s). This usually requires not only the knowledge of the objects being utilized, but also the knowledge of format and syntax for the representational system(s) expressing them." (Haapasalo and Kadijevich, 2000, p. 141, italics added).
- *Conceptual knowledge* is defined as "knowledge of and a skillful "drive" along particular networks, the elements of which can be concepts, rules (algorithms, procedures, etc.), and even problems (a solved problem may introduce a new concept or rule) given in various representation forms."(Haapasalo and Kadijevich, 2000, p. 141, italics added).
- Conception is defined as "a general notion or mental structure encompassing beliefs, meanings, concepts, propositions, rules, mental images, and preferences." (Philipp, R. A, 2007, p.259). Based on the definition proposed by Phillipp (2007), conception of mathematics can be considered as a general notion or mental structure

encompassing beliefs, meanings, concepts, propositions, rules, mental images, and preferences about mathematics.

4.2. Research Questions

This study aims to investigate the following research questions:

(i) Is there a statistically significant difference between 9th grade participants' knowledge of functions before and after attending function instruction based on RME approach?

- Is there a statistically significant difference between 9th grade participants' conceptual knowledge of functions before and after attending function instruction based on RME approach?
- Is there a statistically significant difference between 9th grade participants' procedural knowledge of functions before and after attending function instruction based on RME approach?

(ii) What are the conceptions of mathematics held by 9th grade participants before and after attending function instruction based on RME approach?

(iii) Is there a statistically significant difference between 9th grade participants' gain scores on Function Knowledge Test in terms of their conceptions of mathematics before and after attending function instruction based on RME approach?

(iv) What are the perceptions of 9th grade participants on mathematics learning before and after attending function instruction based on RME approach?

(v) What are the perceptions of 9th grade participants on learning functions after attending function instruction based on RME approach?

5. METHODOLOGY

This chapter provides detailed information about the methodology. Within this context, it includes research design, participants and settings, procedure, instruments, data collection and data analysis.

5.1. Research Design

Action research methodology was used for this study. Mills (2003) defines action research as "any systematic inquiry conducted by teacher researchers, principals, school counselors, or other stakeholders in the teaching/learning environment to gather information about how their particular schools operate, how they teach, and how well their students learn." (p.5). The main difference between action research and other types of research is that action research is done by teachers for their practice (Gay *et al.*, 2009).

Action research has come into prominence in the field of education; since action research provides opportunities to improve educational practices, to bridge the gap between research and practice, and improve the justice of education's impact on society (Gay *et al.*, 2009). Henning, Stone and Kelly (2009) stated that planning, collection of data, analysis and reflection are four steps in describing action research. In detail, based on the work of Kemmis (1982), Herr and Anderson (2005) explained that action research process involves "developing a plan of action to improve what is already happening, acting to implement the plan, observing the effects of action and reflecting on these effects as a basis for further planning" (p.5). Figure 5.1 illustrates the model for action research process. Even though the model seems linear, action research is "a recursive, cyclical process that does not proceed in a linear fashion." (Johnson, 2008, p.17).

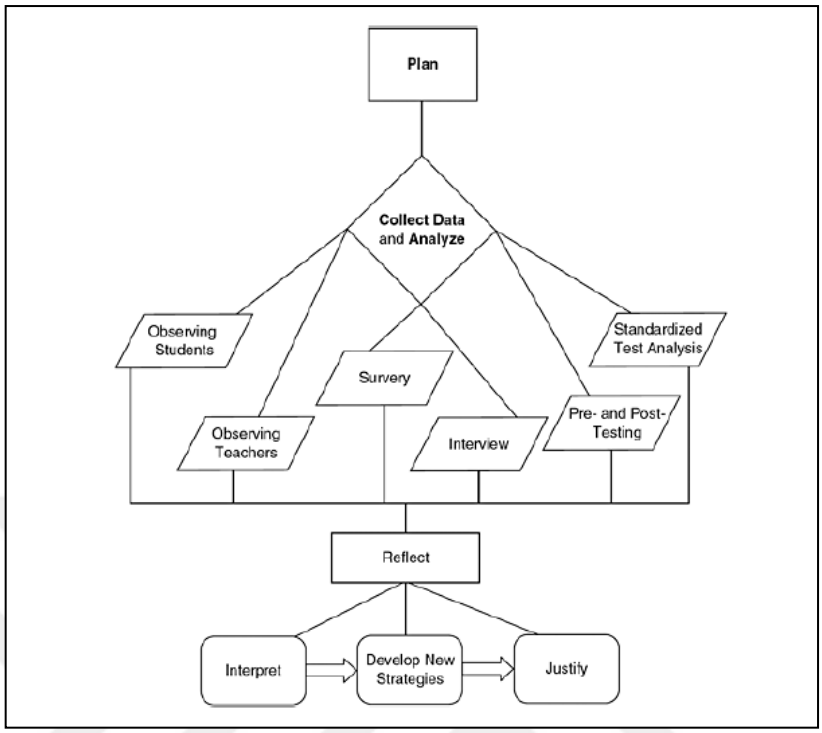


Figure 5.1. A model for action research process (Henning *et al.*, 2009, p.9)

Action research provides teacher researchers with a method to produce solutions to problems that they face in teaching-learning environment (Gay *et al.*, 2009). During my four year teaching experiences, I encountered that some of my students faced difficulties in understanding the function concept and held different misconceptions related to the function concept. For instance, some of my students considered that a function should be given by one single rule or formula and a function should be one to one. Also, some of them struggled with working with piecewise functions or functions whose graphs are not nice or smooth. In addition, they tended to confuse whether they would use vertical or horizontal line test to determine functionality of given relation. This stems from the fact that they have limited understanding of the function concept. Considering my students' difficulties in understanding functions and misconceptions that my students held, I decided to reconsider my teaching practices. In general, I introduced functions by discussing the relationship between the amount to be paid and the amount of gasoline to be bought. I also gave students examples and non-examples of functions to discuss the features of the function concept. Later, I defined the function in a formal way by emphasizing the essential features of the function concept. After I taught the formal definition(s), I guided

them to make practices. However, I realized that teaching functions in that way did not contribute to development of conceptual understanding and procedural fluency of functions of my students to the extent I aimed. Hence, I aimed to improve my instructional practices of functions. Literature concerning teaching and learning of functions suggested the use of Realistic Mathematic Education (RME) approach. After intense literature review on RME, I developed instructional sequences based on RME approach in order to guide students to develop conceptual understanding and procedural fluency of functions. In this study, I sought to investigate the effect of RME approach to teaching functions on 9th grade students' knowledge of functions. I also examined my 9th grade students' conceptions of mathematics and perceptions of mathematics learning. Furthermore, my students' perceptions of learning functions based on RME approach were investigated after they participated into RME based instruction of functions.

5.2. Participants and Settings

This study was conducted at a private science high school in Istanbul. The school is one of three K-12 campuses of foundation schools which have a more than 100 years background in education. There is also a private regular high school in the same building where the science high school takes place. Students enrolled in the science high school have high performance at high school entrance exam. There are 98 students enrolled in grades 9 through 12 in the science high school. Majority of students come from medium or high socioeconomic status groups. I have been working as a mathematics teacher in the science high school and high school, for five years. The participants were a group of 24 9th grade students, age 14 to 15. There are 14 girls and 10 boys in 9th grade.

5.3. Procedure

In the Turkish Mathematics Curriculum for Grade 9-12 (MEB, 2013), it is emphasized that a combination of both conceptual and procedural teaching approaches to teach mathematics is emphasized. In this respect, teachers should encourage their students

to develop informal strategies and to progress to more formal and abstract strategies through the process of guided mathematization. It is aimed that students should develop problem solving, mathematical thinking and communication skills in mathematics. Also, focus should be on understanding of mathematical concepts and making connections between the different concepts (MEB, 2013). In order to achieve aims of Turkish Mathematics Curriculum for functions, *RME principles of teaching functions for understanding* discussed in the section 2.4.2 were generated and implemented in teaching functions. Before and after the function instruction, participants' knowledge of functions was assessed by using an instrument. Also, participants' conceptions of mathematics and perceptions of mathematics learning were examined before and after they attended the instruction. After the instruction, participants' experiences about learning functions based on RME approach were investigated.

Before the implementation of RME based instruction, I adapted and developed tasks based on RME approach for each objective which were specified in the curriculum. Besides each objective, tasks were linked with *big ideas* such as the function concept, covariation and multiple representations of functions. It was essential to use contextual problems or realistic situations within tasks as the starting point in teaching functions. In addition, tasks were designed to involve intertwining lines of learning with other strands; geometry, data and number if appropriate. Multiple examples of different representations were also included within tasks. Lesson plans for RME tasks were also designed in order to specify how *RME principles of teaching functions for understanding* would be implemented before and during the instruction. For example, according to national curriculum students should be able to explain the function concept; which is one of the big ideas involving understanding of the function concept. Qualitative introduction of functions is essential to guide students to explain the function concept. For this purpose, real life and realistic problem situations were investigated. The relationship between the number of a cricket's chirps and the temperature was chosen as a starting point to discuss the relation between dependent and independent variables through a real life example. In order to explain functions not only as a covariation but also as a correspondence, "*Function Wall"* developed by Ronau, Meyer, Crites and Dougherty (2014) were adapted to discuss function as a relation in which each element of the domain corresponds to one and only one element of the range. Lesson plan for this task was provided in Appendix A.

In the lesson plan, how *RME principles of teaching functions for understanding* would be implemented was also specified.

According to Turkish Mathematics Curriculum for Grade 9 (MEB, 2013), 28 lesson periods are allocated to teach the function unit and there are four learning objectives regarding functions. The following table shows sample RME tasks that were used during instruction and brief description of these tasks for each objective.

Table 5.1. Sample RME tasks and brief description of these tasks for each objective

Learning Objectives (MEB, 2013, p.7-8)	**Sample RME Tasks**	**Purpose of Tasks**
O1. Students will be able to explain the function concept.	The Cricket: Nature's Thermometer	The task is used to discuss the relation between dependent and independent variables through a real life example.
	Where do you stand? (Ronau, Meyer, Crites and Dougherty, 2014).	The task is used for students to explain a function as a relation in which each element of the domain corresponds to one and only one element of the range.
O2. Students will be able to represent functions as graphs.	Caterpillar's Location (Clement, 2001)	Real life situations modeled by one to one function is used to discuss a function as the relation between dependent and independent variables. Also, it is used to guide students to consider the definition of function rather than vertical line test in order to determine functionality.
	The Cruise Control System	A real life example is used to discuss constant functions and their graphical representations.
	Demographics of Turkey and Germany (Ortaöğretim Matematik 9. Sınıf 2. Kitap, 2013)	A real life data is used to discuss change and rate of change.
	Parking Fees at Airports (Ronau, Meyer, Crites and Dougherty, 2014).	A real life situation is used to describe piece-wised functions by using word and graphical representation of functions.

Table 5.1. Sample RME tasks and brief description of these tasks for each objective (cont.)

Learning Objectives (MEB, 2013, p.7-8)	**Sample RME Tasks**	**Purpose of Tasks**
O3. Students will be able to sketch the graphs of $f(x) = x^n$ ($n \in Z$).	Representing Graphs of $f(x) = x^n$	Multiple representations of five functions are given students to link among different representations.
O4. Students will able to explain one to one and onto functions.	Where do you stand? (Ronau, Meyer, Crites and Dougherty, 2014).	This task is once used to determine whether given function is one to one or onto.
	One to one? Onto?	Models are used to whether given model represent one to one function / onto function. Students are asked to write models which represent one to one/onto/ not one to one/ not onto functions.

Based on Gravemeijer's work (1994), the RME teacher should be a facilitator, an organizer, a guide and an evaluator in the classroom (as cited in Zulkardi, 1999). From this point, during the implementation of RME based instruction, interaction between students was provided through discussion, negotiations, and collaborations under my guidance. Focus was on doing mathematics and communicating mathematics. While my students engaged in the tasks, I guided them to work individually or in a small group. During the task, I encouraged them to interpret the problem, to share their solutions and to analyze the adequacy and the efficiency of the different solutions.

5.4. Instruments

5.4.1. Function Knowledge Test

In order to assess knowledge of functions, I designed Function Knowledge Test. In the instrument, there are problems which were selected, adapted or designed to assess

conceptual and procedural knowledge of functions. All questions were arranged to cover each learning objective of the function unit in 9th grade; which are determined by MEB (2013). Problems of the instrument were shown to two experienced and one novice mathematics teachers and three mathematics educators to ensure its validity. The instrument was revised according to expert opinions. Then, it was piloted with two 10th grade classes in the high school where I have been working as a mathematics teacher. According to responses from pilot study, the instrument was revised and administered before starting instruction and after completing the instruction of function.

Function Knowledge Test consists of two parts. In the first part, there are eight problems which were used to assess conceptual knowledge of functions. On the other hand, the second part of the instrument consists of 12 problems which were used to assess procedural knowledge of functions (see in Appendix B). In order to score pre-tests and post-tests of participants, I developed a rubric and showed to one experienced mathematics teacher and one mathematics educator (see in Appendix C). In addition, another mathematics teacher evaluated six pre-test and six post-test papers by using the same rubric. Inter-scorer reliability coefficient was found as 0.986.

5.4.2. Open Ended Questionnaires- Conception of Mathematics, Perceptions of Mathematics Learning and Learning Functions Based on RME Approach

The aim of the study was also to investigate the participants' conceptions of mathematics and perceptions of mathematics learning before and after attending the function instruction based on *RME principles of teaching functions for understanding*. In addition, perceptions of learning functions based on RME approach were investigated after attending the instruction. In order to evaluate their conceptions and perceptions, participants completed open ended questionnaires. I asked two open-ended questions about participants' conceptions of mathematics and perceptions of mathematics learning to be answered in the pre-questionnaire and post-questionnaire. Also, I asked one open ended question about participants' perceptions of learning functions based on RME approach which was included only in the post questionnaire. Questionnaire was shown to one

mathematics educator and revised after expert opinions. In order to avoid using improper wording and ambiguity, the pilot questionnaire was administrated to twenty four10^{th}grade students prior to actual one to ensure its validity. All of the participants answered the questionnaire while completing Function Knowledge Test. Open ended questionnaires are in the Appendix D.

5.4.3. Participants' Journals

Participants were asked to keep journals about their learning experiences three times during the instruction of function based on RME approach. Four open ended questions were asked to participants while they were writing their journals (see in Appendix E). Questions involve what participants learn, factors that affect their learning of functions and evaluation of their class participation. Data obtained from the journals were used as a secondary source to reveal participants' perceptions of learning functions based on RME approach.

5.5. Data Collection

Three data sources were used to answer the research questions: (i) Function Knowledge Test (ii) open-ended questionnaires about participants' conceptions of mathematics and perceptions of mathematics learning and learning functions (iii) participants' journals about their experiences during the instruction of function. Function Knowledge Test and open ended questionnaire were administered before starting the instruction and after completing the instruction. The unit of function took four weeks. During the instructional period, participants kept journals three times. The following table shows the process of data collection.

Table 5.2. Process of data collection

Before Instruction	During Instruction	After Instruction
Function Knowledge Test + Questionnaire about • conceptions of mathematics • perceptions of mathematics learning	Journals	Function Knowledge Test + Questionnaire about • conceptions of mathematics • perceptions of mathematics learning • perceptions of learning functions

5.6. Data Analysis

5.6.1. Function Knowledge Assessment Test

Function Knowledge Test was administered to 9th grade participants in order to assess their knowledge of functions before and after attending function instruction based on *RME principles of teaching functions for understanding*. Participants' pre-test and post-test scores on Function Knowledge Test were compared using the Wilcoxon signed-rank test which is the nonparametric test equivalent to the dependent t-test. Wilcoxon signed-rank test was used; since the sample size of participants was less than 30 and normality of the data is not assumed (Privitera, 2012).

5.6.2. Open Ended Questionnaires

5.6.2.1. Conceptions of Mathematics. In order to reveal participants' conceptions of mathematics before and after the function instruction, one open ended question was asked in the pre and post questionnaire. Participants' responses to the question were analyzed qualitatively. First, codes were constructed based on participants' responses. Then, the coded words or phrases were matched with the categories of Crawford *et al.*'s (1994)

classification scheme that they were related to. Crawford *et al.* (1994) investigated conceptions of mathematics held by university students and identified conceptions of mathematics as fragmented and cohesive conceptions of mathematics (as cited in Crawford *et al.*, 1998). The number of occurrence for each category in the pre and post questionnaire responses was tabulated and compared in the result section.

5.6.2.2. Comparison of Participants' Performance on Function Knowledge Test with Respect to Their Conceptions of Mathematics. Participants' conceptions of mathematics before and after the instruction were analyzed by using the categories of Crawford *et al.*'s (1994) classification scheme. It was expected that four groups can be emerged from the results. Participants may demonstrate the fragmented or cohesive conception of mathematics in pre and post questionnaire. Besides, participants may change their fragmented conception of mathematics as the cohesive conception of mathematics or vice versa. According to given answers for pre and post questionnaires, possible groups were generated. Whole group's performance on Function Knowledge Test before and after the instruction was compared; but performances of each group generated with respect to their conceptions of mathematics were also compared. The non-parametric Kruskal Wallis Test was carried out to compare groups' gain scores on Function Knowledge Test. Since sample size for whole group was less than 30 and main assumptions of ANOVA were not satisfied, Kruskal Wallis Test was preferred to carry out (Privitera, 2012).

5.6.2.3. Perceptions of Mathematics Learning. Participants' perceptions of mathematics learning were also investigated. One open ended question was asked in the pre and post questionnaire, and responses to the question were analyzed qualitatively. First of all, I assigned code words or phrases to questionnaire responses and then generated themes from the codes. Themes appearing from the data were meaningful combinations of codes and presented in the following table. As seen from the Table 5.3., themes obtained from the data are diverse. Participants' responses indicated themes related to conventional approach to teaching such as "*Mathematics is learned best by listening to the lecture.*", "*Mathematics is learned best by practicing*.". In spite of that, themes related to teaching principles of RME also occurred in the participants' responses. "*Mathematics is learned*

best by using real life as starting point to learn mathematics." or "*Mathematics is learned best by interacting with others."* are some examples for themes appeared in the data. One theme occurred in the data is *"Matematik mantığını anlayarak en iyi öğrenilir."* in Turkish. Since this theme does not have equivalents in English, I preferred to translate this theme as "Mathematics is learned best by understanding the logic of how mathematical concepts work." These themes were used to categorize the participants' answers and tabulation of theme frequencies was provided for pre and post questionnaire responses in the result section.

Table 5.3. Participants' perceptions of mathematics learning

Themes of Perceptions of Mathematics Learning
Mathematics is learned best by listening to the lecture.
Mathematics is learned best by practicing.
Mathematics is learned best by understanding the logic of how mathematical concepts work.
Mathematics is learned best by using real life as starting point to learn mathematics.
Mathematics is learned best by interacting with others.
Mathematics is learned best by connecting among the topics (or concepts) of mathematics.
Others (by using visual materials, by playing games, etc)

5.6.2.4. Perceptions of Learning Functions Based on RME Principles of Teaching Functions for Understanding. Participants attended function instruction based on RME principles of teaching functions for understanding for four weeks. In order to investigate their learning experiences, one open ended question related to perceptions of learning functions was included in the post questionnaire. In addition, participants were asked to keep journals about their learning experiences three times during the instruction of function. Participants' responses to questionnaire and questions in the journals were once again analyzed qualitatively. Code words or phrases were assigned to the responses.

Themes obtained from questionnaire answers for perceptions of mathematics learning were used to categorize the code words or phrases; since participants' answers for perceptions of mathematics learning and learning functions were found to be similar. For the same reason, same themes were used to categorize the code words or phrases assigned to journal responses. Combining data obtained from the journal questions and post questionnaire, participants' perceptions of learning functions were revealed. Also, in the result section, participants' post questionnaire answers for perceptions of mathematics learning and learning functions were compared in terms of frequencies of the themes.

6. RESULTS

6.1. Function Knowledge Test

Participants' pre-test and post-test scores of Function Knowledge Test which assesses participants' knowledge of functions were compared. Since the sample size of participants was less than 30, non-parametric test for group comparison was required. Wilcoxon signed-rank test was used to investigate whether there is a significant change between pre-test and post-test scores of participants (Privitera, 2012). Table below shows descriptive statistics related to pre-test and post-test scores of Function Knowledge Test.

Table 6.1. Descriptive statistics related to pre-test and post-test scores of Function Knowledge Test

	N	Mean	Std. Deviation	Minimum	Maximum
Pre-test	24	18.12	8.838	2	41
Post-test	24	66.25	11.826	27	80

As can be seen in table 6.1., there is an increase in the mean of post-test scores of participants when compared to the mean of their pre-test scores. However, the following table presents whether the change between pre-test and post-test scores is significant. According to results of Wilcoxon signed-rank test, there is a statistically significant difference between the pre-test and post-test scores of participants on Function Knowledge Test ($Z = -4.287, p = 0.000$). Tables below show ranks and test statistics of Wilcoxon signed-rank test respectively.

Table 6.2. Ranks of Wilcoxon signed-rank test

Ranks				
		N	Mean Rank	Sum of Ranks
Post-test–Pre-test	Negative Ranks	0[a]	.00	.00
	Positive Ranks	24[b]	12.50	300.00
	Ties	0[c]		
	Total	24		
a. Post-test< Pre-test				
b. Post-test>Pre-test				
c. Post-test = Pre-test				

Table 6.3. Test statistics of Wilcoxon signed-rank test

Test Statistics[b]	
	Post-test–Pre-test
Z	-4.287[a]
Asymp. Sig. (2-tailed)	.000
a. Based on negative ranks.	
b. Wilcoxon Signed Ranks Test	

Function Knowledge Test consists of two parts. In the first part, there are eight problems which were used to assess conceptual knowledge of functions whereas the second part consists of 12 problems which were used to assess procedural knowledge of functions. Even though there is a significant difference between pre-test and post-test scores of participants on Function Knowledge Test, it is important to investigate whether there is a significant difference between pre-test and post-test scores of each part of Function Knowledge Test. In order to compare pre-test and post-test scores of each part, Wilcoxon signed-rank test was once again used. Table below presents descriptive statistics related to pre-test and post-test scores of conceptual part of Function Knowledge Test.

Table 6.4. Descriptive statistics related to pre-test and post-test scores of conceptual part of Function Knowledge Test

	N	Mean	Std. Deviation	Minimum	Maximum
Pre-test conceptual	24	11.17	5.164	0	23
Post-test conceptual	24	34.67	5.419	19	41

As seen from the table, there is an increase in the mean of post-test scores of conceptual part when compared to the mean of their pre-test scores of conceptual part. According to results of Wilcoxon signed-rank test, it is found that there is a statistically significant difference between the pre-test and post-test scores of conceptual part on Function Knowledge Test($Z = -4.292, p = 0.000$). Tables below show ranks and test statistics of Wilcoxon signed-rank test respectively.

Table 6.5. Ranks of Wilcoxon signed-rank test for conceptual part of Function Knowledge Test

Ranks				
		N	Mean Rank	Sum of Ranks
Post-test conceptual Pre-test conceptual	Negative Ranks	0[a]	.00	.00
	Positive Ranks	24[b]	12.50	300.00
	Ties	0[c]		
	Total	24		
a. Post conceptual < Pre conceptual				
b. Post conceptual > Pre conceptual				
c. Post conceptual = Pre conceptual				

Table 6.6. Test statistics of Wilcoxon signed-rank test for conceptual part of Function Knowledge Test

Test Statistics[b]	
	Post-test conceptual– Pre-test conceptual
Z	-4.292[a]
Asymp. Sig. (2-tailed)	.000
a. Based on negative ranks.	
b. Wilcoxon Signed Ranks Test	

Table below presents descriptive statistics related to pre-test and post-test scores of procedural part of Function Knowledge Test. In addition, results of Wilcoxon signed-rank test showed that the difference between the means of pre-test and post-test scores of procedural part is statistically significant (Z=−4.288, p=0.000). Tables 6.7 and 6.8 show the results of Wilcoxon signed-rank.

Table 6.7. Descriptive statistics related to pre-test and post-test scores of procedural part of Function Knowledge Test

	N	Mean	Std. Deviation	Minimum	Maximum
Pre-test procedural	24	6.96	5.923	0	26
Post-test procedural	24	31.58	7.83	8	40

Table 6.8. Ranks of Wilcoxon signed-rank test for procedural part of Function Knowledge Test

Ranks				
		N	Mean Rank	Sum of Ranks
Post-test procedural– Pre-test procedural	Negative Ranks	0[a]	.00	.00
	Positive Ranks	24[b]	12.50	300.00
	Ties	0[c]		
	Total	24		
a. Post procedural < Pre procedural				
b. Post procedural > Pre procedural				
c. Post procedural = Pre procedural				

Table 6.9. Test statistics of Wilcoxon signed-rank test for procedural part of Function Knowledge Test

Test Statistics[b]	
	Post procedural–Pre procedural
Z	-4.288[a]
Asymp. Sig. (2-tailed)	.000
a. Based on negative ranks.	
b. Wilcoxon Signed Ranks Test	

6.2. Open Ended Questionnaires

6.2.1. Conceptions of Mathematics

Participants' conceptions of mathematics were analyzed qualitatively. Categories of Crawford *et al.*'s (1994) classification scheme was used to categorize participants' pre and post questionnaire answers. Frequencies of the categories occurred in the pre and post questionnaire answers were compared. Table below shows descriptive statistics related to participants' conceptions of mathematics in pre and post questionnaires.

Table 6.10. Participants' conceptions of mathematics in pre and post questionnaires

Categories	Pre Questionnaire	Pre Frequency	Post Questionnaire	Post Frequency
Fragmented conception of mathematics	14	58.33 %	7	29.16%
Cohesive conception of mathematics	10	41.66%	17	70.83%

More than half of the participants' conceptions of mathematics (58.33%) were categorized as fragmented in the pre questionnaire. After the instruction of function based on *RME principles of teaching functions for understanding*, there was a change in the

frequency of participants' conceptions of mathematics. Post questionnaire results show that only 29.16% of participants held fragmented conception of mathematics. In addition, the percentage of participants with cohesive conceptions of mathematics increased from 41.66% to 70.83%. In addition to change in whole group's categories, numbers for individual change was also investigated. The following table presents descriptive statistics about the number of participants who held the same conception of mathematics and who changed their conception of mathematics after attending instruction of function.

Table 6.11. Individual change in conceptions of mathematics in pre and post questionnaires

		Post Questionnaire	
		Fragmented Conception	**Cohesive Conception**
Pre Questionnaire	**Fragmented Conception**	6	8
	Cohesive Conception	1	9

As Table 6.11.indicates, among 14 participants who were in the category of fragmented conception of mathematics for pre questionnaire, six of them still held fragmented conceptions of mathematics whereas eight participants' conceptions were categorized as cohesive based on their answers for post-questionnaire. These numbers indicated that more than half of these participants' categories improved in terms of conceptions of mathematics. In other words, eight participants with fragmented conceptions of mathematics held cohesive conceptions of mathematics after attending the instruction whereas one student with cohesive conception of mathematics changed his/her conception as fragmented one. In addition, it is important to note the results about participants who conceptualize mathematics as cohesive before the instruction. Nine out of ten participants with cohesive conceptions of mathematics before the instruction were still in the category of cohesive in the post questionnaire. In other words, almost all participants with cohesive conceptions of mathematics tended to hold same conception of mathematics after the instruction.

6.2.2. Perceptions of Mathematics Learning

Participants' perceptions of mathematics learning were analyzed according to themes emerged from participants' answers. It is important to note that participants' answers can be matched with more than one theme. Thus, total number of frequencies of each theme in the pre and post questionnaire responses was more than the total number of participants. The following table gives information about descriptive statistics related to participants' perceptions of mathematics learning in the pre and post questionnaires.

Table 6.12. Participants' perceptions of mathematics learning in pre and post questionnaires

Themes	Pre Questionnaire	Pre Frequency	Post Questionnaire	Post Frequency
Mathematics is learned best by listening to the lecture.	5	20.83%	6	25%
Mathematics is learned best by practicing.	14	58.33%	13	54.16%
Mathematics is learned best by understanding the logic of how mathematical concepts work.	6	25%	6	25%
Mathematics is learned best by using real life as starting point to learn mathematics.	N/A	N/A	7	29.16%
Mathematics is learned best by interacting with others.	1	4.16%	3	12.5%
Mathematics is learned best by connecting among the topics (concepts) of mathematics.	1	4.16%	1	4.16%
Others (by using visual materials, by playing games, etc)	5	20.83%	3	12.5%

As seen from the results of pre and post questionnaires, participants tended not to change their perceptions of mathematics learning such as "*Mathematics is learned best by listening to the lecture*", "*Mathematics is learned best by practicing*", "*Mathematics is learned best by understanding the logic of how mathematical concepts work.*" before and after attending the instruction. However, in the pre questionnaire none of the participants mentioned that *mathematics is learned best by using real life as starting point*, whereas conceptualization of *learning mathematics by using real life as starting point* occurs in seven participants' responses to the post questionnaire. In addition, as seen from the table

above, the number of participants who indicated conceptualization of *learning mathematics by interacting with others* in their post questionnaire was increased slightly compared with the number of participants in the pre questionnaire.

6.2.3. Perceptions of Learning Functions Based on RME Approach

Participants' perceptions of learning functions based on *RME principles of teaching functions for understanding* were investigated by using one open ended question in the post questionnaire and questions included in journals that participants kept about their learning experiences. Participants' responses to questionnaire and questions in the journals were categorized according to the themes for perceptions of mathematics learning. Combining data obtained from the journal questions and post questionnaire, participants' perceptions of learning functions were evaluated. The following table gives descriptive statistics related to participants' perceptions of learning functions based on RME approach.

Table 6.13. Participants' perceptions of learning functions

Themes	**Post Questionnaire and Journals**	**Frequency**
Functions are learned by listening to the lecture.	1	4.16%
Functions are learned by practicing.	4	16.66%
Functions are learned by understanding the logic of how mathematical concepts work.	2	8.33%
Functions are learned by using real life as starting point to learn it.	16	66.66%
Functions are learned by interacting with others.	18	75%
Others	2	8.33%

According to results, conceptualizations such as "*Mathematics is learned best by listening to the lecture*", "*Mathematics is learned best by practicing*", "*Mathematics is best learned by understanding the logic of how mathematical concepts work."* occurred few times in post questionnaire answers. On the other hand, in the post questionnaire more than half of the participants (66.66%) indicated that functions are learned by using real life as starting point to learn it. Those participants stated that using real life as starting point to learn facilitates their learning. They also stated that it is creative and entertaining. In addition to this result, three fourths of the participants (75%) mentioned that functions are learned by interacting with others. Fifteen out of eighteen participants who emphasized learning with interactivity in the post questionnaire stated that interacting with others facilitates their learning. They indicated that it is also an entertaining way to learn.

As seen from the results, post-questionnaire answers for perceptions of mathematics learning and learning functions based on RME approach differ greatly. Hence, it is important to compare them in terms of frequencies of each theme. The following table presents descriptive statistics related to participants' perceptions of mathematics learning and learning functions based on RME approach in the post questionnaire.

Table 6.14. Comparison of participants' perceptions of mathematics learning and learning functions based on RME approach

Themes	**Perceptions of Mathematics Learning Post Questionnaire**	**Perceptions of Mathematics Learning Post Frequency**	**Perceptions of Learning Functions Based on RME Approach Post-Questionnaire And Journals**	**Perceptions of Learning Functions Based on RME Approach Frequency**
Learning by listening to the lecture	6	25%	1	4.16%
Learning by practicing	13	54.16%	4	16.66%
Learning by understanding the logic of how mathematical concepts work	6	25%	2	8.33%
Learning by using real life as starting point to learn it	7	29.16%	16	66.66%
Learning by interacting with others	3	12.5%	18	75%
Others(by using visual materials, etc).	3	12.5%	2	8.33%

Themes of *learning by listening to the lecture, learning by practicing* and *learning by understanding the logic of how mathematical concepts work* tended to occur more in the post questionnaire answers for perceptions of mathematics learning compared to perceptions of learning functions based on RME approach. For example, more than half of the participants (54.16%) emphasized learning by practicing in the post questionnaire answer for perceptions of mathematics learning whereas the theme of *learning by practicing* occurred in only 16.66% of participants' answers to the post questionnaire for perceptions of learning functions. The opposite situation takes place for the themes of *learning by using real life as starting point to learn it* and *learning by interacting with others*. The percentage of participants who emphasized the theme of *learning by using real life as starting point to learn it* increased from 29.16% to 66.66%. The difference between the percentages of participants who indicated learning by interacting with others in their post questionnaire answers is noteworthy. Learning by interacting with others occurred in only 12.5% of participants' responses for perceptions of mathematics learning, whereas 75% of participants emphasized the same theme in their answers related to perceptions of learning functions.

6.3. Comparison of Participants' Performance on Function Knowledge Test With Respect to Their Category of Conception of Mathematics

Participants' conceptions of mathematics before and after the instruction were analyzed by using the categories of Crawford *et al.*'s (1994) classification scheme. According to given answers for pre and post questionnaires, six participants were labeled by those with fragmented conception of mathematics both before and after the instruction. On the other hand, eight participants who demonstrated fragmented conception of mathematics in pre questionnaire demonstrated cohesive conception of mathematics in post questionnaire. It is also found that nine participants with cohesive conceptions of mathematics before the instruction were still in the category of cohesive in the post questionnaire. According to pre and post questionnaire results of conceptions of mathematics, all participants except one can be categorized into three groups; *fragmented-fragmented, fragmented-cohesive* and *cohesive-cohesive* conception of mathematics. The

following table shows descriptive statistics about participants' conceptions of mathematics in pre and post questionnaire.

Table 6.15. Descriptive statistics about participants' conceptions of mathematics in pre and post questionnaires

Pre Questionnaire	Post Questionnaire	Frequency
Fragmented conception	Fragmented conception	6
Fragmented conception	Cohesive conception	8
Cohesive conception	Cohesive conception	9

In order to investigate whether their difference in conception of mathematics in pre and post questionnaires is related to their performance on Function Knowledge Test, gain scores of groups with *fragmented-fragmented, fragmented-cohesive* and *cohesive-cohesive* conception of mathematics were compared. Kruskal Wallis Test which is equivalent to a nonparametric test of one way analysis of variance was used to compare gain scores of these three groups (Privitera, 2012). Table 6.16 shows descriptive statistics about pre-test and post-test scores of groups with *fragmented-fragmented, fragmented-cohesive* and *cohesive-cohesive* conception of mathematics. Table 6.17 shows descriptive statistics related to gain scores between pre-test and post-test scores of groups.

Table 6.16. Descriptive statistics about pre-test and post-test scores of groups with fragmented-fragmented, fragmented-cohesive and cohesive-cohesive conception of mathematics

Conception of mathematics	N	Pre-test scores Mean	Post-test scores Mean
Fragmented-Fragmented	6	13.66	59.00
Fragmented-Cohesive	8	19.75	67.00
Cohesive-Cohesive	9	17.11	69.22

Table 6.17. Descriptive statistics related to gain scores between pre-test and post-test scores of groups fragmented-fragmented, fragmented-cohesive and cohesive-cohesive conception of mathematics

Report			
Gain score between pre-test and post-test scores			
Math Conception	Mean	N	Std. Deviation
Fragmented-Fragmented	45.33	6	16.305
Cohesive-Cohesive	52.11	9	8.207
Fragmented-Cohesive	47.25	8	5.676
Total	48.65	23	10.187

The following table shows results of ranks and test statistics related to gain scores between pre-test and post-test scores of groups.

Table 6.18. Results of Kruskal Wallis Test in terms of mean ranks of gain scores between pre-test and post-test scores on Function Knowledge Test

Ranks			
	Math conception	N	Mean Rank
Gain score between pre-test and post-test scores	Fragmented-Fragmented	6	10.75
	Cohesive-Cohesive	9	14.17
	Fragmented-Cohesive	8	10.50
	Total	23	

Table 6.19. Results of Kruskal Wallis Test in terms of test statistics of gain scores between pre-test and post-test scores on Function Knowledge Test

Test Statistics[a,b]	
	gain score between pre and post scores
Chi-Square	1.517
df	2
Asymp. Sig.	.468
a. Kruskal Wallis Test	
b. Grouping Variable: math conception	

A Kruskal Wallis test results show that when gain scores of these three groups were compared, even though there was a difference in gain scores in the favor of group of *cohesive-cohesive*, $\chi^2(2)=1.517$, p = .468, it was not statistically significant. When mean rank of gain scores of each group compared, group of *fragmented- fragmented* had a mean rank score of 10.75, group of *fragmented-cohesive* had a mean rank of 10.50, and group of *cohesive-cohesive* had a mean rank of 14.17. As seen from the results, all of these three groups improved their pre-test scores. Group of *fragmented-fragmented* scored lower in both pre-test and post-test than the other two groups. In addition, gain scores of participants with *cohesive-cohesive* is higher than others groups' gain scores.

7. CONCLUSION AND DISCUSSION

The design of this study is an action research. Thus, it is essential to reflect on findings of the study and to make discussions based on literature in this part. For the past four years, I have taught 9th grade classes each academic year. I have observed that at the end of the unit, students had limited understanding of functions. On the other hand, I aimed to guide my students to develop conceptual understanding and procedural fluency of functions and to develop positive perceptions of mathematics and mathematics learning. Hence, I reconsidered my instructional practices and developed instructional sequences based on *Realistic Mathematics Education (RME) principles of teaching functions for understanding*. Then I implemented them; which was discussed in previous sections. This section will provide discussion on the results of this study. It will be examined in two parts; first implementation of RME approach and also participants' conceptions of mathematics, and second participants' knowledge of functions.

7.1. Implementation of RME Approach

Participants in this study attended the function instruction based on *RME principles of teaching functions for understanding*. Since I was the teacher researcher, different sources were used to investigate effects of RME approach and principles of teaching functions in order to overcome any possible bias. Participants completed open ended questionnaires about their conceptions of mathematics and perceptions of mathematics learning before and after the instruction. In addition, they were asked about their experiences on learning functions after the instruction. Also, they were asked to keep journals about their learning experiences.

(i) Perceptions of mathematics learning: Responses to questionnaire about perceptions of mathematics learning and learning functions and responses to journal questions give information about the implementation of RME approach with respect to its teaching principle. Pre and post responses to perceptions of mathematics learning revealed

that participants emphasize *reality principle* and *interactivity principle* of RME in their learning process. In the pre questionnaire, none of the participants mentioned that *mathematics is learned best by using real life as starting point*, whereas seven participants emphasized reality principle in the post questionnaire. In addition, the number of participants who indicated conceptualization of *learning mathematics by interacting with others* in their post questionnaire was increased slightly compared with the number of participants in the pre questionnaire. On the contrary to the above, it was found that some perceptions of mathematics learning are slow to change. Participants indicated perceptions such as "*Mathematics is learned best by listening to the lecture*", "*Mathematics is learned best by practicing*", "*Mathematics is learned best by understanding the logic of how mathematical concepts work.*" in the questionnaire before and after the instruction. These findings were in line with the literature of Turkish students' general beliefs about mathematics or mathematics learning. According to Toluk Uçar and her colleagues (2010), elementary school students generally consider mathematics as consisting of calculation, numbers and operations and usually perceive problem solving as solving questions on a test. Aksu, Demir and Sümer (2002) also investigated students' beliefs related to mathematics and mathematics learning. Their results indicated that students in grades 4 to 8 believed that questions should be solved by using teachers' methods and this should be done quickly in order to be successful. They argued that possible reason behind this can be Turkish national system with high school and university placement exams. Students in Turkey have to take several entrance examinations and are required to solve questions correctly and quickly within a specific period. Participants in this study were 9^{th} grade students. However, they passed through grades 6 to 8 in the system explained above. In addition, they have already formed most of their opinions about and attitudes toward mathematics and mathematics learning before they started high school. Results of this study showed that participants tended not to change some of their perceptions. According to McLeod (1992), "mathematical beliefs and attitudes are generally stable and changing them to be a gradual process."(as cited in Wilkins and Ma, 2003, p.59). In this respect, for this current study, four-week instruction may not be enough to see significant changes in some perceptions of mathematics learning held by participants. Since there were significant changes in the results of some perceptions but not all, I inferred that RME approach is promising. If I extended RME focused practices in terms of time and

mathematics topics, it is likely to see wider influence on my students' perceptions of mathematics learning.

(ii) Perceptions of learning functions: When participants were asked to indicate their perceptions of learning functions, responses come up with different results compared to perceptions of mathematics learning. Considering different results between perceptions of mathematics learning and learning functions, it can be deduced that when participants were asked to express their views on learning a specific content, it is more probable to reveal to what extent instruction implemented affects their perceptions of mathematics learning. Also, it is important to note that if there would be more mathematics topics taught with a similar method, it may be expected to see more changes in students' perceptions of mathematics learning.

Before discussing the results related to perceptions of learning functions, it is important to underline that participants were asked to express their views on method(s) used while they were learning functions. They were not asked to reflect on their learning functions based on *RME principles of teaching functions for understanding*. Considering results obtained from questionnaire and journals, it was found that listening to lecture, practicing or understanding the logic of how mathematical concepts work occurred few times in the post questionnaire responses concerning perceptions of learning functions. On the other hand, it was found that reality and interactivity principle of RME were overemphasized by participants. More than half of the participants (66.66%) emphasized importance of reality principle in their learning process whereas 29.16% of participants mentioned reality principle in their post responses concerning perceptions of mathematics learning. Similarly, percentage of participants who stated about interactivity principle in learning functions is much more than the percentage of participants who discussed interactivity in learning mathematics. Three fourths of the participants (75%) mentioned that functions are learned by interacting with others. They stated that interacting with others facilitates their learning and one of entertaining ways to learn. According to Van den Heuvel-Panhuizen and Drijvers (2014), an RME teacher should guide students to interact with each other in order to produce mathematical tools. Furthermore, interaction between students should be provided through discussion, negotiations, and collaborations under the guidance of teacher. During four week RME based instruction, I encouraged my

students to work individually, in pairs or in a group of four according to tasks on which they worked. 75% of students put emphasis on interactivity by highlighting different aspects of interactivity principle. Some of them mentioned that it is possible to learn how the others think and interactivity facilitates their learning. Some of them pointed out group work reduces their stress and they feel more confident while working as a group. Considering these responses to post questionnaire and journals, I planned to extent use of RME-based practices for other mathematics topics.

Considering the above findings, some of studies concerning the effect of RME based instruction reported similar findings. For example; Cansız (2016) examined the students' views about RME approach by using interviews after they attended 16-week RME based instruction. Students in his study answered which teaching method(s) is appropriate to use in mathematics classroom. Most participants stated that they were active through discussion guided by teacher during the RME based instruction. Also, in the study of Ersoy (2013), students' views about RME approach were investigated through interviews. 58.3% of students in the study indicated learning through group work is one of the positive sides of RME approach. Taking students' responses into account, it can be discussed that students emphasized *interactivity principle* of RME approach in their learning process. In addition to above studies, Özdemir (2015) also examined students' views about implementing RME based instruction for the topic of sets. Participants indicated that RME approach facilitates learning, makes learning permanent and uses real life examples. Emphasis on *reality principle* of RME was also found in the study of Cansız (2016). He analyzed views of participants who indicated RME based instruction is appropriate to use in mathematics classroom. Twelve out of twenty-eight participants indicated that RME based instruction makes connections to real life. To sum up, reality and interactivity principles of RME come into the prominence in these studies; which is consistent with the findings of this study.

Since participants mentioned about using real life as starting point to learn and interactivity as one of the major factors enhancing their learning of functions, it can be concluded that instruction based on *RME principles of teaching functions for understanding* does lead positive results in participants' perceptions of mathematics learning. This finding is in agreement with the literature that commented that teachers'

method of instruction in classroom is important in changing students' attitudes and habits towards mathematics (Akinsola and Olowoja, 2008; Wilkins and Ma, 2003;Mason, 2003).According to Wilkins and Ma (2003), students' views of mathematics are strongly affected by the activities and mathematics problems that teachers choose. Mason (2003) also argued that students' construction of beliefs are influenced by how teachers present the subject, and what kind of task, assessment methods, prodecures and criteria they use.

(iii) Conceptions of mathematics: Another purpose of the study was to examine participants' conceptions of mathematics. In order to reveal participants' conceptions of mathematics, open ended questionnaires were once used and participants' responses were categorized according to categories of Crawford *et al.*'s (1994) classification scheme. Results related to conceptions of mathematics showed that one third of participants' categories improved in terms of conceptions concerning mathematics. Nine out of ten participants with cohesive conceptions of mathematics before the instruction demonstrated the same conception of mathematics in the post questionnaire. In this study, instruction based on *RME principles of teaching functions for understanding* may help some participants to improve their conceptions of mathematics or to ensure participants with cohesive conception of mathematics to hold the same conception of mathematics. From this point, it can be concluded that *RME principles of teaching functions for understanding* may have positive effects on participants' conceptions of mathematics in this study. This finding is in agreement with findings of the studies which have reported the effectiveness of RME in improving students' attitudes toward mathematics (Fauzan *et al.*, 2002; Zulkardi, 2002; Üzel and Uyangör, 2006; Cansız, 2016; Çakır, 2011; Ersoy, 2013).

Considering development of cohesive conceptions of mathematics, some suggestions are recommended by researchers. According to Murphy (2016), teachers should teach mathematics as a connected set of topics and concepts which are meaningfully related to students' lives. He further emphasized the applications of concepts in real life situations which can be used to promote deeper learning of mathematics and to promote cohesive conception of mathematics. Reid, Petocz, Smith, Wood and Dortins (2003) also argued the same implication for teaching and learning in order to have broadest conceptions of mathematics. They claimed that learning materials that are set in real context should be used. In addition, they asserted that students should be encouraged to investigate the role of

mathematics in their future and personal lives. Suggestions recommended in above studies have common in teaching principles of RME. Considering my RME-based teaching practices, I put emphasis on use of real life or realistic problem situations as a starting point to teach. I realized that students come up with creative and entertaining examples or solutions which were meaningful to their life and reflected their understanding of the topic. I observed that this way to teach mathematics contribute to their understanding of topics and development of cohesive conception of mathematics. Hence, I aimed to teach more mathematics topics based on RME approach.

7.2. Participants' Knowledge of Function

Development of conceptual understanding and procedural fluency of functions was another aim of this study. For this purpose, I developed instructional sequences based on RME approach. Besides of RME principles, principles of teaching functions discussed in the literature section were also used. For example; realistic or real life situations modeled by one to one and many to one functions were used to introduce the function concept qualitatively and non-algebraically (Cooney *et al.*, 2010). Different teaching approaches such as function as a covariation (Carson *et al.*, 2012) and the function metaphor (Selden and Selden, 1992) were also used during instruction. According to national curriculum, students are expected to explain the function concept. In order to achieve this, I used RME tasks which involve function as a correspondence or as a covariation. After the introduction of function concept, I asked groups of four students to define the function concept. All except one group defined function as a correspondence even though tasks which involve both definitions of functions were used during the instruction. I observed the same tendency in students' journals. In the first journal, nine students defined function as a correspondence whereas only two students mentioned function as a covariation. Even though there is tendency toward defining function as correspondence, responses to Function Knowledge Test showed that all students used the essential features of the function concept.

Before and after the instruction, participants' knowledge of functions was assessed by Function Knowledge Test. Then, participants' performances on pre-test and post-test were compared in order to investigate how instruction based on *RME principles of teaching functions for understanding* affect participants' knowledge enhancement of functions. Results of Wilcoxon signed-rank test showed that participants' performance significantly improved in terms of both conceptual and procedural knowledge of functions. I formally introduced the function concept to participants for the first time during the instruction. However, informal knowledge of functions discussed in literature section was already developed by participants before the instruction; since according to Turkish Mathematics Curriculum for Grade 6-8 (MEB, 2013), students are required to analyze the relationship between dependent and independent variables, investigate linear equations and slopes and to represent real life situations including linear relationship.

In depth analysis of participants' pre-test answers to questions in Function Knowledge Test showed that 11 out of 24 participants (45.83%) demonstrated some misconceptions about the function concept. The most common misconception occurred in the responses is related to that a function should be given by a rule. Also, it was seen that some students believed that a graph of function should be nice or continuous. After the instruction based on RME approach and principles of teaching functions, none of the participants demonstrated misconceptions about the function concept within the scope of Function Knowledge Test. Considering significant differences between participants' pre-test and post-test scores and non-occurrences of misconceptions concerning the function concept in post-test, it can be concluded that function instruction based on *RME principles of teaching functions for understanding* has positive impacts on development of participants' knowledge of functions. Similar argument was supported by the study of Webb, Kooij and Geist (2011). They designed a unit of logarithms to promote understanding of logarithms using RME. In their study, realistic contexts were emphasized during the two-week instruction. The researchers realized that students "hook" the knowledge and develop deep understanding of the connection between exponential growth and logarithms. Another study by Rasmussen and King (2000) also revealed that through RME instruction design, students developed conceptual reasoning about the rate. To conclude, as Dickinson and Hough (2012) pointed out that "teachers using RME report that

it enables more students to understand mathematics and to engage with it." (p.6), the findings of this study were in agreement with the studies discussed above.

In addition to above studies, there are also studies that focused on effectiveness of RME based instruction on students' achievement for a specific mathematics topic(Akyüz, 2010; Gelibolu, 2018; Ersoy, 2013, Özdemir, 2015, Çakır, 2011). Effectiveness of RME approach was investigated through experimental studies which were conducted as a pre-test-post-test design with a control group. Results showed that RME based instruction leads to higher achievement on a specific mathematics topic than the traditional method and experimental group taught by RME based instruction shows significantly more improvement from pre-test to post-test. Even though development of conceptual understanding and procedural fluency of functions is the focus of the study, the findings related to knowledge of functions can be seen as complementary with the findings in the above studies.

7.3. Participants' Knowledge of Function with Respect to Conceptions of Mathematics

Participants' conceptions of mathematics before and after the instruction were analyzed by using the categories of Crawford *et al.*'s (1994) classification scheme. According to participants' pre and post conceptions of mathematics, three groups were formed; participants with *fragmented-fragmented*, *fragmented-cohesive* and *cohesive-cohesive* conception of mathematics. In order to investigate whether their difference in conception of mathematics in pre and post questionnaires is related to their performance on Function Knowledge Test, gain scores of these three groups were compared by using Kruskal Wallis Test. It was found that participants with *fragmented-fragmented* scored lower in both pre-test and post-test than the other two groups. Also, participants with *cohesive-cohesive* conceptions of mathematics performed better than other groups in post-test. This is in agreement with that a cohesive conception of mathematics is related to higher achievement in examinations (Wood *et al.*, 2012). Considering results related to gain scores, participants with *cohesive-cohesive* improved their performance more than other two groups. Even though there were differences between gain scores of these groups,

it was also found that these differences between groups were not statistically significant. One possible reason behind this may be that participants attended the instruction based on RME approach for only four weeks. In order to get significant difference between groups in terms of their performance on the test, longer instructional period based on RME approach may be required.

8. LIMITATIONS AND SUGGESTIONS

In this action research, it was aimed to guide participants to develop conceptual understanding and procedural fluency of functions and to develop positive perceptions of mathematics and mathematics learning. Findings from multiple data sources support the effectiveness of instruction based on *RME principles of teaching functions for understanding*. However, instead of classroom observations which are recommended data source for the action research, I kept journals about my teaching experiences with respect to RME principles and received feedbacks from my advisor on my teaching experiences. Moreover, participants in this study completed open ended questionnaires and kept journals throughout the study.

In this study, participants were categorized according to their pre and post conception of mathematics and groups' performances on Function Knowledge Test were compared. Even though there were differences in their gain scores on Function Knowledge Test, the differences were not statistically significant. As explained before, this may stem from that participants attended the instruction based on RME approach for only four weeks. Also, the number of total participants was 24. In this respect, further research can be conducted in a way that more participants attend longer instructional period based on RME approach.

In 2017 Turkish Mathematics curriculum for grade 9-12, students' mathematical competencies are listed. For example; students are expected to develop spatial reasoning and mathematical reasoning, and to communicate mathematically in order to solve problems that they face in the daily life. Another mathematical competency is that students are expected to develop positive attitudes toward mathematics (MEB, 2017, p.11). In the recent curriculum, it is emphasized that learning is meaningful when it is connected to students' lives. In this respect, teachers are guided to design activities that enable students to use what they learn in their daily life and in different disciplines. Also, by providing students with meaningful learning experiences, they may develop positive attitudes toward learning and become lifelong learners (MEB,2017,p.5). Considering literature related RME including the findings of this study, it can be suggested that RME is a promising approach

to accomplish these aims. In this regard, it can be concluded that RME based practices should be extended in terms of time and mathematics topics.

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APPENDIX A: SAMPLE LESSON PLAN

<table>
<tr><td colspan="2">Where do you stand?
Objective: Students will be able to explain the function concept.
At the end of the task, learners should be able to develop following big ideas.
A function is a relation in which each element of the domain corresponds to one and only one element of the range.
Functions can be defined on any sets of objects which are not necessarily sets of numbers. In other words, the domain and range of the functions do not have to be numbers.</td></tr>
<tr><td>Instructions to teacher:
Place the letters A, B, C, D and E along a wall (write each letter, large and bold, on a separate sheet of paper, taped to the wall, or on the board).
Divide students into group of four.
Select one student from each group.
Give selected students instructions on where to stand based on different criteria. Explain that students will be extracting the definition of a function from this experience by discussing the follow up questions.
Questions for Selected Students
Where do you stand?
(i) Mode of travelling to school
If you walked to school today, stand under A.
If you went to school by school bus, stand under B.
If you went to school by public transportations,</td><td>RME principles of teaching functions for understanding will be implemented in the following ways:
In this task, functions will be introduced qualitatively and non-algebraically in a contextual problem.
Functions can be represented in various ways. In this task, students will use arrow diagram representation of a function.
Students will be expected to develop a sense of operational understanding of function in this task. This is essential for further development of structural understanding of functions.</td></tr>
</table>

stand under C. If you cycled to school, stand under D. If you went to school by shared taxi (in Turkish dolmuş), stand under E. **(ii) Clothing** If you are wearing blue, stand under A. If you are wearing black, stand under B. If you are wearing white, stand under C. If you are wearing brown, stand under D. If you are wearing green, stand under E. **Follow up questions for all students** Ask students • Draw an arrow between the name of students and where they stand. • Determine the variables in each situation and identify each as independent or dependent. • Discuss the differences and similarities in two situations. **Instructions to teacher:** Ask each group share their answers and analyze the adequacy and the efficiency of the different answers. Give formal definitions of variables, dependent and independent variables, and a function.	• Teacher will encourage students to *interact* with each other, discuss, negotiate, and collaborate.

APPENDIX B: FUNCTION KNOWLEDGE TEST

FONKSİYON BİLGİ TESTİ

1) Aşağıdaki ikililer arasındaki ilişkilerden hangisi / hangileri A kümesinden B kümesine bir fonksiyon belirtir?

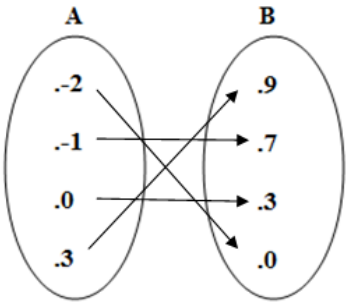
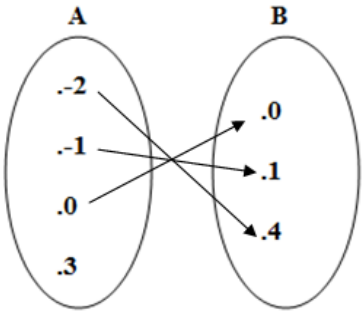
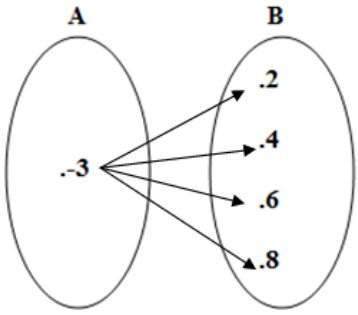
a) Eğer verilen ilişki fonksiyon ise aşağıdaki tablonun ikinci sütuna ✓ , değil ise × koyunuz.
b) Eğer verilen ilişki fonksiyon ise *neden bir fonksiyon olduğunu*, eğer fonksiyon değil ise *neden bir fonksiyon olmadığını* **açıklama** bölümüne yazınız.

	A kümesi	B kümesi	Fonksiyon / Fonksiyon değil	Açıklama
a)	Facebook Kullanıcı Adı	Facebook Kullanıcı Şifresi		
b)	Sınıfınızdaki Öğrenciler	Saç Rengi		
c)	Sınıfınızdaki Öğrenciler	Yaşadığı Gezegen		
d)	Aylar	Ay İçindeki Gün Sayısı		
e)	Ay İçindeki Gün Sayısı	Aylar		
f)	Tarih	Dışarıdaki Sıcaklık		

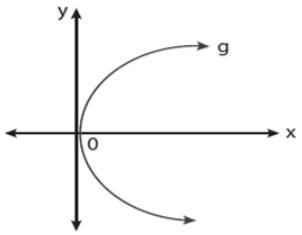
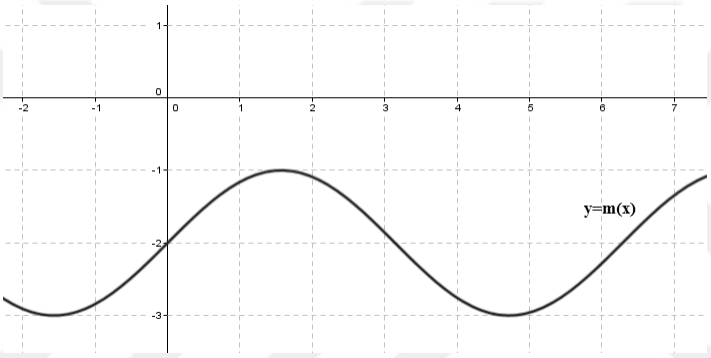
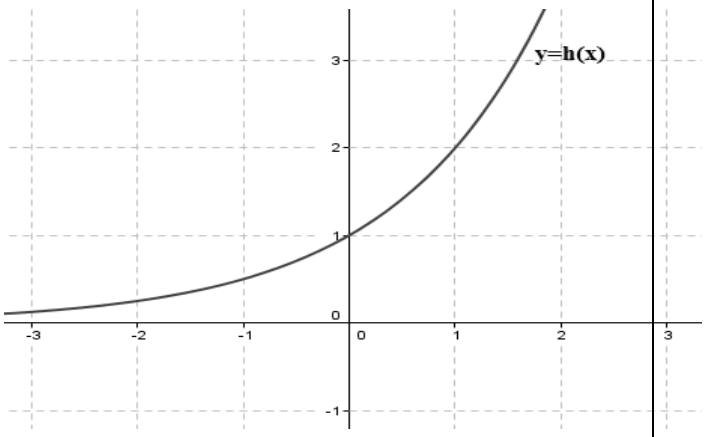
2) Aşağıda verilen kümeler arası eşlemelerden hangisi / hangileri A kümesinden B kümesine bir fonksiyon belirtir?

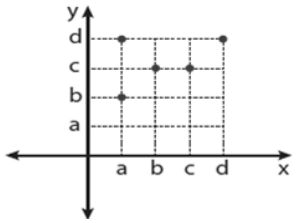
a) Eğer verilen eşleme fonksiyon ise aşağıdaki tablonun ikinci sütuna ✓ , değil ise × koyunuz.

b) Eğer verilen ilişki fonksiyon ise *neden bir fonksiyon olduğunu*, eğer fonksiyon değil ise *neden bir fonksiyon olmadığını* **açıklama** bölümüne yazınız.

	Eşleme	Fonksiyon / Fonksiyon değil	Açıklama
a)			
b)			
c)			

3) Aşağıda verilen grafiklerden hangisi / hangileri fonksiyon belirtir? **Cevabınızı açıklayınız.** Eğer verilen grafik, bir fonksiyon ait ise ✓ , değil ise × koyunuz.

		Fonksiyon/ Fonksiyon değil
a)	g: R→R, y=g(x)  	
	Açıklama:	
b)	m:R⟶R, y=m(x)  	Fonksiyon/ Fonksiyon değil
	Açıklama:	
c)	h: R→R, y=2^x  	Fonksiyon/ Fonksiyon değil
	Açıklama:	

d)	l: {a,b,c,d}→ {a,b,c,d}, y=l(x)  	Fonksiyon/ Fonksiyon değil
	Açıklama:	

4) Park görevlisi, göldeki suyun derinliğini günün aynı saatinde olmak üzere belli bir süre boyunca ölçmüş ve sonuçları aşağıdaki tabloya not almıştır.



Ölçüm Sırası	Gün Sayısı	Gün sayısındaki değişim	Gölün derinliği (m)	Derinlikteki değişim (m)
1.ölçüm	7.gün	-	15.29	-
2.ölçüm	14.gün	7	15.43	0.14
3.ölçüm	28.gün	14	15.57	0.14
4.ölçüm	35.gün	7	15.71	0.14
5.ölçüm	42.gün	7	15.85	0.14

a) Tablodaki bilgilere göre, gün sayısındaki değişime bağlı olarak **gölün derinliğinin** nasıl değiştiğini açıklayınız.

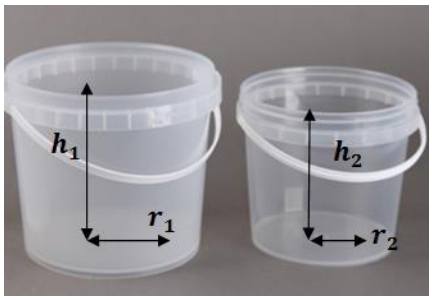
b) Tablodaki bilgilere göre, gün sayısındaki değişime bağlı olarak **gölün derinliğindeki değişimin** nasıl değiştiğini açıklayınız.

c) Tablodaki bilgilere göre, **derinlik-zaman fonksiyonunun** grafiğini çiziniz. (Size verilen gride çiziniz.)

d) Çizdiğiniz grafiği kullanarak, zamana bağlı derinlik *değişim hızını* / *hızlarını* hesaplayınız. (Size verilen grid kâğıdında hesaplamayı gösteriniz.)

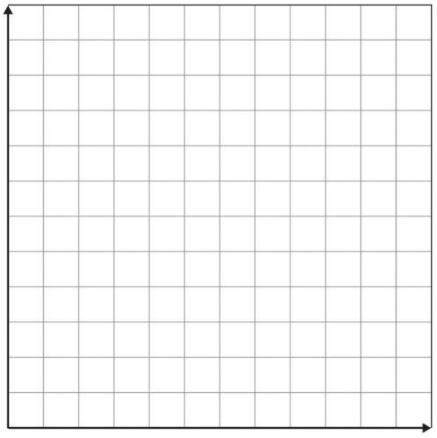
e) Bulduğunuz değişim hızını / hızlarını yorumlayınız.

5) Bahçedeki çiçekleri sulamak için yandaki kovalar ile su taşınacaktır. Kovaların boyutları arasında $h_1 > h_2$ ve $r_1 > r_2$ ilişkisi vardır.



Kovalara birim zamanda eşit miktarda su akıtan çeşmelerden su doldurulacaktır. Kovalar aynı anda doldurulmaya başlanıyor ve kovalardaki suyun yüksekliği ile ilgili gözlem yapıyorsunuz. Buna göre,

a) Her bir kovadaki suyun yüksekliğini zamana bağlı gösteren fonksiyonun grafiklerini çiziniz.



b) Her iki kovadaki suyun yüksekliğinin değişim hızını karşılaştırınız ve yorumlayınız.

6)

……..Lisesi öğrencileri, Sosyal Etkinlik Çalışmaları kapsamında bir kulüp seçmek ve düzenli olarak kulüp çalışmalarına katılmak durumundadırlar.

Yukarıda bir model tanımlanmıştır. Bu modelde, …….Lisesi öğrencileri ve katıldıkları kulüpler arasında bir ilişki verilmiştir. Bu ilişkinin,

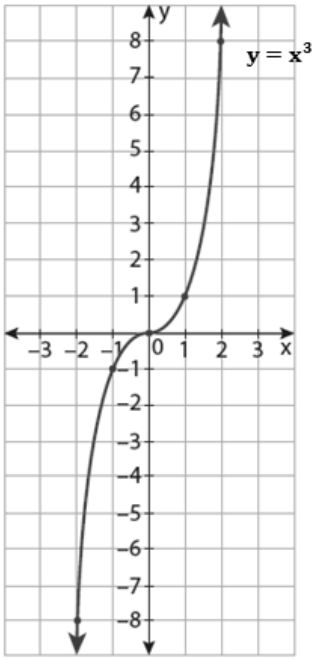
a) bire-bir fonksiyon belirtip belirtmediğini **nedenleri ile açıklayınız.**

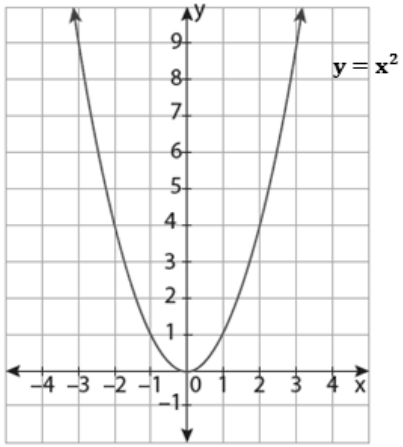
b) örten fonksiyon belirtip belirtmediğini **nedenleri ile açıklayınız.**

7) Marketlerde satılan ürünler ve her birine ait barkod numaraları birer değişken olacak şekilde bir fonksiyon modeli yazınız. Yazdığınız modeldeki ilişkinin bire-bir ama örten olmayan bir fonksiyon belirtmesini sağlayınız.

Model:

8) $y = x^3$ ve $y = x^2$ grafikleri aşağıda verilmiştir. İki grafik arasındaki benzerlikleri ve farklılıkları yorumlayınız.





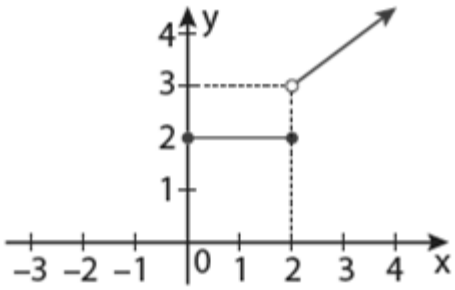
9) $f: R \to R$ ve $f(x) = 5x - 4$ ile verilen f fonksiyonu için ? yerine ne geleceğini bulunuz.

a) $f(2) = ?$

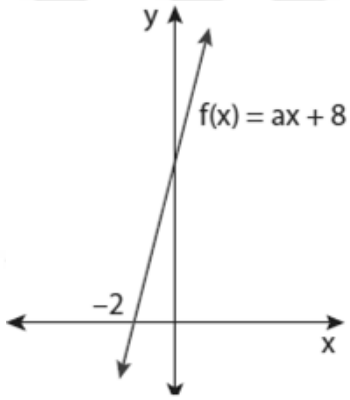
b) $f(?) = 6$

c) $f([-3,9]) = ?$

10) Şekilde grafiği verilen fonksiyonun tanım ve görüntü kümesini bulunuz.



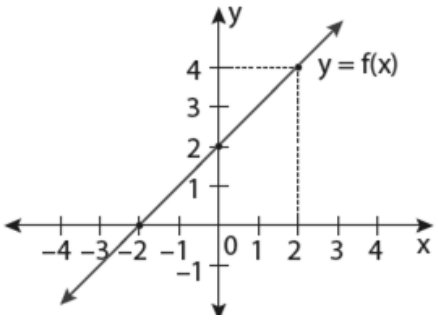
11) Aşağıda gerçek sayılarda tanımlı f(x) = ax + 8 fonksiyonun grafiği verilmiştir. Buna göre a kaçtır?



12) Yanda **doğrusal f** fonksiyonun grafiği verilmiştir. Buna göre,

a) verilen tabloyu doldurunuz.

x	f(x)=y
-2	
0	
1	
2	



b) f fonksiyonunun kuralını yazınız.

13) Aşağıda kuralları verilen doğruların **eğimlerini** bulunuz.

a) $y = \frac{x}{3} + 1$

b) $2x - 3y = 6$

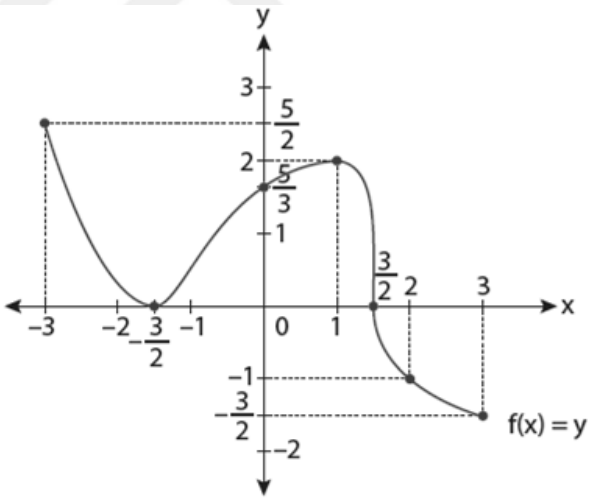
c) $x = 3$

d) $y = 1$

14) g: R → R ve $g(x) = (a-2)x^2 + (b+2)x - a.b$ fonksiyonu **sabit bir fonksiyon** ise $g(2017)$ kaçtır?

15) k: R → R ve $k(x) = ax + b - 4x + 8$ fonksiyonu **birim fonksiyon** ise a ve b'nin değerini bulunuz.

16) f fonksiyonunun grafiği şekildeki gibidir. Buna göre;



a. f fonksiyonunun tanım ve görüntü kümelerini bulunuz.

b. $f(-3)$'ün değerini bulunuz.

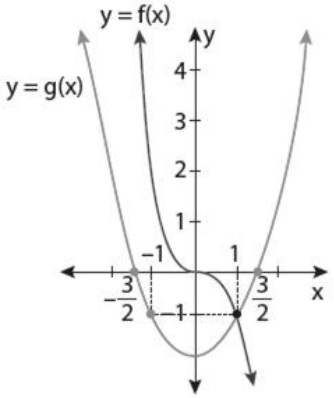
c. $f(2)$'nin değerini bulunuz.

d. $f(x) = \frac{5}{2}$ sağlayan x değerini / değerlerini bulunuz.

e. Aşağıdaki tabloyu doldurunuz. Her biri için sadece bir örnek veriniz.

		Örnek
a)	**Ters görüntüsü tek olan** görüntü kümesinden eleman	
b)	**Ters görüntüsü iki tane** olan görüntü kümesinden eleman	
c)	**Ters görüntüsü üç tane olan** görüntü kümesinden eleman	

17) Gerçek sayılar kümesinde tanımlı f ve g fonksiyonlarının grafikleri verilmiştir. Buna göre;



a. f(x)=0 sağlayan x değerini/değerlerini bulunuz.

b. g(x)=0 sağlayan x değerini/değerlerini bulunuz.

c. f(x) = g(x) eşitliğini sağlayan x değerini / değerlerini bulunuz.

18) Tanım kümesi tüm gerçek sayılar olan bir f fonksiyonu parçalı tanımlı olarak aşağıdaki gibi veriliyor:

$$f(x)=\begin{cases}3x-2 & ,x<-5\\ \dfrac{x+1}{x^2+1} & ,-5\le x<0\\ 2x+4 & ,0\le x\end{cases}$$ Buna göre aşağıda verilenleri bulunuz.

a) $f(-5)$

b) $f(-10)$

c) $f(x^2)$

19) Bir firmanın zamana (yıl) bağlı gelir (milyon TL) fonksiyonu f(x)=12 – 2x, gider (milyon TL) fonksiyonu g(x) = x + 3 dir. $0 \le x \le 6$ verildiğine göre aşağıdaki soruları cevaplayınız.

a. f ve g fonksiyonlarının grafiklerini aynı koordinat düzleminde çiziniz.

b. Gelir ve giderin eşit olduğu değeri ve zamanı bulunuz.

20) f: R → R olmak üzere $f(x) = x^2$ fonksiyonunun

a) bire-bir olup olmadığını belirtiniz.

b) örten olup olmadığını belirtiniz.

APPENDIX C: RUBRIC OF FUNCTION KNOWLEDGE TEST

FONKSİYON BİLGİ TESTİNİN PUANLAMA ANAHTARI

1) Aşağıdaki ikililer arasındaki ilişkilerden hangisi / hangileri A kümesinden B kümesine bir fonksiyon belirtir?

a) Eğer verilen ilişki fonksiyon ise aşağıdaki tablonun ikinci sütuna ✓ , değil ise × koyunuz.

b) Eğer verilen ilişki fonksiyon ise *neden bir fonksiyon olduğunu*, eğer fonksiyon değil ise *neden bir fonksiyon olmadığını* **açıklama** bölümüne yazınız.

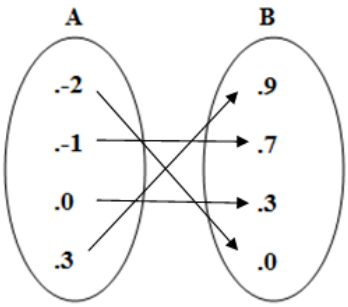
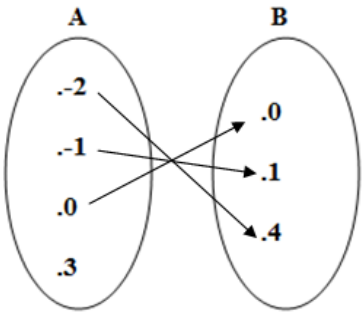
	A kümesi	B kümesi	Fonksiyon / Fonksiyon değil	Açıklama
a)	Facebook Kullanıcı Adı	Facebook Kullanıcı Şifresi		Her facebook kullanıcı adına karşılık bir şifre olmalıdır. Fonksiyondur. 1 Aynı kullanıcı adına sahip kişilerin farklı şifreleri olabilir. Fonksiyon değildir. 1 *Univalence condition* açıklaması verilerek fonksiyon olup olmadığı belirtilmiştir. 1 *Univalence condition* açıklaması verilmemiştir. 0 Soru üzerinde herhangi bir işlem yapılmamış.0
b)	Sınıfınızdaki Öğrenciler	Saç Rengi		Her öğrencinin bir saç rengi vardır. Fonksiyondur. 1 Bir öğrencinin birden fazla saç rengi olabilir. Fonksiyon değildir. 1 *Univalence condition* açıklaması verilerek fonksiyon olup olmadığı belirtilmiştir. 1 *Univalence condition* açıklaması verilmemiştir. 0 Soru üzerinde herhangi bir işlem yapılmamış.0

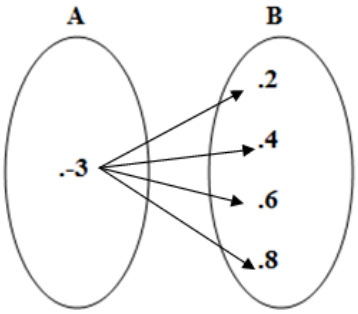
c)	Sınıfınızdaki Öğrenciler	Yaşadığı Gezegen		Sınıftaki her öğrenci dünyada yaşadığı için fonksiyondur. 1 *Univalence condition* açıklaması verilerek fonksiyon olduğu belirtilmiştir. 1 *Univalence condition* açıklaması verilmemiştir. 0 Soru üzerinde herhangi bir işlem yapılmamış.0
d)	Aylar	Ay İçindeki Gün Sayısı		Her ay içindeki gün sayısı bir tanedir. Fonksiyondur. 1 Şubat ayı dört yıl içinde 29 çektiği için Şubat ayı hem 28 hem de 29 ile eşleşir. Fonksiyon değildir.1 *Univalence condition* açıklaması verilerek fonksiyon olup olmadığı belirtilmiştir. 1 *Univalence condition* açıklaması verilmemiştir. 0 Soru üzerinde herhangi bir işlem yapılmamış.0
e)	Ay İçindeki Gün Sayısı	Aylar		30 ve 31 sayıları birden fazla ay ile eşleşecektir. Fonksiyon değildir. 1 *Univalence condition* açıklaması verilerek fonksiyon olmadığı belirtilmiştir. 1 *Univalence condition* açıklaması verilmemiştir. 0 Soru üzerinde herhangi bir işlem yapılmamış.0
e)	Tarih	Dışarıdaki Sıcaklık		Gün içerisindeki sıcaklık değişmez diye var sayarsak ya da ortalama sıcaklık alınırsa fonksiyon belirtir. 1 Gün içerisindeki sıcaklık değişeceği için bir tarih birden fazla sıcaklık derececisi ile eşleşir. Fonksiyon değildir. 1 *Univalence condition* açıklaması verilerek fonksiyon olup olmadığı belirtilmiştir. 1 *Univalence condition* açıklaması verilmemiştir. 0 Soru üzerinde herhangi bir işlem yapılmamış.0

2) Aşağıda verilen kümeler arası eşlemelerden hangisi / hangileri A kümesinden B kümesine bir fonksiyon belirtir?

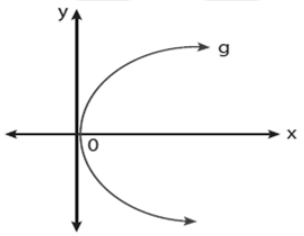
a) Eğer verilen eşleme fonksiyon ise aşağıdaki tablonun ikinci sütuna ✓ , değil ise × koyunuz.

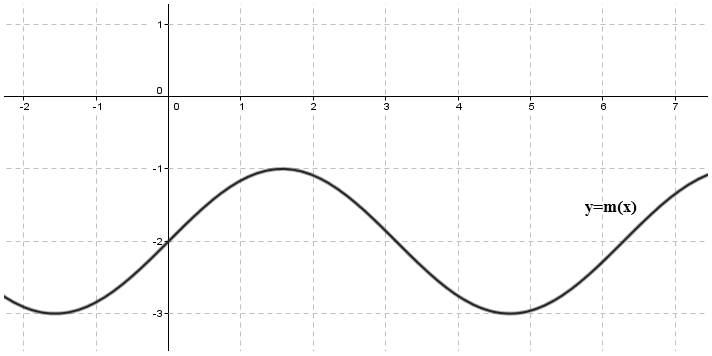
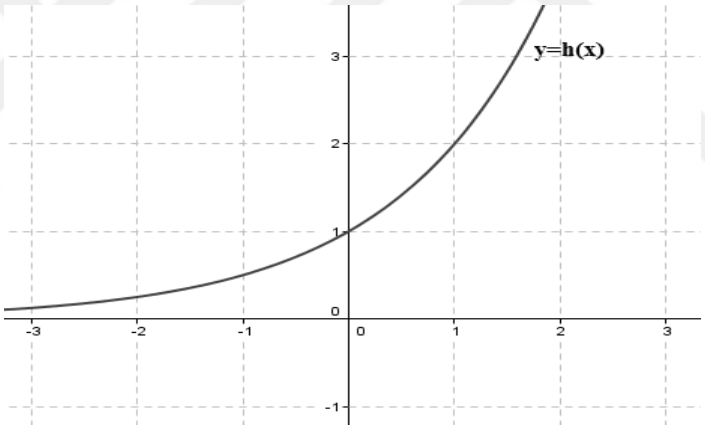
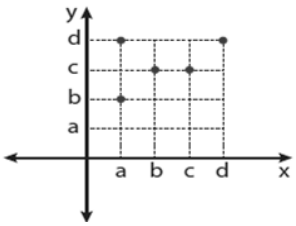
b) Eğer verilen ilişki fonksiyon ise *neden bir fonksiyon olduğunu*, eğer fonksiyon değil ise *neden bir fonksiyon olmadığını* **açıklama** bölümüne yazınız.

	Eşleme	Fonksiyon / Fonksiyon değil	Açıklama
a)			Tanım kümesindeki her eleman değer kümesinde bir ve yalnız bir eleman ile eşleşmiştir. Fonksiyondur. 1 *Univalence condition* açıklaması verilerek fonksiyon olduğu belirtilmiştir. 1 Soru üzerinde herhangi bir işlem yapılmamış.0
b)			Tanım kümesindeki her eleman değer kümesindeki bir ve yalnız bir eleman ile eşleşmemiştir. 1 *Univalence condition* açıklaması verilerek fonksiyon olmadığı belirtilmiştir. 1 Soru üzerinde herhangi bir işlem yapılmamış.0

c)			Tanım kümesindeki bir eleman birden fazla eleman ile eşleşmiştir. 1 *Univalence condition* açıklaması verilerek fonksiyon olmadığı belirtilmiştir. 1 Soru üzerinde herhangi bir işlem yapılmamış.0

3) Aşağıda verilen grafiklerden hangisi / hangileri fonksiyon belirtir? **Cevabınızı açıklayınız.** Eğer verilen grafik, bir fonksiyon ait ise ✓ , değil ise × koyunuz.

		Fonksiyon/ Fonksiyon değil
a)	g: R→R, y=g(x) 	
	Açıklama: Tanım kümesindeki her eleman değer kümesindeki bir eleman ile eşleşmemiştir. Tanım kümesindeki bazı elemanlar değer kümesindeki birden fazla eleman ile eşleşmiştir. Grafik, fonksiyona ait değildir. 2 Tanım kümesindeki her eleman değer kümesindeki bir eleman ile eşleşmemiştir. Grafik, fonksiyona ait değildir. 1 Tanım kümesindeki bazı elemanlar değer kümesindeki birden fazla eleman ile eşleşmiştir. Grafik, fonksiyona ait değildir. 1 Grafik, dikey doğru testinden geçmemektedir. Grafik, fonksiyona ait değildir. 1 Soru üzerinde herhangi bir işlem yapılmamış.0	

b)	m:R⟶R, y=m(x) 	Fonksiyon/ Fonksiyon değil
	Açıklama: Tanım kümesindeki her eleman değer kümesinde bir ve yalnız bir eleman ile eşleşmiştir. Grafik, fonksiyona aittir. 2 Grafik, dikey doğru testinden geçmektedir. Grafik, fonksiyona aittir. 1 Soru üzerinde herhangi bir işlem yapılmamış.0	
c)	h: R→R, $y=2^x$ 	Fonksiyon/ Fonksiyon değil
	Açıklama: Tanım kümesindeki her eleman değer kümesinde bir ve yalnız bir eleman ile eşleşmiştir. Grafik, fonksiyona aittir. 2 Grafik, dikey doğru testinden geçmektedir. Grafik, fonksiyona aittir. 1 Soru üzerinde herhangi bir işlem yapılmamış.0	
d)	l: {a,b,c,d}→ {a,b,c,d}, y=l(x) 	Fonksiyon/ Fonksiyon değil

	Açıklama: Tanım kümesindeki "a" elemanı değer kümesinden birden fazla eleman ile eşleşmiştir. Grafik, fonksiyona ait değildir. 2 Grafik, dikey doğru testinden geçmemektedir. Grafik, fonksiyona ait değildir. 1 Soru üzerinde herhangi bir işlem yapılmamış.0

4) Park görevlisi, göldeki suyun derinliğini günün aynı saatinde olmak üzere belli bir süre boyunca ölçmüş ve sonuçları aşağıdaki tabloya not almıştır.



Ölçüm Sırası	Gün Sayısı	Gün sayısındaki değişim	Gölün derinliği (m)	Derinlikteki değişim (m)
1.ölçüm	7.gün	-	15.29	-
2.ölçüm	14.gün	7	15.43	0.14
3.ölçüm	28.gün	14	15.57	0.14
4.ölçüm	35.gün	7	15.71	0.14
5.ölçüm	42.gün	7	15.85	0.14

a) Tablodaki bilgilere göre, gün sayısındaki değişime bağlı olarak **gölün derinliğinin** nasıl değiştiğini açıklayınız.

Gölün derinliğinin değişimini doğru ifade etmiştir. 1 Gölün derinliğinin değişimini doğru ifade etmiştir; fakat değişim hızını yanlış ifade etmiştir. 1 Soru üzerinde herhangi bir işlem yapılmamıştır. / Gölün derinliğinin değişimini yanlış ifade etmiştir. 0

b) Tablodaki bilgilere göre, gün sayısındaki değişime bağlı olarak **gölün derinliğindeki değişimin** nasıl değiştiğini açıklayınız.

Gölün derinliğindeki artış hızının 1 ve 2, 3 ve 5. ölçümler arası aynı, 2 ve 3. ölçümler arasında artış hızının yavaşladığını belirtmiştir. 2 Gölün derinliğinin artış hızının aynı olduğunu belirtmiştir. 1 Soru üzerinde herhangi bir işlem yapılmamıştır. / Gölün derinliğinin değişim hızını yanlış ifade etmiştir. 0

c) Tablodaki bilgilere göre, **derinlik-zaman fonksiyonunun** grafiğini çiziniz. (Size verilen gride çiziniz.)

Değişkenler uygun eksene girilmiştir. 1 Tablodaki veriler koordinat sistemine girilmiştir. 1 7-14, 28-42 günleri arasında eğimi 0,02 olan, 14-28 günleri arasında eğimi 0,01 olan parçalı tanımlı fonksiyonun grafiği çizilmiştir. 1 Soru üzerinde herhangi bir işlem yapılmamıştır./Fonksiyonunun grafiğini yanlış çizmiştir.0

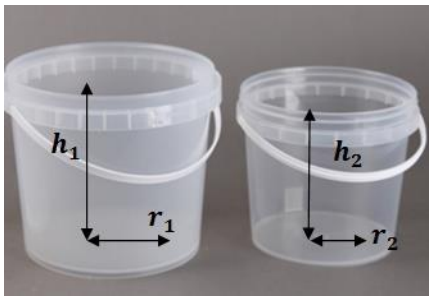
d) Çizdiğiniz grafiği kullanarak, zamana bağlı derinlik *değişim hızını* / *hızlarını* hesaplayınız. (Size verilen grid kâğıdında hesaplamayı gösteriniz.)

Zamana bağlı derinlik değişim hızlarını doğru bulmuştur. 2 7-14, 28-42 arası değişim hızını doğru bulmuştur. 1 14-28 arası değişim hızını doğru bulmuştur.1 Soru üzerinde herhangi bir işlem yapılmamıştır. / Zamana bağlı derinlik değişim hızlarını yanlış hesaplamıştır. 0

e) Bulduğunuz değişim hızını / hızlarını yorumlayınız.

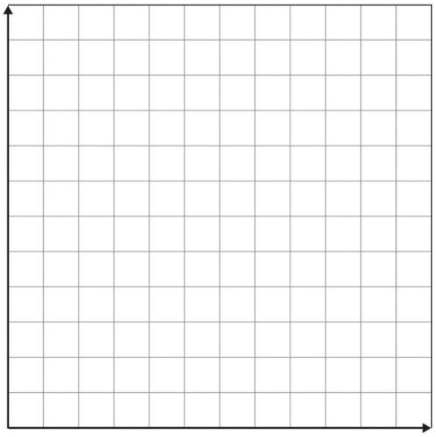
Artış hızının 14-28 günleri arasında yavaşladığını belirtmiştir. 28-42 günleri arasında başlangıçtaki artış hızına döndüğünü belirtmiştir. 1 Soru üzerinde herhangi bir işlem yapılmamıştır. / Verilen cevap yanlıştır. 0

5) Bahçedeki çiçekleri sulamak için yandaki kovalar ile su taşınacaktır. Kovaların boyutları arasında $h_1 > h_2$ ve $r_1 > r_2$ ilişkisi vardır.



Kovalara birim zamanda eşit miktarda su akıtan çeşmelerden su doldurulacaktır. Kovalar aynı anda doldurulmaya başlanıyor ve kovalardaki suyun yüksekliği ile ilgili gözlem yapıyorsunuz. Buna göre,

a) Her bir kovadaki suyun yüksekliğini zamana bağlı gösteren fonksiyonun grafiklerini çiziniz.



Her iki grafiğin başlangıç noktası orijindir. 1
Her iki kova su ile dolana kadar geçen süre için çizilen grafik doğrusal olarak çizilmiştir.1
Her iki kova su ile dolduktan sonraki süre için çizilen grafik sabit olarak çizilmiştir.1
h_1 yüksekliğinin, h_2 yüksekliğinden büyük olduğu grafiğin y ekseninde belirtilmiştir. 1
Küçük kovanın grafiğinin doğrusal kısmının eğimi, büyük kovanın grafiğinin doğrusal kısmının eğiminden daha fazla olarak çizilmiştir.1
Soru üzerinde işlem yapılmamıştır. 0

b) Her iki kovadaki suyun yüksekliğinin değişim hızını karşılaştırınız ve yorumlayınız.

Her iki kovadaki suyun yüksekliğinin değişim hızı doğru şekilde kıyaslanmış ve yapılan yorum doğrudur. 2
Her iki kovadaki suyun yüksekliğinin değişim hızı doğru şekilde kıyaslanmıştır. 1
Soru üzerinde işlem yapılmamıştır. / Verilen cevap yanlıştır. 0

6) …… Lisesi öğrencileri, Sosyal Etkinlik Çalışmaları kapsamında bir kulüp seçmek ve düzenli olarak kulüp çalışmalarına katılmak durumundadırlar.

Yukarıda bir model tanımlanmıştır. Bu modelde, ……. Lisesi öğrencileri ve katıldıkları kulüpler arasında bir ilişki verilmiştir. Bu ilişkinin,

a) bire-bir fonksiyon belirtip belirtmediğini **nedenleri ile açıklayınız.**

İlişkinin bire-bir fonksiyon belirtip belirtmediği nedeni ile verilmiştir. 2 Soru üzerinde işlem yapılmamıştır. / Verilen cevap yanlıştır. 0

B

b) örten fonksiyon belirtip belirtmediğini **nedenleri ile açıklayınız.**

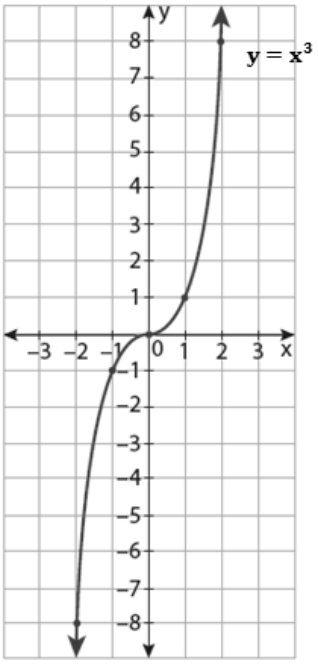
İlişkinin örten fonksiyon belirtip belirtmediği nedeni ile verilmiştir. 2 Soru üzerinde işlem yapılmamıştır. / Verilen cevap yanlıştır. 0

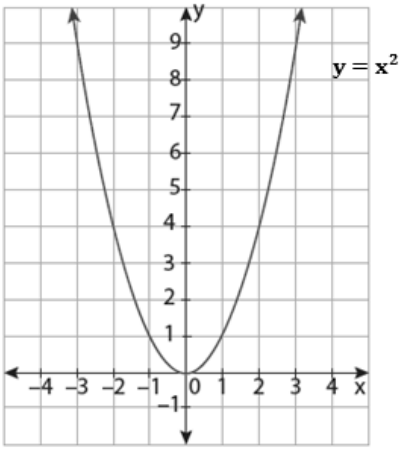
7) Marketlerde satılan ürünler ve her birine ait barkod numaraları birer değişken olacak şekilde bir fonksiyon modeli yazınız. Yazdığınız modeldeki ilişkinin bire-bir ama örten olmayan bir fonksiyon belirtmesini sağlayınız.

Model:

Yazılan modeldeki ilişki bire-bir ama örten olmayan fonksiyon belirtir. 2
Yazılan modeldeki ilişki bire-bir fonksiyon belirtir; ama örten fonksiyon belirtmez.1
Yazılan modeldeki ilişki bire-bir fonksiyon belirtmez; ama örten fonksiyon belirtir.1
Yazılan modeldeki ilişki istenilen koşullara uygun değildir. 0
Soru üzerinde işlem yapılmamıştır. 0

8) $y = x^3$ ve $y = x^2$ grafikleri aşağıda verilmiştir. İki grafik arasındaki benzerlikleri ve farklılıkları yorumlayınız.





İki grafiğin bir fonksiyona ait olup olmadığı doğru şekilde belirtilmiştir. 1
İki grafiğin bire-bir olup olmadığı doğru şekilde belirtilmiştir. 1
İki grafiğin örten olup olmadığı doğru şekilde belirtilmiştir. 1
Grafikleri verilen fonksiyonların tanım kümesinin benzerlikleri doğru şekilde belirtilmiştir. 1
Grafikleri verilen fonksiyonların görüntü kümesinin farklılıkları doğru şekilde belirtilmiştir. 1
Grafiklerin simetri durumları doğru şekilde kıyaslanmıştır. 1
Soru üzerinde işlem yapılmamıştır. / Verilen tüm açıklamalar yanlıştır. 0

9) $f: R \to R$ ve $f(x) = 5x - 4$ ile verilen f fonksiyonu için ? yerine ne geleceğini bulunuz.

a) $f(2) = ?$

Step 1: x yerine 2 yazıp f(2) nin değerini bulmak $f(2) = 5.2 - 4 = 6$	Step 1'i uygulayarak doğru cevabı bulmuştur. 1 Step 1'i uygulamıştır; fakat işlem hatası yaptığından dolayı yanlış cevap bulmuştur. 1 Soru üzerinde işlem yapılmamıştır. 0

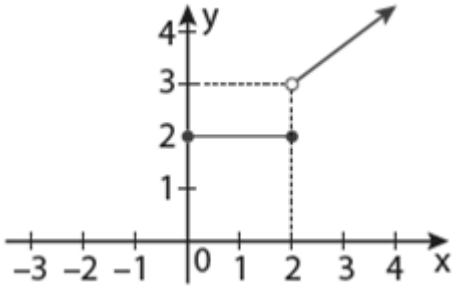
b) $f(?) = 6$

Step 1: Çıktı değerini 6'ya eşitlemek $5x - 4 = 6$ $5x = 10$ $x = 2$	Step 1'i uygulayarak doğru cevabı bulmuştur. 1 Step 1'i uygulamıştır; fakat işlem hatası yaptığından dolayı yanlış cevap bulmuştur. 1 Soru üzerinde işlem yapılmamıştır. 0

c) $f([-3,9]) = ?$

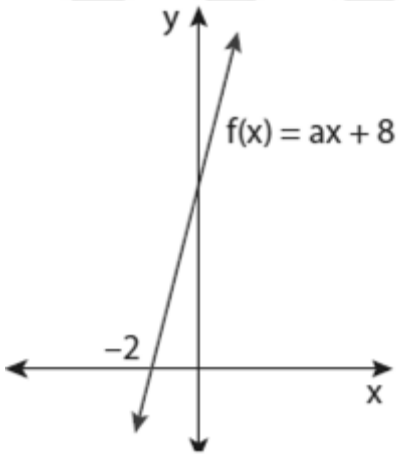
Step 1: $-3 \leq x \leq 9$ ifadesini yazmak Step 2: $5x - 4$ ifadesi için aralık bulmak $-3.5 \leq 5.x \leq 5.9$ $-15 \leq 5x \leq 45$ $-15 - 4 \leq 5x - 4 \leq 45 - 4$ $-19 \leq x \leq 41$ $[-19,41]$	Step 1 ve step 2'yi uygulayarak doğru cevabı bulmuştur. 2 Step 1 ve step 2'yi uygulamıştır; Step 1'i uygulamıştır; fakat işlem hatası yaptığından dolayı yanlış cevap bulmuştur. 2 Step 1'i uygulamıştır; fakat step 2'yi uygulamamıştır. 1 Soru üzerinde işlem yapılmamıştır. 0

10) Şekilde grafiği verilen fonksiyonun tanım ve görüntü kümesini bulunuz.



Step 1: Tanım kümesi [0,∞) Step 2: Görüntü kümesi {2}∪ (3,∞)	Step 1 ve step 2'yi uygulayarak doğru cevabı bulmuştur. 2 Step 1 uygulamıştır.1 Step 2 uygulamıştır. 1 Soru üzerinde işlem yapılmamıştır. 0

11) Aşağıda gerçek sayılarda tanımlı f(x) = ax + 8 fonksiyonun grafiği verilmiştir. Buna göre a kaçtır?

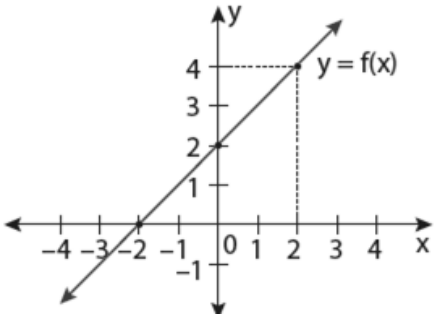


Step 1: Grafik üzerindeki (-2,0) noktası fonksiyonun kuralında yerine yazılarak a değeri bulmak $0 = a.-2 + 8$ $2a = 8$ $a = 4$	Step 1'i uygulayarak doğru cevabı bulmuştur. 1 Step 1'i uygulamıştır; fakat işlem hatası yaptığından dolayı yanlış cevap bulmuştur. 1 Soru üzerinde işlem yapılmamıştır. 0

12) Yanda **doğrusal f** fonksiyonun grafiği verilmiştir. Buna göre,

a) verilen tabloyu doldurunuz.

x	f(x)=y
-2	
0	
1	
2	



Step 1: Grafik üzerindeki noktalardan verilen x değerlerine karşılık y değerlerini yazmak x: -2, 0, 1, 2 f(x)=y: 0, 2, 3, 4	Step 1'i uygulayarak tüm değerleri doğru bulmuştur. 1 Soru üzerinde işlem yapılmamıştır. 0

b) f fonksiyonunun kuralını yazınız.

Step 1: y değerlerinin x değerlerinden iki fazla olduğunu bulmak $f(x) = x + 2$	Step 1'i uygulayarak doğru cevabı bulmuştur. 1 Soru üzerinde işlem yapılmamıştır. 0

13) Aşağıda kuralları verilen doğruların **eğimlerini** bulunuz.

e) $y = \frac{x}{3} + 1$

Step 1: " y = f(x) = mx + b ile verilen doğruların eğimleri m değeridir" kullanmak $m = \frac{1}{3}$	Step 1'i uygulayarak doğru cevabı bulmuştur. 1 Soru üzerinde işlem yapılmamıştır. 0

f) $2x - 3y = 6$

Step 1: Verilen denklemi $y = mx + b$ formunda yazmak $2x - 6 = 3y$ $\frac{2x - 6}{3} = y$ Step 2: " $y = f(x) = mx + b$ ile verilen doğruların eğimleri m değeridir." kullanmak $m = \frac{2}{3}$	Step 1 ve step 2'yi uygulayarak doğru cevabı bulmuştur. 2 Step 1'i uygulanmıştır; fakat step 2'yi uygulamamıştır. Soru üzerinde işlem yapılmamıştır. 0

g) $x = 3$

Step 1: $m = \frac{\text{y değerlerindeki değişim}}{\text{x değerlerindeki değişim}}$ kuralını kullanmak m tanımsız	Step 1'i uygulayarak doğru cevabı bulmuştur. 1 Soru üzerinde işlem yapılmamıştır. 0

h) $y = 1$

Step 1: $y = 1$ doğrusunu $y = 0x + 1$ şeklinde ifade etmek $m = 0$	Step 1'i uygulayarak doğru cevabı bulmuştur. 1 Soru üzerinde işlem yapılmamıştır. 0

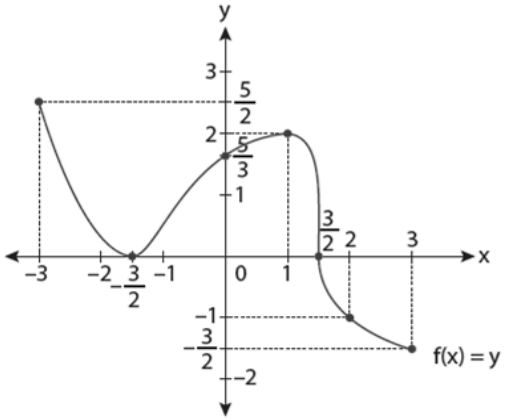
14) g: R → R ve $g(x) = (a - 2)x^2 + (b + 2)x - a.b$ fonksiyonu **sabit bir fonksiyon** ise $g(2017)$ kaçtır?

Step 1: $a-2$ ve $b+2$ ifadelerini 0 değerine eşitlemek $a-2=0 \Longrightarrow a=2$ $b+2=0 \Rightarrow b=-2$ Step 2: a ve b nin değerlerini yerine yazıp g(2017) 'nin değerini bulmak $g(2017)=0.2017^2+0.2017-2.-2=4$	Step 1 ve step 2'yi uygulayarak doğru cevabı bulmuştur. 2 Step 1 ve step 2'yi uygulamıştır; fakat işlem hatası yaptığından dolayı yanlış cevap bulmuştur. 2 Step 1'i uygulamıştır; fakat step 2'yi uygulamamıştır. 1 Soru üzerinde işlem yapılmamıştır. 0

15) k: R → R ve k(x) = ax + b − 4x + 8 fonksiyonu **birim fonksiyon** ise a ve b'nin değerini bulunuz.

Step 1: Fonksiyonun kuralını düzenlemek $k(x)=(a-4)x+b+8$ Step 2: $k(x)=(a-4)x+b+8=x$ eşitlemek $a-4=1 \Rightarrow a=5$ $b+8=0 \Rightarrow b=-8$	Step 1 ve step 2'yi uygulayarak doğru cevabı bulmuştur. 2 Step 1 ve step 2'yi uygulamıştır; fakat işlem hatası yaptığından dolayı yanlış cevap bulmuştur. 2 Step 1'i uygulamış; fakat step 2'yi uygulamamıştır. 1 Soru üzerinde işlem yapılmamıştır. 0

16) f fonksiyonunun grafiği şekildeki gibidir. Buna göre;



a. f fonksiyonunun tanım ve görüntü kümelerini bulunuz.

Step 1: Tanım kümesi $[-3,3]$ Step 2: Görüntü kümesi $[-\frac{3}{2},\frac{5}{2}]$	Step 1 ve step 2'yi uygulayarak doğru cevabı bulmuştur. 2 Step 1'i uygulamıştır.1 Step 2'i uygulamıştır. 1 Soru üzerinde işlem yapılmamıştır./ Yanlış cevap verilmiştir. 0

b. $f(-3)$'ün değerini bulunuz.

Step 1: Grafik üzerindeki noktalardan verilen x değerine karşılık y değerini yazmak $f(-3) = \frac{5}{2}$	Step 1'i uygulayarak doğru cevabı bulmuştur. 1 Soru üzerinde işlem yapılmamıştır./ Yanlış cevap verilmiştir. 0

c. $f(2)$'nin değerini bulunuz.

Step 1: Grafik üzerindeki noktalardan verilen x değerine karşılık y değerini yazmak $f(2) = -1$	Step 1'i uygulayarak doğru cevabı bulmuştur. 1 Soru üzerinde işlem yapılmamıştır./ Yanlış cevap verilmiştir. 0

d. $f(x) = \frac{5}{2}$ sağlayan x değerini / değerlerini bulunuz.

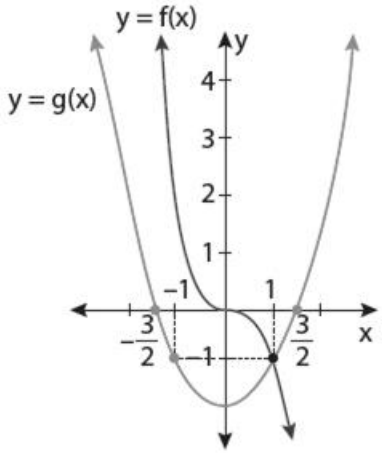
Step 1: Grafik üzerindeki noktalardan verilen y değerine karşılık x değerini/ değerlerini yazmak $x = -3$	Step 1'i uygulayarak doğru cevabı bulmuştur. 1 Soru üzerinde işlem yapılmamıştır./ Yanlış cevap verilmiştir. 0

e. Aşağıdaki tabloyu doldurunuz. Her biri için sadece bir örnek veriniz.

		Örnek
a)	**Ters görüntüsü tek olan** görüntü kümesinden eleman	
b)	**Ters görüntüsü iki tane** olan görüntü kümesinden eleman	
c)	**Ters görüntüsü üç tane olan** görüntü kümesinden eleman	

a)	Verilen örnek istenilen şarta uygundur. 1 Soru üzerinde işlem yapılmamıştır. / Verilen örnek istenilen şarta uygun değildir. 0
b)	Verilen örnek istenilen şarta uygundur. 1 Soru üzerinde işlem yapılmamıştır. / Verilen örnek istenilen şarta uygun değildir. 0
c)	Verilen örnek istenilen şarta uygundur. 1 Soru üzerinde işlem yapılmamıştır. / Verilen örnek istenilen şarta uygun değildir. 0

17) Gerçek sayılar kümesinde tanımlı f ve g fonksiyonlarının grafikleri verilmiştir. Buna göre;



a. f(x)=0 sağlayan x değerini/değerlerini bulunuz.

Step 1: f fonksiyonun grafiği üzerindeki noktalardan y değeri 0 olan noktanın x koordinatını okumak $f(x) = 0 \Rightarrow x = 0$	Step 1'i uygulayarak x değerini bulmuştur. 1 Soru üzerinde işlem yapılmamıştır. / Verilen cevap yanlıştır. 0

b. g(x)=0 sağlayan x değerini/değerlerini bulunuz.

Step 1: g fonksiyonun grafiği üzerindeki noktalardan y değeri 0 olan noktanın x koordinatını okumak $g(x) = 0 \Rightarrow x = -\frac{3}{2}$ veya $x = \frac{3}{2}$	Step 1'i uygulayarak her iki x değerini bulmuştur. 2 Step 1'i uygulayarak bir tane x değerini bulmuştur. 1 Soru üzerinde işlem yapılmamıştır. / Verilen cevaplar yanlıştır. 0

c. f(x) = g(x) eşitliğini sağlayan x değerini / değerlerini bulunuz.

Step 1: f ve g fonksiyonlarının grafiklerinin kesişim noktasının x değerini / değerlerini okumak $f(x) = g(x) \Rightarrow x = 1$	Step 1'i uygulayarak x değerini bulmuştur. 1 Soru üzerinde işlem yapılmamıştır. / Verilen cevap yanlıştır. 0

18) Tanım kümesi tüm gerçek sayılar olan bir f fonksiyonu parçalı tanımlı olarak aşağıdaki gibi veriliyor:

$$f(x) = \begin{cases} 3x-2 & , x < -5 \\ \dfrac{x+1}{x^2+1} & , -5 \le x < 0 \\ 2x+4 & , 0 \le x \end{cases}$$ Buna göre aşağıda verilenleri bulunuz.

a) $f(-5)$

b) $f(-10)$

c) $f(x^2)$

a)	Step 1: $f(-5) = \frac{-5+1}{(-5)^2+1} = \frac{-4}{26}$	Step 1'i uygulayarak doğru cevabı bulmuştur. 1 Step 1'i uygulamıştır; fakat işlem hatası yaptığından dolayı yanlış cevap bulmuştur. 1 Soru üzerinde işlem yapılmamıştır. 0
b)	Step 1: $f(-10) = 3.-10-2 = -32$	Step 1'i uygulayarak doğru cevabı bulmuştur. 1 Step 1'i uygulamıştır; fakat işlem hatası yaptığından dolayı yanlış cevap bulmuştur. 1 Soru üzerinde işlem yapılmamıştır. 0
c)	Step 1: $f(x^2) = 2x^2 + 4$	Step 1'i uygulayarak doğru cevabı bulmuştur. 1 Soru üzerinde işlem yapılmamıştır. 0

19) Bir firmanın zamana (yıl) bağlı gelir (milyon TL) fonksiyonu f(x)= 12 – 2x, gider (milyon TL) fonksiyonu g(x) = x + 3 dir.$0 \leq x \leq 6$ verildiğine göre aşağıdaki soruları cevaplayınız.

a. f ve g fonksiyonlarının grafiklerini aynı koordinat düzleminde çiziniz.

$0 \leq x \leq 6$ aralığı dikkate alınarak bulunan sıralı ikililerden geçen birer doğru çizilmiştir. 1 f(x)=12-2x fonksiyonunun kuralını sağlayan iki tane (x,y) sıralı ikilisini koordinat düzleminde işaretlemiştir.1 g(x)=x+3 fonksiyonunun kuralını sağlayan iki tane (x,y) sıralı ikilisini koordinat düzleminde işaretlemiştir.1 f(x) ve g(x) fonksiyonlarının grafiklerinin zaman değişkeni $0 \leq x \leq 6$ aralığında gösterilmiştir. 1 Soru üzerinde işlem yapılmamıştır. / Grafikler doğru şekilde çizilmemiştir. 0

b. Gelir ve giderin eşit olduğu değeri ve zamanı bulunuz.

Step 1: $f(x) = g(x)$ $12 - 2x = x + 3$ $9 = 3x$ $x = 3$	Step 1'i uygulayarak doğru cevabı bulmuştur. 1 Step 1'i uygulamıştır; fakat işlem hatası yaptığından dolayı yanlış cevap bulmuştur. 1 Grafiklerin kesişim noktasının x koordinatı okunarak doğru cevap bulunmuştur. 1 Soru üzerinde işlem yapılmamıştır. 0

20) f: R → R olmak üzere f(x) = x^2 fonksiyonunun

a) bire-bir olup olmadığını belirtiniz.

Fonksiyonun bire-bir olup olmadığını doğru şekilde belirtmiştir. 1 Fonksiyonun bire-bir olup olmadığını yanlış şekilde belirtmiştir. 0 Soru üzerinde işlem yapılmamıştır. 0

b) örten olup olmadığını belirtiniz.

Fonksiyonun örten olup olmadığını doğru şekilde belirtmiştir. 1 Fonksiyonun örten olup olmadığını yanlış şekilde belirtmiştir. 0 Soru üzerinde işlem yapılmamıştır. 0

APPENDIX D: OPEN ENDED QUESTIONNAIRES

PRE QUESTIONNAIRE

1. Matematiği nasıl tanımlarsınız?
2. Sizce matematik en iyi nasıl öğrenilir?

POST QUESTIONNAIRE

1. Matematiği nasıl tanımlarsınız?
2. Sizce matematik en iyi nasıl öğrenilir?
3. Fonksiyonlar konusunu öğrenirken uyguladığımız yöntem/yöntemler hakkında ne düşünüyorsunuz?

APPENDIX E: JOURNAL QUESTIONS

1. Bu hafta öğrendiklerinden aklında neler kaldı? Açıklayın.
2. Fonksiyonlar konusunu öğrenirken güçlük çektin mi? Hangi konularda güçlükle karşılaştın? Açıklayın.
3. Öğrenmeni kolaylaştıran etkenler nelerdi? Açıklayın.
4. Bu hafta matematik dersine katılımını nasıl değerlendirirsin? Bir örnekle açıklayın.