PRICE DISCRIMINATION IN A NETWORK: A MODEL OF TELECOMMUNICATIONS NETWORKS

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ABSTRACT

Price Discrimination in a Network:
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This thesis considers access pricing and consumer pricing problems of telecommunications firms as a context to analyze the effects of a network structure on firms' equilibrium strategies in Bertrand competition. The ability of competitors to route connections to a given player to replicate a direct link provides a constraint on said player's ability to price discriminate between connections over different links. The thesis shows that, in equilibrium, firms create incentives to originate only direct connections, as long as direct connections have lower physical costs than indirect alternatives which is shown to be a necessary condition. An example is analyzed to illustrate a case where an indirect connection may be optimal. As in the usual price discrimination models, agents with sufficiently low demand will be left out of the market, but if they incur sufficiently low costs from connections they will be routing other agents' connections.

Key Words: Networks, Price Discrimination, Telecommunications, Bertrand Competition.

KISA ÖZET

Ağ İçinde Fiyat Ayrımcılığı:
Bir Telekomünikasyon Ağları Modeli

Selçuk Eren Günay

Bu tez, iletişim firmalarının ara bağlantı ve tüketici fiyatlandırma problemlerini ele alarak, ağ yapısının Bertrand modeli rekabette firmaların denge stratejileri üstündeki etkisini incelemektedir. Rakiplerin bir oyuncuya yönelik bağlantıları yönlendirerek doğrudan bağlantıyı taklit edebilmesi, bu oyuncunun farklı hatlar üzerindeki bağlantıların fiyatlarını ayrıştırmasını sınırlandırmaktadır. Bu tez, doğrudan bağlantılar dolaylı bağlantılardan daha düşük maliyete sahip olduğu zaman, firmaların yalnızca doğrudan bağlantıların gerçekleşmesini teşvik edeceğini göstermektedir; bu koşulun gerekli olduğu gösterilmiştir. Dolaylı bir bağlantının optimal olabileceği bir durumu sergilemek için bir örnek incelenmiştir. Bilindik fiyat ayrımcılığı modellerinde olduğu gibi, talebi yeterince düşük olan oyuncular dengede pazar dışında kalmakta; buna karşın eğer bağlantı maliyetleri yeterince düşükse, diğer oyuncuların bağlantılarını yönlendirdikleri görülmektedir.

Anahtar Kelimeler: Ağlar, Fiyat Ayrımcılığı, Telekomünikasyon, Bertrand Modeli Rekabet.

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1 Introduction

This thesis considers the effects of the network setup that define network industries on the individual firms' pricing decisions. For the most part, firms are considered to be outside the scope of regulation and only the effects of competition, as influenced by the existence of the network, are considered.

The literature on network games began with taking into consideration the effects of the connections among players that form coalitions. In one of the earliest articles associated with that literature, Myerson (1977) considered an enrichment of cooperative game theory and determined an allocation which he showed to be the unique rule satisfying a fairness criteria. That rule is a variation on Shapley value, which allocates each player in a network a payoff based not on the average contribution the player makes when added to each coalition, but the average contribution the player makes by adding a new link to each network. This allocation rule, called the Myerson value, characterized a different payoff in comparison with cooperative games which demonstrated the need to consider potential effects of a network structure. However, in Myerson's setup stable allocation rules always benefit players forming a link, therefore complete network is the only stable structure; this limitation reduces the applicability of Myerson's setup, nevertheless, the article is an important contribution in drawing attention to a richer setup.

The lacking component in Myerson's model which led to the complete network to emerge as the only stable network, was that links only had benefits but not costs. In other words, he did not endogenize the choice of network. By specifying benefits and costs to creating (and keeping) links, Jackson and Wolinsky

(1996) formally modeled stable networks, that is which networks would occur at equilibrium. They introduced the concept of pair-wise stability, which is a weak criterion that relies on single player deviations on a link-by-link basis to characterize the equlibrium. Instead of considering network formation, they characterized stable networks using that criterion at steady state. They noted that there was a trade-off between stability and efficiency and showed that there does not exist an allocation rule that guarantees both stability and efficiency.[§]

Network industries feature structural aspects that have been already considered in network games such as the connections between players that are directional and of varying intensities. In fact, these aspects, which are exactly the refinements from network games, are unalienable parts of the structure of such industries. Despite this complimentarity with network games, network industries have not been modeled within a network game framework. This thesis will attempt to consider the effects of a network game framework on network industries.

Models of telecommunication networks tend to consider the regulation of a monopolist incumbent and how it will set access prices. Taking into consideration the large fixed costs of networks, these models derive appropriate optimal Ramsey prices. Determining conditions on the incumbent's pricing so as to make entry viable is a major aspect of the literature as that gives insight into how regulation should be setup. Laffont, Rey and Tirole (1998) is an example of this literature and Laffont and Tirole (1999) provides an extensive survey of the general concerns and challenges.

This thesis considers an oligopolistic network industry where each player has

[§]For an extensive survey of network literature, see Jackson (2004).

demand for connections to other players; we consider connections to be directional and have endogenously determined intensity. Connections occur over links in a complete network which has already been established and each connection incurs the terminating party a unit cost. Each player has complete information on the demands and physical costs faced by every player. We model demand for connections to arise from the existence of customers of each firm who have demand for connections to customers of other firms.

Without the possibility of indirect connections, each player is a monopolist in fulfilling demand for connections on each link terminating at its node. However, indirect connections over the network provides incentives for players to exploit the connection of another player to reach any given player. While each firm could achieve full price discrimination as a monopolist without indirect connections, the network effects caused by indirect connections act as constraints on the level of price discrimination available.

Note that this article considers a model where the network structure is always the complete network, however; the intensity of links will be determined by the realization of demand for the connection when prices are revealed. The intensity of the link will then shape the profit generated from that link. Depending on the demands for each link and prices, some links may have zero intensity, which seems conceptually equivalent to the removal of that link for the consideration of how communication takes place. However, the existence of that link provides a threat point which influences the prices for indirect alternatives to that direct connection as well as being part of an indirect alternative connection to other direct connections. Therefore, even though a link may not have any connections

occurring over it in an equilibrium, that the network structure is the complete network has implications on the equilibrium.

This trade-off between direct connections and indirect connections causes each firm to distort prices upward and tightening the range of prices, when full price discrimination cannot be sustained given the demands for connections. When physical costs are uniform, each firm prefers to receive only direct connections. Thus, some players may be excluded from the market if their demand is sufficiently low; i.e. they originate or route zero connections. When physical costs are non-uniform, there is room for indirect connections, and some players may be excluded from the market completely or only perform routing for other players while they make no connections of their own. If a player routes connections for others, then the price faced by that player is distorted to a greater extent in the non-uniform case.

Section 2 introduces the model and notation, Section 3 considers the case of uniform costs of links and Section 4 generalizes to the case of links with non-uniform costs. Section 5 considers deregulation of a telecommunications market within a network structure. Section 6 concludes.

2 The Model

Consider consumers in distinct markets, each of whom have demand for connections to consumers in the other markets. Each market is served by a monopoly and it is unto these firms to provide consumers in their market (i.e. their customers) with connections. Since communication is bi-directional, firms may originate or terminate connections for their customers. The infrastructure is assumed to exist

and firms only incur variable costs according to the quantity of connections they serve and no fixed costs. This means that the setup corresponds to a complete network with links of varying intensity.

The index ji will be used in association with a variable to refer to a connection originated at j and terminated at i.

Firm j charges its customers p_{ji} for a connection towards a customer in market i. The demand for connections of consumers in market j to consumers in market i is given by $q_{ji}(p_{ji})$ for all markets $i \neq j$. q_{ji} is assumed to be a non-increasing function of p_{ji}.

Communication occurs through the network over costly links. Links are directed and a link from firm j to firm i is denoted (j, i). A set g of links that form a sequence of players $x_1, \cdots, x_K$ such that $(x_k, x_{k+1}) \in g$ for each $k \in \{1, \cdots, K-1\}$ with $x_1 = j$ and $x_K = i$ is a path from j to i. j is connected to i by the path g. Define the set of all such paths G_{ji}. For $g \in G_{ji}$, if $g = \{(j, i)\}$, then g is said to be a direct path; otherwise g is said to be an indirect path. It will be necessary to refer to the last link within a path, as well as the routed part of a path. For a path g, these will be denoted g^D and g^R, respectively.

If firm i is the destination for the connection it receives from firm j, then firm i charges firm j a direct connection charge of d_{ji} for terminating the connection. If, on the other hand, i is not the destination and the connection is to be relayed to some other player, firm i will charge firm j a routing charge that may depend on g, the path communication occurs over; this charge will be denoted $r_{ji}(g)$. In either case, the physical cost of each link is born by the receiving firm and is a constant marginal cost of c_{ji}.

At each step of the connection, without additional information, the receiver is unable to distinguish whether the connection is originated the previous firm on the path or if it has been routed. Therefore, the receiver cannot price discriminate to discourage routing. Conversely, routers must necessarily have information on the destination of the connection and may choose to price discriminate based on that.

For notational simplicity, some additional variables will be defined. Define the total routing charges for a connection over g, $R(g) = \sum_{(m,n)\in g^R} r_{mn}(g)$ and the direct connection charges collected by the terminating player, $D(g) = d(g^D)$. Note that if $g = \{(j,i)\}$, $D(g) = d_{ji}$ and $R(g) = 0$. Thus, total cost of a connection is $T(g) = R(g) + D(g)$. In similar fashion, define $C(g) = \sum_{(m,n)\in g} c_{mn}$, the total actual cost incurred by all agents involved in establishing the connection using the path g.

The game has two stages. In the first stage, each firm i chooses and announces d_{ji} and $r_{ji}(g)$, for all $j \neq i$ and for all $g \in G_{mn}$ where $m \neq n$. That is, firms choose a routing charge for all paths of communication. In the second stage, each firm chooses p_{ji} for all $j \neq i$ to charge its customers for connections, taking into account prices announced in the first stage. The game is played once and perfect and complete information is assumed.

Each monopolist faces no competition when serving its customers, as its market is isolated from the other players and the firm can use its monopoly power fully. Then, the second-stage of the game is the trivial monopoly pricing problem. Each monopolist needs to decide on price, given a demand function and cost. As the solution to the second stage of the game, each monopolist would

charge its customers the monopoly price, which leads to the monopoly quantity. Since the equilibrium quantity depends solely on the cost of connection over the path chosen by the monopolist, equilibrium demand can be denoted by $Q_{ji}(T(g))$ a non-increasing function of the cost of connection $T(g)$ over the path g. Notice that Q_{ji} is not defined as a function of the price the monopolist charges its customers, but as a function of the cost the monopolist actually faces. P_{ji} is the price the monopolist chooses, as a function of the cost of connection. The path chosen will be the one with the lowest total cost, that is $g^* = \min_{g \in G_{ji}} T(g)$. Thus, there is a Bertrand competition between the receiver offering a direct connection and a set of players who replicate that with an indirect connection.

When serving other firms with connections to its customers, firms are no longer free from competition as the other firms may establish indirect connections as alternatives to the direct connection offered by the firm. If routing was not possible, each firm could act as a monopolist in that market as well and set connection charges to maximize its profit without regard to what other firms are doing. The main distinguishing feature of this network setup, then, is that other firms may offer to route the connection.

Proceeding with backward induction, first stage payoffs need to be determined. Each monopolist has three sources of profit: Π^i_{ji} denotes profit from terminating connections from firm j and Π^i_{ij} denotes profits from originating connections to firm j and Π^i_R denotes profits from routing connections. Then total profits for firm i is the sum:

$$\Pi^i = \sum_{j \in N \setminus \{i\}} (\Pi^i_{ji} + \Pi^i_{ij}) + \Pi^i_R,$$

where

$$\Pi^i_{ji} = \begin{cases} Q_{ji}(d_{ji})(d_{ji} - c_{ji}), & \text{if } g^*_{ji} = \{(j,i)\}, \\ Q_{ji}(T(g^*_{ji}))(D(g^*_{ji}) - c(g^{*D}_{ji}), & \text{if } g^*_{ji} \neq \{(j,i)\}, \end{cases}$$

$$\Pi^i_{ij} = Q_{ij}(T(g^*_{ji}))(P_{ij}(T(g^*_{ji})) - T(g^*_{ji})),$$

$$\Pi^i_R = \sum_{\substack{\{k|\{k,i\}\in g^*_{mn} \\ \text{and } m,n\notin\{k,i\}\}}} Q_{mn}(T(g^*_{mn}))(r_{ki}(g^*_{mn}) - c_{ki}).$$

The objective of each firm i is to choose d_{ji} and $r_{ji}(g)$ for all $j \neq i$ and $g \in G_{mn}$ for all $m \neq n$ to maximize Π^i.

3 Uniform Costs

In order to arrive at some insights about the effects of the network, first consider the simplifying assumption of uniform physical costs, that is $c_{ji} = c$, $\forall i, \forall j \neq i$. Insights learned in the analysis of this setup will prove valuable when the analysis is generalized to consider non-uniform physical costs.

Whether indirect paths will be utilized at equilibrium can be easily analyzed comparing how direct and indirect paths are evaluated by the terminating player. Profits from connections between each pair of agents are separable and therefore can be considered individually. When the direct path from a player is compared to any alternate indirect path, the direct path proves more profitable to the terminating player.

Lemma 3.1. *Only direct connections will be observed at equilibrium and firms' equilibrium strategies are constrained by:*

$$d_{ji} \leq T(g), \qquad \forall g \in G_{ji}, \qquad \forall i \in N, \forall j \neq i. \tag{1}$$

Proof. Assume that d_{mn} and $r_{mn}(g)$ $\forall m, n \in I$ and $\forall g$ constitute an equilibrium and that $d_{ji} > T(g)$ for some j and $g \in G_{ji}$ (then, player j prefers g to the direct path $\{(j,i)\}$). Consider the deviation strategy $\hat{d}_{mi} = d_{mi}, \forall m \neq j$ and $\hat{d}_{ji} = T(g)$. Compare the profits from g and $\{(j,i)\}$ given the new price strategy.

$$Q_{ji}(T(g))(D(g) - c),$$

$$Q_{ji}(T(g))(T(g) - c),$$

Then it is trivial to establish that

$$Q_{ji}(T(g))(D(g) - c) < Q_{ji}(T(g))(T(g) - c),$$

since $T(g) > D(g)$ when g is an indirect path and routing is not costless. Firm i would prefer that the direct path is chosen since the profit is greater. Then, it follows that no equilibrium strategy exists such that $d_{ji} > T(g)$ for some j and $g \in G_{ji}$, thus equilibrium strategies are constrained as stated above in (1). □

When terminating connections to its subscribers, each monopolist can only capture the direct connection charge related to the final link in the path chosen. While this charge is the entire cost of the connection when a direct path is taken, it is only a part of the total cost of an indirect path. Then, if an indirect path is ever preferred, a direct path of the same cost will bring greater profit to the terminating party and will lead to deviation. In any equilibrium, therefore, direct connection charges must be chosen so that direct paths are no more expensive than indirect paths.

The key insight is that connections to customers of a player are in the monopoly of that player and the player influences the cost of any connection to them, direct or indirect. It should be noted that connection through a direct path is efficient

in this case. The terminating player can capture the increase in efficiency by taking advantage of its monopoly position in this context and increase its profit. which helps increase the profit of the terminating player. Thus, it is intuitive that each player would set prices so that only direct paths are taken for connections towards its customers.

Since the cheapest indirect path creates the constraint on the pricing of direct connections, this best alternative needs to be characterized. Any indirect path should be short and simple, as will be established below.

Lemma 3.2. *Within an equilibrium involving an indirect path, there can be only one firm routing the indirect connection.*

Proof. Consider an indirect path $g \in G_{ji}$, such that $g = \{(x_k, x_{k+1})\}_{k=0}^{s}$, where $x_0 = j$ and $x_{s+1} = i$. This path costs $T(g)$ to player j. Each firm x_k, $k \in \{1, s\}$ incurs c in physical costs while routing and charges the previous player in the path x_{k-1}, $r_{(x_{k-1},x_k)}(g)$ to route connections. In this arrangement, x_s receives

$$Q_{ji}(T(g))(r_{(x_{s-1},x_s)}(g) - c).$$

Now consider the path $\hat{g} = \{(j, x_s), (x_s, i)\}$ with the cost $T(\hat{g}) = r_{(j,x_s)}(\hat{g}) + d(x_s, i)$. If x_s sets $r_{(j,x_s)}(\hat{g}) < R(g)$, then $T(\hat{g}) < T(g)$ and firm j will prefer $\hat{g}$ to g. Now, x_s receives

$$Q_{ji}(T(\hat{g}))(r_{(j,x_s)}(\hat{g}) - c),$$

instead. It follows that $r_{(j,x_s)}(\hat{g}) \geq r_{(x_{k-1},x_k)}(g)$ is sufficient to have

$$Q_{ji}(T(\hat{g}))(r_{(j,x_s)}(\hat{g}) - c) \geq Q_{ji}(T(g))(r_{(x_{k-1},x_k)}(g) - c),$$

which holds with strict inequality when $Q_{ji}(T(\hat{g})) > 0$. Thus, x_s would prefer that j chooses path $\hat{g}$.

It is easy to see that firm j also prefers path $\hat{g}$. Consider firm j's profits under the two schemes. Over the path g, j receives

$$Q_{ji}(T(g))(p_{ji}^{M}(T(g)) - T(g)),$$

whereas over the path $\hat{g}$, setting the possibly non-optimal price of $P_{ji}(T(g))$, j would instead receive

$$q_{ji}(P_{ji}(T(g)))(P_{ji}(T(g)) - T(\hat{g})),$$

which is higher. Notice that the equilibrium quantity and price would be the same, but firm j would have lower costs per connection and therefore higher profits. Thus, firm j would also prefer $\hat{g}$ to g. □

It should be repeated that firms do not know how a connection has been routed before it reaches the player who relays it to her. Therefore, x_s cannot deviate from being a part of g_{ji}, for the very reason that she does not know she is a part of it. However, given each firm will choose the lowest path connection between itself and its destination, firms can determine which path will be chosen for an indirect connection. Thus, at equilibrium, x_s will choose its routing pricing to be so that firm j will prefer $\hat{g}$. Thus, at equilibrium, any indirect connection cannot involve more than one player who is routing it. This result is analogous to the conclusion from the perspective of the terminating party; each routing player would prefer that the originating party takes the direct path to reach it.

Given the conclusion above, now it should be characterized which player could route the connection and at what price.

Lemma 3.3. *Connections may be routed only at physical cost.*

Proof. Under the assumption of perfect and complete information, each firm is equally able to solve each others' optimization problem. Firms would anticipate that their counterparts will also make their decisions given the constraints (1), therefore when choosing their routing charges, they will take into account these constraints that their counterparts face. Consider that firm x might route connections between j and i such that $g = \{(j,x),(x,i)\}$ at a price $r_{jx}(g)$ and as a best response, firm i chooses prices according to (1) given the routing cost is $T(g) = r_{jx}(g) + d_{xi}$. Firm x will want to deviate from $r_{jx}(g)$ so that (1) no longer hold for i.

Consider, then, as case 1, that in addition to (1), $d_{ji} \leq c + d_{xi}$. For routing to occur, then, x needs to set $r_{jx}(g) < d_{ji} - d_{xi} \leq c$. However, if routing were indeed to occur, firm x would generate a profit of $r_{jx}(g) - c$ for each connection routed, which is clearly negative. Thus, any strategy $r_{jx}(g) < c$ is weakly dominated by the strategy $r_{jx}(g) = c$. This provides a lower limit to routing charges.

Consider as case 2, that $d_{ji} > c + d_{xi}$. Now firm x stands to improve its profit by changing its strategy to $\hat{r}_{jx}(g) \in (c, r_{jx}(g))$ such that (1) no longer holds given firm i's choices of prices. That would imply that there are positive profits from routing, however, then firm i's strategy is not optimal since it was already established that direct connections generate more profit than indirect connections. Thus, firm i should change its strategy to satisfy (1), given the new routing cost. It should be obvious that, as established in case 1, unless $d_{ji} \leq c + d_{xi}$, any firm x can set $\hat{r}_{jx}(g) > c$ to generate positive routing profits. □

Then, the constraint at equilibrium on direct connection charges, given by (1),

becomes:

$$d_{ji} \leq c + d_{xi}, \qquad \forall j \neq i \neq x. \tag{2}$$

When these constraints are satisfied, at the resultant prices no firm would find indirect connections preferable, as they must necessarily cost at least as much as direct connections. Thus, in equilibrium, firms will never use strategies that involve routing of calls.

Characterizing the equilibrium is significantly simplified, since firms should eliminate strategies involving routing of calls. Thus, the problem is reduced to optimizing the prices charged to each firm given the constraint of no routing given by (2). Constraints (2) imply that the difference between the lowest and highest connection charges needs to be no greater than the physical connection cost c. This constraint provides a limit to the degree of price discrimination.

Remark. *The network limits the price discrimination power of each firm. As the costs of links over the network decrease, the ability to price discriminate is further restricted.*

Lemmas 3.1, 3.2 and 3.3 lead to the following proposition.

Proposition 3.4. *In equilibrium only direct connections will be observed. Direct connection charges are can differ no more than the physical cost of links:*

$$\max_{x \in N \setminus \{i\}} \{d_{xi}\} - \min_{x \in N \setminus \{i\}} \{d_{xi}\} \leq c.$$

Corollary 3.5. *If links do not impose physical costs, then the receiver chooses uniform pricing.*

3.1 Characterizing the Equilibrium

The monopolist's problem, in its role of receiving connections, is then to determine the optimal pricing that can be sustained in a non-cooperative equilibrium.

$$\max_{d} \sum_{j \in N \setminus \{i\}} Q_{ji}(d_{ji})(d_{ji} - c) \text{ subject to } \max_{x \in N \setminus \{i\}} \{d_{xi}\} - \min_{x \in N \setminus \{i\}} \{d_{xi}\} \leq c.$$

Consider firm i chooses a lower-bound $\underline{d}$ and an upper-bound $\bar{d} = \underline{d} + c$ and chooses $d_{xi} \in [\underline{d}, \bar{d}]$, $\forall x \in N \setminus \{i\}$. Then, firm i has available a strategy d_{xi} only if $d_{xi} \in [\underline{d}, \bar{d}]$. Denote d_{xi}^M the optimal monopoly price for a direct connection when firms are unable to route connections for each other. d_{xi}^M is the optimal price when there are no constraints, thus $d_{xi}^* = d_{xi}^M$ if d_{xi}^M is available. If there exists $y \in N \setminus \{i\}$ such that d_{yi}^M is unavailable, then it must be that $d_{yi}^M < \underline{d}$ or $d_{yi}^M > \bar{d}$. By concavity of Π_{yi}^i, if the former holds, then $d_{yi}^* = \underline{d}$ and if the latter holds, then $d_{yi}^* = \bar{d}$.

Now firm i will need to choose $\underline{d}$ to maximize its profits. This simplified problem can be expressed as:

$$\max_{\underline{d}} \sum_{j | d_{ji}^M \leq \underline{d}} Q_{ji}(\underline{d})(\underline{d} - c) + \sum_{j | d_{ji}^M \in (\underline{d}, \underline{d}+c)} Q_{ji}(d_{ji}^M)(d_{ji}^M - c) + \sum_{j | d_{ji}^M \geq \underline{d}+c} Q_{ji}(\underline{d}+c)(\underline{d}+c-c).$$

The optimal $\underline{d}$ will solve the following first-order condition:

$$\sum_{j | d_{ji}^M \leq \underline{d}} Q'_{ji}(\underline{d})(\underline{d} - c) + Q_{ji}(\underline{d}) + \sum_{j | d_{ji}^M \geq \underline{d}+c} Q'_{ji}(\underline{d}+c)(\underline{d}+c-c) + Q_{ji}(\underline{d}+c) = 0,$$

which reduces to the condition given in the following proposition.

Proposition 3.6. *Optimal choice of direct connection charges satisfy*

$$\sum_{j | d_{ji}^M \leq \underline{d}} Q'_{ji}(\underline{d})(\underline{d} - c) + Q_{ji}(\underline{d}) + \sum_{j | d_{ji}^M \geq \underline{d}+c} Q'_{ji}(\underline{d}+c)(\underline{d}) + Q_{ji}(\underline{d}+c) = 0. \quad \text{(FOC)}$$

Since Π_{ji}^i is concave for all j, so is $\sum_{j \neq i} \Pi_{ji}^i$ and the SOC is trivially satisfied. It follows that the above FOC is sufficient for optimality.

3.2 Case of Linear Demand

For an analytical solution of the model, consider the case of linear demand functions. Consider a simplified setting where the linear demand is given by:

$$q_{ji} = \begin{cases} a_{ji} - p_{ji}, & \text{if } a_{ji} \geq p_{ji}, \\ 0, & \text{if } a_{ji} < p_{ji}. \end{cases}$$

Demand functions are differentiated by the parameter a_{ji} which completely describes the demand from each market to every other market. Given the result of Proposition 3.4, there will be no routing and each originating peer that firm i faces will choose the strategy given below

$$Q_{ji}(d_{ji}) = \begin{cases} \frac{a_{ji}-d_{ji}}{2}, & \text{if } a_{ji} \geq p_{ji}, \\ 0, & \text{if } a_{ji} < p_{ji}. \end{cases}$$

leading to the equilibrium price of

$$P_{ji}(d_{ji}) = \frac{a_{ji} + d_{ji}}{2}.$$

As mentioned before, routing were not possible and the monopolist were able to have the other agents always establish direct connections, then the optimal price the monopolist would charge would be $d_{ji}^M = \frac{a_{ji}+c}{2}$ and each of its peers would charge their customers $P_{ji} = \frac{3a_{ji}+c}{2}$ for originating connections.

When there is the possibility of alternate routing, however, there is also the possibility of a different equilibrium. However, it should be noted that for some levels of demand, monopoly pricing could be the only equilibrium, even when routing is possible. Example 1 below illustrates such a case.

Proposition 3.7. *Direct connection charges d_{ji} are ranked by the demand parameters a_{ji}, in the same order.*

Proof. For $a_{ji} \geq a_{ki}$, compare an optimal strategy with $d_{ji} < d_{ki} = \hat{d}$ and another $d_{ji} = d_{ki} = \hat{d}$. If conditions (2) hold for the former, then they hold for the latter, since setting two fees equal trivializes the system of constraints by replicating some of them. By optimality of the former strategy

$$\frac{a_{ki} - \hat{d}}{2}(\hat{d} - c) \geq \frac{a_{ki} - d_{ji}}{2}(d_{ji} - c).$$

Trivial manipulation of this inequality leads to

$$\begin{aligned}
\frac{a_{ji} - \hat{d}}{2}(\hat{d} - c) &= \frac{a_{ki} - \hat{d}}{2}(\hat{d} - c) + \frac{a_{ji} - a_{ki}}{2}(\hat{d} - c), \\
&> \frac{a_{ki} - d_{ji}}{2}(d_{ji} - c) + \frac{a_{ji} - a_{ki}}{2}(\hat{d} - c), \\
&> \frac{a_{ki} - d_{ji}}{2}(d_{ji} - c) + \frac{a_{ji} - a_{ki}}{2}(d_{ji} - c), \\
&= \frac{a_{ji} - d_{ji}}{2}(d_{ji} - c),
\end{aligned}$$

which means that profits of agent i is higher in the latter strategy and thus, $d_{ji} < d_{ki}$ cannot be part of an optimal strategy. □

Now, without loss of generality, consider a reordering of agents $j \neq i$ such that $a_{1i} \leq a_{2i} \leq \cdots \leq a_{(n-1)i}$. Given this reordering, conditions (2) become:

$$d_{ti} \leq d_{(t+1)i}, \qquad \forall t \in \{1, 2 \cdots, n-1\} \qquad \text{and} \qquad d_{(n-1)i} - d_{1i} \leq c.$$

With the reordering above, the Lagrangian representation of the maximization problem agent i faces becomes:

$$\mathcal{L} = \sum_{t=1}^{n-1} \max\left\{0, \frac{a_{ti} - d_{ti}}{2}\right\}(d_{ti} - c) + \sum_{t=1}^{n-2} \gamma_t (d_{(t+1)i} - d_{ti}) + \delta(c - d_{(n-1)i} + d_{1i}).$$

We can use (FOC) to specify the relevant first-order condition:

$$\sum_{\{j | d_{ji}^M \leq \underline{d}\}} Q'_{ji}(\underline{d})(\underline{d} - c) + Q_{ji}(\underline{d}) + \sum_{\{j | d_{ji}^M \geq \underline{d}+c\}} Q'_{ji}(\underline{d} + c)(\underline{d} + c - c) + Q_{ji}(\underline{d} + c) = 0.$$

Note that if $\underline{d} \geq a_{ji}$, $Q_{ji} = 0$ and $Q'_{ji} = 0$. Then, given $\underline{d} \geq c$, if $a \leq \underline{d}$, then $d_{ji}^M = \frac{a_{ji}+c}{2} \leq \underline{d}$. This will reduce the expression to:

$$\sum_{\{j|d_{ji}^M \leq \underline{d} \text{ and } a_{ji} \geq \underline{d}\}} \frac{a_{ji} - 2\underline{d} + c}{2} + \sum_{\{j|d_{ji}^M \geq \underline{d}+c\}} \frac{a_{ji} - 2(\underline{d}+c) + c}{2} = 0,$$

which is equivalent to:

$$\sum_{\{j|d_{ji}^M \leq \underline{d} \text{ and } a_{ji} \geq \underline{d}\}} \frac{a_{ji} - 2\underline{d} + c}{2} + \sum_{\{j|d_{ji}^M \geq \underline{d}+c\}} \frac{a_{ji} - 2\underline{d} - c}{2} = 0.$$

3.3 Examples of Equilibria in the Case of Linear Demand

In our model each firm already has links to every other firm, but some of the firms may find that they can not have connections to certain firms. We can characterize equilibria from the perspective of each firm and consider which of the other firms will have connections to that firm.

Example 1 (Non-Binding Constraints Leading to Unrestrained Monopoly Pricing)**.** Consider $\max_{x \in N\setminus\{i\}}\{a_{xi}\} - \min_{x \in N\setminus\{i\}}\{a_{xi}\} \leq 2c$. This implies

$$\begin{aligned} \max_{x \in N\setminus\{i\}}\{d_{xi}^M\} - \min_{x \in N\setminus\{i\}}\{d_{xi}^M\} &= \max_{x \in N\setminus\{i\}}\left\{\frac{a_{xi}+c}{2}\right\} - \min_{x \in N\setminus\{i\}}\left\{\frac{a_{xi}+c}{2}\right\}, \\ &= \frac{1}{2}\left[\max_{x \in N\setminus\{i\}}\{a_{xi}\} - \min_{x \in N\setminus\{i\}}\{a_{xi}\}\right], \\ &\leq c. \end{aligned}$$

Then, firm i optimally chooses $d_{xi} = d_{xi}^M$, $\forall x \in N \setminus \{i\}$. Here the demand from each firm is sufficiently close that the existence of the network does not limit price discrimination. The firm can set prices as if there were no competition, i.e. $d_{ji} = \frac{a_{ji}+c}{2}$.

When demand parameters are differentiated to a greater extent, monopoly pricing may not be feasible in equilibrium. Such examples will be demonstrate

how the FOC stated above captures the trade-off between serving firms of low and high demand.

Examples with Three Players

The simple case with three players will be instructive to understand the tradeoffs each firm faces. Since there are only three players, each firm has to meet demands from two other firms and the relevant trade-off is about the demands of those two firms.

Example 2 (Binding Constraints, Deviation from Monopoly Pricing)**.**
Assume that demand parameters are no longer close to each other. Specifically, $\max_{x\in N\setminus\{i\}}\{a_{xi}\} - \min_{x\in N\setminus\{i\}}\{a_{xi}\} = a_{23} - a_{13} \leq 2c$ no longer holds. Here, the FOC implies that

$$\frac{a_{13} - 2\underline{d} + c}{2} + \frac{a_{23} - 2\underline{d} - c}{2} = 0,$$

$$\underline{d} = \frac{a_{13} + a_{23}}{4}.$$

Firm 1 is charged $d_{13} = \underline{d}$ and Firm 2 is charged $d_{23} = \underline{d} + c$. Then, $\underline{d} = a_{13}$ is the critical level of price below which Firm 1 will no longer originiate connections. That Firm 1 makes connections requires,

$$\begin{aligned} a_{13} &> \underline{d}, \\ a_{13} &> \frac{a_{13} + a_{23}}{4}, \\ 3a_{13} &> a_{23}. \end{aligned}$$

As long as the firm with the greater demand does not have a demand parameter that exceeds three times the demand parameter of the other firm, both firms

will have connections. If there are more firms with low demand, then this statement will be weakened; if there are more firms with high demand, this statement will be strengthened.

In the previous case, both firms were able to have connections, despite the difference in their levels of demand. The limiting condition also provides the measure of difference that should exist for one of the firms to be excluded. $3a_{13} < a_{23}$ is sufficient for Firm 3 to exclude Firm 1 and only serve Firm 2. Then, Firm 3 will charge Firm 2 $d_{23} = d_{23}^M$, since that is the optimal price at which to serve Firm 2. Firm 1 may be charged any price such that $|d_{13} - d_{23}| \leq c$ at equilibrium.

Examples with Five Players

Now consider how the number of agents changes the equilibrium. Consider a market with five firms, then each firm will have to consider four firms connecting to itself and itself connecting to four firms. Given the greater number of firms, the trade-offs involved also become richer. In order to facilitate the analysis, reindex agents such that $a_{4i} \geq a_{3i} \geq a_{2i} \geq a_{1i}$.

Example 3 (Duplicate Players)**.**

If $a_{4i} = a_{3i}$ and $a_{2i} = a_{1i}$, by the discussion given above for the case of 3 agents, players 1 and 2 will able to connect only if $a_{4i} = a_{3i} < 3a_{2i} = 3a_{1i}$. Players 3 and 4 will always be able to connect.

Example 4 (Duplicates at the Lower End)**.**

If $a_{4i} > a_{3i} > a_{2i} = a_{1i}$, the FOC becomes

$$2\left(\frac{a_{1i} - 2\underline{d} + c}{2}\right) + \frac{a_{4i} - 2\underline{d} - c}{2} = 0,$$

$$\underline{d} = \frac{2a_{1i} + a_{4i} + c}{6}.$$

This example reduces to the three player case, as a pair of agents are replicas of the other pair and the objective function is simply doubled. Player 1 or 2 will be able to connect only if $\underline{d} < a_{1i}$ which implies $a_{4i} + c < 4a_{1i}$; Player 3 will be able to connect only if $a_{4i} < 3a_{3i}$. Player 4 will always be able to connect.

Example 5 (Duplicates at the Higher End)**.**

If $a_{4i} = a_{3i} > a_{2i} > a_{1i}$, the FOC becomes

$$\frac{a_{1i} - 2\underline{d} + c}{2} + 2\left(\frac{a_{4i} - 2\underline{d} - c}{2}\right) = 0,$$

$$\underline{d} = \frac{a_{1i} + 2a_{4i} - c}{6}.$$

Player 1 will be able to connect only if $\underline{d} < a_{1i}$ which implies $2a_{4i} - c < 5a_{1i}$ and Player 2 will be able to connect only if $\underline{d} < a_{2i}$ which implies $2a_{4i} - c < 5a_{2i}$. Players 3 and 4 will always be able to connect.

Example 6 (No Duplication or No Duplication at Extremes)**.**

Since only the players at either extreme of demand enter the FOC, both cases can be analyzed in the same manner. If $a_{4i} > a_{3i}$ and $a_{2i} > a_{1i}$, the FOC becomes

$$\frac{a_{1i} - 2\underline{d} + c}{2} + \frac{a_{4i} - 2\underline{d} - c}{2} = 0,$$

$$\underline{d} = \frac{a_{1i} + a_{4i}}{4}.$$

Player 1 will be able to connect only if $a_{4i} < 3a_{1i}$. Player 2 will be able to connect only if $a_{4i} < 3a_{2i}$ when $a_{3i} > a_{2i}$ and only if $a_{4i} + c < 4a_{2i}$ when $a_{3i} = a_{2i}$. Player 3 will be able to connect only if $a_{4i} < 3a_{3i}$. Player 4 will always be able to connect.

Example 7 (Triplicates at the Lower End)**.**

If $a_{4i} > a_{3i} = a_{2i} = a_{1i}$, the FOC becomes

$$3\left(\frac{a_{1i} - 2\underline{d} + c}{2}\right) + \frac{a_{1i} - 2\underline{d} - c}{2} = 0,$$

$$\underline{d} = \frac{3a_{1i} + a_{4i} + 2c}{8}.$$

Players 1, 2 and 3 will be able to connect only if $\underline{d} < a_{1i}$ which implies $a_{4i} + 2c < 5a_{1i}$

Example 8 (Triplicates at the Higher End)**.**

If $a_{4i} = a_{3i} = a_{2i} > a_{1i}$, the FOC becomes

$$\frac{a_{1i} - 2\underline{d} + c}{2} + 3\left(\frac{a_{4i} - 2\underline{d} - c}{2}\right) = 0,$$

$$\underline{d} = \frac{a_{1i} + 3a_{4i} - 2c}{8}.$$

Player 1 will be able to connect only if $\underline{d} < a_{1i}$ which implies $3a_{4i} - 2c < 7a_{1i}$

It is obvious to see that equilibria of setups with fewer players may be characterized by assuming the extraneous players to have $a_{ji} < c$, which would reduce to one of the cases described above.

Note that the network structure introduces incentives to distort prices upward (downward) at lower (higher) levels of demand.

3.4 Comparison to Collusion

Consider the collusive equilibrium where connection paths and pricing are determined to maximize total profits. It follows that firms extract from consumers the highest level of surplus possible. Since direct paths are efficient and Lemma 3.1 still applies, firms choose only direct connections and maximize their total profits

with respect to consumer pricing. Since connection charges between players do not affect overall profit, the maximization problem is as below:

$$\max_{p} \sum_{i} \sum_{j} q_{ji}(p_{ji})(p_{ji} - c).$$

Note that double marginalization does not apply here. Thus, we have the optimal prices satisfy:

$$q'_{ji}(p_{ji})(p_{ji} - c) + q_{ji}(p_{ji}) = 0 \qquad \forall i, j.i \neq j.$$

The consequences of joint profit maximization will be analyzed in an example.

Example 9. Consider the case of linear demand analyzed in the previous section. Recall that demand for connections were given by $q_{ji} = a_{ji} - p_{ji}$ and demand parameters satisfied $\max_{x \in N \setminus \{i\}}\{a_{xi}\} - \min_{x \in N \setminus \{i\}}\{a_{xi}\} \leq 2c$. In this case, the condition stated above becomes:

$$-(p_{ji} - c) + a_{ji} - p_{ji} = 0,$$

$$p_{ji} = \frac{a_{ji} + c}{2}.$$

whereas in the network setup, firms would charge $p_{ji} = \frac{3a_{ji}+c}{4}$ as determined in Example 1. Assuming that $\min_{x \in N \setminus \{i\}}\{a_{xi}\} > c$, this means that the competitive effects brought upon by the network structure distort prices higher than even a collusive price in all markets.

The reason for such distortion is the issue of double marginalization in the non-cooperative setup, which the cartel resolves given its objective of joint profit maximization. Recalling the optimal price $P_{ji} = \frac{3a_{ji}+c}{2}$ when routing is not possible, it can be observed that double marginalization causes greater distortion at the top. Recall that non-cooperative play in the network structure causes downward distortion at the top compared to the non-cooperative case without routing.

However, upward distortion caused by double marginalization acts at both ends and counteracts such downward distortion. It would only be markets with high demand among low demand markets that might face lower prices in the non-cooperative network structure compared to the collusive case. Most consumers would find collusive prices lower. Network structure does not create competition that improves consumer welfare, since competitive effects are limited by each firm's monopoly on access to its own customers.

4 Non-Uniform Costs

Now consider the more general case of non-uniform physical costs of connections. We will consider whether indirect connections may provide incentives for deviation from an assumed equilibrium where there are only direct connections. Consider that d_{mn} and $r_{mn}(g)$ $\forall m, n \in I$ and $\forall g$ are prices in an equilibrium which only has direct connections. Consider that firm i wants to have firm j make an indirect connection, instead. Firm j will find an indirect connection $g \in G_{ji}$ acceptable if and only if

$$d_{ji} \geq T(g),$$

where necessarily $T(g) - D(g) \geq C(g^R)$ for the indirect connection to be acceptable to the firms routing it. Then, as in the case of uniform costs, firm i would set d_{ji} and $D(g)$ such that routing will occur at cost; that is

$$d_{ji} = C(g^R) + D(g).$$

Assume that given the same cost, firm i can manipulate firm j to choose either of direct or indirect connections (alternatively consider that the firm makes the

preferred type connection cost arbitrarily less than the other).

Lemma 4.1. *Firms will set connection charges such that, if at all, routing occurs at cost.*

Proof. Assume that i finds it more profitable to have the connection occur over the indirect path g rather than the direct path $\{(j,i)\}$. For the indirect path to be acceptable to firm j as well, it must hold that

$$d_{ji} \geq T(g).$$

Players in the path will choose to route the connection as long as $R(g) \geq C(g^R)$. Now consider the two alternatives $d_{ji} > C(g^R) + D(g)$ and $d_{ji} = C(g^R) + D(g)$ both of which lead to indirect connections yielding the respective profits:

$$Q_{ji}(R(g) + D(g))(D(g) - C(g^D),$$

$$Q_{ji}(C(g^R) + D(g))(D(g) - C(g^D).$$

Since $R(g) \geq C(g^R)$ and Q_{ji} is a non-increasing function of the cost of the connection, it is trivial to show:

$$Q_{ji}(R(g) + D(g))(D(g) - C(g^D) \leq Q_{ji}(C(g^R) + D(g))(D(g) - C(g_D).$$

Thus, if firm i prefers the indirect connection over a direct connection, it should choose

$$d_{ji} = C(g^R) + D(g).$$

This implies that firms will choose $r_{mn}(g)$ so that $T(g) = d_{ji}$. Then, $R(g) = C(g^R)$ which means any routing will be at cost. □

Lemma 4.2. *If $c_{ki} \leq c_{ji}$, then no equilibrium exists where firm j routes connections for firm k.*

Proof. Now assume that the connection from k to i will be indirect, routed through j, while $c_{ji} \geq c_{ki}$. Compare the indirect connection $\hat{g}$ with cost $T(\hat{g}) = r_{kj}(\hat{g}) + d_{ji}$ with a direct connection with the price $d_{ki} = d_{ji}$. Since the profit to the receiver on an indirect connection is $d_{ji} - c_{ji}$ and $\hat{d}_{ki} = T(\hat{g})$,

$$(d_{ji} - c_{ji})Q_{ki}(T(\hat{g})) \leq (d_{ji} - c_{ki})Q_{ki}(d_{ji}),$$

since $Q_{ki}(d)$ is a non-increasing function of d, $c_{ki} \leq c_{ji}$ and $d_{ji} \leq \hat{d}_{ki} = d_{ji} + r_{kj}(\hat{g})$; direct connection is at least as profitable as the indirect connection. Thus, if $c_{ji} \geq c_{ki}$, it will never be optimal to set prices so that j routes k's connections. □

The following proposition states a necessary condition for optimality of indirect paths.

Proposition 4.3. *An indirect connection between players j and i may be optimal only if physical costs of links display superadditivity, as stated below*

$$\exists x, g_{jx} \text{ such that } c_{ji} \geq C(g_{jx}) + c_{xi}.$$

Proof. Assume that firm i is considering whether to induce j to connect via a path $\hat{g}_{ji}$, which has x as the last agent in the path before i. Define $N^{i,D}$ and $N^{i,R}$ give the set of players other than j, connected to i directly and indirectly, respectively. Thus, $N^{i,D} \bigcup N^{i,R} = N \setminus \{i\}$. Then, the relevant maximization problems for i are

$$\begin{aligned} Q_{ji}(d_{ji})(d_{ji} - c_{ji}) + \sum_{m \in N^{i,D} \setminus \{j\}} & Q_{mi}(d_{mi})(d_{mi} - c_{mi}) \\ + \sum_{m \in N^{i,R}} & Q_{mi}(T(g_{mi}))(D(g_{mi}) - C(g^D_{mi})), \end{aligned} \tag{3}$$

subject to

$$d_{mi} \leq T(g_{mi}), \forall m \in N^{i,D}, \forall g_{mi},$$

$$d_{mi} > T(g_{mi}), \forall m \in N^{i,R}, \exists g_{mi},$$

$$d_{ji} \leq T(g_{ji}), \forall g_{ji},$$

when j is to make a direct connection and

$$Q_{ji}(T(\hat{g}_{ji}))(D(\hat{g}_{xi}) - c_{xi}) + \sum_{m \in N^{i,D} \setminus \{j\}} Q_{mi}(d_{mi})(d_{mi} - c_{mi})$$

$$+ \sum_{m \in N^{i,R}} q^{M}_{mi}(T(g_{mi}))(D(g_{mi}) - C(g^{D}_{mi})),$$

subject to:

$$d_{mi} \leq T(g_{mi}), \forall m \in N^{i,D}, \forall g_{mi},$$

$$d_{mi} > T(g_{mi}), \forall m \in N^{i,R}, \exists g_{mi},$$

$$d_{ji} = T(\hat{g}_{ji})$$

$$d_{ji} \leq T(g_{ji}), \forall g_{ji},$$

when j is to connect over $\hat{g}_{ji}$.

The difference between the two problems is that the latter problem has the additional constraint

$$d_{ji} = T(\hat{g}_{ji}),$$

and i has j choosing the indirect connection via x instead of the direct connection by means of manipulations already discussed.

If d_{ji} is a solution of the latter problem where j has an indirect connection, then it will also satisfy the constraints of the former problem even though it may not be optimal. Since other players will receive the same prices and therefore choose the same equilibrium quantities and paths, it is enough to compare the profits obtained from j's connections when j connects directly and via x.

$$Q_{ji}(C(g_{jx}) + d_{xi})(d_{xi} - c_{xi}) \geq Q_{ji}(C(g_{jx}) + d_{xi})(C(g_{jx}) + d_{xi} - c_{ji}),$$

$$d_{xi} - c_{xi} \geq C(g_{jx}) + d_{xi} - c_{ji},$$

which reduces to superadditivity of physical cost of links

$$c_{ji} \geq C(g_{jx}) + c_{xi}.$$

□

This result shows that when costs of connection are subadditive, making direct connections are always more profitable. Therefore, for any equilibrium to exist where indirect connections are employed, physical costs of connection must be superadditive, at least for some players.

Note that this is a necessary condition and the optimal profits from j making a direct connection would be higher than when j is making a direct connection given the optimal pricing for an indirect connection. This means that this necessary condition may not be sufficient and there may be cases where direct connections are preferred even when costs of connection are superadditive.

Since choosing prices also changes the feasible set over which prices need to be chosen, characterizing a sufficiency condition is difficult. The remainder of this section considers the analytical solution for an example of a network with non-uniform costs to illustrate when an indirect connection may become optimal.

Example 10 (Some Players May Only Route Connections)**.**

Consider a setup with three agents who face linear demand for connections to other markets, given by $q_{ji} = a_{ji} - p_{ji}$, as considered in the case of uniform costs. As determined in that case, these agents will set $p_{ji} = \frac{a_{ji}+T(g^*_{ji})}{2}$ where $T(g^*_{ji})$ is the lowest cost of reaching player i. Assume that physical costs of connection are symmetric, that is $c_{ji} = c_{ij}$ and $c_{ji} \leq c_{ki}$

Transformation into a Problem with Uniform Costs

Consider player i's problem when both players are to make direct connections:

$$\max_{d_{ji},d_{ki}} \frac{a_{ji}-d_{ji}}{2}(d_{ji}-c_{ji})+\frac{a_{ki}-d_{ki}}{2}(d_{ki}-c_{ki})$$

subject to

$$d_{ji}-d_{ki}\leq c_{jk}$$

$$d_{ki}-d_{ji}\leq c_{jk}$$

We can transform this problem into a problem with uniform physical costs. Denote $c=c_{jk}$ and rewrite the demand functions and prices

$$\begin{aligned}\max_{d_{ji},d_{ki}} &\frac{(a_{ji}-c+c_{ji})+(d_{ji}+c-c_{ji})}{2}[(d_{ji}+c-c_{ji})-c]\\ &+\frac{(a_{ki}-c+c_{ki})+(d_{ki}+c-c_{ki})}{2}[(d_{ki}+c-c_{ki})-c]\end{aligned}$$

subject to

$$(d_{ji}+c-c_{ji})-(d_{ki}+c-c_{ki})\leq c$$

$$(d_{ki}+c-c_{ki})-(d_{ji}+c-c_{ji})\leq c$$

We can now make use of previous results obtained under uniform costs.

If $a_{ji}+c_{ji}\leq a_{ki}+c_{ki}$, then firm i should set $d_{ji}-c_{ji}\leq d_{ki}-c_{ki}$. Since $c_{ji}\leq c_{ki}$, we have $d_{ji}\leq d_{ki}$. We can then eliminate one of the constraints, namely that $d_{ji}-d_{ki}\leq c_{jk}$, since it would be satisfied trivially.

Direct Connections for Both Players

Now the maximization problem becomes, written as a Lagrangian:

$$\max_{d_{ji},d_{ki}} \mathcal{L}=\frac{a_{ji}-d_{ji}}{2}(d_{ji}-c_{ji})+\frac{a_{ki}-d_{ki}}{2}(d_{ki}-c_{ki})+\lambda(c_{jk}+d_{ji}-d_{ki})$$

If $a_{ki} + c_{ki} - a_{ji} - c_{ji} \leq 2c_{jk}$, the constraint is satisfied and the solution has $d_{ji} = \frac{a_{ji}+c_{ji}}{2}$ and $d_{ki} = \frac{a_{ki}+c_{ki}}{2}$.

If the above inequality does not hold, then the constraint must be satisfied with equality, and $d_{ki} = d_{ji} + c_{jk}$. The maximization problem then becomes the following unconstrained maximization:

$$\max_{d_{ji}} \frac{a_{ji} - d_{ji}}{2}(d_{ji} - c_{ji}) + \frac{a_{ki} - d_{ji} - c_{jk}}{2}(d_{ji} + c_{jk} - c_{ki})$$

which has the solution $d_{ji} = \frac{a_{ji}+c_{ji}+a_{ki}+c_{ki}-2c_{jk}}{4}$ and $d_{ki} = \frac{a_{ji}+c_{ji}+a_{ki}+c_{ki}+2c_{jk}}{4}$.

As long as $d_{ji} \leq a_{ji}$, firm j will make non-zero connections. However, otherwise firm j effectively cannot connect to firm i. Then, firm i would only maximize profits from k's connections and the price becomes $d_{ki} = \frac{a_{ki}+c_{ki}}{2}$. $d_{ki} \in (d_{ki} - c_{jk}, d_{ki} + c_{jk})$ is optimal.

Indirect Connection for Player with Higher Cost Direct Path

Now consider that $c_{ki} > c_{ji} + c_{jk}$, then indirect connections may be preferable for k, but never for j by Proposition 4.3. Consider that i sets $\hat{d}_{ki} = \hat{d}_{ji} + c_{jk}$ and coerces firm j to choose the indirect connection as discussed before. The problem, then, becomes

$$\max_{\hat{d}_{ji}} \frac{a_{ji} - \hat{d}_{ji}}{2}(\hat{d}_{ji} - c_{ji}) + \frac{a_{ki} - \hat{d}_{ji} - c_{jk}}{2}(\hat{d}_{ji} - c_{ji})$$

which has the solution $\hat{d}_{ji} = \frac{a_{ji}+a_{ki}+2c_{ji}-c_{jk}}{4}$ and $\hat{d}_{ki} = \frac{a_{ji}+a_{ki}+2c_{ji}+3c_{jk}}{4}$. Note that $\hat{d}_{ji} = d_{ji} + c_{ji} + c_{jk} - c_{ki}$ and $\hat{d}_{ki} = d_{ki} + c_{ji} + c_{jk} - c_{ki}$. Comparing profits obtained under direct and indirect connections, (assuming $a_{ki} + c_{ki} - a_{ji} - c_{ji} > 2c_{jk}$) indirect connection is preferred when

$$a_{ji} - 3aki + 14c_{jk} - 11c_{ki} \leq 0$$

which implies that for some sets of parameters, indirect connection will indeed be optimal.

Summary of Conclusions

Since $c_{ki} > c_{ji} + c_{jk}$, $\hat{d}_{ji}$ is lower than d_{ji}. Then, the first important conclusion is that, when k is connected indirectly, firm j may be able to make non-zero connections in some cases when it could not with k connected directly. This occurs when a_{ji} is in the range $(a_{ki} + 2c_{ji} - c_{jk}, a_{ki} + c_{ji} + c_{ki} - 2c_{jk})$.

The second conclusion is that this result may prevent firm j from having non-zero connection even when it would be assured of that under uniform costs. Consider that $a_{ki} + c_{ki} - a_{ji} - c_{ji} \leq 2c_{jk}$ which is the condition analogue to the condition for uniform costs. Then, assuming $d_{ji} > a_{ji}$ and the above condition holds:

$$a_{ji} \geq a_{ki} + c_{ki} - c_{ji} - 2c_{jk}$$

$$a_{ki} + 2c_{ji} - c_{jk} > 3a_{ji}$$

which together imply:

$$a_{ki} + 2c_{ji} - c_{jk} > 3a_{ki} + 3c_{ki} - 3c_{ji} - 6c_{jk}$$

$$5c_{ji} + 5c_{jk} - 3c_{ki} > 2a_{ki}$$

If the above holds, then firm j will have no connections to firm i even when there is little difference between the levels of demand.

The third conclusion, as highlighted in this section, is that firm j will be routing connections for firm k even when it will have no connections of its own. This is a significant difference from the case of uniform costs and extends to a different

class of problems. Many economic problems with fixed costs imply superadditive costs. This result concurs that in such problems indirect connections may be preferable to direct connections. Indirect connections involve cooperation of agents, which is similar to cooperation or integration to overcome superadditive costs.

5 Behavior Under Regulation: Excluding Incumbent from New Technology

In the previous discussion, customers were implicitly assumed to be captive to their monopoly service providers. In some setups this is exactly the case, as it might be when there is a regional or perhaps even a national monopoly serving a market. However, increasing unbundling of communication services has created competition and consumers might have the option of switching to a different operator. In such a context, one can consider the customers weighing the cost of their connections when served by one firm against that when served by another and the cost of switching between operators. The setup of captive customers can be modeled by arbitrarily large switching costs.

Assume that after deregulation, there are two telecommunications companies who want to enter the market utilizing new technology that offers lowers marginal costs, but is not yet embraced by the incumbent. This may be the case because the regulator desires to have multiple viable companies operating in the market and denies the incumbent the option of implementing the new technology.

Let N_i denote the set of consumers served by firm i and N all the consumers

in the market. $|N_i|$ is the number of consumers served by firm i. No consumer may be served by more than one firm, therefore $N_i \bigcap N_j = 0, \forall i, j, i \neq j$. Then, $|N| = \sum_{i=1}^{k} |N_i|$ is the number of consumers served by all k firms in the market. Normalizing the number of consumers to 1 will only change the scale of revenues and profits and therefore will call for the same optimal list of prices. Thus $\{n_i\}$ denote a partition of $[0, 1]$ such that $\bigcup_{i=1}^{k} n_i = [0, 1]$, $\bigcap_{i=1}^{k} n_i = 0$ and $|n_i| = \frac{|N_i|}{|N|}$, the fraction of consumers served by firm i. $|n_i|$ gives the share of the market held by each firm, so denote that $s_i = |n_i|$.

Assume that consumers are identical and each has demand for connections from their operator i to each consumer served by operator j, given by some function $q(p_{ij})$ that depends only on the price of the connection. Since firm j has s_j subscribers, each subscriber of firm i has demand $s_j q(p_{ij})$ for calls to operator j. Since operator i has s_i, subscribers, total demand from i's network to j's network is $Q_{ij} = s_i s_j q(p_{ij})$.

Consumers are no longer captive to the incumbent when the entrants do enter the market and may switch to the entrants' services if they find the perceived cost of the service to be lower. Assume that the consumers' perception of cost is the average of the prices weighted by the number of consumers served by each operator. This average is given by

$$\sum_{k=1}^{n} |s_k| p_{i,k}$$

for each firm.

Consumers are also aware that they will face costs to switching between operators, w_{ij}, which depends on which operators the consumer is switching between. Therefore, the perceived cost associated with subscribing to an alternate

operator j is:

$$\sum_{k=1}^{n} |s_k| p_{j,k} + w_{ij}$$

Proposition 5.1. *In equilibrium, average cost of communication of each firm, weighted by distribution of consumers, must be no greater than that of its competitor plus a switching cost. This constraint may be stated as:*

$$\sum_{k=1}^{n} |s_k| p_{i,k} \leq \sum_{k=1}^{n} |s_k| p_{j,k} + w_{ij}$$

Proof. Now, assume that some share of the consumers, ϵ, served by firm i finds it worthwhile to switch to an operator j which implies that given the above costs

$$\sum_{k=1}^{n} |s_k| p_{i,k} > \sum_{k=1}^{n} |s_k| p_{j,k} + w_{ij} \tag{4}$$

Without immediate changes in the prices, after the loss of those consumers to firm i, the left- and right-hand sides of that inequality become, respectively,

$$\begin{aligned} \sum_{k=1}^{n} |s_k| p_{i,k} &+ \epsilon(p_{i,j} - p_{i,i}) \\ \sum_{k=1}^{n} |s_k| p_{j,k} &+ w_{ij} + \epsilon(p_{j,j} - p_{j,i}) \end{aligned}$$

Given lower prices for in-network connections than external connections, $p_{i,j} - p_{i,i}$ is positive and $p_{j,j} - p_{j,i}$ is negative. Therefore, the inequality continues to hold as before, for any given ϵ. If inequality (4) is satisfied for any $j \neq i$, then firm i loses all of its customers; thus it must hold for all i, j, $i \neq j$ that

$$\sum_{k=1}^{n} |s_k| p_{i,k} \leq \sum_{k=1}^{n} |s_k| p_{j,k} + w_{ij}$$

□

Example 11 (A Case One Incumbent and Two Innovative Entrants)**.**

Assume that all firms are capable of routing connections at cost c. Further assume

that the new technology introduced by both entrants enables them to connect their customers at zero marginal cost whereas the incumbent incurs $\hat{c}$ to connect its customers. When the total physical costs incurred by the relevant operators when an entrant's customer calls within the network, towards the other entrant and the incumbent are 0, c and $c + \hat{c}$, respectively. On the other hand when a customer of the incumbent initiates the call, the costs are $2\hat{c}$ within the network and $c + \hat{c}$ towards the entrants. Thus, depending on the how significant $\hat{c}$ is compared to c determines the cost advantage of the entrants. Assume that for marketing reasons, the incumbent prefers to set intra-network prices no higher than inter-network prices despite the higher cost it faces. Denote the incumbent as I and the entrants as E and $\hat{E}$.

The regulator desires to determine how long to withhold the incumbent, that is how much market share should the entrants have before the incumbent is allowed to switch to the new technology.

Without further regulation, it should be obvious that the incumbents cannot gain marketshare at all. To see this result, consider that the incumbent sets $p_{II} < d_{EI}$ which implies that neither entrant can offer connections at a cost lower than that from the incumbent. We will assume that the regulator introduces a regulatory rule that will change the setup to enable competition. An acceptable regulatory rule could be the efficient component pricing rule (ECPR), which would hold $d_{EI} = c+\hat{c}+k_{EI}$ and $d_{IE} = c+k_{IE}$, where k_{EI} and k_{IE} are opportunity costs of the incumbent in losing the sale to the entrant.

Proposition 5.2. *The entrants can gain marketshare if and only if $c < \hat{c}$.*

Proof. At first, any customer of the entrant or the incumbent will be having con-

nections with customers of the incumbent; the relevant comparison for a customer considering the three options would be p_{EI} and p_{II}. Therefore, the incumbent needs to set $p_{II} \leq p_{EI}$. The opportunity cost of the incumbent needs to be determined. The incumbent could offer the connection from the customer of the entrant to its customer, by incurring the cost $2\hat{c}$, if both were in its network. Thus, at price p_{II}, $k_{EI} = p_{II} - 2\hat{c}$. Then, no equilibrium may have $p_{II} > c + \hat{c}$, because the entrant could reduce its pricing until $p_{EI} = c + \hat{c} + k_{EI} = p_{II} + c - \hat{c} < p_{II}$, since $c < \hat{c}$. It then follows that $p_{II} \leq p_{EI}$ as long as $2\hat{c} \leq c + \hat{c}$ which implies $\hat{c} \leq c$. Thus, unless $\hat{c} > c$, the entrants will not be able to capture any share of the market.

If $\hat{c} > c$, then the entrants are able to capture the entire market by setting prices $p_{EI} = c + \hat{c}$. However, the existence of two entrants leads to $p_{EE} = 0$ and $p_{EX} = c$ at equilibrium. □

Depending on the cost savings introduced by the new technology, in the first case delaying the incumbent does not allow for entrants to capture marketshare, while in the second case allows for the entrants to capture the entire share if the incumbent is prevented from implementing the same technology. The regulator should allow the incumbent when a desirable market structure has arisen, assuming that customers switch operators in progression and not all at once.

6 Conclusion

When physical costs of connections are uniform, routing does not occur at equilibrium. In this setting, then, physical cost provides a limit to price discrimination since only price differences up to the physical cost can be supported and large

differences in demand cause upward distortions of price, which may lead to the exclusion of lower demand firms.

When physical costs are non-uniform, however, indirect connections may become optimal. The necessary condition for optimality of an indirect connection is that costs of connection are superadditive over that path. Determining sufficient conditions for optimality of indirect connections is difficult and such a condition has not been included in this thesis.

However, when costs are indeed superadditive and there are indirect connections, players that route connections face prices that have been distorted even higher when compared to the uniform cost case. In fact, such players may find that they are only able to route connections for others, but that they do not have enough demand to make their own connections, whereas they would have under the case of uniform cost.

One shortcoming of this model is that it only considers constant marginal costs and no fixed costs. With the introduction of avoidable fixed costs, firms may have the incentive to consider a greater number of indirect connections. However, some intuition can be gained from consideration of the non-uniform costs case. If instead of marginal costs, average costs at equilibrium demand levels are considered, we may claim that links, that have relatively high demand for connections and relatively low associated fixed costs, would be good candidates to use in routing other connections, as average costs would be lower than other links. If direct demand is not high, but fixed costs are relatively low, the low demand player may only be routing connections and not originate them.

Another refinement of the model could be to consider network structures other

than the complete network. Since each link removed also removes a threat point, this should increase (or at least not decrease) price discrimination. However, if network structure is endogenous, it seems reasonable that a missing link would likely be one where demand is relatively low and fixed costs are high and would not be high intensity link even if it existed in the network. Therefore, it might be reasonable to assume that the introduction of a missing link would not have significant effects on the equilibrium and considering only the complete network does not detract from the conclusions of the model. On the other hand, if network structure, and therefore existence of a link, is exogenous, then the consequences might be significant.

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