ASSESSING TAX EVASION WITHIN AN ENDOGENOUS GROWTH MODEL WITH PRODUCTIVE PUBLIC EXPENDITURE

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ASSESSING TAX EVASION WITHIN AN ENDOGENOUS GROWTH MODEL WITH PRODUCTIVE PUBLIC EXPENDITURE

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Assessing Tax Evasion Within An Endogenous Growth Model
With Productive Public Expenditure

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Thesis Abstract

Mehmet Emre Tokgöz, "Assessing Tax Evasion Within An Endogenous Growth Model With Productive Public Expenditure"

By setting up an endogenous growth model with productive government expenditures, this thesis analyzes effects of tax rate, deterrence policies, corruption and government externality level on tax evasion decision of economic agents. Moreover, within a Ramsey-type problem, I investigate optimal tax policy, audit rate, and level of public expenditures within an economy that is characterized by tax evasion. The thesis incorporates analytical solutions and supplements them by quantitative analysis by calibration of the model for Turkey. The results show that under the existence of tax evasion, optimal level of productive public expenditure would be less than the efficiency level stated in Barro (1990). In addition, existence of tax evasion would not necessarily require higher optimal tax rates compared to the case of no evasion. In fact, quantitative analysis suggests a lower tax rate. Moreover, optimal conditions do not necessitate complete elimination of tax evasion. The results also show that optimal policies have potential to bring about significant gains on growth rate depending on the positive externality level of public expenditures and the level of corruption in the economy.

Tez Özeti

Mehmet Emre Tokgöz, "Üretken Kamu Harcması İçeren Bir Endojen Büyüme Modeli Çerçevesinde Vergi Kaçırmanın İncelenmesi "

Bu tez, üretken devlet harcaması içeren bir endojen büyüme modeli kurarak vergi oranlarinin,caydırıcı politikaların,yolsuzluğun ve devlet dışsallık seviyesinin ekonomik ajanların vergi kaçırma tercihlerine etkilerini incelemektedir. Buna ek olarak, Ramsey-tip bir problem çerçevesinde,vergi kaçırma ile karakterize olmuş bir ekonomi içerisinde optimal vergi politikası,optimal denetim oranı ve optimal kamusal harcama oranı incelenmektedir. Tez analitik çözümleri içermekle birlikte, bu analitik çözümleri kantitatif analizle de desteklemektedir. Sonuçler göstermektedir ki vergi kaçırma altındaki bir ekonomide optimal kamu harcaması seviyesi Barro(1990)'da belirtilen verimlilik seviyesinden düşük olmaktadır.Buna ek olarak, vergi kaçırmanın varlığı vergi kaçırmanın olmadığı bir ekonomiyle karşılaştırıldığında daha yüksek bir vergilendirmeyi illa ki gerektirmemektedir. Aslına bakıldığında, kantitatif analiz daha düşük bir vergi oranına işaret etmektedir.Dahası, optimal koşullar vergi kaçırmanın tamamen yok edilmesini gerektirmemektedir.Sonuçlar ayrıca göstermektedir ki, optimal politikalar kamu harcamalarının pozitif dışsallığının seviyesine ve ekonomideki yolsuzluk seviyesine bağlı olarak büyüme oranına kayda değer bir katkı sağlama potansiyeline sahiptir.

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CHAPTER 1
INTRODUCTION

Income tax evasion is one of the most important problems of countries all over the world. Especially in developing countries, non-compliance in taxation of individuals and firms is very high and poses severe problems on the revenue side of public authorities. As governments can not collect income tax efficiently, they use indirect taxation as the main source of government revenue. This situation results in enormous distortion in the economy with effects on relative prices. In addition as Penalosa and Turnovsky (2005) suggests, developing countries need additional tax revenues to improve public services that are crucial for development. Since indirect taxes are already at a high level, increasing tax revenue will require higher personal income tax revenues. Therefore, studying the income tax evasion problem in a dynamic framework by considering the development and growth effects is crucial.

Since the seminal work of Allingham and Sandmo (1972), in which tax-evasion was simply an isolated decision of an individual tax-payer facing uncertainty of an audit, theoretical analysis of income tax-evasion has become a widely studied subject(See Slemrod and Yitzhaki (2000) and Slemrod (2007) for review of literature). Most of the studies in this line of research have explored dynamics of tax-evasion decision in a partial equilibrium setting. However, this kind of a setting can be hardly classified as public economics as it does not take into account wide effects and externalities of evasion decision. In addition, such analysis does take into account only negative side of tax compliance associated with additional tax burden. Modeling tax evasion behavior must also take into account positive returns of taxes. With taxes, government finances its expenditures which can be used as a direct input in production process or it can lead to positive externalities on the economic activity. In other words, as Cowell

and Gordon (1988) asserts "...while government taketh away, it also giveth back, and the latter activity surely exerts some influence on evasion"(p.2). As individual taxpayers evade, government revenues that are used for provision of public goods become scarce which in turn decreases the welfare of the taxpayers. Therefore, more genuine treatment of the tax evasion problem must pose and analyze general equilibrium effects in a dynamic setting with rigorously exploring theoretical link between government expenditure and tax evasion.

Studies that are analyzing income tax evasion within an endogenous growth set up by considering effect of the public expenditures came about only in 2000s.In one of the pioneering studies, Lin and Yang (2001) examines effects of tax evasion on economic growth and welfare within framework of endogenous growth. Their main result indicates that in the presence of tax evasion growth rate is U-shaped with respect to tax rate which is in contrast to monotonically decreasing relation without evasion. In their model, they incorporate public goods into utility function to illustrate the interaction between government and taxpayers. However, they do not consider positive effects of government spending on production function. Moreover, government is taken only as a passive supplier of public goods as they do not consider optimization of tax rates by government as a response to optimal tax evasion of households. In an influential study, Chen (2003) analyzes tax evasion in an endogenous growth setting with productive government. In the model, he explores optimality conditions for government polices of taxation, penalty rates and probability of audits. He also calibrates model to quantitatively assess the effects of government policies. He finds that these policies are quantitatively effective in discouraging tax evasion, but with small growth effects, unless the relative importance of productive government expenditure is very high. He also concludes that optimal tax rate in the presence of tax evasion is higher than the optimal rate in an economy with no evasion.

These studies incorporate productive public expenditure into analysis of tax evasion problem but neither of them associates tax evasion decision of the individuals directly to the level of government externality. As Cowell and Gordon's proposition suggests, the take-and-give policies of government must certainly effect tax evasion decision directly. This purpose can be attained by letting individuals taking government expenditure level not as given but endogenized to their decisions. Nevertheless, this line of reasoning will be flawed as for all individuals, evading taxes when others will not evade will be the best response which will lead to free-rider problem. By taking this fact into account, this thesis connects tax evasion decision to the government externality level directly with a different set-up.

An inspiring mechanism for this purpose can be found in formulation of Loayza (1997). In his model, Loayza models informal firms as utilizing only some part of the productive public expenditure. He asserts that "because of their illegal status, informal agents do not fully enjoy public services, particularly those that allow them full, enforceable property rights over their capital and output"(p.25). As their contractual relations are flawed, they are poorly protected by police and courts. In addition informal firms "find obstacles to use other public services such as social welfare, skill training programs, and government-sponsored credit facilities"(p.26). Therefore, presence of government as a productive factor in the economy gives incentives to the firms to be formal. Addition to Loayza, Penalosa and Turnovsky (2005) models government as purchasing "some of the final good in order to provide the infrastructure required to operate the formal sector technology"(p.8). By this way Penolasa and Turnovsky also suggests that being informal makes firms unable to utilize positive externality of public expenditure.

This thesis does not model firms by dividing them into formal and informal ones. Dividing firms into formal and informal types misses point that

all firms evade some part of their output. Therefore in this thesis, tax evasion level is used for approximation to informality size.

With similar logic to Loayza (1997) and Penalosa and Turnovsky (2005), in the model the part of the public expenditure that can be utilized properly is made endogenous by tax evasion decision of the agents. Agents take the general level of productive government expenditures that have positive externalities on its production function as given. However, the level of this expenditure that can be utilized for own purposes depends on the tax evasion decision. As agents evade, they get into illegal activity that takes them to the road of informality. The more they evade, the more they try to set their economic activity away from sites of government. As a result, they can not utilize productive expenditure properly; they can only utilize some part of it. Therefore, there is an exclusion factor of public expenditures that is increasing with respect to evaded part of income.

In addition, the model incorporates probability of audit and fines with respect to tax evasion behavior. When agents get caught, they face the fines as an additional burden. Moreover, tax evasion also involves transaction costs. In the process of evasion, agents waste some part of their income. Therefore, when agents evade tax, they get rid of some part of the tax burden, but at the same time they face an expected burden of fines, transaction costs and underutilization of productive public expenditures. These gains and burdens result in a trade-off between evading or not evading taxes for agents of the economy. This trade-off, as it will shown, results in internal solution to tax evasion problem.

In the model of this thesis, central authority has the power to set tax rates and to determine the parts of the revenues that will be spend on productive government expenditures or spend on costly audits. Therefore, there is a trade-off for the government on both revenue and expenditures side of the

government. On revenue side, defining tax rates must take into account rising revenues to supply productive public expenditures, at the same time government must bear in mind the tax evasion and its negative externalities. On revenue side, government must spend some part of its revenues for audit purposes to deter tax evasion. Nevertheless, these costly audits crowds out the part of revenues to be spend on productive public expenditures.

Main aim of this thesis is conceptualizing optimal policies for the part of central authority under the existence of tax evasion with negative externalities mentioned above. For this purpose, optimization of tax rate and optimization of audit probability will be analyzed in a Ramsey-type formulation. That is, government will figure out optimal tax rate and optimal audit probability by taking the competitive behavior of the agents of the economy as given. Within this set up, sequence of optimal decisions actually starts with optimal decisions of the government. Government announces optimal tax Rate and audit probability by taking into account best responses of representative agents. Given the statutory tax rate and audit probability, representative agents choose the optimal level of consumption-saving and the level of reported income. These sequential decisions of the model will be solved backwards. First, we will present competitive equilibrium without taking into account externalities of productive government expenditure and tax evasion. Than, we will analyze government optimization by taking into account negative externalities of tax evasion and competitive equilibrium. These aims will be achieved by both analytical solutions and also quantitative analysis by calibration of the model for Turkey.

The thesis is organized as follows: Chapter 2 will lay out the economic environment and the model. Chapter 3 will characterize decentralized equilibrium without optimization on the part of government. Chapter 4, will present comparative dynamics in terms of effects of the deterrence policies, tax policy, and level of government externality. Chapter 5 will lay out and analyze

optimal policies of the government. Chapter 6 will calibrate the model for Turkey and will supplement theoretical findings with quantitative simulations. Chapter 7 will conclude.

CHAPTER 2

THE MODEL

The model is an extension of standard AK growth model of Barro (1990). Time is taken as continuous. In the economy, there exist identical infinitely lived households and identical firms.

The representative household maximizes following discounted life-time utility;

$$\int_0^\infty e^{-\rho t} \frac{c(t)^{1\ -\gamma} - 1}{1-\gamma} dt \tag{1}$$

where c(t) is the private consumption expenditure, $\rho > 0$ is the time-preference rate and γ defines the intertemporal elasticity of substitution. Utility solely depends on consumption level as labor endowment is supplied inelastically.

The representative firm has the following AK production technology;

$$y(t) = Ak(t)^{\sigma} \hat{g}(t)^{1-\sigma} \tag{2}$$

in which $0 < \sigma < 1$, y(t) is output level, k(t) is the private capital input per capita , $\hat{g}$ is the effective level of productive government expenditure and $A > 0$ is the exogenously given technology parameter.

For simplicity, it is assumed that private capital is not depreciating. It is also assumed that firm side is competitive in input and output markets. The technology level is positively affected by level of productive government services by externality level of $1 - \sigma$.It is assumed that government takes some part of production and uses it to supply public expenditures free of charge. Here, it is also assumed that there exists no congestion effect on public services. But, the level of public expenditure that can be utilized effectively depends on the level

of tax compliance.

Productive expenditures are taken as flow of goods and services including infrastructure expenditures, research and development, education, property right protecting services such as police and court, to name just a few. That is, which is meant by productive expenditures is not narrow definition of physical public infrastructure. Like in the case of broad definition of human capital, g(t) includes all public goods and services that can be thought as effecting production positively.

The output of the firm is taxed by government with an income tax rate of τ. As representative firm is faced with tax burden, it conceals some part of the output from government authorities therefore evades tax. The firm reports $\alpha \epsilon$ [0,1] part of output to the tax authorities and faces as tax burden of $\alpha\tau$y. The firm does not pay tax on the unreported part. But, with probability p ϵ [0,1], there is a possibility of audit. If a firm is audited and it is revealed that it evades taxes, it is faced with a fine rate of $\theta > 1$ times of the evaded tax liability.

In addition, tax evasion involves transaction costs. These costs includes efforts of ensuring to be successful in evading tax such as hiring accountant to dodge tax , bribing to officials and even foregone opportunities that results from accepting transactions only in cash to make transaction nontaxable. The transaction costs assumed to be take form as $h(1-\alpha)^2 y$ with $h\epsilon[0,1]$ as a cost parameter. Transaction costs are taken as a convex function of level of tax evasion and it is increasing with the level of income concealed. Cost parameter h reflects the easiness level of tax evasion and it can be thought as a proxy for level of corruption in the economy. As corruption decreases, h increases and transaction costs associated with tax evasion becomes larger; therefore tax evasion activity becomes harder.

Moreover, tax evasion makes a firm unable to take full advantage of

productive public expenditures in the following way;

$$\hat{g} = \alpha^{\epsilon} g \tag{3}$$

in which g is per capita public expenditures and ϵ is the exclusion parameter associated with tax evasion.

When $\epsilon < 1$, $\hat{g}$ becomes a convex function of $(1 - \alpha)$,the evasion rate, which means that underutilization of public expenditures is increasing with an increasing rate when a firm evades taxes. In other words, when firm evades little, it can utilize most part of the public expenditure. But, when it evades much, which is a proxy that it is close to the characteristics of informality, it can not utilize much of the public expenditure.

As g is taken as an essential unit of production, $\hat{g}$, the level of public expenditure that a firm can utilize, can not be zero. This can sound unrealistic, but in representative agent framework as we look at the whole economy that is represented by a single firm, essentializing g is legitimate.With such a formulation, the model suggests that α can not be zero at all. This fact is reasonable as we can not expect full informality on a general level economy.

As households own the shares of firms, their disposable income is;

$$y_d = (1 - p)(1 - \alpha\tau)y + p((1 - \alpha\tau)y - \theta\tau(1 - \alpha)y) - h(1 - \alpha)^2 y \tag{4}$$

With some manipulation, disposable income of the individual can be formulated as;

$$y_d = (1 - \tau_E - h(1 - \alpha)^2)y \tag{5}$$

where $\tau_E = \tau(\alpha + (1 - \alpha)p\theta)$ represents effective tax rate that is imposed on individual income which includes the punishment that results from tax evasion

It should be noted that effective punishment $p\theta$ must be less than one to have an interior level of tax evasion.If $p\theta > 0$, there occurs no incentive to evade any part of the income and tax evasion level becomes zero.

The representative household consumes some part of the disposable income and saves the rest to augment private capital.Hence, the capital accumulation process of the model can be formulated as follows;

$$\dot{k} = \left(1 - \tau_E - h(1-\alpha)^2\right) y - c \tag{6}$$

Government collects the income tax and spends its revenues to either for productive government expenditures or for efforts of audit to reveal evaded incomes. Audits have costs of fp^2y in which $f > 0$ is unit cost of audits. Cost of audits are assumed to be convex function of probability of audit which means that there is diminishing returns to audit technology of the government.[1] In order to increase the part of output audited, government must increase its expenditures more than the desired increase in probability. In addition, costs of audits are assumed to be linear function of level of output.[2]

Within this set up government budget constraint becomes;

$$\tau_E y \equiv g + fp^2 y \tag{7}$$

It is assumed that government budget is balanced in all periods.From government's budget constraint, we can deduce the level of productive

[1]This form is chosen to reflect empirical reality of audit technology. As government tries to reach more parts of the economy, it must reach to the more underground part of it which requires more efforts than before. In addition, as probability gets larger, marginal audits will be directed toward smaller firms with excessive number of audits.

[2]In calibration part, we will see that costs of audits as a percentage of GDP is actually constant over time.

government expenditures as follows;

$$g = (\tau_E - fp^2)y \tag{8}$$

Therefore, higher efforts of audits result in less supply of productive government expenditures.At the same time, as audit efforts increases the cost of tax evasion, the level of evasion decreases;hence effective tax rate increases. Thus, government faces the trade-off between spending on audit efforts and productive expenditures. This trade of results in various effects in a dynamic general equilibrium setting. These effects will be studied extensively within the section which analyzes the optimal policies of government under the presence of tax evasion.

CHAPTER 3
HOUSEHOLD OPTIMIZATION AND COMPETITIVE EQUILIBRIUM

As Barro (1990)suggests the representative household's problem is equivalent to the second-best welfare maximization problem, where the planner is constrained to ignore the externalities. Constrained planner chooses consumption, saving, and tax evasion level to maximize discounted life-time utility by taking audit probability, tax rate and fine rate given.

That is; $\max_{c,k,\alpha} \int_0^\infty e^{-\rho t} \frac{c(t)^{1-\gamma}-1}{1-\gamma} dt$ subject to

$$\dot{k} = (1 - \tau_E - h(1-\alpha)^2)\, y - c$$

$$\tau_E = \tau(\alpha + (1-\alpha)p\theta)$$

$$y(t) = Ak(t)^\sigma \hat{g}(t)^{1-\sigma}$$

$$\hat{g} = \alpha^\epsilon g$$

Optimization problem leads to following current value Hamiltonian equation;

$$H = \frac{c^{1-\gamma}-1}{1-\gamma} + \lambda\left(\left(1 - \tau_E - h(1-\alpha)^2\right) Ak^\sigma \hat{g}^{1-\sigma} - c\right) \tag{9}$$

in which λ is the current-value shadow price of capital formation.

Optimization leads to following relations;

$$\frac{\dot{c}}{c} = \frac{1}{\gamma}\left(A\sigma\left(1 - \tau_E - h(1-\alpha)^2\right)\left(\alpha^\epsilon \frac{g}{k}\right)^{1-\sigma} - \rho\right) \tag{10}$$

$$\frac{g}{k} = A^{1/\sigma}\alpha^{\epsilon(\frac{1-\sigma}{\sigma})}(\tau_E - fp^2)^{1/\sigma} \tag{11}$$

$$\left(\frac{\epsilon(1-\sigma)}{\alpha}\right)\left(1 - \tau_E - h(1-\alpha)^2\right) + 2h(1-\alpha) = (1-p\theta)\tau \tag{12}$$

$$\lim_{t\to\infty} e^{-\rho t}\lambda(t)k(t) = 0 \tag{13}$$

Equation (10) is growth rate of consumption which is also growth rate of income in endogenous growth set up. Equation (12) determines degree of tax evasion. And equation (13) is the transversality condition.Equation (12) solves α with respect to exogenous parameters.Given α, from (12) $\frac{g}{k}$ is determined.If we put this $\frac{g}{k}$ into (10), $\frac{\dot{c}}{c}$ is solved.Within this dynamics,the model constitutes exogenous growth rate $\lambda = \frac{\dot{c}}{c} = \frac{\dot{k}}{k}$ (See Appendix for formal solution of the model)

Definition of the Competitive Equilibrium

A perfect-foresight competitive equilibrium with tax evasion is characterized by quadruple $\{\frac{\dot{c}}{c}, \frac{\dot{k}}{k}, \frac{g}{k}, \alpha\}$

Such that

i) the representative household-firm optimizes ((10) and (12) holds)

ii)household budget is balanced ((5) holds)

iii) government budget is balanced ((7) holds)

In endogenous growth set-up, as there is no transitional dynamics, the economy is always at its steady-state balanced growth path such that consumption growth rate and growth rate of capital stock is equal to each other; and government expenditure to capital stock ratio and evasion rate is constant.

CHAPTER 4
ASSESSMENT OF OPTIMAL DEGREE OF TAX EVASION AND COMPARATIVE DYNAMICS

As stated before, equilibrium dynamics lead to the following equation for optimal decision of tax evasion;

$$\left(\frac{\epsilon(1-\sigma)}{\alpha}\right)(1-\tau_E-h(1-\alpha)^2)+2h(1-\alpha)=(1-p\theta)\tau$$

Left-hand side of the equation represents marginal cost of tax evasion and right-hand side of the equation represents marginal gains associated with the decreasing effective tax rate. With some manipulation, optimal condition becomes;

$$\frac{e(1-\sigma)}{\alpha}+2h(1-\alpha)+p\theta\tau=\tau+\frac{\epsilon(1-\sigma)}{\alpha}\alpha\tau+\frac{\epsilon(1-\sigma)}{\alpha}(1-\alpha)p\theta\tau+\frac{\epsilon(1-\sigma)}{\alpha}h(1-\alpha)^2 \tag{14}$$

This equation presents the dynamics affecting choice of tax evasion. First term in left-hand side, represents direct cost of tax evasion via change in productive government expenditures that can be utilized effectively. This effect positively depends on level of externality of the public expenditures on production function and also depends positively on the exclusion parameter. It is also increasing function of tax evasion level. Second term represents the effect of transaction cost of evasion, which is positively related to evasion level as it is a convex function of tax evasion. Third term represents the indirect cost of tax evasion. As household evades more, the part of the income that is subject to fine when get caught increases; therefore cost of evasion increases.

First term in the right-hand side represents the direct gain from evasion via decreasing tax burden. Rests of the terms in right hand side represent indirect gains from tax evasion. As households evade, their income decreases via decrease in effectively used public expenditures. This decrease in income results

in decrease in tax liability, decrease in fines when get caught and decrease of transaction cost as income subject to it decreases.

Equation (12) leads to following second order equation for degree of tax compliance;

$$-(2h+\epsilon(1-\sigma)h)\alpha^2+(2h-(1-p\theta)\tau)(1+\epsilon(1-\sigma))\alpha+\epsilon(1-\sigma)(1-h-p\theta\tau)=0 \quad (15)$$

Under circumstances $\epsilon > 0$, $(1-\sigma) > 0$, $p\theta < 1$, $\tau > \frac{\epsilon(1-\sigma)}{(1-p\theta)+\epsilon(1-\sigma)}$, there exists interior solution and optimal degree of tax compliance becomes[3] ;

$$\alpha^* = \phi(h,p,\theta,\tau,\epsilon,\sigma) + \sqrt{\psi(h,p,\theta,\tau,\epsilon,\sigma)} \quad (16)$$

where

$$\phi = \frac{(2h-(1-p\theta)\tau)(1+\epsilon(1-\sigma))}{2h(2+\epsilon(1-\sigma))} > 0$$

$$\psi = \frac{((2h-(1-p\theta)\tau)(1+\epsilon(1-\sigma)))^2 + 4(2h+\epsilon(1-\sigma)h)\epsilon(1-\sigma)(1-h-p\theta\tau)}{4h^2(2+\epsilon(1-\sigma))^2} > 0$$

Relationship between Income Tax Rate and Tax Compliance

In the static case of tax evasion, as Yitzhaki (1974) shows, relationship between tax rate and tax compliance is positive when evasion fines are imposed upon evaded tax. This result is in contrast to empirical reality, and takes severe criticisms.

In the model of this thesis, assessing effect of tax rate on the level of tax

[3]Certainly α has two roots but the other root is always negative in plausible values of parameters.

compliance is not straightforward as there are several dynamics in place. Nevertheless, all of these dynamics results in following proposition;

Proposition 1: There is unambiguously negative relationship between income tax rate and tax compliance. Increase in taxes makes households conceal more part of the income.

The proof of proposition 1 can be found in Appendix C.

Therefore, the counterintuitive result of static case models are reversed in the dynamic framework. As gains from avoiding higher tax burden overcome costs of it, households react to increase in tax rates by evading more.

Relationship between Audit Probability, Fine Rate and Tax Compliance

Empirical studies show that enforcement policies of audit efforts and penalty rates have positive impact on degree of tax compliance.Higher probability of audit and more severe penalties associated with tax evasion leads to higher marginal cost of evading income.Therefore,the increase in the effective cost of tax evasion leads higher compliance.The set up of this thesis's model,too,leads to this expected result.

Proposition 2: A positive change in audit probability and fine rate increases tax compliance.

The proof of proposition 2 can be found in Appendix C.

Relationship between Unit cost of Tax Evasion and Tax Compliance

Unit cost of evasion can be thought as representing the level of corruption in an economy. When unit cost rises, this means that corruption in the economy decreases in which case we can expect that tax evasion decreases. Following proposition reflects the relationship between unit cost and tax compliance.

Proposition 3: A rise in the unit cost of tax evasion (i.e. fall in corruption level)

decreases tax evasion rate. Effect of decrease in corruption is most effective in moderate levels of tax evasion. In an economy that is characterized by extreme level of tax evasion, combating corruption to increase tax compliance rate may not be fruitful.

The proof of proposition 3 can be found in Appendix C.

In high level of evasion, increasing h (decreasing corruption) may not mean so much which means that in a country with extreme tax evasion, combating corruption may not be fruitful as expected at start. Nevertheless, as some gains achieved in the long run, within moderate level of tax evasion, gains from combating corruption will increase.

Relationship between Government Externality and Tax Compliance

The degree of government externality on production function indicates the end value of the taxes collected by government. If the services supplied by government become more productive, accessibility to these services becomes more crucial. As tax evasion leads to underutilization of these services, we can expect that higher externality level of public expenditures gives incentive to be compliant to tax system in order to utilize public services properly. Therefore, dynamics of this paper's model leads to following proposition:

Proposition 4: There is a positive relationship between public expenditure externality level and tax compliance. Higher externality increases the value of utilizing public expenditures hence being compliant and results in less tax evasion.

The proof of proposition 4 can be found in Appendix C.

Summary Results

If we summarize comparative dynamics results;

$$\frac{\partial \alpha}{\partial \tau} < 0, \frac{\partial \alpha}{\partial p} > 0, \frac{\partial \alpha}{\partial \theta} > 0, \frac{\partial \alpha}{\partial h} > 0, \frac{\partial \alpha}{\partial \sigma} < 0$$

Tax compliance is positively related to probability of audit, fine rates, higher transaction costs and higher government externality. It is negatively correlated with tax rates.

These conditions lead to very crucial results. They reinforce the arguments about decreasing tax rates and increasing audit efforts and punishment rates. They are also proposing the important role of increasing unit cost of evasion, hence decreasing corruption in an economy and making the sustainability of high rates of tax evasion implausible.

More importantly, these results suggest that struggle with tax evasion does not only about deterrence policies. It is also about giving households positive incentives for being compliant to tax structure. If government supplies productivity enhancing services to citizens, they will comply voluntarily to tax structure without need of harsh punishments and extreme audit efforts.

Special Cases

Case 1: $\epsilon = 0$ or $(1 - \sigma) = 0$.

When exclusion parameter or externality level of public expenditures is zero, we come across the case which is studied by Chen (2003).In other words, Chen's article is a special case of this thesis's model with $e = 0$ or $(1 - \sigma) = 0$. In this

special case, optimal level of tax compliance becomes;

$$\alpha = 1 - \frac{(1 - p\theta)\tau}{2h} \tag{17}$$

This formulation suggests that tax compliance does not depend on externality level of productive government expenditures. Taking derivative of this formulation with respect to tax rate results in;

$$\frac{\partial \alpha}{\partial \tau} = -\frac{(1 - p\theta)}{2h} \tag{18}$$

This result suggests that relationship between tax rate and compliance is a linear one which is equal in economies with high and low externality level of public expenditures. This result is unrealistic in the sense that in an economy in which government plays a crucial role, taxation may lead to less incentive to evade than in an economy in which government supplies almost nothing. In addition, derivative of compliance level with respect to tax rate also suggests that response of tax compliance to one unit change in tax rate is equal when tax rate is low or high which is unrealistic, too.

In addition, derivative also suggests that response of tax compliance to one unit change in tax rate is equal when tax rate is low or high which is unrealistic, too.

More importantly, above formulation suggests that $\alpha = 0$ if $\tau = \frac{2h}{(1-p\theta)}$ .With benchmark values of Chen [4] when income tax rate becomes above 34.6%,households conceals all of the income from the tax authorities. This result is unrealistic, as income tax rate is even higher in some countries such as Sweden, Belgium. To put it differently, exclusion of effect of tax evasion on the effective level of public expenditures leads to such dynamics in which the relationship between tax rates and tax evasion becomes exaggerated.

[4] h=0.15, p=0.089 and $\theta = 1.5$

Case 2: $p\theta > 1$.

As in classical static models, when effective punishment becomes greater than one, tax evasion vanishes because expected gain on tax evasion becomes certainly negative.

Case 3: $\tau < (\epsilon(1-\sigma))/((1-p\theta)+\epsilon(1-\sigma))$.

This condition upon tax rate results in $\alpha = 1$,i.e. full compliance on income tax.In other words, this level of tax can be thought as threshold level that eliminates tax evasion problem completely.That is the level of tax which equalizes tax burden of an individual and the gain from being completely compliant to the tax system.

Existence of such a level of tax rate can make non-sense when thought about macro perspective. One can think that existence of an even very small level of a tax brings tax evasion with it. However, existence of such a level is can be thought as an exaggerated manifestation of an important micro reasoning: exchange equity. The concept of exchange equity is one of the widely studied subjects in tax compliance literature. In many studies, exchange equity is found significantly affecting tax compliance decision of economic agents. When people think about evade or not-evading, they think about its costs as well as its benefits.

The structure of the model of this study incorporates one aspect of this phenomenon. Tax evasion makes them unable to use up benefits of productive public expenditures. Therefore paying taxes honestly brings them additional positive incentive apart from negative consequence of facing tax burden. And in some threshold level[5] representative agent decides not to evade anymore.

[5]In numeric analysis in optimal taxation part, we will see this tax rate is very low and suboptimal within perspective of central authority

CHAPTER 5
OPTIMAL POLICIES

Up to now, optimization on the side of government was missing in the dynamics of the model. Supplementing the model with optimal decision making of central authority would lead to more genuine general equilibrium effects between choices of the agents of the model. When defining optimal policies, government has to take into account direct and indirect general equilibrium effects of tax and audit policies on growth.Tax rates on the one hand decrease the rate of return to capital and on the other raises public spending to capital ratio which has direct effect on growth in our model. Moreover, tax rates causes evasion which results in under-utilization of productive government expenditures and also results in diversion of resources from productive uses to tax evasion costs. Audit policies leads to reduction in tax evasion rate but at the same time they divert resources from productive public expenditures.All of these effects results in complicated dynamics in defining the optimal policies.

In this section, within a Ramsey-type formulation we will try to figure out optimal policies of the central authority. That is, government tries to figure out optimal tax rate and optimal audit probability by taking the competitive behavior of the agents of the economy as given. Within this set up, sequence of optimal decisions actually starts with optimal decisions of the government. Government announces optimal Tax Rate and audit probability by taking into account best responses of representative agents.Given the statutory tax rate and audit probability, representative household-firm chooses the optimal level of consumption-saving and the level of reported income.These sequential decisions of the model are solved backwards. First, we have found competitive equilibrium without taking into account externalities of productive government expenditure and tax evasion.Now, we will try to figure out government

optimization by taking into account externalities and competitive equilibrium.

In following subsections, by following the literature , we will firstly analyze optimal tax problem. Than, we will try to figure out characteristics of optimal policy on audit probability and will look at the problem of joint optimization of audit probability and tax rate.

Optimal Taxation

Government decides optimal τ to maximize lifetime utility of the representative individual subject to government budget constraint; with taking into consideration competitive behavior of households characterized by equations (10) and (12). As shown in Barro (1990), in endogenous growth set up with productive public expenditures, maximization of discounted lifetime utilities is equivalent to maximization of the consumption growth rate, if the utility is bounded and the elasticity of substitution between g and k in production is unity. Cobb-Douglass production function enables the second condition and condition for bounded utility is plausible within the parameters of the model. (Formal treatment of this issue can be found in Appendix)

When optimizing growth rate by choosing optimal tax rate, government will certainly take into account externality of public expenditures. Therefore, contrary to optimization process of households, it will not take public expenditure level as given. So, we should put actual level of public expenditures to capital ratio into the Euler equation.

By (2) and (7);

$$\frac{g}{k} = A^{1/\sigma}(\alpha^{\epsilon(1-\sigma)}(\tau_E - fp^2)^{1/\sigma} \tag{19}$$

If we put (37) into (2) growth rate becomes;

$$\frac{\dot{c}}{c} = \frac{1}{\gamma}\left(A^{\frac{1}{\sigma}}\sigma\left(\alpha^{\epsilon}(\tau_E - fp^2)\right)^{\frac{1-\sigma}{\sigma}}(1-\tau_E - h(1-\alpha)^2\right) - \rho) \tag{20}$$

Government maximizes (20) with respect to τ by taking into account (12) which leads to following Ramsey-Rule relation;

$$\frac{1-\sigma}{\sigma}\frac{\frac{\partial\tau_E}{\partial\tau}}{\tau_E - fp^2} = \frac{\frac{\partial\tau_E}{\partial\tau}}{1-\tau_E - h(1-\alpha)^2} - \frac{2h(1-\alpha)\frac{\partial\alpha}{\partial\tau}}{1-\tau_E - h(1-\alpha)^2} - \epsilon\frac{1-\sigma}{\sigma}\frac{\frac{\partial\alpha}{\partial\tau}}{\alpha} \tag{21}$$

Left-hand side of the equation shows marginal benefit of taxation. When tax rate changes, tax revenues that are spend for productive public expenditures are affected which in turn affects output. This effect of tax via government expenditure is augmented by externality level of productive public expenditures. The more public expenditures are important for production, the more marginal benefit from taxation. Marginal benefit is also adjusted by level of government expenditures which reflects the diminishing returns.

First term in Right-hand side (RHS) reflects the effect of taxes on disposable income which in turn affects capital accumulation. This effect is adjusted by the level of disposable income. When effective tax rate is high, the marginal effect of taxation on capital accumulation is becomes more severe. Second term, in RHS is the effect of taxation on transaction cost. As taxation affects tax evasion choice, it indirectly affects the level of transaction cost that is foregone in the process of evasion. Reflecting the convex structure of transaction cost, this marginal cost effect is increasing function of the level of evasion. Third term reflects the effect of taxation on effectively utilized level of government expenditure via its effect on tax evasion decision. It is adjusted by level of government externality and exclusion parameter.

As stated in Barro (1990), in an economy without tax evasion, optimal tax

rate is equal to the externality level of productive government expenditures on production function.In other words, optimal taxation is defined by the motive to supply necessary public expenditures to attain full productive efficiency.In Judd (1999) it is also shown that if public good affects only output , optimal provision of productive public goods satisfies full productive efficiency independent of the efficiency costs of taxation.Whether existence of tax evasion and the negative externalities associated with it alters these results is very important problem.

Equation (21) defining the level of optimal tax level is not solvable in algebraic way. Nevertheless, some analysis on (21) can lead us to figure out some characteristics of optimal taxation and the associated supply level of productive government expenditures.The first remark that can be concluded from (21) is that optimal tax level is not the level that maximizes effective tax rate of the government. Because if $\frac{\partial \tau_E}{\partial \tau} = 0$, marginal cost of taxes exceeds marginal benefits, hence $\frac{\partial \dot{c}/c}{\partial \tau} < 0$.

In addition,$\frac{\partial \tau_E}{\partial \tau} = (\alpha + (1-\alpha)\mathrm{p}\theta) + \tau(1-p\theta)\frac{\partial \alpha}{\partial \tau} = (1-p\theta)\left(\alpha + \tau\frac{\partial \alpha}{\partial \tau}\right) + p\theta$ is not always positive as $\frac{\partial \alpha}{\partial \tau} < 0$.

It is more likely that it is negative when α is low and τ is high. These two conditions are parallel and complimentary to each other as we know that increase in tax rate leads to more tax evasion. Therefore this scenario happens within an economy with high taxes and high informality. Within that situation, informality is extremely severe and sensitive to increases in taxes such a way that, when tax authority raises income tax rate, its revenues declines. This situation makes non-sense for a level of tax that is optimal[6] . Therefore optimal tax rate solutions lies in a region where $\frac{\partial \tau_E}{\partial \tau} > 0$ This fact leads to a very important result.

[6]Quantitative simulations also suggests that for Turkey there is no such level of tax that makes $\frac{\partial \tau_E}{\partial \tau} < 0$

Proposition 5: Optimal tax rate satisfies

$$(1-\sigma)+X>\tau_E^* \tag{22}$$

in which $X=\sigma fp^2-(1-\sigma)h(1-\alpha(\tau*))^2$

The proof of proposition 5 can be found in Appendix C.

X is composed of audit costs fp^2 and transaction cost $h(1-\alpha)^2$. Depending on the magnitude of audit costs and transaction cost we can assert that effectively collected tax lies above or under the externality level of productive government expenditures. Proposition 5 suggests that when higher audit costs necessitates government to collect more taxes, higher transaction cost associated with corruption results collecting less tax in optimal condition. Higher audit costs require more revenues for government to supply effectively necessary public expenditures. In other words, audit costs crowd outs some part of tax revenues that can be spend to productive expenditures, so government needs more tax revenues. Higher transaction costs mean that higher fraction of income is gone to unproductive purposes. To put it differently, higher transaction costs crowds out some part of the income that can be saved and used as capital or consumed or given as taxes.Proposition 5 suggests that when higher audit costs necessitates government to collect more taxes, higher transaction cost associated with corruption results collecting less tax in optimal condition.Higher audit costs require more revenues for government to supply effectively necessary public expenditures. In other words, audit costs crowd outs some part of tax revenues that can be spend to productive expenditures, so government needs more tax revenues.Higher transaction costs mean that higher fraction of income is gone to unproductive purposes. To put it differently, higher transaction costs crowds out some part of the income that can be saved and used as capital or consumed or given as taxes.

Moreover, further manipulation of (22) results in following proposition;

Proposition 6: Under the existence of tax evasion with transaction and audit costs, optimal level of productive public expenditure to be supplied by central authority is less than the externality level of public expenditures on production.

Existence of tax evasion by itself does not lead to under-provision of productive public expenditures.In other words, without any negative externality, central authority still tries to attain production efficiency by supplying necessary public expenditures in line with its externality effect.Nevertheless, negative externality associated with tax evasion via transaction cost and exclusion effects leads central authority to supply public expenditures less than its externality level.That is, existence of negative externality of tax evasion leads to productive inefficiency in terms of supply of public expenditures.

Furthermore, manipulation of (21) leads to following condition for the level of optimal tax;

Proposition 7: Optimal level of income tax is given by;

$$\tau^* = \frac{1}{Z}((1-\sigma) + X + W)$$

in which $Z = (1 - p\theta)\alpha(\tau^*) < 1$

$$W = \frac{\partial\alpha(\tau^*)/\partial\tau^*}{\partial\tau_E(\tau^*)/\partial\tau^*} g\left(\tau^*\right)\left(2h(1-\alpha\left(\tau^*\right)) + \frac{\epsilon}{\alpha(\tau^*)}\left(\frac{1-\sigma}{\sigma}\right) y_d\left(\tau^*\right)\right) < 0$$

If $\alpha(\tau^*)$ is small, Z is small therefore τ^* is high. At the same time, if transaction cost of evasion and/or underutilization of public expenditures are high, X and W can become negative which leads to low τ^* .This fact can occur even if ϵ or $(1-\sigma)$ is zero.

Therefore, optimal tax rate depends on several dynamics.On the one hand, existence of informality makes central authority to increase taxes to collect necessary revenues. In addition to this effect, existence of cost of audits makes

central authority to increase taxes further.On the other hand, negative externality of tax evasion via transaction costs (i.e. corruption) and the effect of tax evasion on underutilization of public expenditures forces central authority decrease taxes.

Proposition 8: Existence of informality and audit costs makes government to increase taxes. In contrast, existence of corruption in the economy and exclusion from public expenditures forces government to decrease tax. The net effect is ambiguous, therefore optimal tax rate can become higher, lower or equal to the externality level [7] .This result holds even when there is not any exclusion effect from public expenditures (i.e $\epsilon = 0$).

Proposition 8 is in stark contrast with Chen (2003) who asserts that optimal tax is higher than externality level of public expenditures in an economy with tax evasion. He asserts in the paper that "First, tax evasion forces the government to set a lower tax rate, but in order to maintain optimal public services, the government needs to set a higher tax rate to collect enough tax revenues. As the latter effect dominates the former effect, the net effect is a higher tax rate"(p.12). When proposing in this way, Chen skips the effect of transaction cost on optimal level of tax rate. He does not consider the fact that increasing taxes higher to maintain optimal level of public services increases informality and associated the transaction costs, hence the part of income that is foregone. In addition, he skips the fact that distortion of tax evasion and corruption makes optimal level of productive government expenditures less compared to the situation of an economy with no evasion and no corruption.

In the light of above conditions we can conclude that existence of evasion does not necessarily legitimize taxing more.In opposite, tax evasion and corruption can lead a government to decrease tax rate in optimality compared to case with no evasion.

[7]Following quantitative simulations will also suggest that indeed optimal tax rate is less than externality level.

Optimization of Audit Probability

Another policy option for the central authority in our model is defining how much to spend for audit purposes in order to deter agents of the economy from tax evasion. Spending on audits certainly decreases tax evasion as it is studied in previous section on comparative dynamics. Nevertheless, spending on audits has adverse effect on the economy as it crowds out the amount that is spend on productive public expenditures. In other words, spending on audit purposes has the opportunity cost of foregone supply of public expenditures which directly affects production of the economy.

In previous section, we have studied revenue side of the optimal policy of the central government with emphasis on optimization of tax rate. In this section, we will abstract from revenue side and will study the trade-off on the spending side of the central authority by trying to figure out optimal audit probability.

With a similar logic to previous section, government decides optimal p to maximize lifetime utility of the representative individual subject to government budget constraint; with taking into consideration competitive behavior of households characterized by equations (10) and (12). And like in previous section, the problem can be conceptualized as maximization of the consumption growth rate by choosing the optimal audit probability with taking into account externality of public expenditures on the production function. Therefore, we should put actual level of public expenditures to capital ratio into the Euler equation of the competitive equilibrium and maximize (20) with respect to p by taking into account (12) which leads to following Ramsey-Rule relation;

$$\frac{1-\sigma}{\sigma}\frac{\partial\tau_E/\partial p}{\tau_E - fp^2} + \frac{2h(1-\alpha)\partial\alpha/\partial p}{(1-\tau_E - h(1-\alpha)^2)} + e\frac{1-\sigma}{\sigma}\frac{\partial\alpha/\partial p}{\alpha} = \frac{\partial\tau_E/\partial p}{(1-\tau_E - h(1-\alpha)^2)} + \frac{1-\sigma}{\sigma}\frac{2fp}{\tau_E - fp^2} \quad (23)$$

We can infer from optimality condition of audit probability the fact that it is very similar to the equation (21) that defines optimal tax level. Nevertheless, it includes an additional term $\frac{1-\sigma}{\sigma}\frac{2fp}{\tau_E - fp^2}$ in right hand side of the equation and some terms in RHS of the (21) are in LHS.

Left-hand side of the equation (23) shows marginal benefit of increasing the probability of audit. First term in LHS reflects the marginal benefit of increasing audit probability by its effect on evasion rate and hence on effectively collected tax rate. As these effectively collected taxes affects the supply of public expenditures, this marginal effect of audit probability is augmented by externality level of productive public expenditures. The more public expenditures are important for production, the more marginal benefit from increasing audit probability. This marginal benefit is also adjusted by level of government expenditures which reflects the diminishing returns. Second and third term, in LHS reflect the effect of audit probability on transaction cost and effectively utilized level of government expenditure via its effect on tax evasion decision. These terms were included as a marginal cost in eqrefoptimalite as tax evasion rate and tax rate were inversely related. As the level of tax evasion and probability of audit are positively related, the second and third terms in LHS of the equation (23) reflects the marginal benefit of increasing probability rate via decreasing the negative externality of tax evasion with diminished transaction cost and underutilization of public expenditures.

First term in Right-hand side reflects the effect of audit probability on disposable income with its effect on effective tax rate which in turn affects capital accumulation. Second-term in RHS reflects costly structure of audits as an additional burden on the economy. Observe that, this marginal cost is adjusted by the term in denominator which reflects the supply level of productive government expenditures. This fact suggests that naturally, marginal cost of increasing audit probability depends on its relative cost within

the total revenue of government. Therefore, with given unit cost of audit, we will expect lower level of optimal audit probability when we have lower level of effectively collected tax revenue.

Equation (23) defining the level of optimal audit probability is not solvable in algebraic way. Nevertheless, some analysis on equation (23) can lead us to figure out some characteristics of optimal audit probability rate. To attain this goal, we will exploit one of the special cases that was presented in the Section 4 that have studied comparative dynamics on optimal tax evasion rate which have talked about the level of tax rate that have eliminated tax evasion rate. In that section, we have asserted that when tax rate satisfies $\tau = \frac{\epsilon(1-\sigma)}{(1-p\theta)+\epsilon(1-\sigma)}$, tax evasion is completely eliminated and we will have $\alpha = 1$. For a given tax rate if we manipulate this equation we will have

$$\bar{p} = \frac{1}{\theta}(1 - \epsilon(1-\sigma)\frac{(1-\tau)}{\tau}) \tag{24}$$

$\bar{p}$ is the level of audit probability that eliminates tax evasion completely for given level of tax rate and other structural parameters. Notice that it is inversely related to punishment rate, degree of government externality and exclusion parameter and positively related on tax rate. In addition, if there is no exclusion effect (i.e $\epsilon = 0$) this level simply defined as $\bar{p} = \frac{1}{\theta}$ which is simply the condition that effective punishment $p\theta$ is equal to one or not. At this point, the question is whether $\bar{p}$ is the optimal level of audit probability to be attained. In other words, the problem is whether eliminating tax evasion completely with spending on audit efforts as much as needed is optimal or not. With inspecting the desirability of this condition we can infer the level of optimal audit probability with given parameters.

To achieve this aim, suppose that in optimal condition $\alpha = 1$.Than τ_E will simply becomes τ and $\frac{\partial \tau_E}{\partial p}$ becomes zero. Therefore, in equation (23) ,first and

second term in LHS and first term in RHS is eliminated, our equation defining optimal level of audit probability becomes very simplified and we can reach easily the following proposition;

Proposition 9: If

$$\left.\frac{\partial \alpha}{\partial p}\right|_{p=\bar{p}} \geq \frac{2f\bar{p}}{\epsilon(\tau - f\bar{p}^2)} \tag{25}$$

$$\text{than } p^* = \bar{p} \text{ which leads to } \alpha^* = 1,\ \tau_E = \tau \text{ and } g^* = \tau - f\bar{p}^2$$

Proposition (9) suggests that increasing audit efforts up until to have full compliance level can be optimal depending on the sensitivity of tax compliance rate to probability of audit, on unit cost of audits, on level of exclusion parameter and on the tax rate.

Inspection of the proposition leads us to the fact that LHS of the equation (25) depends on unit cost of tax evasion as $\frac{\partial \alpha}{\partial p}$ is negatively related to h. On the other hand, RHS of (25) does not depend on h. Therefore, given the levels of other parameters, we are more likely to have $\bar{p}$ as optimal level of audit probability if we have smaller h. More generally, we can assert that smaller values of h are associated with high optimal values of audit probability. In other words, audits efforts become more valuable in an economy with small unit cost of tax evasion (i.e. with high corruption). In addition, RHS of the (25) suggests that with given f we will more likely to have $\bar{p}$ as optimum with higher levels of t. This fact suggest that audit costs are not evaluated with its absolute cost but evaluated with respect to its relative cost on revenues of the government.

Joint Optimization of Tax Rate Audit Probability

Optimization of growth rate with respect to tax rate and probability rate simultaneously leads to have both equation (21) and equation (23).These

equations together are analytically insolvable as they include complexities of both former optimal tax and optimal audit probability problems. In addition, within these equations there are additional complexities resulting from endogenous interactions between probability of audit and tax rate. Higher audit probability leads to have higher optimal tax rate and higher optimal tax rate leads to have higher optimal audit probability as relative cost of audits decreases. Therefore, there is positive synergetic relationship between these two optimal policy options. The magnitude of this synergetic relationship and its effect on growth rate of the economy will be investigated in the following sections with quantitative analysis.

CHAPTER 6
CALIBRATION OF THE MODEL AND SIMULATIONS ON OPTIMAL POLICIES

Calibration

In this section, we attribute values to the parameters of our benchmark model to present quantitative results of the model for the case of Turkey. For this end, we use a combination of figures from previous studies and facts about empirical realities of Turkish economy. Benchmark parameter values for calibration are given in Table (1). To find a figure of tax burden before evasion process, we use data of the Public Revenue Administration (PRA). The data suggests that average tax that is paid by households that are registered to tax system is around 25%. That is, when a household complies to tax system fully, it will face a 25% tax burden. This level is in line with Saracoglu (2008) who chooses 24% as benchmark value as average tax burden. For the value for audit probability, one can conceptualize the problem in two ways. Either one can take the actual number of audits and divide it by potential number of tax payer or one can take the part of the national income that is scanned by audits as an average figure. Former approach does not take into account a weighting process between large and small firms. To make calibration in a representative agent framework, one should take figures as overall weighted average for the whole economy. Therefore, as latter approach gives a sounder picture for overall audit probability in an economy, we will use this approach. Data of the Public Revenue Administration (PRA) suggests that with its audits, on average 10% of the GDP of Turkey is audited or revealed as evaded in an audit. In other words, ministry of finance reaches 10% of income on average with its audits. We take this value as probability of audit. When compared to literature, this level can

Table 1. Benchmark Parameter Values

Parameter	Symbol	Value
Unit cost of audit	f	0.15
Probability of audit	p	0.10
Punishment Rate	θ	1.5
Average tax rate	τ	0.25
Coefficient of risk aversion	γ	1.5
Time preference rate	ρ	0.08
Degree of government externality	σ	0.15
Unit cost of tax evasion	h	0.27
Coefficient of productivity	A	0.29
Exclusion Parameter	ϵ	0.5

be seen as reasonable. Chen (2003) takes p as 0.089 for Taiwan and Hepburn (1992) calculates p as 0.11 for Australia with the same approach on probability of audit. Data of PRA also suggests that costs of audits in Turkey constitutes around 0.15% of the GDP. Within model's formulation, this fact leads to $f = 0.15$ for calibration. For the figure of punishment rate, the law in Turkey states $\theta = 2$, but not all of the fines are paid to the government. Usually some part of fine is discounted by compromises with tax authorities. Therefore I follow the literature (e.g. Chen (2003)) and take $\theta = 1.5$. In terms of preferences, coefficient of risk aversion and time preference rate is set to 1.5 and 0.08, respectively, following Alper, Ardic, and Hatipoglu (2010). The figure for coefficient of risk aversions reflects the level for developing countries. The high time preference rate of 0.08 reflects the high real interest rates in Turkey which was around about $12\% - 16\%$ in 90s and even in low inflation era of recent years it could barely decrease to 7%.

In terms of technological parameters, selecting a figure for degree of government externality on production function is not straightforward. There is no direct study to estimate this figure for Turkey. In addition, for this parameter, there is ongoing debate in the literature. Estimates can be found in

wide ranges which are dependent upon the assumptions about productive part of public expenditures. Ayogu (2007) gives a comprehensive literature review for this issue. As stated before, this study takes broad definition of public expenditures including most of the goods and services supplied by government. As share of capital income from national output is mostly taken for approximation of elasticity of capital stock to production function, share of government spending can be used as a figure for government externality. Nevertheless, there are certain figures in government expenditures which can not be considered as productive expenditure such as interest payments. As a result, for Turkey, we take government externality as 0.15 in benchmark scenario by inspecting actual national accounts data and subtracting interest payments and pure consumption expenditures of government from total public expenditure. Moreover, different values for government externality will be also used in simulations on optimal policies in order to elaborate effects of different levels of government externality. The other technological parameter, the coefficient of productivity is chosen to calibrate the log-run per capita growth for Turkey. In Altug and Pamuk (2008), average per capita growth rate for Turkey for the period of 1980-2005 is given as 2.5%. In 2008, Turkish Statistical Institute started to declare new series for GDP data with a new base year. According to this new series, the negative growth rates of Turkey in 1999 and 2001 crisis is much lower than previous figures. With this respect, 2.5% average growth rate figure underestimates the long-run trend of Turkey. On the other hand, taking the end year of 2005 to calculate long-run growth rate for Turkey does not include the effects of recent financial crisis, therefore that figure overestimates the long-run growth rate for Turkey. Including growth figures up until 2009 and including estimates of recovery in 2010, we reach to a level of 2.4% average per capita growth rate for Turkey which is used in calibration process.

In terms of parameters that constitutes negative externalities of tax

evasion, unit cost of evasion is calibrated to reach informality level of 33.3% for Turkey that is average of the 2000-2007 data given by Schneider and Montenegro (2010). In Schneider et. al. (2010), this figure is the magnitude of the shadow economy in terms of the official GDP figures. And, it is defined as the magnitude of market-based legal activities that are concealed from public authorities to avoid taxes and other burdens. This definition is in line with the evasion process of our model. To mimic the fact that shadow economy part of the output produced is 33.3% of the official GDP, we must calibrate unit cost of evasion such a way to reach $\alpha = 0.75$. In terms of calibrating the exclusion parameter ϵ, the theoretical assumption that relation between tax evasion and underutilization of public expenditures is convex is maintained and as a result ϵ is taken as 0.5.To assess the validity of this assumption about ϵ we will simulate the calibrated economy with different levels of ϵ .

When calibrating the model for Turkey, the only missing parameters that are needed to calibrate tax evasion level are unit cost of evasion and exclusion parameter. Therefore, we have simulated the level of tax compliance for all levels of h under assumption of $e = 0.5$ than simulated economy to calibrate actual case with different levels of e. Results can be seen in Figure (1), Table (2) and Table (3). As it can be observed, the level of h that calibrates the level taken from Schneider and Montenegro (2010)) is 0.27 which leads to a transaction cost level about 1.7% of the GDP. The calibrated h value of 0.27 is close to the more corruption part of $(0, 1)$ continuum. Therefore, in order to explain the informality level of Turkey, the model needs high level of corruption in Turkey.

Than we calibrate economy to reach 33% evasion level for different levels of e. Notice that when $e = 0$, h needed for calibration is 0.43 and transaction cost is 2.7% of GDP which becomes quite high. In addition, with a linear trend of exclusion with $e = 1$, level of h is very low which states that the needed level

Table 2. Calibration of Unit Cost of Tax Evasion(h)

$\dot{c}/c$	h	α	Transaction Cost	τ_E
2.4%	0.10	0.53	2.2%	14.9%
2.4%	0.27	0.75	1.7%	19.7%
2.4%	0.50	0.86	1.1%	21.9%
2.4%	0.75	0.90	0.7%	22.9%
2.4%	1.00	0.92	0.6%	23.4%

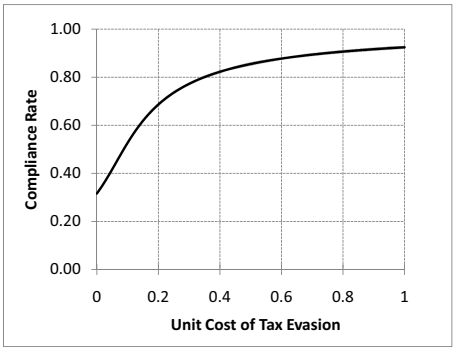


Figure 1. Unit cost of tax evasion and compliance rate

of corruption is extremely high. What is more , when $e = 2$ which implies even small levels of evasion leads to high exclusion from public expenditures, the actual level of informality can not be reached by the model. Therefore, these results support the assumption that $e = 0.5$.

Table 3. Checking for Assumption $e = 0.5$

e	h	Transaction Cost
0	0.43	2.7%
0.5	0.27	1.7%
1	0.11	0.7%
2	-	-

In Figure (2), we can see simulation of informality level for continuum of tax rates for benchmark parameter values. The results states that the threshold level of tax that model states for initiation of problem of informality is 7.8%. After that rate, informality starts to rise in a decreasing rate as income tax rate increases.

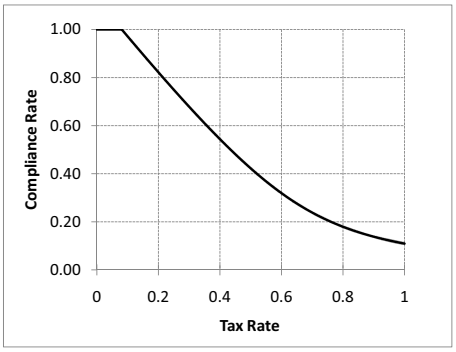


Figure 2. Income tax rate and growth

In figures (3) to (6), the simulation results of informality with respect to probability of audit, punishment rate, externality level and exclusion parameter when taking all other parameters constant is presented. These figures illustrate theoretical implications of the model in terms of effects of deterrence policies and government externality on tax compliance.

Figures (5) and (6) suggests that even with zero level of audit and punishment rates there would occur 60% of compliance .This result can be thought as extreme and actually it reflects the fact that in the model public expenditures are given as essential for the production of firms. Nevertheless, it suggests qualitatively that major part of tax compliance in Turkey arises as a result of positive benefits of being compliant via taking advantage of public expenditures. In order to utilize public expenditures properly, most of firms would comply with the tax system even if audits and punishments are minimal.

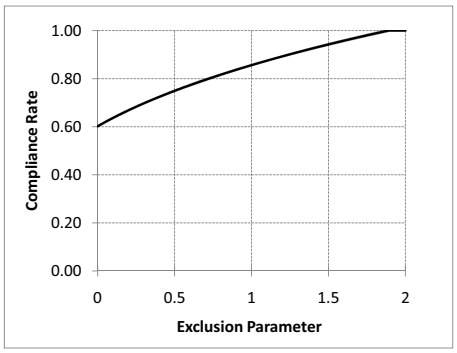


Figure 3. Exclusion parameter and compliance rate

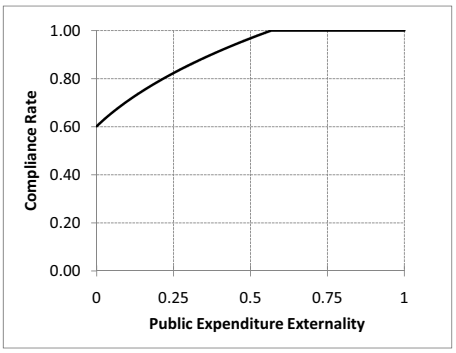


Figure 4. Public expenditure externality and compliance rate

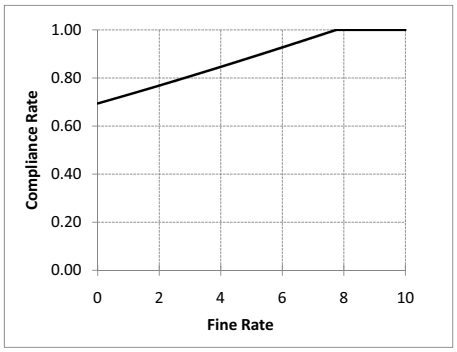


Figure 5. Fine rate on evaded tax and compliance rate

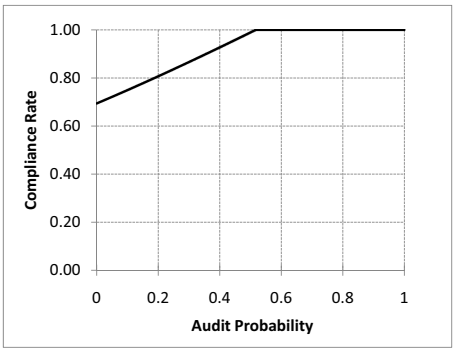


Figure 6. Audit probability and compliance rate

On the other hand, extreme values of $p = 0.50$ or $\theta = 8$ are needed for eliminating tax evasion when other parameters are taken as constant. This situation indicates that simply extending audits or punishments would not result complete elimination of informality. In fact, as other figures suggests, efforts of audits and punishments must be supplemented by other policies such as;

i) increasing externality level of public expenditures (i.e. better of productivity enhancing expenditures)

ii) policies that makes exclusion parameter higher(i.e. extending public expenditures which can only be utilized by compliant firms)

iii) increasing unit cost of evasion (i.e. combating corruption)

iv) decreasing tax rates(i.e. making them compatible with externality level)

As solution of optimal tax problem is intractable, we try to figure out level of it by quantitative analysis. Simulation of optimal tax rate with benchmark parameter values that has used before leads following results presented in Figure (7) and Table (4) As it can be seen from Figure (7), optimal tax problem has a unique interior solution. With benchmark values, optimal income tax rate for Turkey is found as 13.40% with growth rate of 2.73% which means more than 10% improvement in the growth rate. Remark that optimal tax rate and associated optimal productive public expenditure 12.34% is below externality rate of 0.15.

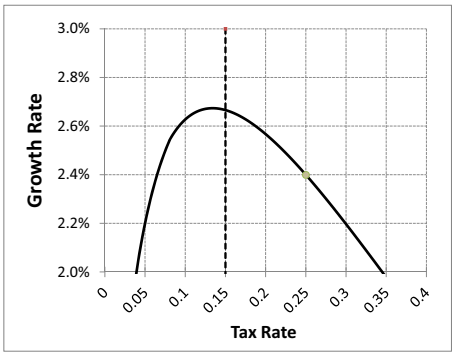


Figure 7. Income tax rate and growth

As stated before, debates about externality level of productive public expenditures results in wide range of estimates. With taking into account this fact, to examine sensitivity in results, we have simulated optimal tax level for different levels of externality level from 0.10 to 0.3.When simulating with different externality levels, we have changed h and A at every turn to calibrate the initial state of the economy. As it can be seen in Table (4) ,different externality levels changes the optimal tax levels but the fact that optimal tax level is below the externality level does not change. In addition, the wedge between externality level and optimal tax widens with increasing externality

Table 4. Simulations on Optimal Tax Rate

$(1-\sigma)$	h	$\tau*(\%)$	Growth Rate(%)	$\alpha(\tau*)(\%)$	$g(\tau*)(\%)$
0.15	0.27	13.40	2.73	92.02	12.34
0.10	0.32	9.49	3.00	94.89	8.93
0.2	0.21	16.46	2.59	89.47	14.84
0.25	0.16	18.31	2.53	88.27	16.34
0.3	0.11	18.58	2.54	90.39	16.91

levels. The reason for this fact can be infer from the fact that high values of 0.25 and 0.3 results in very low figures of 0.16 and 0.11 for unit cost of tax evasion. In other words, with such high levels of government externality, current level of tax evasion can e achieved by extreme levels of easiness of tax evasion. This fact increases associated negative externality of tax evasion and results in wider wedge between externality level and optimal tax. As a result, the increase in optimal tax level associated with increased externality level diminishes very much and optimal tax level does not go over 18.58%. The extreme low levels of h and very wide wedge of optimal tax level and externality level proposes that calibrating Turkish economy with externality levels of 0.25 and 0.30 is not convenient. The growth effect of optimal taxation, too, significantly changes with different externality levels. The more is the wedge between the taken externality level and current level of tax as 0.25, the more is the growth gain with optimal taxation. Gain from optimal taxation reaches to 25% with 0.10 externality level and it is below 10% with externality levels above 0.20.

It can also be observed from Table (4) that level of tax evasion in optimal tax rate with benchmark parameter values is almost 8%. Therefore, the model implies some level of tax evasion even in optimality. Therefore, with a relatively low level of h within [0,1] continuum, it is optimal for government to tolerate some level of tax evasion. More genuine treatment of this issue can be traced from Figure (8) which presents simulation of optimal tax rate and associated

optimal evasion level in Turkey for all possible levels of transaction cost parameter. As Figure (8) suggests, as h increases (i.e. corruption decreases) optimal tax rate approaches to the level of government externality in a decreasing rate. More importantly, this simulation suggests that optimal evasion rate associated with optimal taxation is low when h is low, it increases and makes a peak at $h = 0.11$ with $evasion(\tau*) = 10.3\%$ and then it starts to decrease. It can be observed that $h = 0.27$ makes Turkey to be close to the peak point. Therefore, with level of $h = 0.27$, existence of tax evasion is optimal even in low levels of optimal tax rate.

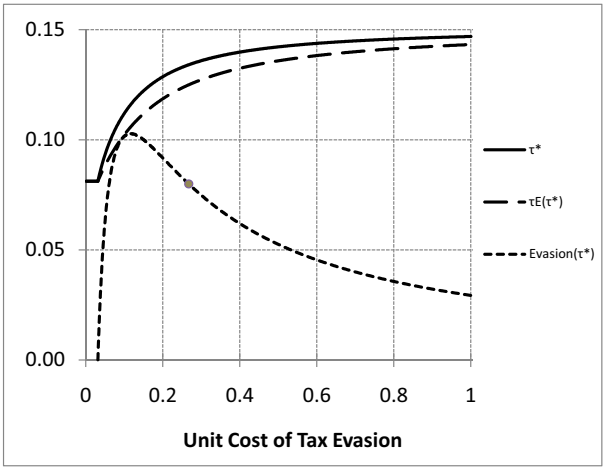


Figure 8. Effect of unit cost of evasion on optimal tax rate and tax evasion

This fact actually represents a general result. We can approximate level of h to the corruption level of economy and we can assume that corruption in an economy will decrease as it develops. Therefore we can assert that when a country is at the low level of development, tax evasion is optimally very low. When it attains moderate level of development, optimal level of tax evasion becomes higher. And, when it attains higher level of development with low level of corruption, optimal level of tax evasion again becomes very small. In other words, the relationship between optimal level of tax evasion associated with optimal choice of government and the level of development in an economy is

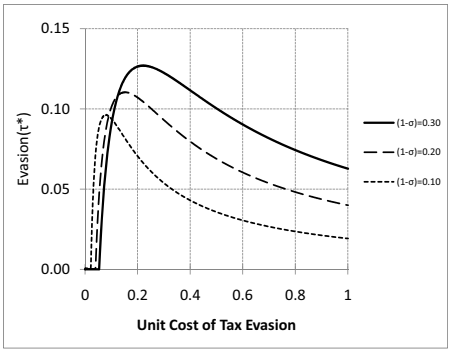


Figure 9. Effect of externality on the relationship between unit cost of evasion and optimal tax evasion

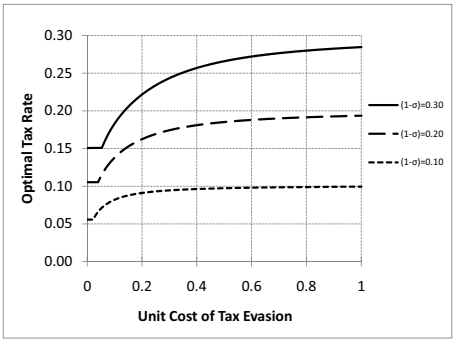


Figure 10. Effect of Externality on The Relationship between Unit Cost of Evasion and Optimal Tax Rate

inverted-U shaped. Turkey is close to the peak of this inverted-U shaped relationship. This result supports the idea of some that in Turkey tax evasion is tolerated by government. Indeed, tolerating some level of tax evasion is optimal even in an optimized income tax scheme for Turkey with existing structure in which unit cost of tax evasion is very low (i.e. corruption high). If we look at this relation with different levels of government externality, we can see that inverted-U shaped relation survives. As it can be observed from Figure (9) it is even strengthened with higher levels of government externality. In addition, as seen in Figure (10), higher levels of unit cost of tax evasion results in optimal tax rate to approach to government externality level. Nevertheless, supporting the argument stated before, wedge between optimal tax rate and externality level is always higher for increased level of government externality for all levels of unit cost of tax evasion.

Another parameter that effects the dynamics of the model is ϵ, the exclusion parameter. Changes in the exclusion parameter, too, (as changes in h does) will certainly affect the level of optimal tax. For instance, an increase in ϵ results in more severe underutilization of public expenditures which results in lower tax evasion. To illustrate general equilibrium effects of this fact, we have simulated comparative dynamics of tax evasion and optimal taxation for

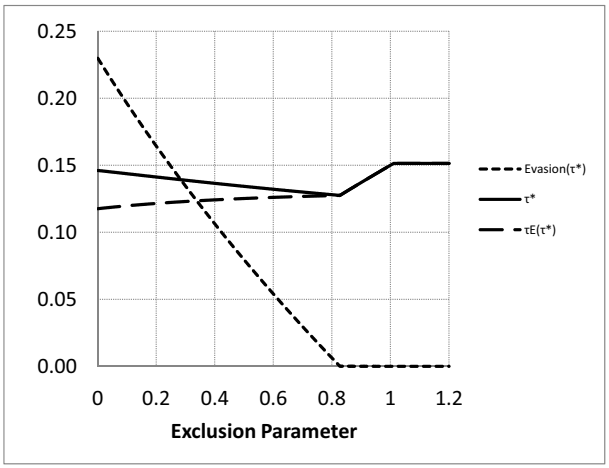


Figure 11. Effect of exclusion parameter on optimal tax rate and tax evasion

different levels of ϵ with give benchmark parameters in Figure (11). As it can be observed from Figure (11), when $\epsilon = 0$ (i.e no exclusion effects) there occurs almost 30% tax evasion even in optimal tax level which indicates the importance of including some level of ϵ into the model. In addition, as ϵ increases, effectively collected taxes increases even with decreasing optimal tax rate.If ϵ approaches close to 1, which means linear relation between compliance and effectively used government expenditure, tax evasion vanishes in optimal tax level. These simulations signify the fact that we have two additional supportive policies to optimization of tax rate. One is increasing h (i.e. decreasing corruption).The other is increasing ϵ that is policies that prevents tax evaders to utilize public expenditures effectively or policies to increase supply public expenditures which can only be utilized by non-evaders.

Simulations on Optimal Audit Probability

In the section that have analyzed optimal audit probability, we have asserted that depending on the parameters of the model we can have such an optimal audit probability that can lead us to eliminate tax evasion. As explicit solution of optimal audit probability is intractable, we will try to figure out level of it by

Table 5. Simulations on Optimal Audit Probability

$(1-\sigma)$	$p*(\%)$	Growth Rate(%)	$\alpha(p*)(\%)$	$g(p*)(\%)$	$fp^2(\%)$	fp^2/τ_E
0.15	23.34	2.43	82.70	21.37	0.82	0.037
0.10	3.87	2.41	71.86	18.37	0.02	0.001
0.2	33.03	2.53	90.24	22.13	1.64	0.069
0.25	38.76	2.70	97.47	22.48	2.25	0.091
0.3	36.66	2.97	100.00	22.98	2.02	0.081

quantitative analysis. Simulation of optimal audit probability with benchmark parameter values that has used before and sensitivity analysis with respect to government externality level leads following results presented in Table (5) Table (5) suggests that optimal level of audit probability with benchmark values is 23.34% and associated tax compliance level is 82.7% which is far from having full compliance in optimality. This figure necessitates a five-fold increase in cost of audits compared to benchmark level. Nevertheless, this policy results in almost no growth gains.

When we look at simulations with different externality levels, we can infer better ideas about optimal audit probability. Calibrating the economy with externality level of 0.10 results in decreasing audit probability in optimal polices with increase in associated tax evasion. In contrast, when government externality is taken as higher up to the level 0.25, optimal audit probability increases. When we take externality level greater than 0.25 , optimal audit probability decreases. Therefore, the more we take government externality far from the current level of tax rate 0.25, the lesser is the optimal audit probability.

Intuition behind this fact is simple. If we take externality level far from current tax rate, the reason behind the tax evasion becomes the divergence between externality level (i.e what government gives) and the tax rate (i.e. what government takes). In that case, there is no meaning to increase probability of audit up to the point that eliminates tax evasion. Because, the reason behind

the tax evasion does not disappear with increased the audit probability. In contrast, if we calibrate the economy with externality level that is close to the current tax level, the reason behind tax evasion becomes low level of unit cost of tax evasion. In other words, the matter behind evasion becomes its easiness and low cost. As increasing audit probability is a way to increase the cost of tax evasion, it is more optimal to have high levels of audit probability. As a result, even previously mentioned probability level $\bar{p}$ can become optimal. As increasing audit probability becomes more effective on decreasing tax evasion with high levels of government externality, its effect on growth becomes enormous.optimization of audit probability has better effects on growth within higher government externality levels and lower unit cost of tax evasion.

Simulations on Joint Optimization of Tax Rate and Audit Probability

Previous two subsections have analyzed optimal policies of tax rate and audit probability separately. In other words, we have made quantitative analysis of optimal policies on the revenue side and expenditure side of the government with abstraction from each other. By this way, we gained insights about possible growth effects of policy options within different parameterizations. Main result in terms of growth effects of optimal taxation was the fact that it is more effective if the wedge between the taken externality level and current level of tax is high. That is, calibrating the economy with lower government externality resulted in more growth gains. On the other side, we have proposed that optimization of audit probability has better effects on growth within higher government externality levels and lower unit cost of tax evasion. As we can see, these two policy options have complementary effects with different parameterizations of the economy. Therefore, we can expect that joint optimization of audit probability and tax rate can result in significant effects on growth rate.

Table 6. Simulations on Joint Optimization of Tax Rate and Audit Probability

$(1-\sigma)$	$p*(\%)$	$\tau*(\%)$	Growth Rate(%)	$\alpha(p*)(\%)$	$g(p*)(\%)$
0.15	13.94	13.70	2.73	92.88	12.64
0.10	6.32	9.35	3.00	94.33	8.80
0.2	23.98	18.22	2.63	93.39	16.58
0.25	36.04	23.22	2.71	97.53	21.01
0.3	41.62	28.53	3.00	100.00	25.93

The figures presented in Table (6) supports this fact. Joint optimization with different government externality levels results in at least 10% improvement in growth rates. In benchmark case, optimal taxes and associated optimal supply of productive government expenditures increases and gets closer to the government externality level. In addition, as optimal audit probability supports effectively optimal taxation, optimal supply of government expenditures increases significantly compared to the case in which only tax is optimized.

CHAPTER 7
CONCLUSION

This thesis analyzed the problem of income tax evasion and optimal taxation in a model of endogenous growth with productive public expenditures. The results about tax compliance behavior suggest that deterrence policies would not be effective by themselves on decreasing tax evasion. They must be supplemented by positive incentives for households by policies on the supply of productive public expenditures. Quantitative analysis for Turkey supports this fact as major part of tax compliance in Turkey is found as a result of positive benefits of being compliant via taking advantage of public expenditures.

With the inclusion of optimization on the part of government, it is shown that under the existence of tax evasion with transaction and audit costs, optimal level of productive public expenditure would be less than the externality level of public expenditures on production. In addition, it is theoretically stated that existence of tax evasion would not necessarily require higher optimal tax rates compared to no evasion case. This result is reinforced by quantitative analysis in which optimal tax rate is found below externality level for a wide range of parameter values. Therefore, this thesis shows that incorporating tax evasion and negative externalities of it into the dynamics of an endogenous growth model have considerable effects on optimal tax problem and it can force the government to divert from attaining productive efficiency in terms of public expenditures.

The thesis also incorporated optimization of audit probability along with optimization of income tax rate. Theoretical analysis suggested that depending on the parameters of the model, there is a possibility that optimal audit probability can be increased optimally such that associated tax evasion becomes zero. Nevertheless, quantitative analysis suggested that such levels of audit

probability are hard to be prevailed as optimal. As the audit efforts of government are taken as costly, the part of tax revenue that would be allocated to audit efforts to increase the probability of audit has some limits as marginal cost of audits exceeds marginal gains at some point. In terms of growth effects, optimal policies have variable effects depending on the externality level of public expenditures and the level of unit cost of tax evasion. Nevertheless, complementary and synergetic effects of optimization of audit rate and tax rate results in at least 10% improvement in growth rate.

The analysis of this thesis incorporates a representative agent framework and gives a general picture of interplay between negative reinforcement and positive incentives in terms of defining tax compliance pattern of general economy.An alternative path would be including heterogeneities of agents into the dynamics of the model.For example, one can account for different rates of probability of audit in terms of ex-ante heterogeneities between firms or households.Moreover, one can account for differences between agents' ability to utilize productive public expenditures and exclusion resulting from tax evasion in terms of proper utilization of public expenditures.In addition,incorporating ex-post heterogeneities between agents such as stochastic technological shocks or incomplete asset markets framework can be very fruitful.Therefore, extending this thesis's model by above mentioned ways would lead to interesting paths for further debates.By these extensions, one can incorporate both efficiency considerations and also distributional and equity consequences of optimal policies within the context of tax evasion.

APPENDICES

APPENDIX A:FORMAL SOLUTION OF THE COMPETITIVE EQUILIBRIUM

The current value Hamiltonian of the model is;

$$H = \frac{c^{1-\gamma} - 1}{1-\gamma} + \lambda \left(\left(1 - \tau_E - h(1-\alpha)^2\right) A k^{\sigma} \hat{g}^{1-\sigma} - c \right) \tag{26}$$

First order conditions are;

$$\frac{\partial H}{\partial c} = c^{-\gamma} - \lambda = 0 \tag{27}$$

$$\frac{\partial H}{\partial k} = \lambda(1 - \tau_E - h(1-\alpha)^2) A\sigma k^{\sigma-1} \hat{g}^{1-\sigma} = \rho\lambda - \dot{\lambda} \tag{28}$$

$$\frac{\partial H}{\partial \alpha} = \lambda(-\tau(1-p\theta) + 2h(1-\alpha)) A k^{\sigma} \hat{g}^{1-\sigma} + (1 - \tau_E - h(1-\alpha)^2) A k^{\sigma} \hat{g}^{1-\sigma} (1-\sigma) \frac{e}{\alpha} \tag{29}$$

From equation (27) we have $c^{-\gamma} = \lambda$. If we take natural logarithm of both side and take derivative with respect to time, we will have

$$\frac{\dot{\lambda}}{\lambda} = -\gamma \frac{\dot{c}}{c} \tag{30}$$

If we put (30)into (28) and rearrange the terms we will get the growth rate of consumption;

$$\frac{\dot{c}}{c} = \frac{1}{\gamma} \left(A\sigma \left(1 - \tau_E - h(1-\alpha)^2\right) \left(\alpha^{\epsilon} \frac{g}{k}\right)^{1-\sigma} - \rho \right) \tag{31}$$

and from (29) we will get the optimal level of tax compliance;

$$\left(\frac{\epsilon(1-\sigma)}{\alpha}\right)\left(1-\tau_E-h(1-\alpha)^2\right)+2h(1-\alpha)=(1-p\theta)\tau \tag{32}$$

To see that equation (31)generates endogenous growth rate,we have to analyze the term $fracgk$ explicitly.In order to do that,we must write the equation for output with taking into account the positive externality of government expenditure.

We know that $y=Ak^{\sigma}\hat{g}^{1-\sigma}$ in which $\hat{g}=\alpha^{\epsilon}g$ and $g=\tau_E-fp^2$.With taking all of this into account,production function becomes;

$$y=(A^{1/\sigma}(\tau_E-fp^2)^{\frac{1-\sigma}{\sigma}}\alpha^{e\frac{1-\sigma}{\sigma}})k \tag{33}$$

Therefore,it is now more clear that production function is AK type.By using (33) can find explicit formulation for $\frac{g}{k}$;

$$\frac{g}{k}=(\tau_E-fp^2)\frac{y}{k}=(\tau_E-fp^2)(A^{1/\sigma}(\tau_E-fp^2)^{\frac{1-\sigma}{\sigma}}\alpha^{e\frac{1-\sigma}{\sigma}}\sigma)=(A^{1/\sigma}(\tau_E-fp^2)^{\frac{1}{\sigma}}\alpha^{e\frac{1-\sigma}{\sigma}}) \tag{34}$$

which does not depend on k.Therefore $\frac{\dot{c}}{c}$ does not depend on k.So, dynamics of the model does not exhibit transitional dynamics which generates endogenous growth rate $\Lambda_c=\frac{\dot{c}}{c}$ for consumption.

From capital accumulation process we have growth rate for capital as follows;

$$\Lambda_k=\frac{\dot{k}}{k}=(1-\tau_E-h(1-\alpha)^2)\frac{y}{k}-\frac{c}{k} \tag{35}$$

As we know from (33) that $\frac{y}{k}$is constant, $\frac{c}{k}$ must be constant to have a steady state growth rate for capital stock.Having $\frac{c}{k}$ is equal to a constant means that

consumption and capital stock grows at same rate that is;

$$\Lambda_k = \Lambda_c = \Lambda \tag{36}$$

APPENDIX B:FORMAL PROOF FOR EQUALITY OF MAXIMIZING UTILITY AND GROWTH RATE FOR GOVERNMENT WITHIN BALANCED GROWTH PATH

Government's problem is maximizing the utility level subject to competitive behavior of the agents of the economy.

Hence the optimal tax problem is; $\max_{c,k,\tau} \int_0^\infty e^{-\rho t} \frac{c(t)^{1-\gamma}-1}{1-\gamma} dt$ subject to

$$\dot{k} = (1 - \tau_E - h(1-\alpha)^2)y \text{ - c}$$

$$\frac{\dot{c}}{c} = \frac{1}{\gamma}\left(A\sigma\left(1 - \tau_E - h(1-\alpha)^2\right)\left(\alpha^e \frac{g}{k}\right)^{1-\sigma} - \rho\right)$$

In optimization process, government also takes into account optimal tax evasion behavior;

$$\left(\frac{e(1-\sigma)}{\alpha}\right)(1 - \tau_E - h(1-\alpha)^2) + 2h(1-\alpha) = (1 - p\theta)\tau$$

In addition, when optimizing by choosing optimal tax rate, government will certainly take into account externality of public expenditures. Therefore, contrary to optimization process of households, it will not take public expenditure level as given. So, we should put actual level of public expenditures to constraints of the problem.

By 2 and 3;

$$\frac{g}{k} = A^{\frac{1}{\sigma}}(\alpha^{e(1-\sigma)}(\tau_E - fp^2))^{\frac{1}{\sigma}} \tag{37}$$

and

$$y = A^{\frac{1}{\sigma}}(\alpha^{e(1-\sigma)}(\tau_E - fp^2))^{\frac{(1-\sigma)}{\sigma}} k \tag{38}$$

If we define

$$\triangle(\tau) = (1 - \tau_E - h(1-\alpha)^2)$$

$$a(\tau) = A^{\frac{1}{\sigma}}(\alpha^e(\tau_E - fp^2))^{\frac{1-\sigma}{\sigma}}$$

The optimization problem becomes; $\max_{c,k,\tau} \int_0^\infty e^{-\rho t} \frac{c(t)^{1-\gamma}-1}{1-\gamma} dt$ subject to

$$\dot{k} = \triangle(\tau) a(\tau) k - c \tag{39}$$

$$\frac{\dot{c}}{c} = \frac{1}{\gamma}(\sigma a(\tau) \triangle(\tau) - \rho) \tag{40}$$

As economy lies in balanced growth path we have

$$\frac{\dot{c}}{c} = \frac{\dot{k}}{k} = \Lambda \tag{41}$$

Therefore we have

$$c(t) = c(0) e^{(\frac{1}{\gamma}(\sigma a(\tau) \triangle(\tau) - \rho))t} \tag{42}$$

and from (39), (40) and (41) we have ;

$$c(0) = \frac{k(0)}{\sigma}(\rho + \Lambda(\gamma - \sigma)) \tag{43}$$

If we put (42) into the objective function we will have;

$$U = \int_0^\infty e^{-\rho t} \frac{c(0) e^{(\frac{1-\gamma}{\gamma}(\sigma a(\tau) \triangle(\tau) - \rho))t} - 1}{1-\gamma} dt \tag{44}$$

Provided that utility is bounded i.e. $\rho > \frac{1-\gamma}{\gamma}((\sigma a(\tau) \triangle(\tau) - \rho)$ objective function after evaluation of the integral becomes;

$$U = \frac{c(0)^{1-\gamma}}{1-\gamma} \frac{1}{\rho - \frac{1-\gamma}{\gamma}((\sigma a(\tau) \triangle(\tau) - \rho)} - \frac{1}{\rho(1-\gamma)} \tag{45}$$

If we put (43) into (45)

$$U = \frac{k(0)^{1-\gamma}}{\sigma^{1-\gamma}} \frac{(\rho - (1-\gamma)\Lambda + \Lambda(1-\sigma))^{1-\gamma}}{\rho - (1-\gamma)\Lambda} - \frac{1}{\rho(1-\gamma)} \tag{46}$$

It can be seen from (46) that effect of Λ on U is positive for all values of $\gamma > 0$ provided that utility is bounded, so that $\rho > (1-\gamma)\Lambda$

Therefore, when utility is bounded, maximization of growth rate coincides with maximization of utility.To retain bounded utility condition together with positive growth we must have following condition with respect to parameters of the model;

$$(\frac{\rho}{\sigma(1-\gamma)})^{\sigma} \frac{1}{(1-\tau_E - h(1-\alpha)^2)^{\sigma}(\alpha^e(\tau_E - fp^2))^{1-\sigma)}} > A > (\frac{\rho}{\sigma})^{\sigma} \frac{1}{(1-\tau_E - h(1-\alpha)^2)^{\sigma}(\alpha^e(\tau_E - fp^2))^{1-\sigma)}} \tag{47}$$

APPENDIX C:PROOF OF PROPOSITIONS IN THE TEXT

Proof of Proposition 1.

Taking derivative of the tax compliance rate with respect to tax rate leads to;

$$\frac{\partial \alpha}{\partial \tau} = -\frac{(1-p\theta)(1+\epsilon(1-\sigma))}{2(2h+\epsilon h(1-\sigma))} - \frac{4\epsilon p\theta(2h+\epsilon h(1-\sigma))(1-\sigma)+2(1-p\theta)(1+\epsilon(1-\sigma))^2(2h-(1-p\theta)\tau)}{4(2h+\epsilon h(1-\sigma))^2\sqrt{4\psi}} \tag{48}$$

With some manipulation we get;

$$\frac{\partial \alpha}{\partial \tau} = -\frac{(1-p\theta)(1+\epsilon(1-\sigma))}{2h(2+\epsilon(1-\sigma))}\left(1+\frac{(1+\epsilon(1-\sigma))(2h-(1-p\theta)\tau)}{\sqrt{C}}\right) - \frac{\epsilon(1-\sigma)p\theta}{(2h+\epsilon h(1-\sigma))\sqrt{C}} \tag{49}$$

where

$$C = ((2h-(1-p\theta)\tau)(1+\epsilon(1-\sigma)))^2 + 4(2h+\epsilon(1-\sigma)h)\epsilon(1-\sigma)(1-h-p\theta\tau) > 0$$

To have ambiguity we must have;

$$\frac{((1+\epsilon(1-\sigma))(2h-(1-p\theta)\tau))}{\sqrt{C}} < -1 \Rightarrow 2h > (1-p\theta)\tau$$

But this condition results in

$$\sqrt{C} > (2h-(1-p\theta)\tau)(1+\epsilon(1-\sigma))$$

which leads to

$$\frac{((1+\epsilon(1-\sigma))(2h-(1-p\theta)\tau))}{\sqrt{C}} > -1$$

Therefore, we can conclude that

$$\frac{\partial \alpha}{\partial \tau} < 0$$

Proof of Proposition 2.

If we take derivative of tax compliance rate with respect to audit probability and fine rate; and perform some straightforward algebra, we find;

$$\frac{\partial \alpha}{\partial p} = \frac{\tau\theta(1+\epsilon(1-\sigma))}{2h(2+\epsilon(1-\sigma))} + \frac{\theta\tau\left(2h+\tau(1-p\theta)(1+\epsilon(1-\sigma))^2\right)}{2h(2+\epsilon(1-\sigma))^2\sqrt{4\psi}} > 0$$

$$\frac{\partial \alpha}{\partial \theta} = \frac{p\tau(1+\epsilon(1-\sigma))}{2h(2+\epsilon(1-\sigma))} + \frac{p\tau\left(2h+\tau(1-p\theta)(1+\epsilon(1-\sigma))^2\right)}{2h(2+\epsilon(1-\sigma))^2\sqrt{4\psi}} > 0$$

which confirms the proposition.

Proof of Proposition 3.

When we take partial derivative of tax compliance rate with respect to unit cost of tax evasion, we get;

$$\frac{\partial \alpha}{\partial h} = \frac{1}{2h^3(2+\epsilon(1-\sigma))^2\sqrt{\psi}}((1-p\theta)(1+\epsilon(1-\sigma))\tau\sqrt{C} - 2\epsilon h(2+\epsilon(1-\sigma))(1-\sigma)$$
$$+ 2h(1-p\theta+\epsilon(2+\epsilon(1-\sigma))(1-\sigma))\tau - (1-p\theta)^2(1+\epsilon(1-\sigma))^2\tau^2)$$

The denominator is certainly positive. The numerator becomes negative when

$$\tau < \frac{\epsilon(1-\sigma)}{1-p\theta+\epsilon(1-\sigma)} \text{ and } \tau > \frac{2+\epsilon(1-\sigma)}{1+p\theta+\epsilon(1-\sigma)}$$

First condition implies $\alpha \geq 1$.Therefore it is implausible. With the assumption $p\theta < 1$, second condition implies that tax rate is greater than one. Therefore we can conclude that

$$\frac{\partial \alpha}{\partial h} > 0$$

As h increases the economy becomes less corrupt so compliance increases.

As two roots suggests, positive effect of decreasing corruption on tax compliance is low when tax rate is low. It becomes higher up to some point and than it starts to decrease again. Therefore combating corruption to decrease tax evasion is most effective in moderate levels of tax rates which are associated with medium level of tax evasion rate.

Proof of Proposition 4.

Taking partial derivative of tax compliance rate with respect to σ leads to;

$$\frac{\partial \alpha}{\partial \sigma} = \frac{1}{2h^2(2+\epsilon(1-\sigma))^2\sqrt{C}}(\epsilon(-2h+\tau(1-p\theta))h\sqrt{C}$$
$$+\epsilon(-4h^2+(1-p\theta)^2(1+\epsilon(1-\sigma))\tau^2-4h(2+\epsilon(1-\sigma)(1-\tau)+\tau(1-p\theta)))$$

Denominator of this equation is positive. With some algebra,numerator becomes;

$$-2h^2\sqrt{C}-4h^2-8h-4h\epsilon(1-\sigma)(1-\tau)$$

$$- \tau(1-p\theta)h\left(4-\sqrt{C}-(1-p\theta)(1+\epsilon(1-\sigma))\tau\right)$$

which is negative because $4 > \sqrt{C} + (1-p\theta)\tau(1+\epsilon(1-\sigma))$ Therefore;

$$\frac{\partial \alpha}{\partial \sigma} < 0$$

Proof of Proposition 5.

Some manipulation on (21) leads to;

$$\frac{\partial \tau_E}{\partial \tau}\left(\frac{\frac{1-\sigma}{\sigma}\left(1-\tau_E-h(1-\alpha)^2\right)-\tau_E+fp^2}{\left(\tau_E-fp^2\right)\left(1-\tau_E-h(1-\alpha)^2\right)}\right) = - \frac{2h(1-\alpha)\frac{\partial \alpha}{\partial \tau}}{1-\tau_E-h(1-\alpha)^2} - \epsilon\frac{1-\sigma}{\sigma}\frac{\frac{\partial \alpha}{\partial \tau}}{\alpha} \quad (50)$$

$\frac{\partial \tau_E}{\partial \tau} > 0$ and denominator of LHS is positive as it represents level of public expenditures and disposable income.RHS of the equation is also positive as $\frac{\partial \alpha}{\partial \tau} < 0$.

These conditions lead to $\frac{1-\sigma}{\sigma}\left(1-\tau_E-h(1-\alpha)^2\right)-\tau_E+fp^2 > 0$ which results in $(1-\sigma)+\sigma X > \tau_E^*$

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