AMBIGUITY, HIGH-FREQUENCY DATA, AND ASSET PRICES

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Ambiguity, High-Frequency Data, and Asset Prices

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ABSTRACT

Ambiguity, High-Frequency Data, and Asset Prices

Many theoretical studies have long argued that ambiguity is associated with stock price, but only a limited number of studies can provide empirical evidence about the form and magnitude of the hypothesized relationship. This thesis consists of two separate empirical studies aiming to fill this gap by using a recent methodology proposed by Brenner and Izhakian (2018) as a measure of ambiguity. In the first study, we quantify ambiguity and ambiguity attitudes for Turkish market and test their impact on the stock prices. Our results show that there is an ambiguity premium different than the risk premium in Turkish market. We also find that Turkish investors are ambiguity neutral in lateral markets but tend to avoid ambiguity as the probability of favorable returns moves away from the historical average. In the second study, we examine the effect of ambiguity exposure on the cross-section of returns in the US equity market. Results show that there is a U-shaped association between ambiguity exposure and expected return. This pattern is more pronounced for smaller stocks. Besides, U-shaped pattern is left-skewed on average; meaning that expected return increases as absolute ambiguity exposure increases but the absolute effect of the positive exposure is larger than the absolute effect of negative exposure. U-shaped trend is essentially same when controlling for widely known cross-sectional predictors, other ambiguity proxies, different holding and formation periods, and different states. Overall, our results provide substantial empirical evidence that indicates that ambiguity is priced in stock returns.

ÖZET

Belirsizlik, Yüksek Frekanslı Veriler ve Varlık Fiyatları

Birçok teorik çalışma, belirsizliğin hisse senedi fiyatı ile ilişkili olduğunu uzun süredir tartışmaktadır, ancak yalnızca sınırlı sayıda çalışma, hipotezlenen ilişkinin biçimi ve büyüklüğü hakkında ampirik kanıtlar sunabilmektedir. Bu tez, belirsizlik ölçüsü olarak Brenner ve Izhakian (2018) tarafından önerilen yeni bir metodoloji kullanarak bu boşluğu doldurmayı amaçlayan iki ayrı ampirik çalışmadan oluşmaktadır. İlk çalışmada, Türkiye piyasası için belirsizlik ve belirsizlik tutumları nicelleştirilmekte ve etkileri hisse senedi fiyatları üzerinde test edilmektedir. Sonuçlar, Türkiye pazarında risk priminden farklı bir belirsizlik primi olduğunu göstermektedir. Ayrıca, Türk yatırımcılarının yanal piyasalarda belirsizliğe karşı nötr oldukları, ancak pozitif getiri olasılığı tarihsel ortalamadan uzaklaştıkça belirsizlikten kaçınma eğiliminde oldukları gözlenmektedir. İkinci çalışmada, belirsizliğe maruz kalmanın ABD hisse senedi piyasasındaki getirilerin kesiti üzerindeki etkisini incelenmektedir. Sonuçlar belirsizliğe maruz kalma ile beklenen getiri arasında U şeklinde bir ilişki olduğunu göstermektedir. Bu ilişki küçük hisse senetleri için daha belirgindir. Ayrıca U şeklindeki desenin genelde sola eğik olduğu görülmüştür. Yani, mutlak belirsizliğe maruz kalma arttıkça beklenen getiri artmaktadır, ancak pozitif maruziyetin mutlak etkisi, negatif maruziyetin mutlak etkisinden daha büyüktür. U şekilli eğilim, yaygın olarak bilinen enine kesit tahmincileri, diğer belirsizlik vekilleri, farklı portfolyo oluşturma ve tutma dönemleri ve farklı zaman dilimleri kontrol edildiğinde de önemli ölçüde aynıdır. Genel olarak, sonuçlarımız belirsizliğin

hisse senedi getirilerinde fiyatlandığını gösteren önemli ampirik kanıtlar sunmaktadır.

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to my daughter,
Defne Açar

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ABBREVIATIONS

AIC	Akaike Information Criterion
ARMA	Autoregressive Moving Average
BIST	Borsa Istanbul
CAPM	Capital Asset Pricing Model
CEU	Choquet Expected Utility
EUUP	Expected Utility with Uncertain Probabilities
GDP	Gross Domestic Product
I/B/E/S	Institutional Brokers' Estimate System
MEU	Maxmin Expected Utility
MPP	Multiple Prior Preferences
SEU	Subjective Expected Utility Theory
US	United States
VNM	Von Neumann–Morgenstern

CHAPTER 1

INTRODUCTION

In real life, due to factors such as lack of reliable data, noise, or factors not foreseen, decisions often have to be made under circumstances that precise probability distributions over the possible outcomes are not available. Decision making theory distinguishes such a case as from risk and defines it as ambiguity. Risk corresponds to the situations that (subjective or objective) unique probability distributions exist and are known by the decision makers, whereas, ambiguous situations do not afford the decision maker the chance to build a precise probability distribution. Financial markets, by their nature, abound with situations where investors are compelled to make decisions under ambiguity, where incomplete information and noise prevent them from forming precise probabilistic models for the future outcomes.

Since decision making under ambiguity is inevitable in real world, this topic is a central concern in decision science and psychology. Since the earliest identification of the phenomenon, the consensus has been that a rational decision maker evades ambiguous situations and prefers known risks over unknown risks. In financial setting, disinclination of investors towards ambiguous markets, assets or instruments, is branded ambiguity aversion, which is recently drawing significant attention in financial economics literature, especially with the advent of behavioral finance questioning the dominant classical models.

In classical finance models, investors are assumed to know their utility preferences in each state and maximize their subjective expected utility (SEU) accordingly. This assumption removes away the distinction between risk and ambiguity, making the uncertainty equal only to risk. Therefore, modern asset

pricing theories mainly examine the relationship between risk and return. However, in reality the investors do not know the precise probability distributions and they are exposed to not only risk but also ambiguity. Although the theoretical studies recently started incorporating ambiguity into the asset pricing models, the empirical studies are very limited in number. This lack of empirical work is due to the lack of consensus on a reliable method for measuring ambiguity.

The main objective of this thesis is to contribute to the literature on ambiguity measurement and empirical tests on impact of it on intertemporal and cross-sectional asset prices. We take as our starting point and guide the natural measure of ambiguity as probability perturbations (uncertain probabilities) and aversion to ambiguity as aversion to mean-preserving spreads in these probabilities, recently developed by Izhakian (2017) and empirically tested on asset prices by Brenner and Izhakian (2018). This measure is simple, intuitive, and applicable for the empirical measurement of the degree of ambiguity. Second, the measure is risk-independent. These qualities are critical to bring ambiguity into empirical tests on asset prices.

Our empirical analysis covers both emerging and developed markets by regarding Turkey and US as one of the most important representatives of their kinds. We aim to address the main gap in the empirical literature for each market. Therefore, our first analysis quantifies ambiguity and ambiguity attitudes for Turkish market and examines how ambiguity is priced along with risk. Our results indicate that market ambiguity is priced in the Turkish market with a distinct premium than the market risk. Besides, we find that investors trading Turkish markets are ambiguity averse under positive and negative expectation but they are ambiguity neutral for lateral markets, where direction is not clear. This is quite different than the ambiguity attitudes US-market investors who are ambiguity averse (lover) when

they expect a gain (loss) as reported by Brenner and Izhakian (2018). Controlling for higher moments, recessions, and sentiment do not change our results. Secondly, we examine how market ambiguity calculated aforementioned methodology impacts the cross-section of returns in US not only because cross-section effect of the market ambiguity is not examined in the US market but also the existing cross-section analysis with differing ambiguity proxies provide conflicting results. Our findings show that market ambiguity exposure is non-linearly related with the expected stock return. There is a left-skewed U-shaped pattern between ambiguity beta and expected return, as the stocks that are most negatively and most positively exposed to market ambiguity yield higher expected returns, while the latter is larger than the former. Our results are essentially robust to other cross-sectional return predictors, other ambiguity proxies, different holding or formation periods, and different states. However, two of the cross-sectional predictor, momentum and idiosyncratic volatility, is able to decrease the slope of the U-shaped trend to some extent, fail to completely explain non-linear association though. Finally, our analysis of the test portfolio reveals that the stocks with most negative market ambiguity exposure tend to be long-term losers, speculative (low-priced) stocks with more desperate fundamental and intangible growth. Moreover, they have higher distance from efficient market frontier, higher return persistency, higher revenue growth compared to inventory growth, higher cash-flow volatility, and higher advertising expense.

The remainder of the thesis is organized as follows. Chapter 2 provides related literature on related concepts. Chapter 3 empirically estimates the ambiguity and ambiguity attitude for Turkish market by following the recent methodology introduced by the Brenner and Izhakian (2018). Chapter 4 examines how ambiguity

is priced in the cross-section of stock returns in US market. Finally, Chapter 5 concludes the thesis.

CHAPTER 2

RELATED LITERATURE

The purpose of this chapter is to provide the formal definition for the related concepts and to provide a general literature review on ambiguity and asset pricing. It starts with the definition of ambiguity and ambiguity aversion, continues with modeling decision making under ambiguity, followed by implications of ambiguity in asset pricing models, and finally ends with ambiguity and related issues. Each subsequent chapter will include a more focused literature review on the relevant topic covered in the chapter.

2.1 Definition of ambiguity and ambiguity aversion

Although uncertainty and risk are sometimes used interchangeably in the finance literature, the difference between the two has been clearly demonstrated in early studies.

Knight (1921) argues that two types of uncertainty exist. First one is called risk and it refers to the situations in which each relevant event is associated with a unique probability assignment. The second type is Knightian uncertainty (ambiguity), corresponding to the situations in which some of the relevant events do not have unique and obvious probability assignments. A very similar distinction is also implied by Keynes (1937) who argues the following:

> "By 'uncertain' knowledge, let me explain, I do not mean merely to distinguish what is known for certain from what is only probable. The game of roulette is not subject, in this sense, to uncertainty; nor is the prospect of a Victory bond being drawn. Or, again, the expectation of life is only slightly uncertain. Even the weather is only moderately uncertain. The sense in which I am using the term is that in which the prospect of a European war is uncertain, or the price of copper and the rate of interest twenty years hence, or the obsolescence of a new invention, or the position of private

> wealth- owners in the social system in 1970. About these matters there is no scientific basis on which to form any calculable probability whatever. We simply do not know" (pp. 213-214)

The Ellsberg's experiment, on the other hand, is a turning point in the Wider recognition of the two types of uncertainty with appropriate definitions in the literature. Conducting a set of experiments, Ellsberg (1961) examines the decision making process under the risky and ambiguous situations. In first experimental design, there are two urns and each urn contains various color of balls. Urn A includes 50 red and 50 black balls while the urn B includes 100 red and black balls with unknown distribution. There are two gambles and participants are asked to choose between following bets:

Gamble 1: You receive $100 if you draw a red ball.

Bet 1A: draw a ball from urn A, get $100 if Red; zero otherwise

Bet 1B: draw a ball from urn B, get $100 if Red; zero otherwise

Gamble 2: You receive $100 if you draw a black ball.

Bet 2A: draw a ball from urn A, get $100 if Black; zero otherwise

Bet 2B: draw a ball from urn B, get $100 if Black; zero otherwise

Results show that participants are participants strictly choose 1A over 1B and choose 2A over 2B. Choosing 1A over 1B implies that participants think that probability of drawing a red ball from urn B is lower than 50% (probability of drawing a red ball from urn A). Choosing 2A over 2B implies, on the other hand, implies participants think probability of drawing a black ball from urn B is lower than 50% (probability of drawing a black ball from urn A). This is known as two urns paradox.

In another experiment, there is one urn which contains 90 balls, 30 of which are known to be red, remaining 60 balls are either yellow or black with an unknown distribution. The participants are first asked to choose between following bets:

Gamble A: You receive $100 if you draw a red ball

Gamble B: You receive $100 if you draw a black ball

Then, in the second game, participants are asked to choose between following bets within the same situational parameters:

Gamble C: You receive $100 if you draw a red or yellow ball

Gamble D: You receive $100 if you draw a black or yellow ball

In the first game, participants strictly choose Gamble A over Gamble B, which implies that they think probability of drawing a black ball is lower than 1/3 (probability of drawing a red ball). In the second game, participants choose Gamble D over Gamble C, meaning that they see additive probability of drawing a red or yellow ball is lower than additive probability of drawing a black or yellow ball. Since the probability of drawing a yellow ball is same in both Gamble C and D, we can infer that choosing Gamble D over Gamble C means probability that more than 1/3 of the balls are black. Therefore, the results of first and second game are contradicting and known as one urn paradox.

The two urns and one urn paradoxes show that conventional choices are inconsistent with Savage‘s (1954) theory of subjective expected utility based on additive probabilities. The experiments basically show that people are averse to ambiguous situations and usually prefer to bet on known odds rather than on ambiguous odds of equal size. Starting with Ellsberg experiment, Knightian uncertainty is widely known as ambiguity and dislike of ambiguous situations are known as ambiguity aversion.

2.2 Modeling decision making under ambiguity

Modelling the complicated nature of the human decision-making under uncertainty is one of the utmost important interest of the literature starting from earliest studies to the most recent ones. The St. Petersburg paradox reveals that expected value cannot be sole criterion to model the human decisions. More specifically, the paradox shows that a game of flipping coin where the expected value theoretically goes to infinity is of little value to participants. Solution offered by Bernoulli (1954)[1] to St. Petersburg paradox lays the foundations of expected utility theorem. More specifically, people do not decide based on expected monetary gain but instead they consider the expected utility which is a function of decision makers' wealth. Besides, there is a diminishing marginal utility which means that each additional wealth yields smaller marginal utility. The law of diminishing marginal utility is the building block of the concave utility function assumed by many economical and finance theories.

Von Neumann and Morgenstern (1944) formulize the modest conditions under which expected utility hypothesis holds. Von Neumann–Morgenstern (VNM) utility theory assumes that decision-makers are rational and main criterion of decision-making is to maximize expected utility under some certain axioms. The probability of each event is assumed to be known with an objective probability distribution. Subjective utility yield by each event is retrieved by the utility function of each decision markers. Expected utility to be maximized is then quantified by the multiple sum of the subjective utility and objective probability assigned to each event. VNM utility theorem is followed by Savage's subjective expected utility theory (SEU). Savage (1954) argues that not only the utility function but also the provability distribution is subjective. Under certain axioms, a decision-maker defines

[1] Reference is made to the English translation of the original Latin text written in 1738.

the set of possible outcomes and assesses the probabilities of each outcome subjectively. While subjective utility function is analogous to the tastes, subjective probability distribution reflects the beliefs of the decision-makers.

VNM and SEU theories provide great insights for understanding decision making uncertainty; however, they only deal with the risk. No matter subjectively or objectively assigned, decision-makers are assumed to assess a unique probability distribution. However, as implied by Keynes (1937), real life is full of ambiguous situations in which we cannot assign a quantified or unique probabilities to certain events. After empirical failure of expected utility theorem revealed by Ellsberg's experiment, some studies propose the earliest primitive models that provide some room for the ambiguity aversion. Fellner (1961) introduces uncorrected subjective probability concept which is analogous to psychological weights used in the decision-making process. Fellner (1961) argues that ambiguity aversion observed in Ellsberg's experiment can be explained by non-additive uncorrected probabilities. Similarly, Kahneman and Tversky (1979) challenge expected utility theory with prospect theory which associates utility to gains and loss rather than the final wealth and which relates each probability with a decision weight. Although, Kahneman and Tversky (1979) state that their analysis assumes that respondents adopted stated probabilities, they add that ambiguity may affect the decision-weight referring to event.

Proceeding studies offer more concrete theoretical models accounting for ambiguity along with risk. Although ambiguity is a known and intuitive observation, modeling the decision-making process under ambiguity is a complex issue that leads to the emergence of many models that support or compete with each other. For example, Schmeidler (1989) introduces Choquet expected utility (CEU) which

models the ambiguity aversion based on the non-additive probability (or capacities) and Choquet integral. The main intuition behind CEU is that an ambiguity averse decision-maker considers the most unfavorable probability distributions among the possible distribution set. More specifically, decision-maker initially assigns a capacity to the worst outcome among probable set of outcomes. She considers rest of the outcomes as the increments of the worst outcome and assigns capacities to other outcomes based on her overall belief placed on the outcomes.

Gilboa and Schmeidler (1989) extend CEU by suggesting that an ambiguity averse decision maker considers the most unfavorable prior, which is known as maxmin expected utility (MEU) or multiple prior preferences (MPP). To elaborate, under certain axioms, an ambiguity averse individual behaves as if she has multiple prior subjective probability distributions and prefers the option that maximizes the minimum expected utility. Although MPP and CEU imply same type of pessimism when capacity is convex, neither method is a special case of the other (Guidolin & Rinaldi, 2013). Ghirardato, Maccheroni, and Marinacci (2004) provide α-MEU model by introducing the heterogeneity in ambiguity aversion. By including a new term corresponding to the level of ambiguity aversion, they relax the extreme ambiguity aversion assumed by MPP and CEU. Chen and Epstein (2002) introduce an intertemporal version of MPP that is time-consistent and time-consecutive.

Hansen and Sargent (2001) formulate MPP in the setting of robust control theory used in engineering. Robust control theory deals with deep uncertainty in which agents have a set probability distributions and one reference model. Being aware of that probable errors, agents seek to robustness against probable model misspecifications. Hansen and Sargent (2001) argue that the concern for the probable model misspecification may be regarded as ambiguity aversion.

Some of studies use multi-stage approach for modeling the ambiguity. Among the earliest studies of multi-stage approach, Ergin and Gul (2009) and Nau (2006) argue that SEU model does not hold as decision makers compare the bets which are uncertain because of totally independent reasons. The authors define ambiguous bets in two-stage lotteries where uncertainty arises for two different reasons. More specifically, while some of the choices are based upon the ball, some of them are related with both the color of the ball as well as the composition of the urn. They report that ambiguity aversion is associated with only the bets based on the color of the ball.

Inspired by relaxing reduction between first and second-order probabilities offered by Segal (1987) and the previous multi stage ambiguity models, Klibanoff, Marinacci, and Mukerji (2005) develop smooth ambiguity model in which the ambiguity and the attitude toward ambiguity is separated. By this way, decision-maker does not have to be assumed as too pessimistic to consider the most unfavorable distribution. Some of the other studies on smooth ambiguity model are Klibanoff, Marinacci, and Mukerji (2009), Klibanoff, Marinacci, and Mukerji (2012), Hayashi and Miao (2011), and Chew and Sagi (2008).

More recently, Izhakian (2017) introduces expected utility with uncertain probabilities (EUUP) theory which provides a very intuitive definition of decision-making model under ambiguity. Similar with Rothschild–Stiglitz attitude towards risk is defined as attitude toward mean-preserving spreads in outcomes, attitude toward ambiguity is described as attitude toward mean-preserving spreads in probabilities. The model improves the distinction between taste and belief as well as the distinction between risk and ambiguity, making the empirical assessment of the model easier.

2.3 Implications of ambiguity in asset pricing models

One of the main focuses of the literature is to provide a simple theoretical framework for how relevant traditional models work when ambiguity is introduced. This section gives a brief survey of the theoretical implications of ambiguity in asset pricing models.

2.3.1 Portfolio theory under ambiguity

According to modern portfolio theory introduced by Markowitz (1959), a rational risk averse investor under SEU preferences chooses mean – variance efficient portfolios which maximizes the expected return (mean) for the given level of risk (variance). Garlappi et. al. (2007) extend Markowitz's mean-variance framework by accounting for the ambiguity modeled under MPP preferences. In this framework, optimum portfolio selection is determined by an adjustment factor which incorporates ambiguity to the model. Adjustment factor argued to be determined by the precision of parameters' estimation, the length of the data included, and ambiguity aversion of investor. Boyle, Garlappi, Uppal and Wang (2012) argue that asset familiarity is a sound proxy for ambiguity and examine the impact of ambiguity on flight to familiarity and under-diversification. Using a similar framework in Boyle et al. (2012), Garlappi, Uppal and Wang (2007) argue that investors tend to hold familiar assets; however, they hold a portfolio of less familiar assets as well in order to balance their investments. It is the relative ambiguity that determines the relative investment between the unfamiliar portfolio and the familiar assets. When the number of assets goes to infinity, the investment weight for the familiar assets

approaches a positive constant while the investment weight for the unfamiliar portfolio approaches to zero.

Fei (2009) derive optimal consumption and optimal portfolio under a model characterized by α-MEU which enables separating ambiguity (subjective beliefs) and ambiguity attitudes (subjective tastes). Maccheroni, Marinacci, and Ruffino (2013) work on a problem including one risky, one ambiguous, and one riskless asset. Their results indicate that it is the alpha of ambiguous asset that determines the degree of the portfolio rebalancing. As alpha measures the portion of performance that cannot be attributable to risk, positive alpha means an increase in the unmeasurable uncertainty for ambiguity-averse investors and vice versa. Huang and Tzeng (2018) explores under what conditions higher ambiguity results in a lower demand for an uncertain asset. Their results indicate that ambiguity and risk have similar comparative statistics under smoot ambiguity model while they have fundamentally different comparative statistics under α-MEU model. More specifically, when risk aversion is lower than unity, only an increase in ambiguity may decrease the demand for uncertain asset. A recent work by Makarov (2021) argues that investors may also have more than one prior about second-order distributions, which means that they are also exposed to ambiguous ambiguity. Results show that under ambiguous ambiguity, two-fund separation theorem fails because heterogeneous attitudes towards ambiguity or ambiguous ambiguity address different portfolios of risky stocks. On the other hand, Berger and Eeckhoudt (2021) find that ambiguity aversion may decrease the value of diversification opposed to economic intuition that both risk and ambiguity aversion should increase the value of diversification by serving for hedging. One of the reasons cited for this counter intuitive results is that ambiguity averse investors have low motivation for diversification which would

make them exposed to various ambiguous assets. Lin and Riedel (2021) examine the ambiguity in the interest rate market and find when interest rates are sufficiently ambiguous, the investors abstain from the bond market and invest fully in the asset market.

Extension of the mean-variance framework deals with the portfolio rebalancing in order to have the optimum portfolio under ambiguity. Some studies argue robust portfolio approach is more relevant for modelling the optimal portfolio choice under ambiguity. Similar with the analogy used by Hansen and Sargent (2001) in the context of ambiguity modeling under robust control theory, robust portfolio is defined as a portfolio that may be sub-optimal but is robust to model misspecifications (Guidolin & Rinaldi, 2013).

The earliest important example of robust portfolio framework is provided by Trojani and Vanini (2002) who develop the robust version of intertemporal portfolio choice offered by Merton (1971). Assuming there is one risk-free asset and one risky asset and applying the preference for robustness, Trojani and Vanini (2002) report that investors will consume more of their wealth and buy less of risky asset than they would buy under SEU. Under a continuous time and different robust control optimizations, inclusion of ambiguity aversion changes the optimal portfolio in a similar way of an increase in risk aversion does. Maenhout (2004) runs a set of simulation tests and shows that seeking robustness increases the demand for hedging while it deceases the demand for equity market, which numerically confirms theoretical implications addressed by Trojani and Vanini (2002). In their subsequent study, Trojani and Vanini (2004) examine implications of ambiguity in two different robustness setting and conclude that the specific method of modeling ambiguity does not qualitatively have a significant impact on optimal consumption decisions, but it

does have a variety of impacts on portfolio rules. Under a continuous time and different robust control optimizations, inclusion of ambiguity aversion changes the optimal portfolio in a similar way of an increase in risk aversion does. Pflug and Wozabal (2007) use a maxmin approach to model optimum portfolio selection in case when underlying probability model is uncertain. Their results imply that there is a tradeoff between return, risk, and robustness. By measuring the ambiguity via the Kantorovich distance, Pflug, Pichler, and Wozabal (2012) study on robust optimization and show theoretically and empirically that the optimum portfolio weights approach to uniform distribution as ambiguity becomes higher. Ameur and Prigent (2013) provide general characteristics of optimal portfolio under ambiguity using the Maccheroni, Marinacci and Rustichini (2006) approach which contains MPP as well as multiplier preferences. Lin and Riedel (2014) consider optimal consumption problem under ambiguity in continuous-time parameter and show that under some robust parameter constellations, optimal portfolio is achieved by investing all of the wealth into the asset market without lending or borrowing at all.

More recently, Jang and Park (2016) work on optimal portfolio problem under a model having Value-at-Risk restrictions and accounting ambiguity aversions and find that risk aversion determinates dynamic portfolio decision more dominantly than ambiguity aversion. Zhang Jin, and An (2017) use robust optimization methods to model optimal portfolio under ambiguity in the presence of transaction costs. Their results show that robust portfolio, which attaches less weights to highly volatile return-predicting factors and loads less on the securities having costly and large existing positions, outperforms the non-robust portfolio. Bergen, Escobar, Rubtsov and Zagst (2018) examine the robust multivariate portfolio choice with stochastic covariance when ambiguity exists in the market and report that ambiguity aversion

strongly affects the optimal exposure to stock as well as covariance risk. By providing data calibrations on stocks returns, long-term bonds, cash, real estate, and gold, Dangl and Weissensteiner (2020) show that ambiguity averse long-term investors invest in a portfolio which seems inefficient but has the strongest robustness against the estimation errors. Wang and Niu (2020) study on portfolio choice of hedge fund managers in the presence of ambiguity aversion which is modeled by three distinct robust control optimization problem. While in the first setting optimal portfolios are found to be negatively impacted by volatility of risky asset and ambiguity aversion, second and third settings lead to a U-shaped pattern between investment strategy and volatility of risky assets. In particular, as ambiguity increases, manager exhibits two opposite investment decision. (i) over-investment because of decrease in the risk attitude, (ii) under-investment because of the decrease in the uncertainty-adjusted market price of risk, later being dominated by the former resulting in excess investment.

2.3.2 Ambiguity and equilibrium prices

Since the conventional theories regard risk as the only source of the uncertainty, an equity is evaluated as being over or under priced based mainly on the risk it includes. On the other hand, incorporation of ambiguity addresses a different equilibrium prices that have a great potential to explain some of the systematic mispricing observed in the market. In this regard, inter-temporal asset pricing model under MPP preferences provided by Epstein and Wang (1994) is one of the earliest asset pricing models that accounts for ambiguity. Epstein and Wang (1994) extend Lucas (1978) general equilibrium and show that inclusion ambiguity may result in indeterminate equilibria, which may ultimately lead to excess volatility observed in the stock prices. Later, Epstein and Wang (1995) also report that ambiguity aversion might be

associated with the market booms and crashes in such a way that even a small change in the market fundamental results in some considerable variations in the security prices due to the ambiguity aversion. Recently, Werner (2022) argues that common ambiguous beliefs may lead to speculative bubbles and adds that while learning about the true probabilities make bubbles deflated under heterogeneous beliefs, learning does not necessarily change the common ambiguous beliefs and speculative bubble may persist.

Chen and Epstein (2002) extend the model of Epstein and Wang (1994) and build CAPM in a dynamic and recursive framework under MPP preferences. Their model shows that correction on the parameters capturing ambiguity aversion might cause a change in the weights addressed by standard mean-variance model. More explicitly, if all investors have the same exact parameters, then mutual fund separation holds and optimal portfolio is mean-variance efficient. However, if parameters are not common for all investors, then there is a heterogeneity in the ambiguity perceived which leads to difference in the optimal portfolio weights hold.

Kogan and Wang (2003) extend the risk premium in the CAPM by adding the ambiguity component. More specifically, their representative agent model covers one-period and the market has one risk free and multiple risky assets to invest. The model accounts for ambiguity as it is assumed that investors are uncertain about the distribution of returns. Under this setting, Kogan and Wang (2003) indicates that the risk premium can decomposed into two factors: (i) first one is standard CAPM term which is multiplication of risk premium (λ) and a vector of betas quantifying with correlation of the asset returns with market portfolio (β), (ii) second one is a new ambiguity term which is the multiplication of risk premium on ambiguity (λu) and a vector of betas quantifying the exposure of assets to market ambiguity. Therefore,

two assets having the exactly same beta based on classical CAPM may have economically and statistically different returns if their exposure to ambiguity is different. Similarly, Izhakian (2020) provides an ambiguity extended CAPM which reveals that there is an ambiguity premium related to systematic ambiguity which is distinguishable from systematic risk.

2.3.3 Dynamic ambiguity models: information and learning under ambiguity

One of the important contributions is provided by the recent studies which extend static portfolio choice and equilibrium models under ambiguity into a dynamic setting, enabling the information processing and learning. Epstein and Schneider (2007), for example, provide one of earliest form of dynamic portfolio choice model which accounts for both ambiguity and the learning. Their model reveals that learning not only affects the optimal weight but also affects the length of investment horizon. More specifically, the model includes $\bar{\vartheta}$ which measures the amount of information that the investor is expecting to learn. $\bar{\vartheta}$ goes infinity when investor expects no learning about the equity premium and takes value of zero when she is fully confident that she will learn the equity premium. First implication of $\bar{\vartheta}$ is that optimal weights of risky asset of an ambiguity-averse investor increases as $\bar{\vartheta}$ goes to zero, i.e. the confidence is higher. Second implication is that as $\bar{\vartheta}$ higher, there is more participation in the longer horizon motivated by intertemporal hedging. Epstein and Schneider (2008) further extend this model by interpreting how quality of information plays a role on the pricing under ambiguity.

Chen, Ju, and Miao (2014) works on a consumption and investment problem in a discrete time – infinite horizon framework. There are two probable stock return models to be considered: identically and independently distributed return model and

vector autoregressive model. The agent does not know which one of the model is correctly specified. Assuming that effective measures considered are chosen based on a state variable Xt standing for the predictive variable, the beliefs of the agents are shaped as follows: When Xt is higher than zero (meaning that predictive variable is above the average), vector autoregressive model predicts above average excess return which causes agents to downward their beliefs about the identically and independently distributed return model and vice versa.

The other considerable studies using dynamic settings is from Liu (2011) who describes the optimal portfolio and consumption characteristics for an ambiguity-averse investor in a regime switching investment opportunity setting. Accordingly, optimal demand consists of four different components. First term (standard mean and variance efficient myopic demand for risky asset) and second term (perceived ambiguity) quantify the ambiguity-adjusted myopic demand for risky asset. Together with this, third term (hedging demand for incomplete information risk) and fourth term (hedging demand for ambiguity) compose the total hedging demand.

Caskey (2009) explores the effect of asymmetric information on the equilibrium prices by building a model in which both ambiguity averse and SEU investors trade. Their model reveals that ambiguity averse investors may try to decrease their level of perceived ambiguity by using the aggregate signals although this results in losing some portion of the available information, which ultimately causes prices to fall short of reflecting all of the available information. In a similar spirit, Condie and Ganguli (2017) indicate that it is important which segment of the investors owns the private information. In particular, combined with the findings that ambiguity aversion causes limited participation, private information obtained by ambiguity-averse investors will not be reflected fully in the prices.

2.3.4 Ambiguity and related issues

One of the important implication of the ambiguity aversion is that it is argued to be associated with market microstructure (limited participation, illiquidity, higher bid-ask spread,), some puzzles (equity premium puzzle, risk free rate puzzle, home-bias puzzle), and anomalies (size, value, momentum, excess volatility), This section will provide the major relevant studies.

2.3.4.1 Ambiguity and market microstructure

Starting from the earliest studies, incorporation of ambiguity to optimal consumption and the portfolio choice models reveals that ambiguity may be related with the limited participation and trade disruptions. Dow and Werlang (1992) work on a two-period problem in a hypothetical market including one riskless and one ambiguous asset. They find that portfolio trade in the market generically continues under SEU assumptions; however, ambiguity causes gaps in the prices in such a way that no one buys or sells the asset at some price intervals. i.e. trade for that specific asset totally stops. A similar intuition is reported by Easley and O'Hara (2009) for a standard portfolio choice problem under ambiguity. Under constant absolute risk aversion framework, Easley and O'Hara (2009) show that ambiguity averse investors do not trade for some sets of price level. Therefore, there is a limited participation equilibrium in which the ambiguity-averse investors do not trade and the market participation is limited to only investors with SEU. Further extension of limited participation model is provided by Guidolin and Rinaldi (2010) who divide the uncertainty as systemic ambiguity (uncertainty that impact the overall market) and idiosyncratic ambiguity (uncertainty that impact the specific stocks/sectors etc.). Accordingly, limited participation is found to be associated with ambiguity on the

idiosyncratic payoffs rather than ambiguity on the overall market payoffs. Huang, Zhang, and Zhu (2017) argue that limited participation may stem from the rational decision made by naive investors for the sake of avoiding the correlation ambiguity and find that lower quality assets yields positive excess returns in equilibrium.

Inspired by the intuition that ambiguity may result in limited trade and trade disruptions in the market, some studies examine the relationship between illiquidity and the ambiguity aversion. Easley and O'Hara (2010) offer a model in which the spread between bid and ask price are mainly caused by the ambiguity aversion. They also discuss the implication of their model in the crisis times and discuss the illiquidity issue during 2007-2009 global crisis time by associating it with the ambiguity aversion. Ozsoylev and Werner (2011) state that ambiguity decreases for those who can reach private information; however, arbitrageurs who do not have specific private information still make their decisions under the higher level of ambiguity. In this setting, limited participation occurs when arbitrageurs prefer not to trade, which lowers the liquidity as well as the market depth. Together with this, Routledge and Zin (2009) work on a market maker's optimization problem under MPP framework in which liquidity is measured by the bid-ask spread of the European call. Results show that though being reasonably sensible, it is difficult to strictly state that ambiguity aversion always causes to market breakdowns by increasing the illiquidity. For instance, an ambiguity averse market maker may offer more aggressive bid or ask for an optimal portfolio that has no position in derivative yields but hedges ambiguity.

2.3.4.2 Ambiguity and puzzles

Starting from Mehra and Prescott (1985), empirical studies consistently find a higher equity premium and a lower risk free rate than the levels implied by the theoretical models. In theory, extremely high risk aversion could explain the large equity premium and low risk-free rate in the market. The results found by Chen and Epstein (2002), however, have some implications on the risk free rate puzzle and equity premium puzzle. Under certain conditions, ambiguity incorporated consumption-based CAPM implies relatively low risk-free rate and relatively high equity premium consistent with the systematic empirical observation for years. In a similar spirit, Trojani and Vanini (2004) shows that ambiguity aversion plays a similar role with increased risk aversion on optimal portfolio, which means that ambiguity can be accounted for at least some part of the high equity premium and low interest rates. Besides, Trojani and Vanini (2004) reports that when volatility is low, equity premium is mainly due to ambiguity aversion rather than risk aversion.

More recently, Ju and Miao (2012) argue that ambiguity averse investors relate the recessions with the higher stochastic discount factor and have a lower continuation value during recessions. The pessimistic decisions of ambiguity averse investor decrease the stock prices, which eventually increases the stock returns. Besides, since the ambiguity averse investors would want to save more for the future, they go to bond market, which eventually decreases the risk-free rate. Although the data calibration results show that model of Ju and Miao (2012) has a good fit with the market data, it seems that the results are driven by the separation between risk aversion and the elasticity of intertemporal substitution because without such separation the model only captures the means and the volatilities of asset returns. Collard, Mukerji, Sheppard, and Tallon, (2018) provide a model that has a good fit

with the mean and volatility of the asset returns even when a separate elasticity of intertemporal is not defined. The main difference between models of Collard et. al (2018) and Ju and Miao (2012) is that former uses exponential ambiguity function while later uses power specification. Besides, Collard et. al (2018) assume that investors are uncertain not only about the average dividend payment, but also about the size of the parameters that determine the persistence of a temporary shock at such an average.

Based on the results of a questionnaire distributed to professional money managers, Olsen and Troughton (2000) also support the argument that equity premium puzzle is observed because the decision-makers are mostly ambiguity-averse. Recently, Dimmock, Kouwenberg, Mitchell, and Peijnenburg (2016) find that ambiguity aversion – which is quantified by a survey including Ellsberg-type questions – leads to limited stock market participation. Together with this, Dimmock, Kouwenberg, and Wakker (2016) add that ambiguity aversion is inversely tied to the stock market participation for only those perceiving stock returns as very ambiguous. Gollier (2011), on the other hand, shows that increased aversion to ambiguity does not always lead to a demand decrease in the risky or ambiguous asset and concludes that it would be an over-statement to conclude that ambiguity always related to observed equity premium puzzle in the market.

By using smooth ambiguity attitudes, Mukerji, Ozsoylev, and, Tallon (2022) provide a model of financial markets with heterogeneously ambiguous assets and heterogeneously ambiguity averse investors. Their model produces consistent implication with the numerous puzzling observations in the literature such as the asset allocation puzzle, large trade volume with a marginal price change after a post earnings announcement period, and positive association between increase in

uncertainty and high trade activity and portfolio rebalancing toward safer assets. Other studies that provide further insights about the association between the ambiguity aversion and the equity as well as the risk-free rate puzzle are Hansen, Sargent, and Tallarini (1999), Hansen and Sargent (1998), Maenhout (2004), Hansen (2007), Barillas, Hansen, and Sargent (2009), Bekaert, Engstrom, and Xing (2009).

Ambiguity is also a strong candidate for explaining the home-bias puzzle, which is the observation that investors' tendency to include local assets in their portfolio with a higher weight implied by SEU. In an intertemporal setting that assumes preference for robustness, Uppal and Wang (2003) argue that ambiguity may lead to home-bias puzzle observed in the market. An ambiguity parameter is defined as $\varrho \geq 0$ which approaches to infinity as ambiguity declines and approaches to zero as ambiguity increases. Uppal and Wang (2003) shows that as ϱ approaches to infinity (zero ambiguity), the optimal portfolio will be equivalent to the portfolio addressed by Merton's standard model. At the other extreme, as ϱ approaches to zero (highest ambiguity), the optimal portfolio will include no risky assets. In between two extremes, which is defined as finite and positive ϱ, the weight of risky assets will not be zero but will be lower than the weights invested in the case of zero ambiguity. Therefore, when the there is a low confidence for joint stock distribution, even a very small differences in the level of confidence for marginal payoff distribution may lead to considerable under diversification in comparison with the classical mean-variance portfolio, which result in home-bias puzzle. Data calibrations also addresses a similar conclusion. For example, French and Poterba (1991) conclude that the observed under-diversification stems from investors choice rather than institutional restrictions, implying that even a small heterogeneity in the perceived ambiguity

across local and foreign market may result in the serious under-diversification in the final portfolio.

A recent study form Liu and Zeng (2017) shows that ambiguity in the correlations of asset return may be result for the under-diversification. Dimmock Kouwenberg, Mitchell et al. (2016) measure ambiguity aversion by a survey including Ellsberg-type questions and report that ambiguity aversion is inversely related with stock market participation and foreign stock ownership while it is positively associated with own-company stock ownership. These results imply that ambiguity aversion is related with home-bias puzzle in addition to equity premium puzzle. Similarly, Bianchi and Tallon (2019) measure ambiguity attitudes with survey data and show that ambiguity averse investors exposed to more risk of local market in comparison to the international markets because of home bias they exhibit. Guidolin and Liu (2016) also provides further evidence on role of ambiguity aversion on the home-bias, which is even stronger in the regime-switching setting.

2.3.4.3 Ambiguity and anomalies

As it is strongly validated by empirical studies, CAPM's beta is related with market return too flatly to explain the variation of the cross-section of returns. A set of studies done by Fama and French show that small (stock with lower market capitalization) and value stocks (stocks with higher book to market ratio) outperform and have an explanatory power in explaining returns (Fama & French, 1992, 1993, 1996). Later, stock returns are also found to be related with momentum (Carhart, 1997; Jegadeesh & Titman, 1993) and liquidity (Amihud, 2002; Pástor & Stambaugh, 2003). Although there is a strong empirical relationship between asset returns and these factors, there is no strong theoretical explanation behind why these

factors are able to explain the variations in the cross-sectional returns. Some studies argue that one of the theoretical reasons of these empirical results may be ambiguity.

Some studies report that ambiguity may have some role on explaining the higher risk-adjusted returns of small and value stocks. For example, results of the questionnaire distributed to that professional investors by Olsen and Troughton (2000) show that return distribution of small stock are found to be more speculative and harder to estimate correctly. The results also indicate that when professional investors perceive higher ambiguity, they seem to make investment decisions based on their personal judgement rather than formal quantitative metrics. Therefore, ambiguity aversion may explain the systematically higher risk- adjusted returns of small stocks that CAPM's beta fails to capture. A similar interpretation is also provided by Antoniou, Galariotis, and Read (2014) who argue that unconditional size effect in returns around earnings announcements may stem from the ambiguity aversion. In particular, because the quality of information - measured by analyst forecasts- is more ambiguous for smaller firms, ambiguity-averse investors tend to price small stocks pessimistically. When ambiguity starts to be resolved at the end of the quarter, market corrects pessimism which leads to size effect.

Based on experimental data, Bossaerts, Ghirardato, Guarnaschelli, and Zame (2010) find a positive correlation between ambiguity aversion and risk aversion and they suggest that this may explain the value effect. More particularly, value and growth stock stocks can be assumed as risky and ambiguous respectively and market can be assumed to be segmented for value and growth stocks because of the heterogeneity in risk and ambiguity aversion. Growth stocks, which are more likely to be held by less ambiguity averse investors, should also be held by the less risk averse investors because of the positive correlation between risk aversion and

ambiguity aversion. Consequently, growth stocks include less risk premiums and give lower rate of returns compared to value stocks, explaining the underperformance of growth stocks. Viale, Garcia-Feijoo, and Giannetti (2014) argue that value premium may stem from larger preference for value stocks by ambiguity averse investors due to the their relatively higher performance when the confidence of investor in return generating model is lower.

Zhang (2006) finds that ambiguity (information uncertainty) has a considerable role on momentum which is widely tied to the gradual reaction to news in the market ((L. K. C. Chan, Jegadeesh, & Lakonishok, 1996). Combining under-reaction hypothesis with well-established argument that uncertainty increases the behavioral biases (Daniel, Hirshleifer, & Subrahmanyam, 1998, 2001; Hirshleifer, 2001), Zhang (2006) argue that if the momentum stems from underreaction, then it should be observed even further underreaction (greater momentum) in cases when ambiguity is larger. The results show that when information uncertainty is higher, a good news is preceded by higher expected returns and vice versa, supporting the hypothesis that high ambiguity inflated the underreaction. Similarly, Caskey (2009) offers that accounting for trading behavior of ambiguity averse investor would explain the anomalies tied to underreaction and overreaction to in the market including momentum. More specifically, Caskey (2009) hypothesizes that tendency of ambiguity-averse investors is to reduce ambiguity exposure at the cost of information loss by trading based on aggregate information. As a result, prices reflect not all but aggregate signals which causes lagged price adjustments towards the correct level and ultimately to momentum. Recently, Gong, Wang, and Dlugosch (2019) examine joint effect of ambiguity attitude and information uncertainty on momentum strategy across 39 countries and find that momentum effect is highest for

firms with high information uncertainty and in countries with less level of ambiguity aversion.

One of other the interesting empirical observation is that volatility in financial markets is considerably larger than predicted by the financial theories. Starting from seminal paper by Shiller (1981) showing that asset price is more volatile to be justified by subsequent changes in dividends, a lot of studies attempt to find a theoretical explanation for that phenomenon. Although the excess volatility is seen as one of the strong evidence against the rational investor hypothesis assumed by most of the theoretical model, the implications of ambiguity on information provides an alternative story for excess volatility. Epstein and Schneider (2008) examine the how quality of information plays a role in the asset price given that investors are exposed to a wide number of signals and they have to process them presumably based on their quality. In particular, some signals come from a reliable source while some signals come from a source quality of which is difficult to evaluate, making the latter ambiguous information. Epstein and Schneider (2008) show that ambiguous information leads to larger price volatilities than implied by the fundamentals because ambiguity avoidance agent has a large range of accuracy to consider and thus puts more weight on the signals than an agent who knows the true accuracy. The implications of model are also consistent with the skewness in the cross-section of returns reported by Harvey and Siddique (2000). More specifically, because models predict that ambiguity averse investors respond asymmetrically to news, even if the fundamental has a symmetrical distribution, the distribution of the stocks return can be skewed. Illeditsch (2009) extends the model of Epstein and Schneider (2008) by accounting for three important aspects. First investors are assumed to be not only ambiguity-averse but also risk-averse. Second, investors are heterogeneous in their

attitude towards risk and ambiguity. Third, investors are able to learn from ambiguous signal over time. Under this set-up, Illeditsch (2009) reports that heterogeneous risk and ambiguity aversion have very different implications; and more importantly, risk aversion exaggerates the impact of the ambiguous signal on the stock returns. In a subsequent study, Illeditsch (2011) examines the effect of information that is difficult to tie with the fundamentals and finds that surprising news may lead to zero price change because of the desire of hedging against ambiguity. This ultimately may lead to the excess volatility which is more pronounced in the case of small shocks to cash flow news, asset betas, or market risk premium.

CHAPTER 3

QUANTIFYING AMBIGUITY AND AMBIGUITY ATTITUDE IN TURKEY

This study empirically documents the effect of ambiguity on stock returns in a major emerging market along with the ambiguity attitude under various market conditions. Our results show that ambiguity is a priced factor in Turkish stock market returns with a positive premium that is distinct from risk premium. In contrast with the findings in the US market, the investors in Turkey show an increasing level of ambiguity aversion as expected probability of favorable returns deviate from the mean value. The investors are effectively ambiguity neutral in lateral markets. The results are robust to testing with higher moments, sentiment measures and under recession conditions. This study is a major contribution to documenting international differences in ambiguity attitudes.

3.1 Introduction

Conventional asset pricing theories have several assumptions enabling them to be simple and isolated at the cost of being unrealistic to be hold in the real-life. However, presumably one of the major strengths of these theories is that they are built upon well-defined assumptions which can be relaxed straightforwardly when necessary. One of those assumptions is that asset prices reflect SEU preferences. Under SEU model, having their own subjective probability distribution based on additive probabilities, each investor is assumed to make decisions in order to maximize their subjective utility function. Under this assumption, there is a unique probability distributions assessed by each investors and the uncertainty is derived from the only outcomes (i.e. the volatility of stock returns). However, Knight (1921)

and Keynes (1937) argue that decision makers are also uncertain about probabilities due to informational or cognitive restrictions, which is known as ambiguity (or Knightian uncertainty). Beside, Ellsberg (1961) shows that decision makers are ambiguity averse because they strictly evade the ambiguous situations.

According to traditional asset pricing theories assuming that SEU model holds, there is a tradeoff between risk and return, which dates back to ICAPM provided by Merton (1973). However, a considerable body of the previous literature show that theoretical and intuitive expectation of positive risk and return relationship is insignificant (Baillie & DeGennaro, 1990; Campbell & Hentschel, 1992; French, Schwert, & Stambaugh, 1987). Some studies even report a negative association between risk and return (Black, 1972, 1976; Campbell, 1987; Harvey, 2001; Nelson, 1991). Offered reasons for this counter intuitive observations are time varying risk aversion (Campbell & Cochrane, 1999; Whitelaw, 2000), alternative risk measures (Ghysels, Santa-Clara, & Valkanov, 2005), and econometric models (Glosten, Jagannathan, & Runkle, 1993; Harvey, 2001). Welch and Goyal (2008), however, report that accounting for the previously offered factors does not produce a stronger relationship between risk and return. In a behavioral perspective, deviations of risk and return trade-off are also tied with noise traders (De Long, Shleifer, Summers, and Waldmann, 1990; Shleifer and Summers 1990) or irrational investor sentiment (Shiller, 1981; Yu & Yuan, 2011).

Another proposed explanation for the confusing risk-return relationship is the absence of ambiguity in asset price models. Although conventional asset pricing models ignore ambiguity, ambiguous situations in financial markets are too severe to be ignored. Factors such as noise, incomplete or unreliable information, asymmetric information dissemination, and unforeseen regulations etc. force investors to make

trading decisions under ambiguity. Incorporation of ambiguity to conventional asset pricing models reveals that ambiguity has several important implications in asset prices as excellently summarized by Guidolin and Rinaldi (2013). However, literature on ambiguity and asset prices is dominated by mostly theoretical studies and partly data calibrations probably because it is difficult to quantify ambiguity and ambiguity aversion. Recently, Bühren, Meier, and Pleßner (2021) conduct bibliometric analysis on ambiguity and report that only seven per cent of the 456 studies published in high-ranking journal are empirical studies. While majority of the studies (60 per cent) are model applications without data, 23 per cent of them is experimental studies.

While some studies measure ambiguity based on a survey including Ellsberg-type questions (Bianchi & Tallon, 2019; Dimmock, Kouwenberg, Mitchell, et al., 2016; Olsen & Troughton, 2000), some studies use proxies such as VIX (Bekaert, Hoerova, & Lo Duca, 2013; Bloom, 2009; Williams, 2015), entropy of inflation (Ulrich, 2013), uncertainty index (Bali, Brown, & Tang, 2017), asset familiarity (Boyle et al., 2012), variance of mean (Maccheroni et al., 2013) or variance of variance (Faria & Correia-da-Silva, 2014). Dispersion in beliefs is also among offered ambiguity proxies. For instance, disagreement among the analysts (Anderson, Ghysels, & Juergens, 2009; Antoniou, Harris, & Zhang, 2015; Diether, Malloy, & Scherbina, 2002) or dispersion in fund managers' beliefs (Jiang & Sun, 2014) are found to be priced in the assets. Recently, Hsiao (2022) argue that correlation between trading volume and absolute price change is a negative indicator for investor disagreement because if there is a high investor disagreement, they trade in the exact opposite directions, leading to only a small price change. Lee, Min, and Kim (2019) find a significant cross-sectional relation between return and

macroeconomic ambiguity which is measured by the dispersion in real-time forecasts of real gross domestic product (GDP) growth. On the other hand, Andreou, Kagkadis, Maio, and Philip (2014) argue that the dispersion in the volume-weighted strike prices of the S&P 500 Index options may measure the dispersion in the beliefs about future prices.

Some studies use option-based implied ambiguity. For example, Driouchi, Trigeorgis, and So (2018) compute implied ambiguity based on a similar methodology with computing implied volatility from Black and Scholes formula and find that lead-lag relation between realized volatility and implied volatility of options may be attributed to the ambiguity. So, Driouchi, and Trigeorgis (2016) also study with implied ambiguity and report that ambiguity contributes to risk factors in explaining the excess returns. Changes in some fundamental indicators are also used for quantifying ambiguity. Drechsler (2013) find that there is an option ambiguity premium stemming from investors' desire to hedge against model uncertainty which can be measured by three components (i) expected growth in the cash flows, (ii) stochastic volatility, (iii) jumps in the expected growth and growth volatility processes. Similarly, Bekaert et. al. (2009) define uncertainty as the conditional variance of fundamentals such as consumption growth and dividend growth and find that variation in equity premium is related with both uncertainty and risk aversion, latter being more dominant.

One recent proxy for ambiguity is offered by Brenner and Izhakian (2018), which use the variations of the price distribution for measuring the ambiguity. Based on theoretical model developed by Izhakian (2017), Brenner and Izhakian (2018) offer an ambiguity proxy which is both derived from a theoretical setting and computed from the real market data. This measure is simple, intuitive, risk

independent, and applicable for the empirical measurement of the degree of ambiguity. Brenner and Izhakian (2018) measures ambiguity for S&P 500 index and report three main results: (1) when ambiguity is included in the model alongside with the risk, risk and return are found to be significantly and positively related (2) investors are ambiguity averse when they expect favorable returns; and ambiguity-lover when they expect unfavorable returns (3) the level of aversion of (love for) ambiguity increases with expected probability of favorable (unfavorable) returns. Their results are consistent with a set of behavioral studies implying people avoid from (seek for) ambiguity when the probability of gain (loss) is high (Abdellaoui, Vossmann, & Weber, 2005; Du & Budescu, 2005; Mangelsdorff & Weber, 1994; Viscusi & Chesson, 1999). Their measure is further tested by the authors in other contexts in a series of papers as well (Augustin & Izhakian, 2020; Brenner & Izhakian, 2022; Izhakian, Yermack, & Zender, 2021).

The measure is used by Kim and Byun (2021) to test ambiguity on international ETF's trading in the US. Their results show that ambiguity aversion is uniform across markets, albeit attitudes do not display linearity, especially for emerging markets. Luo, Mishra, Yarovaya and Zhang (2021) test Bitcoin returns and conclude crytpo investors are also ambiguity averse. There are a few empirical studies on ambiguity in Borsa Istanbul, ambiguity and ambiguity attitude in Turkish equity market is not comprehensively examined following aforementioned methodology. For example, Altay (2015) finds that ambiguity is significantly and negatively related with excess return by following the measure offered by Brenner and Izhakian (2011). This methodology could be considered as the primitive version of the methodology in Brenner and Izhakian (2018) as the variance of the loss probabilities are taken as the ambiguity rather than variance of price distributions.

Şahin and Danışoğlu (2022) also find a negative and significant relationship between ambiguity and excess returns by measuring ambiguity for Turkish equity market with different proxies including the one offered by Brenner and Izhakian (2018). The study, however, does not track on ambiguity attitudes of Turkish investors. To the best of our knowledge, international studies with returns denominated in local currencies and traded in local markets are very limited. The main aim of this chapter is to quantify ambiguity and ambiguity attitude in Turkey by a recent methodology and to test impact of it on asset prices. The study at hand would be the first of its kind documenting international evidence with such rigor and detail and naturally provide a comparison of a large emerging market's ambiguity and local investors' attitude toward ambiguity vis-a-vis developed markets. Our results show that ambiguity is priced in Turkish stock market with a positive premium that is distinct from risk premium. Besides, we find that attitude of investors in Turkey towards ambiguity exhibit different properties compared with the investors in US. Turkish investors are ambiguity averse under positive and negative market expectations, but they remain ambiguity neutral for lateral markets, where direction is not clear.

3.2 Model and methodology

In this study, we follow the recently developed methodology by Brenner and Izhakian (2018) to measure volatility of probabilities as a consistent measure of ambiguity. Denote by $\varphi(r)$ the marginal probability (density function) associated with the cumulative probability $P \in \mathcal{P}$ of r. The expected marginal and cumulative probability of r, taken using the second-order probability measure ξ, which is defined as the probability of a subset of the state space, are then respectively defined by:

$$E[\varphi(r)] = \int_{\mathcal{P}} \varphi(r) d\xi \tag{1}$$

$$E[P(r)] = \int_{\mathcal{P}} P(r) d\xi \tag{2}$$

and the variance of the marginal probability is defined by

$$Var[\varphi(r)] = \int_{\mathcal{P}} (\varphi(r) - E[\varphi(r)])^2 d\xi. \tag{3}$$

We assume investors treat ambiguity as they treat risk via a concave utility function (reflecting their ambiguity aversion). Using the notion of the volatility of probabilities above, the measure of ambiguity becomes

$$\mho^2[\mathrm{r}] = \int E[\varphi(r)] Var[\varphi(r)] dr \tag{4}$$

Assuming that returns are normally distributed but not independent and identically distributed, degree of ambiguity can be written as:

$$\mho^2[r] = \int E[\phi(r; \mu, \sigma)]\, Var[\phi(r; \mu, \sigma)] dr \tag{5}$$

where $\phi(.)$ is normal probability density funtion.

The idea behind measure $\mho^2$ is that ambiguity can be measured by the volatility of probabilities, just as the degree of risk can be measured by the volatility of returns. In general, $\mho^2$ encompasses not only an ambiguous variance and an ambiguous mean but also the ambiguity of the higher moments of the probability distribution (i.e., skewness, kurtosis, etc.) through the uncertainty of probabilities. This measure, $\mho^2 \in [0, \infty)$, attains its minimum value 0, only when all probabilities are known. Another key property of this measure is that it is independent of risk measure.

As a practical approach to building a time series of ambiguity and volatility measures, we use intraday data of Borsa Istanbul 100 index (BIST 100) and calculate

returns for every five minutes from opening to closing hours in each day. Our data covers the period between January 1998 and February 2021. We next compute the parameters of the empirical distribution of returns within each day, assuming they are normally distributed. Then we aggregate them monthly to compute the mean and variance of probabilities, hence the ambiguity measure.

In order to compute the monthly degree of ambiguity, as defined in Equation (5). we divide the range of daily returns into intervals (bins), calculate the chance of the return being in each bin for each day as implied by the distribution with parameters computed from 5-minute returns. Returns are assumed to be normally distributed. We then calculate the mean and variance of probability of the return being each bin across the days of the month. Then, using the discrete formula below, we quantify the degree of ambiguity for each month:

$$\begin{aligned}\mho^2[r] = \frac{1}{w(1-w)} + (&E[\phi(r_0;\mu,\sigma]Var[\phi(r_0;\mu,\sigma] \\ &+ \sum_{i=1}^{60} E[\phi(r_i;\mu,\sigma) - \phi(r_{i-1};\mu,\sigma)]Var\,[\phi(r_i;\mu,\sigma) \\ &- \phi(r_{i-1};\mu,\sigma)] \\ &+ E[1-\phi(r_{60};\mu,\sigma)]Var[1-\phi(r_{60};\mu,\sigma)])\end{aligned} \tag{6}$$

where r_0 and r_{60} are the lowest and highest returns respectively. The range we set the cut points is ±10% since this range covers the most of the observation in our dataset with only a limited number of outliers. w corresponds to $r_i - r_{i-1}$ and 1/w(1 −w) scales the weighted-average volatilities of probabilities to the bin size. This scaling, which is analogous to Sheppard's correction, is tested to verify that it minimizes the effect of the bin size on the values of $\mho^2$. As robustness check, we try different scaling and bin size to ensure that our results are not sensitive the scaling and the bin size chosen. Our results are essentially the same.

We then use a standard ARMA(p; q) model to estimate a forward looking measure of volatility and ambiguity from the time series via

$$ln\sqrt{v_t} = \psi_0 + \epsilon_t + \sum_{i=1}^{p} \psi_i \cdot ln\sqrt{v_{t-1}} + \sum_{i=1}^{q} \theta_i \cdot \epsilon_{t-1} \quad (7)$$

$$ln\mho_t = \psi_0 + \epsilon_t + \sum_{i=1}^{p} \psi_i \cdot ln\mho_{t-1} + \sum_{i=1}^{q} \theta_i \cdot \epsilon_{t-1} \quad (8)$$

Natural logarithms are taken to ensure stationarity of the variables. The ARMA models are estimated for each p = 1,….,10 and q = p = 1,….,10, i.e. in total p x q = 100 combinations. Then the coefficients that attain the minimal AIC (i.e. the highest-quality model) are used to estimate the expected volatility which is given by:

$$v_{t+1}^E = E_t[v_{t+1}] = exp(2\widehat{ln\sqrt{v_t}} + 2Var[u_t]) \quad (9)$$

$$(\mho_{t+1}^2)^E = E_t[\mho_{t+1}^2] = exp(2\widehat{ln\mho_t} + 2Var[u_t]) \quad (10)$$

where $Var[u_t]$ is the minimal predicted variance of the error term. The estimations of other variables in the model are computed via similar econometric methods.

Testing the impact of ambiguity as measured above requires eliciting numerical attitudes towards ambiguity, a measure of ambiguity aversion, which is not constant across varying probabilities. As a practical approach, the probabilities are divided into a discrete number of bins (10 is a reasonable number as tested), and the following regression model is proposed by Brenner and Izhakian (2018):

$$r_{t+1} - r_{f,t+1} = \alpha + \gamma v_t^E + \eta((\mho_t^2)^E \times \vartheta_t^E) + \sum_{i=2}^{10} \eta_i \left(D_{i,t} \times P_t^E \times (\mho_t^2)^E \times \vartheta_t^E\right) + \varepsilon_t \quad (11)$$

where excess returns are regressed against estimated volatility (v_t^E), the average absolute deviation of returns from the expected return (ϑ_t^E), the probability of favorable returns (P_t^E) and ambiguity $(\mho_t^2)^E$. The returns higher than zero are

counted as favorable and otherwise as unfavorable. $D_{i,t}$ are dummy variables for each probability bin. (ϑ_t^E) could be expressed as $E[|r - E[r]|]$. The notable feature of equation (11) is that it distinguishes the beliefs and tastes for both risk and ambiguity. More specifically, while v_t^E and γ stand for risk and risk attitude respectively, $(\mho_t^2)^E$ and η measures the ambiguity and ambiguity attitude. $\eta((\mho_t^2)^E \times \vartheta_t^E)$ implies that tastes for ambiguity depends on expectations of probability of gains. More details for the theoretical foundation behind the model is given by Brenner and Izhakian (2018), Izhakian (2017), and Izhakian (2020).

The coefficient γ is the measure of risk aversion and η_i are the coefficients of ambiguity for the range of probabilities considered. More specifically, as the range of P_t^E is between [0.43, 0.53] and as we divide this range into ten bins, the width of each probability bin is 0.01. Therefore, η_1 presents the ambiguity attitude in the range of probability in [0.43, 0.44], η_2 in the range of probability in [0.44, 0.45] and so on. Based on equation (11), the attitudes toward ambiguity conditional on the expected probability of the favorable return being in the bin i can be shown as $\eta(P_i^E) = \hat{\eta} + \hat{\eta_i}$. Negative value of this expression gives a negative ambiguity premium and signals ambiguity-loving behavior and vice versa. Besides a higher $|\hat{\eta_i}|$ in low probability of favorable return implies an increasing love for ambiguity and higher $\hat{\eta_i}$ in high probability of favorable return signals an increasing aversion for ambiguity.

In addition to model addressed in equation (11), which is a discrete model, we will also run the continuous model:

$$r_{t+1} - r_{f,t+1} = \alpha + \gamma v_t^E + \eta((\mho_t^2)^E \times \vartheta_t^E) + \eta_s\left(D_{i,t} \times P_t^E \times (\mho_t^2)^E \times \vartheta_t^E\right) + \varepsilon_t \quad (12)$$

where v_t^E, ϑ_t^E, P_t^E, $\mho_t^2$ correspond the same the variables as discrete model. The continuous model implicitly assumes that the ambiguity attitudes have a linear relation with the expected probability of favorable returns.

3.3 Empirical findings

3.3.1 Estimating ambiguity

Figure 1 shows the monthly return, monthly risk, and monthly ambiguity for BIST 100 index between January 1998 and February 2021. The x-axis stands for the timeline and y-axis stands for the realized return in the upper plot, realized risk in the middle plot, and realized ambiguity in the lower plot for BIST 100 index. Monthly risk is measured by the monthly standard deviation of daily returns. Monthly ambiguity is calculated by the monthly standard deviations of the daily probabilities of five-minute returns of BIST 100, i.e. $\mho$, which is square root of the equation in (6). The shaded areas show the recession periods for Turkey[2].

[2] Organization of Economic Development (OECD) based Recession Indicators for Turkey from the Peak through the Period preceding the Trough [TURRECP], retrieved from FRED, Federal Reserve Bank of St. Louis; https://fred.stlouisfed.org/series/TURRECP.

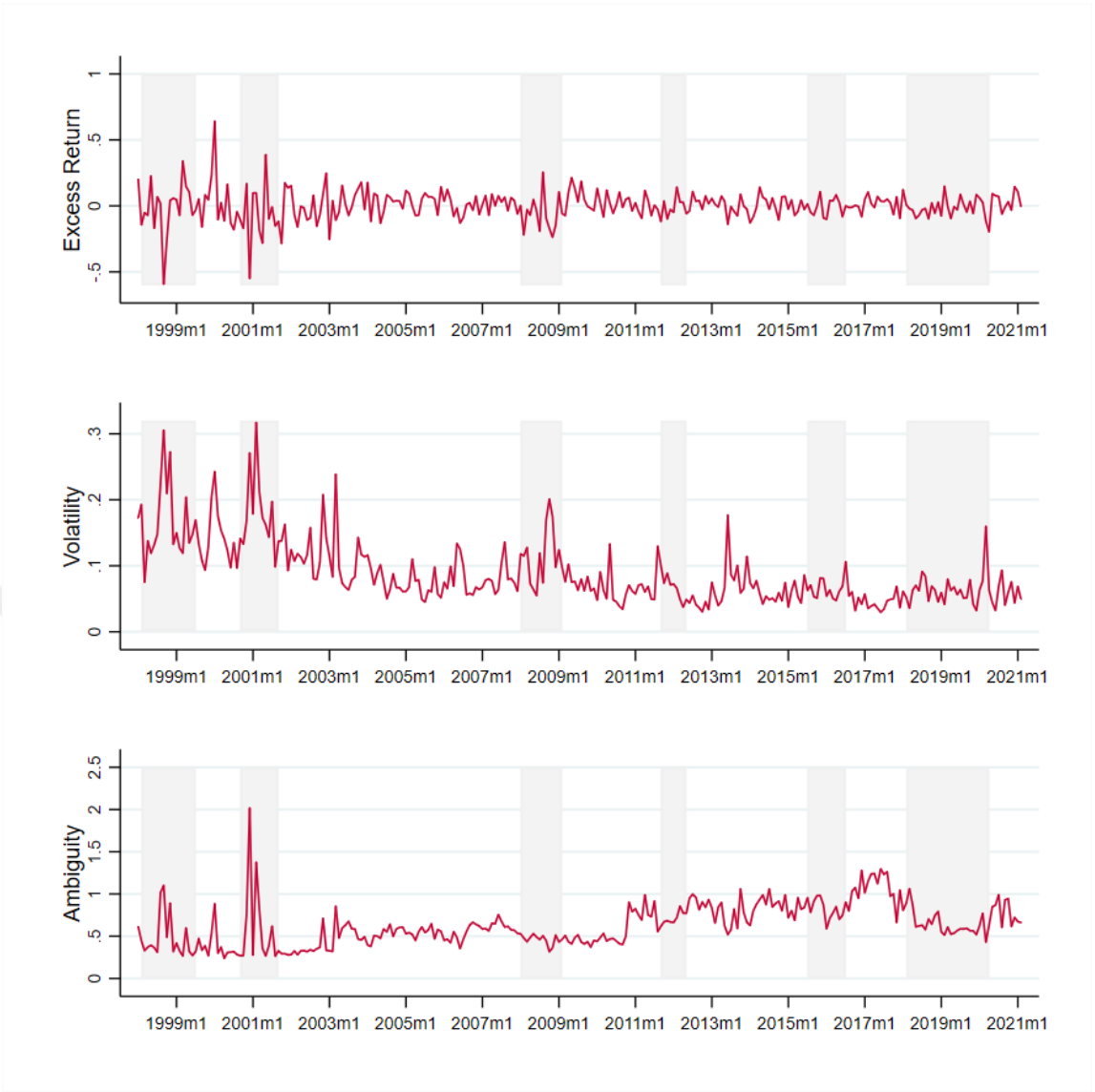


Figure 1. Time series of monthly realized risk (volatility), monthly realized ambiguity, and monthly realized excess return of BIST 100

The peak of ambiguity is observed during December of 2000 which corresponds to the period that overnight interest rates have dramatically increased leading to an eventual liquidity crisis. Followed by worsening economic indicators and political distress, banking crisis has started on February 2001 which is the month that the second highest ambiguity is observed. Ambiguity is also relatively high in the recession period during 1998 and 1999. In the other recession periods, ambiguity is not necessarily higher than average while almost all of the recession periods are accompanied with the relatively high volatility. As Brenner and Izhakian (2018) also

argue, ambiguity seems to be related with how market participants interpret the uncertainty not solely in the equity market but in the market as a whole. Therefore, occasional jumps in ambiguity levels might be tied to political and economic uncertainty as well as previous run-ups in prices that investors may expect a correction. The relation of ambiguity with other factors require further examination that is beyond the scope of this study.

Panel A of Table 1 reports the daily descriptive statistics. n is the number of five-minute returns per day, which has an average of 67.179 and changes between 19 and 96. Since the opening and closing hours of BIST 100 have changed throughout the time period we covered, the number of five-minute returns per day is not constant. The minimum observation numbers belong to the days on which BIST 100 is open for only half of the day. μ is the daily average of five-minute returns, which has a mean close to zero and a standard deviation of 0.021. σ is the daily standard deviation of five-minute returns, with mean of 0.016 and standard deviation of 0.010. P is the daily probability of favorable returns, where a favorable return is defined as five-minute return higher than zero. Mean of P is 0.478. Brenner and Izhakian (2018) report that on daily terms probability of favorable return has a mean of 0.51 and standard deviation of 0.32 for the US market. They argue that this result is reasonable because on average probability of having a favorable or unfavorable return should be almost the same. The riskier nature of Turkey, on the hand, leads to fatter tails and a negative skewness (kurtosis for 5 minute daily return is more than 11 and skewness is -0.304) and making the mean of probability of favorable returns lower.

Panel B of the Table 1 shows the monthly descriptive statistics between January 1998 and February 2021, making 278 months in total. r_f is risk free rate and

calculated as 1-month US Treasury Bill plus local inflation minus the US inflation. We use this proxy as risk-free to alleviate the fact that the Turkish government bond yields are not necessarily risk-free and they in fact exceed stock market returns for extensive periods leading to negative equity premium. The approximation is a natural choice as it proxies an international investor's parity adjusted or hedged risk-free return in Turkey. r is the dividend adjusted return on BIST 100. Consistent with the features of emerging markets, both risk-free rate and the total return on BIST 100 has a higher mean and standard deviation compared to developed markets. $r - r_f$ stands for excess return with a mean of 0.002. Excess return for BIST 100 is highly volatile with standard deviation of 0.118, negatively skewed and has fat tails. $\sqrt{v}$ stands for volatility, measured by the standard deviation of the daily return on BIST 100 and has a monthly average of 0.089. ϑ is the average of absolute daily deviations of returns from the monthly average daily returns, which is denoted as $E[|r - E[r]|]$ and has a mean of 0.069. $\bar{P}$ is monthly average of the daily probabilities of favorable returns, and also has a mean around 0.478. Finally, $\mho$ denotes the monthly ambiguity calculated as the monthly standard deviations of the daily probabilities. Between January 1998 and February 2021, $\mho$ ranges between 0.238 and 2.018, with a mean of 0.633 and standard deviation of 0.254.

Panel C shows the cross-correlations among the variables in the regression model showed in equation (11). In the regression estimation, monthly variance of the daily returns and monthly variance of daily probabilities are used as a proxy for risk and ambiguity respectively. The excess return and risk are negatively related contradicting with the risk and return association hypothesized by main classical finance theories. Although counterintuitive, similar patterns have been documented by several studies dating back to Black (1972). There is no high correlation detected

except correlation between volatility (ν) and average absolute deviation (ϑ). Brenner and Izhakian (2018) also report a high and positive correlation between volatility and average absolute deviation for the US market. The correlation between risk and ambiguity is negative and low, showing that ambiguity is risk-independent.

Table 2 reports the descriptive statistics of the estimated variables. Each variable presented in Table 2 is out-of-sample fitted value of the relevant ARMA(p; q) model i.e. ARMA model with the lowest AIC. The first 10 months of data is an exception, because we chose to estimate them in-sample and add to analysis to avoid losing data. Dropping them does not change the results. As expected, Panel A shows that the estimated expected values of all variables have lower range and standard deviation than their realized values, meaning that they have a smoother trend. Panel B shows the cross-correlation of estimated expected variables. The correlation between expected ambiguity and expected risk are still negative but higher compared to Panel C of Table 1. The magnitudes of correlation coefficients are comparable to the US counterparts.

Table 1. Summary Statistics

		N	Mean	Median	Min	Max	Std. Dev.	Skewness	Kurtosis
Panel A. Daily Descriptive Statistics									
n	Num. of obs.	5813	67.179	67.000	19.000	96.000	14.290	0.183	2.506
μ	Mean return	5813	-0.002	-0.001	-0.195	0.166	0.021	-0.304	11.092
σ	Std. dev.	5813	0.016	0.013	0.003	0.151	0.010	2.879	20.005
μ/σ	Mean/Std	5813	-0.068	-0.092	-3.944	3.652	1.054	0.124	2.841
P	Probability	5813	0.478	0.463	0.000	1.000	0.300	0.114	1.743
Panel B. Monthly Descriptive Statistics									
r_f	Risk-free	278	0.014	0.009	-0.013	0.103	0.017	1.629	6.258
r	Return	278	0.016	0.013	-0.522	0.691	0.117	0.336	9.528
$r - r_f$	Excess return	278	0.002	0.003	-0.593	0.641	0.118	-0.151	9.347
$\sqrt{v}$	Volatility	278	0.089	0.073	0.030	0.317	0.050	1.727	6.521
ϑ	Avg. absolute dev.	278	0.069	0.056	0.024	0.230	0.039	1.678	6.068
$\bar{P}$	Mean Prob.	278	0.478	0.483	0.315	0.639	0.070	0.029	2.374
$\mho$	Ambiguity	278	0.633	0.591	0.238	2.018	0.254	1.042	5.458
Panel C. Cross Correlations									

	$r - r_f$	v	ϑ	$\bar{P}$	$\mho^2$
$r - r_f$	1.000				
v	-0.178	1.000			
	(0.003)				
ϑ	-0.155	0.977	1.000		
	(0.010)	(0.000)			
$\bar{P}$	-0.031	-0.214	-0.214	1.000	
	(0.612)	(0.000)	(0.000)		
$\mho^2$	-0.096	-0.115	-0.106	0.104	1.000
	(0.109)	(0.055)	(0.078)	(0.085)	

p values in parentheses

Table 2. Expected Values

Panel A. Descriptive statistics of forecasted variables								
	N	Mean	Median	Min	Max	Std. Dev.	Skewness	Kurtosis
$\sqrt{\upsilon^E}$	278	0.096	0.083	0.041	0.224	0.040	1.094	3.427
ϑ^E	278	0.070	0.060	0.032	0.161	0.029	1.076	3.288
$\bar{P}^E$	278	0.487	0.487	0.407	0.550	0.027	-0.318	2.995
$\mho^E$	278	0.652	0.620	0.259	1.341	0.200	0.569	3.090

Panel B. Cross-correlations of expected values of the variables				
	υ^E	ϑ^E	$\bar{P}^E$	$(\mho^2)^E$
υ^E	1.000			
ϑ^E	0.960	1.000		
	(0.000)			
$\bar{P}^E$	-0.179	-0.195	1.000	
	(0.003)	(0.001)		
$(\mho^2)^E$	-0.448	-0.559	0.197	1.000
	(0.000)	(0.000)	(0.001)	

p values in parentheses

3.3.2 Risk, return and ambiguity attitudes

Table 3 presents the results of the regression model in equation (11), with standard errors computed via Newey-West estimators. The model in the first column includes only volatility as the independent variable and reports the relationship between risk and return. Starting from the second column the estimated ambiguity is incorporated to the model. Second column examine the relationship with the return and the unconditional ambiguity.

The risk coefficient is significant and negative. This is a contradicting observation with the positive risk and return relationship offered by the classical finance theories. As previously noted, there are previous empirical studies documenting this seemingly anomalous risk-return relationship. Coefficient for unconditional ambiguity is positive as hypothesized but is not significant on its own, in line with the literature.

Column-3 includes conditional ambiguity attitudes in the model. As specified in the equation (11), the model divides the expected probability of favorable returns into ten ranges and assign dummy variables to each range. Therefore, the model in the columns 3 and 4 include nine dummies. Five out of nine dummies are significant at different levels. Column-4 adds the estimated risk to the model in column-3. The risk coefficient is still negative and dummy coefficients remain significant. These results indicate that ambiguity is priced in the Turkish equity market. The weighted least squares (WLS) regression results on Panel B confirm the results of ordinary least squares (OLS) regressions. In the WLS regressions, the weights are inversely proportional to the sum of the estimated risk and ambiguity. For robustness check, we also divide the probability range to 5, 15, and 20 bins. The results are essentially the same.

Table 3. Main Regression Results - Discrete Ambiguity Attitudes

	Panel A. OLS				Panel B. WLS			
	(1)	(2)	(3)	(4)	(1)	(2)	(3)	(4)
$\hat{\gamma}$	-1.286			-0.779	-1.534			-0.997
	(0.456)			(0.551)	(1.077)			(1.029)
$\hat{\theta}$		0.008				0.012		
		(0.014)				(0.021)		
$\hat{\eta}$			-1.778	-1.436			-1.548	-1.120
			(0.929)	(0.992)			(1.121)	(1.315)
$\widehat{\eta_2}$			7.833	7.622			7.615	7.369
			(3.344)	(3.207)			(4.126)	(4.305)
$\widehat{\eta_3}$			4.201	3.517			2.345	1.505
			(4.912)	(5.127)			(3.325)	(3.599)
$\widehat{\eta_4}$			2.289	1.668			1.880	1.068
			(2.029)	(2.187)			(2.123)	(2.399)
$\widehat{\eta_5}$			3.573	2.933			2.994	2.221
			(2.302)	(2.250)			(2.189)	(2.479)
$\widehat{\eta_6}$			1.262	0.906			0.753	0.343
			(1.869)	(1.922)			(2.134)	(2.237)
$\widehat{\eta_7}$			4.317	3.652			3.451	2.613
			(2.155)	(2.148)			(2.038)	(2.329)
$\widehat{\eta_8}$			4.216	3.511			3.695	2.801
			(1.773)	(1.731)			(1.915)	(2.243)
$\widehat{\eta_9}$			6.439	5.852			5.724	4.974
			(1.913)	(1.873)			(1.902)	(2.115)
$\widehat{\eta_{10}}$			6.567	5.992			5.814	5.061
			(2.202)	(2.199)			(1.882)	(2.113)
$\hat{\alpha}$	0.015	-0.002	-0.003	0.004	0.017	-0.004	-0.001	0.006
	(0.006)	(0.011)	(0.014)	(0.013)	(0.009)	(0.015)	(0.018)	(0.016)
N	277	277	277	277	277	277	277	277
R^2	0.011	0.000	0.086	0.090	0.016	0.001	0.094	0.099
Adj. R^2	0.007	-0.003	0.052	0.052	0.013	-0.002	0.060	0.062

Newey-West standard errors in parentheses

Figure 2 displays the attitude of Turkish stock investors toward ambiguity conditional on the expected probability of the favorable returns. The solid black line depicts the ambiguity attitude, and the solid red line is created by a polynomial of third degree. The dotted lines depict the 5% confidence interval of ambiguity attitudes. Attitude towards ambiguity coefficients are attained by the sum of the coefficients of η and η_i as explained in the methodology section. Recalling that η_1 presents the ambiguity attitude in the range of probability in [0.43, 0.44], η_2 in the range of probability in [0.44, 0.45] and so on, Figure 2 presents the relation between the expected probabilities and probabilistically contingent coefficients of ambiguity attitudes obtained from OLS results reported in the fourth column in Panel A of Table 3. The trend of attitudes reproduced with WLS results are mostly parallel.

The results of the experiment-based behavioral studies report that people are ambiguity averse for gains and ambiguity lover for losses (Abdellaoui et al., 2005; Du & Budescu, 2005; Mangelsdorff & Weber, 1994; Viscusi & Chesson, 1999). As such, the expected attitude for our study would be negative coefficient $\eta + \eta_i$ for expected probabilities below 0.50, implying ambiguity-loving when the investors expect losses; and a positive coefficient $\eta + \eta_i$ for expected probabilities above 0.50, implying ambiguity aversion when the investors expect favorable returns. The US investors do indeed show such an attitude toward ambiguity.

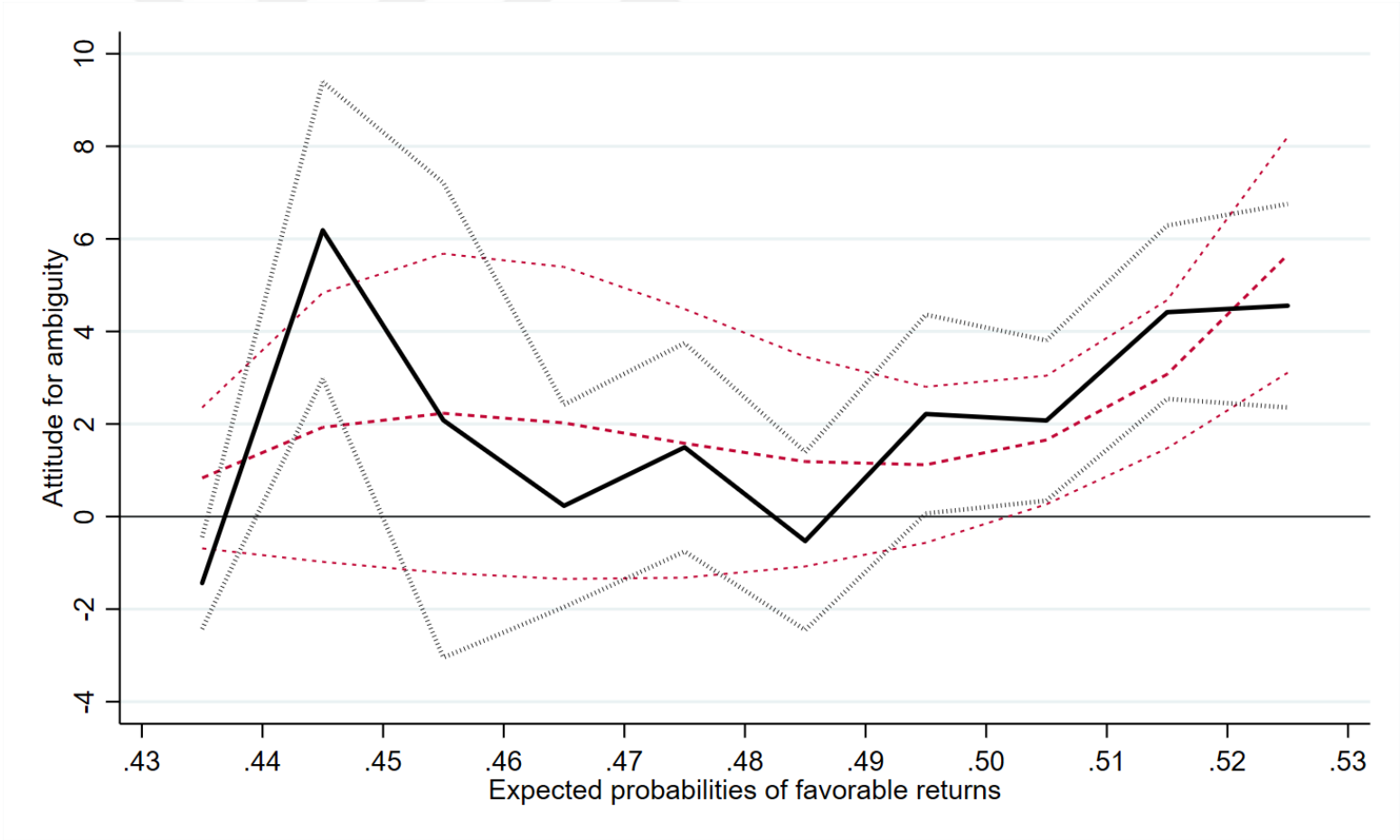


Figure 2. Coefficient of ambiguity attitudes for BIST 100

The trend in Figure 2, however, does not really support these hypotheses for Turkish investors. The coefficients are negative only when the probability of favorable of return is extremely low [0.43-0.44] and almost zero when the probability of favorable return is between [0.46-0.50], making not an S-shape as in US market but a hump-shaped attitude curve. While interpreting the behavior of ambiguity attitude coefficients in Figure 2, recall that the mean of expected probability of favorable return is 0.487. Therefore, the investors traded in BIST 100 are ambiguity neutral when the probability of favorable return is around its historical mean. As the probability of favorable return moves away from the historical mean, investors become more ambiguity averse until ambiguity aversion reaches peak in the both sides. On the upper side, the peak value is between [0.52-0.53], the highest bin. On the lower side, ambiguity aversion has the maximum value between [0.44-0.45]. The general trend of the ambiguity attitude shows that Turkish investors are ambiguity-averse when faced with potential losses and gains, and they demand a premium for ambiguity. However, they are essentially ambiguity-neutral in lateral markets when the direction of the market is not clear. The extreme case, where the expected probability of favorable returns is at its lowest, investors become ambiguity loving. It could be due a strong demand for contrarian trading, however, it needs further investigation.

Panel A and Panel B of Table 4 reports the results of the OLS and WLS estimation of continuous model presented in Equation (12) respectively. The coefficient of volatility is still negative. However, the magnitude is lower and it is not significant in WLS estimation and barely significant in OLS estimation. Similar with discrete model η is negative, η_s is positive, and $\eta + \eta_s$ is positive consistent with discrete model, showing that investors traded in BIST 100 are ambiguity averse.

Table 4. Main regression results - Continuous ambiguity attitudes

	Panel A. OLS				Panel B. WLS			
	(1)	(2)	(3)	(4)	(1)	(2)	(3)	(4)
$\hat{\gamma}$	-1.286			-0.889	-1.534			-1.118
	(0.456)			-0.462	(1.077)			(0.755)
$\hat{\theta}$		0.008				0.012		
		(0.014)				(0.021)		
$\hat{\eta}$			-10.737	-9.397			-11.197	-9.578
			(3.240)	(3.656)			(4.131)	(4.265)
$\widehat{\eta_s}$			21.594	19.097			22.372	19.358
			(6.666)	(7.374)			(8.384)	(8.609)
$\hat{\alpha}$	0.015	-0.002	0.007	0.013	0.017	-0.004	0.010	0.016
	(0.006)	(0.011)	(0.011)	(0.010)	(0.009)	-0.015	(0.015)	(0.015)
N	277	277	277	277	277	277	277	277
R^2	0.011	0.000	0.022	0.027	0.016	0.001	0.026	0.034
Adj. R^2	0.007	-0.003	0.015	0.016	0.013	-0.002	0.019	0.023

Newey-West standard errors in parentheses

The continuous model, unlike discrete model, forces ambiguity attitudes to change across probability of favorable returns in a linear way. As it is straightforward from Figure 2, the attitudes toward ambiguity is not linearly contingent upon probability of favorable returns. Hence, the explanatory power of the continuous model is lower. More importantly, because of the linearity assumption, continuous model forces the attitude coefficients to intersect the x-axis (expected probability) at only one point. η/η_s shows intersection point, i.e. expected probability of favorable return threshold that ambiguity attitude turns to aversion from loving. It is around 0.50 for the both of models with or without expected volatility and for both OLS and WLS estimations. Although this is more consistent with hypothesis and findings for the US market, the presented results of discrete model are able to tell a subtler story. Continuous regressions are still able to verify the general model, in which investors are ambiguity averse on average.

3.3.3 Higher moments

In the next two sections, we aim to examine whether our results are robust to inclusion of other factors that might be proxied by the proposed measure of ambiguity. We begin by incorporating higher moments of returns into our regression models, namely skewness, kurtosis, volatility of mean and volatility of volatility. The estimated values of each of these factors are obtained using the same ARMA methodology used on other variables. These parameters are computed from returns, as such they are related to returns and risks. Ambiguity, on the other hand, is not risk-dependent. In unreported correlation calculations, the relations are corroborated as these four have significant and high correlations with volatility, but not with ambiguity.

Table 5 reports the results of continuous regression models with expected skewness ($SKEW^E$), kurtosis ($KURT^E$), volatility of mean (VOL_M^E) and volatility of volatility (VOL_V^E). Each model introduces one of the four variables. Skewness and kurtosis have no effect on returns. Furthermore, ambiguity coefficients are unaffected by introduction of these higher moments of returns, confirming that ambiguity is indeed a distinct factor from higher moments of returns. Volatility of mean and volatility of volatility are two of the measures that are used as proxies for ambiguity. These factors are highly correlated with volatility itself. Introduction of them to the regression model impacts the risk coefficient and likely causes multicollinearity issues. However, they do not impact ambiguity coefficients and their significance. These findings attest that the ambiguity measure cannot be replicated by these popular proxies.

Table 5. Robustness Tests - Higher Moments

	Panel A. OLS				Panel B. WLS			
VARIABLES	(1)	(2)	(3)	(4)	(1)	(2)	(3)	(4)
$\hat{\gamma}$	-0.935	-0.882	-6.193	-2.472	-1.165	-1.112	-7.301	-3.028
	(0.512)	(0.459)	(2.272)	(0.964)	(0.757)	(0.756)	(1.754)	(1.190)
$\hat{\eta}$	-9.508	-9.314	-10.799	-11.293	-9.719	-9.551	-11.258	-11.309
	(3.619)	(3.655)	(3.204)	(3.244)	(4.270)	(4.273)	(4.181)	(4.321)
$\widehat{\eta_s}$	19.351	18.925	20.959	21.824	19.643	19.303	21.631	21.663
	(7.311)	(7.365)	(6.554)	(6.593)	(8.621)	(8.627)	(8.416)	(8.631)
$SKEW^E$	0.029				0.033			
	(0.033)				(0.039)			
$KURT^E$		0.004				0.003		
		(0.007)				(0.010)		
VOL_M^E			0.225				0.263	
			(0.077)				(0.068)	
VOL_V^E				6,663.012				7,790.932
				(2,758.580)				(3,771.701)
$\hat{\alpha}$	0.014	0.002	0.054	0.039	0.018	0.009	0.062	0.044
	(0.010)	(0.022)	(0.020)	(0.015)	(0.015)	(0.032)	(0.019)	(0.020)
N	277	277	277	277	277	277	277	277
R^2	0.028	0.027	0.065	0.039	0.037	0.034	0.085	0.049
Adj. R^2	0.014	0.013	0.051	0.025	0.022	0.020	0.071	0.035

Newey-West standard errors in parentheses

3.3.4 Investor sentiment and recessions

As a final robustness test, we aim to incorporate sentiment measures so as to inspect if ambiguity attitudes change accordingly. Recessions add another dimension to the robustness as well, as time of financial and economic distress are expected to impact investor attitude and behavior. We use three different measures of sentiment and create dummy variables for negative or low sentiment periods. Two of the measures are based on survey data collected by Central Bank of the Republic of Turkey. Real Sector Confidence Indicator (RCI) and Business Confident Indicator (BCI) are two headline survey-based sentiment indicators published for Turkey[3].The dummy variables are formed when they take negative values. We further add an emerging market equity investor sentiment compiled by Sentix[4]. We form dummy variables for this sentiment index based on sign, but also on the median value as the magnitude and sign are not absolute for this particular measure. Recession indicator is taken from OECD database as previously noted.

Table 6 reports the results of continuous regression model with each behavioral factor and recession variable are added one by one along with their ambiguity interaction factors. Business Confidence Indicator dummy has a weak significance at 10% level with a negative sign. Investor sentiment dummy based on median value also has a significance at 5% level with a negative sign, which is lost in WLS regression. Sentiment interactions with ambiguity are not significant in any of the regression models. However, it should be noted that investor sentiment does cause ambiguity coefficient to become larger in magnitude while making risk coefficient lose its significance. We conjecture that this is due to elevated risk levels

[3] 3Monthly data is retrieved from Central Bank of the Republic of Turkey Electronic Data Delivery System; https://evds2.tcmb.gov.tr.
[4] Monthly data is retrieved from Refinitiv Datastream starting on April 2007.

in periods of negative sentiment. The recession indicator has a negative and significant coefficient. It, too, changes risk coefficient to weaker significance. The tests show us that ambiguity is not necessarily related to sentiment or behavior change in recessions.

Table 6. Robustness Tests - Negative Sentiment and Crises

	Panel A. OLS					Panel B. WLS				
	(1)	(2)	(3)	(4)	(5)	(1)	(2)	(3)	(4)	(5)
$\hat{\gamma}$	-1.013	-1.146	-0.666	-0.475	-0.752	-1.324	-1.423	-0.842	-0.870	-1.037
	(0.586)	(0.452)	(2.123)	(2.528)	(0.408)	(0.883)	(0.786)	(2.212)	(2.133)	(0.831)
$\hat{\eta}$	-9.247	-8.719	-16.405	-15.760	-8.121	-9.751	-9.332	-15.683	-15.576	-8.668
	(3.980)	(3.609)	(4.446)	(4.170)	(3.734)	(4.331)	(4.296)	(3.982)	(3.976)	(4.434)
$\widehat{\eta_s}$	19.237	16.812	32.262	30.560	15.927	19.520	17.616	31.154	30.383	16.590
	(7.574)	(7.469)	(8.714)	(8.373)	(7.295)	(8.645)	(8.603)	(7.782)	(7.760)	(8.728)
$SENT^{-}_{RCI}$	0.020					0.003				
	(0.028)					(0.031)				
$SENT^{-}_{RCI}\ x\ (\mho^2)^E x\ \vartheta^E$	-0.355					0.170				
	(0.837)					(0.968)				
$SENT^{-}_{BCI}$		-0.059					-0.060			
		(0.035)					(0.032)			
$SENT^{-}_{BCI}\ x\ (\mho^2)^E x\ \vartheta^E$		1.070					1.243			
		(1.172)					(1.009)			
$SENT^{-}_{EM}$			-0.020					-0.001		
			(0.030)					(0.043)		
$SENT^{-}_{EM}\ x\ (\mho^2)^E x\ \vartheta^E$			-0.048					-0.598		
			(0.855)					(1.357)		
$SENT^{M}_{EM}$				-0.061					-0.046	
				(0.031)					(0.036)	
$SENT^{M}_{EM}\ x\ (\mho^2)^E x\ \vartheta^E$				1.089					0.715	
				(0.939)					(1.107)	
REC					-0.059					-0.068
					(0.027)					(0.031)
$REC\ x\ (\mho^2)^E x\ \vartheta^E$					0.711					0.985
					(0.972)					(0.990)
$\hat{\alpha}$	0.006	0.039	0.029	0.042	0.032	0.018	0.046	0.025	0.041	0.041
	(0.017)	(0.026)	(0.014)	(0.016)	(0.022)	(0.024)	(0.025)	(0.024)	(0.025)	(0.023)
N	277	277	166	166	277	277	277	166	166	277
R^2	0.029	0.045	0.112	0.135	0.052	0.035	0.051	0.121	0.134	0.064
Adj. R^2	0.011	0.028	0.085	0.108	0.034	0.017	0.033	0.094	0.107	0.047

Newey-West standard errors in parentheses

3.4 Conclusion

Classical finance theories usually examine the relationship between return and risk assuming that investors know precise probability distributions in each state and maximize their expected utility accordingly. However, since precise probability distributions are impossible to know for the market participants, majority of the investors are exposed to ambiguity in addition to risk. Most of the studies on ambiguity are theoretical and the empirical studies that offer a proxy for the ambiguity is very limited. In this study, inspired by a recently developed method, we aim to empirically measure the ambiguity in Turkish market, one of the major emerging markets. The methodology that we have followed offer an ambiguity proxy that is independent of risk and calculated based on the local market data denominated in local currency. The study at hand joins among the limited literature in empirical studies on ambiguity, particularly for emerging markets.

In our study, we analyze the excess equity returns under ambiguity in Turkish market by using the high frequency data on headline index BIST 100 as the market return. Our results show that ambiguity is priced in the stock market with a positive premium. Ambiguity attitude in Turkey, however, differs from US based studies in equity market. Turkish investors are ambiguity averse under positive and negative market expectations, but they remain ambiguity neutral for lateral markets, where direction is not clear. The results are robust to higher moments of returns, essentially confirming that ambiguity is a distinct factor and cannot be proxied by such measures. Incorporating behavioral factors such as sentiment and controlling for extreme market conditions under recessions do not change our results, once again confirming ambiguity's distinct nature from other behavioral factors. This study sheds further light on investors' attitudes towards risk and ambiguity, and how

national and cultural factors do yield differences across markets. The determinants of those differences remain to be explored in further studies.

CHAPTER 4

MARKET AMBIGUITY EXPOSURE AND CROSS-SECTION OF RETURNS

4.1 Introduction

Unlike assumed by SEU theory, investors are not certain about the probabilities of the future returns. Rather, they are exposed to ambiguity which is distinct from risk and refers to the situations in which some of the relevant events do not have unique and obvious probability assignments (Keynes, 1937; Knight, 1921). Combined with ambiguity-averse nature of the most of the decision-makers indicated by Ellsberg (1961), various theoretical studies offer asset pricing models which predict that investors demand a premium for bearing the ambiguity, and thus, there exists an ambiguity premium distinct from risk premium in asset returns (Epstein & Schneider, 2010; Izhakian, 2020; Ju & Miao, 2012; Kogan & Wang, 2003; Mukerji et al., 2022; Ui, 2011).

Empirical evidence on the relationship between ambiguity and the cross-section of expected returns, however, is very scant and mixed to arrive a consensus. Besides, the role of market-driven ambiguity on expected return is neglected by the literature. Current studies mostly explore the cross-sectional relationship between stock-based ambiguity and stock returns by quantifying the ambiguity with disagreement in future performance of the relevant stock (Anderson, Ghysels, & Juergens, 2005; Anderson et al., 2009; Diether et al., 2002; Hsiao, 2022; Jiang & Sun, 2014). However, recently Kim and Zapatero (2022) imply that disagreement may not be a good proxy for measuring ambiguity because forecasters have some strategic incentives to misreport their beliefs. Some studies report exposure to aggregate ambiguity explains the cross-sectional variation in the stock returns (Bali

et al., 2017; Lee et al., 2019). However, aggregate ambiguity is mainly quantified by conditional volatility or dispersion in forecast of macroeconomic variables. This results in either ignoring the ambiguity hidden at the higher moments or ignoring ambiguity driven by market itself.

In this study, we aim to test whether the market ambiguity is priced in the cross-section of returns. We quantify the market ambiguity by the monthly standard deviation of daily market return distribution following Brenner and Izhakian (2018) who show that market ambiguity and attitude on market ambiguity contingent on favorable return can explain the variation in the market return. This study shows that market ambiguity, denoted by $\mho$, is also priced in the cross-section of returns. We find that expected return increases as absolute exposure to market ambiguity increases, being absolute effect of positive ambiguity exposure is larger compared to absolute effect of negative ambiguity exposure. In other words, unlike economic ambiguity and disagreement, market ambiguity is non-linearly related with expected return and exhibits a left-skewed U-shaped pattern. U-shape pattern in more obvious for the smaller stocks

Our result is robust to cross-sectional return predictors (size, value, momentum, reversal, market beta, idiosyncratic volatility, synchronicity, market liquidity exposure, analyst coverage etc.), other ambiguity proxies (exposure to dispersion in forecast of GDP growth, exposure to uncertainty index, exposure to VIX, disagreement among analyst recommendation), different holding or formation periods, and different states (high and low ambiguity periods, recessions, and sentiment). Among different robustness test, only momentum and idiosyncratic volatility are able to reduce, but fail to fully explain, the U-shaped pattern representation. Moreover, our result shows that exposure to market ambiguity

exhibits a certain degree of time-varying behavior with respect to sentiment, as the U-shaped pattern becomes partly asymmetrical during the high and low sensitivity period. Finally, estimated ambiguity betas for test portfolios shows that high negative beta stocks are more likely to be long-term losers, low-priced stocks with more desperate fundamental and intangible growth. Besides, they tend to have higher distance from efficient market frontier, higher return persistency, higher revenue growth compared to inventory growth, higher cash-flow volatility, and higher advertising expense.

4.2 Literature review

The effect of ambiguity on returns has been addressed in some studies using various variables that can be measures of ambiguity. For example, a number of studies explore the relationship between expected stock returns and disagreement with or without explicit links to ambiguity. However, the evidence is mixed. Anderson Ghysels, and Juergens (2005) measure the disagreement by dispersion in earning forecast and report that a factor that shorts stocks with low dispersion and long stocks with high dispersion has a positive premium that is beyond market premium as well as SMB, HML, and momentum factors. Similarly, Qu, Starks, and Yan (2003) find that a factor for disagreement, calculated from the annual volatility of a firm's earnings dispersion, is priced in the asset returns with a positive premium. Focusing on aggregate measures of disagreement rather than disagreement about individual stocks Anderson et. al. (2009) measure ambiguity by dispersion of predictions of mean market return forecasts constructed from the forecasts of aggregate corporate profits and report that ambiguity exposure is positively priced in the cross-section of returns.

Some recent studies using different measure for quantifying disagreement also find that stocks with higher disagreement deliver higher returns. For example, Jiang and Sun (2014) find that if there is a high dispersion in funds managers' belief about a stocks' future returns (or high ambiguity in a sense), this stock brings higher return than its otherwise equal counterpart. They argue that "differentially informed managers" hypothesis can explain their empirical results. Basically, if some fund managers retrieve positive private information about a stock, they place a large bet on that stock which increases the dispersion in funds managers. However, short sale constraints prevent them from exploiting negative private info fully. Therefore, high dispersion is related high future returns on average. By arguing that high investor disagreement results in trading in opposite directions and eventually lead to only a small price change, Hsiao (2022) quantifies the investor disagreement with the contemporaneous correlation coefficient between trading volume and absolute price change scaled by minus 1 and reports that there is a positive relationship between investor disagreement and the expected returns. Based on the interpretation of the extended version of market-trading model with disagreement by Kandel and Pearson (1995), Hsiao (2022) argues pessimistic valuation is reflected more in price of stocks with higher disagreement (higher expected returns) because investors interpret other type's interpretation asymmetrically due to ambiguity aversion. More specifically, while optimistic investors attach relatively greater weight to pessimistic valuation, pessimistic investors attach lower weights to optimistic view.

On the other hand, Diether et. al. (2002) report that stocks with higher dispersion in analyst' earning forecast deliver lower expected returns, being even more pronounced for smaller and low momentum stocks. Their result is counterintuitive because investors seem to pay a premium for ambiguity rather than

demanding a discount for it. As an explanation, Diether et. al. (2002) state when there is dispersion in beliefs, the optimists can reflect their beliefs in prices; however, short sale constraints prevent pessimists from fully reflecting their beliefs in the price. As a result, higher dispersion leads to higher prices and lower expected returns. Arguing that dispersion in forecast can be regarded as a proxy for unpriced information risk arising in cases when asset value is unobservable, Johnson (2004) theoretically and empirically show that negative relationship reported by Diether et. al. (2002) may be explained by general options-pricing framework. More specifically, Johnson (2004) shows that since the equity of a levered firm is analogous to a call option with a strike price equals to outstanding leverage value, an increase in idiosyncratic asset risk should always increases the price and decreases the return for a levered firm.

However, recently, Kim and Zapatero (2022) argue that disagreement may not be a sound proxy for measuring the ambiguity because forecasters have some strategic incentives for misreporting their beliefs, resulting in understatement or overstatement behaviors. As stressed by many studies, understatement is expressed as form of herding behavior mainly to avoid unfavorable reputation when wrong while overstatement is observed when the forecasters would like to distinguish themselves from her peers because they believe they have a forecasting advantage (Aharoni, Einhorn, & Zeng, 2017; Bernhardt, Campello, & Kutsoati, 2006; Beyer & Guttman, 2011; Chen & Jiang, 2006; Croushore, 1997; Graham, 1999; Hong, Kubik, & Solomon, 2000; Laster, Bennett, & Geoum, 1999; Ottaviani & Sørensen, 2006; Pfeifer, Grushka-Cockayne, & Lichtendahl, 2014; Trueman, 1994; Williams, 2013). Kim and Zapatero (2022) show that dispersion is low for stocks with high volatility because analysts have greater herding incentives for those stocks. Therefore,

quantifying uncertainty with the dispersion in beliefs may give counterintuitive results.

Exposure to aggregate ambiguity and the expected returns generally argued to be reversely related, meaning that average future return on stocks with low-ambiguity exposure is higher than average future return on stocks with high-ambiguity exposure. For example, Ju and Miao (2012) develop a consumption-based asset pricing model allowing for ambiguity and ambiguity aversion. This model implies that investors demand a compensation for holding stocks with low ambiguity beta (i.e. stock bringing lower return when ambiguity is high). On the other hand, high ambiguity beta stocks serve for a good hedging and have lower rate of expected returns. Lee et. al. (2019) provide empirical evidence supporting that predictions of the model. More specifically, by measuring the economic ambiguity as the dispersion in real-time forecasts of real GDP growth, Lee et. al. (2019) report that stock with lower exposure to economic ambiguity deliver higher expected return compared to their otherwise equal counterparts.

A similar finding is also provided by study of Bali et. al (2017) which measures economic ambiguity by the conditional volatility of the unforecastable component of a large number of economic indicators supplied by Jurado, Ludvigson, and Ng (2015). Bali et. al (2017) argue that economically significant higher expected return on low ambiguity beta stock is consistent with ICAPM framework of Merton (1973). Because increase in economic ambiguity can be regarded as an unfavorable shift in the investment opportunity set, investors would like to hedge for this unfavorable shift by investing in stock having positive correlation with economic ambiguity. One other explanation offered by Bali et. al (2017) is limited participation caused by ambiguity. When ambiguity is high enough, ambiguity-averse investors

may make limited investment in stocks subject to high ambiguity or even stop investing in them altogether. Therefore, stocks with high ambiguity beta are held by only those who are sufficiently optimistic, making them requiring lower rate of returns.

Previous empirical evidence on cross-sectional effect of aggregate ambiguity primarily measures the effect of ambiguity driven by macroeconomic uncertainty given that ambiguity is quantified by computations of macroeconomic variables. The role of the market-driven ambiguity is neglected in the literature. There are some studies using VIX or volatility of VIX as a proxy to market ambiguity (Bekaert et al., 2013; Bloom, 2009), however, the first or second moment of the volatility of returns may not fully reflect all properties of Knightian uncertainty. Andreou, et. al. (2014) derive stock market ambiguity by the dispersion in the volume-weighted strike prices of the S&P 500 Index options which argued to reflect the dispersion in the beliefs about future prices. Another market-based ambiguity measure is option implied ambiguity provided by So et. al (2016). Recently, a proxy for ambiguity based on stock market data is provided by Thimme and Volkert (2015) who test the smooth ambiguity model by introducing a novel methodology to estimate the conditional expectation and report that ambiguity aversion is priced in the cross-section of returns.

A recent, simple, intuitive, and risk-independent proxy for market ambiguity is presented by Brenner and Izhakian (2018) as the variance of the probability distribution of the returns. Although the measure (denoted by $\mho$) is tested in other contexts such as credit risk (Augustin & Izhakian, 2020) or capital structure (Izhakian, Yermack, & Zender, 2021), its cross-sectional effect on expected stock

return is not yet inspected. Hence, this study addresses a major yet neglected point, and examine the role of $\mho$ on cross-sectional returns.

4.3 Data and variables

Our stock sample consists of common stocks traded on the NYSE, Amex, and Nasdaq exchanges from February 1993 through December 2020. We eliminate stocks with a price per share less than $5 or more than $1000. We retrieve the monthly return and volume data from the CRSP and relevant accounting variables from CRSP-Compustat database. Analyst coverage and recommendations come from Institutional Brokers' Estimate System (I/B/E/S). The VIX index is retrieved from Refinitive terminals.

In our study, we set a fixed rolling window of a 60-month for all of the rolling regressions and we require minimum of 24 monthly observations for estimating relevant coefficients. The excess return on market (MKT), the risk free rate (r_f), the factors small-minus-big (SMB), high- minus-low (HML), and winner-minus-losers (WML) are from data library of Kenneth French. The liquidity factor (LIQ) is from data library of Professor Lubos Pastor[5]. While testing whether ambiguity beta is priced in cross-section of returns, we use a set of control variables that have long been used for predicting cross-section of stock returns. Besides, we also control for some other ambiguity proxies in order to explore whether market ambiguity is distinct from them. The rest of this section will give the details on computation of ambiguity beta as well as of other control variables. Appendix A (Table A1, Table A2) provides a summary table for the all of the variables used in the analysis in an alphabetic order.

[5] https://faculty.chicagobooth.edu/lubos-pastor/data

4.3.1 Definition of market ambiguity and other ambiguity proxies

We calculate market ambiguity for US market following Brenner and Izhakian (2018) as specified in equation (6)[6]. For each month, we then calculate ambiguity exposure for each stock ($\widehat{\beta_{i,t}^{\mho}}$) from the monthly rolling regressions of excess stock returns ($r_{i,t} - r_f$) on the ambiguity ($\mho$) after controlling for market (MKT):

$$r_{i,t} - r_f = \alpha_{i,t} + \beta_{i,t}^{\mho} . \mho_t + \beta_{i,t} . MKT_t + \varepsilon_{i,t} \quad (13)$$

For examining whether exposure to market ambiguity is distinct from the exposure to other ambiguity proxies, we calculate the betas for dispersion in beliefs in GDP growth (Lee et al., 2019), economic uncertainty index (Bali et al., 2017), VIX (Bekaert et al., 2013; Bloom, 2009), and dispersion in recommendation in analyst coverage. For calculating GDP based ambiguity, we follow Lee et. al. (2019) and retrieve forecasts of economists on real GDP from the Survey of Professional Forecasters (SPF) conducted quarterly by the Federal Reserve Bank of Philadelphia since 1969. Forecasted real GDP growth from quarter t to quarter t+1 is calculated as shown in equation (14) where $RGDP_{t,t+1}^{i}$ is the forecast made by economist i at quarter t for the level of real GDP at quarter t + 1.

$$g_{t,t+1}^{i} = log \frac{(RGDP_{t,t+1}^{i})}{(RGDP_{t,t}^{i})} \quad (14)$$

We then compute the GDP based ambiguity as the cross-sectional standard deviation of the forecasted GDP growth, denoted as GDP_{AMB}. Since we work with monthly data, we convert quarterly GDP_{AMB} by cubic spline interpolation method which

[6]As a reminder, the ambiguity specified in equation (6) is as follows: $\mho^2[r] = \frac{1}{w(1-w)} + (E[\phi(r_0; \mu, \sigma]Var[\phi(r_0; \mu, \sigma] + \sum_{i=1}^{60} E[\phi(r_i; \mu, \sigma) - \phi(r_{i-1}; \mu, \sigma)]Var\,[\phi(r_i; \mu, \sigma) - \phi(r_{i-1}; \mu, \sigma)] + E[1 - \phi(r_{60}; \mu, \sigma)]Var[1 - \phi(r_{60}; \mu, \sigma)])$. However, tracking the small changes in the calculation of ambiguity, in this chapter we use $\sqrt{\frac{1}{w(1-w)}}$ as correction. (Izhakian, 2020)

connects two points by drawing a unique third degree polynomial. As GDP_{AMB} cannot be lower than zero by definition, we interpolate on logarithm of GDP_{AMB} to avoid negative values, and then reverse the transformation. Finally, monthly exposure to GDP_{AMB} for each stock ($\widehat{\beta_{i,t}^{GDP}}$) is computed from following monthly rolling regressions.

$$r_{i,t} - r_f = \alpha_{i,t} + \beta_{i,t}^{GDP} . GDP_{AMB\,t} + \beta_{i,t} . MKT_t + \varepsilon_{i,t} \quad (15)$$

As for economic uncertainty, following Bali et. al. (2017), we use economic uncertainty index of Jurado, Ludvigson, and Ng (2015), defined as the conditional volatility of the unforecastable component of a large number of economic indicators and denoted as UNC[7]. Monthly stock exposure to economic uncertainty is defined as ($\widehat{\beta_{i,t}^{UNC}}$) which is estimated from the following rolling regression equation:

$$r_{i,t} - r_f = \alpha_{i,t} + \beta_{i,t}^{UNC} . UNC_t + \beta_{i,t} . MKT_t + \varepsilon_{i,t} \quad (16)$$

Similarly, monthly stock exposure to VIX ($\widehat{\beta_{i,t}^{VIX}}$), retrieved from Refinitiv terminals, is computed from the following regression:

$$r_{i,t} - r_f = \alpha_{i,t} + \beta_{i,t}^{VIX} . VIX_t + \beta_{i,t} . MKT_t + \varepsilon_{i,t} \quad (17)$$

Finally, we use monthly cross-sectional standard deviation in the analyst recommendation to measure the dispersion in the analysts' recommendation, which is directly retrieved from I/B/E/S and denoted as STDEV. More specifically, I/B/E/S standardizes recommendations as 1 (strong buy), 2 (buy), 3 (hold), 4 (sell) and 5 (strong sell) and provides monthly cross-sectional standard deviation of five-tier recommendation scale for each stock.

[7] We retrieve the index from Professor Sydney Ludvigson's website: www.sydneyludvigson.com.

4.3.2 Definition of cross-sectional return predictors

We use a set of stock level cross-sectional return predictors as control variables to investigate whether role of ambiguity on expected return is subsumed under these predictor variables.

Size (SIZE) is known to have a considerable return predictive power since Banz (1981) who reports that smaller stocks outperform larger stocks. Following Fama and French (1992), we define size as market capitalization which is the product of price per share and shares outstanding. Similar with Fama and French (1992), if size is used as control variable in the regression equations, then we use natural logarithm of market capitalization, denoted as ln(SIZE), in order to rescale it.

Value (BM) has been long reported to be positively related with expected returns (Rosenberg, Reid, & Lanstein, 1985; Stattman, 1980). Following Fama and French (Fama & French, 1992, 1993, 2000), we calculate book-to-market ratio as the book value of stockholder equity plus deferred taxes and investment tax credit minus the book value of preferred stock at the end of the last fiscal year $t-1$, divided by the market value of equity at the end of December of year $t-1$. We use natural logarithm of the book-to-market ratio ln(BM) when we control for value variable in the regressions.

Jegadeesh and Titman (1993) find that there is a stock return continuum in one year called as momentum effect, meaning that past winner continues to be winner and vice versa. On the other hand, Jegadeesh (1990) observes a significant negative first-order serial correlation in monthly stock returns. Therefore, we include momentum (MOM) and short-term reversal (REV), which are defined as the stock return in the past year skipping the most recent month (11 months total) and stock return over prior month respectively.

One of the fundamental return determinants is the stock market beta, which is the source of the systematic risk and have a positive relationship with return according to classical CAPM (Lintner, 1965; Sharpe, 1964). We calculate the market beta for stock i in month by estimating a rolling CAPM: [8]

$$r_{i,t} - r_f = \alpha_{i,t} + \beta_{i,t}^{MKT} . MKT_t + \varepsilon_{i,t} \quad (18)$$

Harvey and Sıddıque (2000) find that since under-diversified investors prefer right-skewed return distribution, stocks with positive co-skewness have lower required rate of return. Following Harvey and Sıddıque (2000) and Mitton and Vorkink (2007), we obtain co-skewness (COSKEW) of stock i in month t as estimated slope of the market excess return square ($\widehat{\gamma_{i,t}}$) in the following rolling regression:

$$r_{i,t} - r_f = \alpha_{i,t} + \beta_{i,t} . MKT_t + \gamma_{i,t} . MKT_t^2 + \varepsilon_{i,t} \quad (19)$$

Pastor and Stambaugh (2003) demonstrate that stocks with high liquidity betas have high average returns. Ang, Hodrick, Xing, and Zhang (2006) find that stocks with high idiosyncratic volatility deliver abysmally low returns. Many studies stress that stock price synchronicity - refers to stock co-movement with aggregate factors - has a considerable role on stock selection and diversification (Brockman, Liebenberg, & Schutte, 2010; Durnev, Morck, Yeung, & Zarowin, 2003; Engle, 2002; Morck, Yeung, & Yu, 2000; Roll, 1988). For each month and each stock, we compute the liquidity beta (β^{LIQ}), idiosyncratic volatility (IVOL), and synchronicity (SYNC) from the rolling regression specified in equitation (20). Liquidity beta is estimated slope of the liquidity factor $\left(\widehat{\beta_{i,t}^{LIQ}}\right)$, idiosyncratic volatility is standard

[8] In order to account for non-synchronous trading effects, we also compute market beta (β^{MKT}) by regressing the monthly excess stock return on monthly market premium and lagged market premium on a rolling basis as shown. The $\beta_{i,t}^{MKT}$ is then calculated as the sum of estimated coefficients of market premium and its lag. Unreported results show that results are essentially same when accounting for non-synchronous trading.

deviation of residuals, and synchronicity is R-square square of the regression specified in equation (20).

$$r_{i,t} - r_f = \alpha_{i,t} + \beta_{i,t}^{1} . MKT_t + \beta_{i,t}^{2} . SMB_t + \beta_{i,t}^{3} . HML_t + \beta_{i,t}^{4} . WML_t + \beta_{i,t}^{LIQ} . LIQ_t + \varepsilon_{i,t} \quad (20)$$

We also control for analyst coverage (ANALYST) as stocks with lower analyst coverage have higher required rate of return (Arbel & Strebel, 1982, 1983; Elfakhani & Zaher, 1998; Lee, Sharma, & Cai, 2011). If analyst coverage is used as control variable in the regression equations, then we use natural logarithm of it, denoted as ln(ANALYST) to rescale it. Finally, we also check whether ambiguity of market beta has a role on the relationship between ambiguity beta and cross-section of returns. We measure ambiguity of market beta (SE) of stock i in month t as absolute value of the standard error of market beta scaled by effect size (coefficient) of market beta for the relevant stock and month.

4.4 Results

4.4.1 Characteristics of market ambiguity

Panel A of Table 7 presents the monthly descriptive statistics of market ambiguity (ʊ) as well as the other ambiguity measures used by previous studies. From February 1993 through December 2020, market ambiguity is between 0.079 and 0.564 with a mean of 0.198 and standard deviation of 0.091. The means for other ambiguity measures are 0.002 (GDP_{AMB}), 0.645 (UNC), 19.667 (VIX). As their ranges are very different, it is more convenient to compare their standardized statistics such as skewness, kurtosis, and coefficient of variation (standard deviation over mean). All of the ambiguity measures have a positive skewness. This makes sense because, by definition, minimum possible ambiguity level is zero while maximum possible

ambiguity level is limitless for all four ambiguity measures. However, both skewness and kurtosis are lowest for ʊ and highest for GDP_{AMB}, which means that market ambiguity has comparably thinner tails and lower asymmetry towards to the larger ambiguity levels. Market ambiguity has second highest coefficient of variation after GDP_{AMB}. This may imply that ʊ dispersed enough to catch change in the ambiguity level though different months.

Panel B of Table 7 shows Pearson correlation among the ambiguity measures, recession dummies, and sentiment indices. We got the recession series from federal reserve bank of St. Louis, which provides an interpreted recession series using the data provided by OECD and NBER.[9] Recession periods differs in such a way that NBER use a set of indicators[10] while OECD's main reference variable is GDP while deciding the turning points. We obtain sentiment from data library of Professor Jeffrey Wurgler's website (available until at the end of 2018).[11] We use two sentiment indices provided, SENT^ and SENT, former being the orthogonalized with respect to a set of six macroeconomic indicators. Interestingly, market ambiguity is negatively correlated with all other ambiguity proxies, sentiment indices, recession dummies, correlation coefficient is being insignificant for UNC and OECD recession dummy though. Larger absolute coefficient between ʊ and VIX implies a stronger contemporaneous association with VIX. The other three proxies are positively and

[9] OECD based Recession Indicators for the United States from the Period following the Peak through the Trough (USAREC), https://fred.stlouisfed.org/series/ USAREC

NBER based Recession Indicators for the United States from the Period following the Peak through the Trough (USREC) retrieved from FRED, Federal Reserve Bank of St. Louis; https://fred.stlouisfed.org/series/ USREC

[10] Some of the main indicators used by NBER are real personal income less transfers (PILT), nonfarm payroll employment, real personal consumption expenditures, wholesale-retail sales adjusted for price changes, employment as measured by the household survey, and industrial production.

[11] https://pages.stern.nyu.edu/~jwurgler/

significantly correlated with each other; however, UNC has a comparably higher association with both VIX and GDP_{AMB} (coefficient is more than 0.50 in both cases).

Table 7. Summary Statistics of Ambiguity Measures

Panel A. Monthly Descriptive Statistics

Variable	Observation	Mean	Median	Min	Max	Std. Dev.	Skewness	Kurtosis	Coefficient of variation
ʊ	335	0.198	0.179	0.079	0.564	0.091	0.968	3.808	0.459
GDP_{AMB}	335	0.002	0.002	0.001	0.031	0.003	8.042	77.205	1.072
UNC	335	0.645	0.612	0.530	1.177	0.117	2.450	9.528	0.181
VIX	335	19.667	17.750	9.510	59.890	7.938	1.691	7.083	0.404

Panel B. Correlation

	ʊ	GDP_{AMB}	UNC	VIX	SENT^	SENT	NBER	OECD
ʊ	1							
GDP_{AMB}	-0.130	1						
	(0.018)							
UNC	-0.040	0.533	1					
	(0.468)	(0.000)						
VIX	-0.399	0.230	0.597	1				
	(0.000)	(0.000)	(0.000)					
SENT^	-0.272	-0.075	-0.118	-0.063	1			
	(0.000)	(0.190)	(0.037)	(0.268)				
SENT	-0.338	-0.006	-0.071	0.028	0.973	1		
	(0.000)	(0.919)	(0.210)	(0.627)	(0.000)			
NBER	-0.153	0.081	0.588	0.471	0.113	0.197	1	
	(0.005)	(0.140)	(0.000)	(0.000)	(0.046)	(0.001)		
OECD	-0.037	0.233	0.309	0.235	0.128	0.159	0.374	1
	(0.501)	(0.000)	(0.000)	(0.000)	(0.024)	(0.005)	(0.000)	

p values in parentheses

Figure 3 depicts the market ambiguity (ʊ), GDP_{AMB}, UNC, and VIX. The shaded areas indicate the (National Bureau of Economic Research) NBER recessionary periods. Market ambiguity begins to rise just before the recession periods, reflecting the uncertainty arising from the recession in advance. High market ambiguity before recessions may be due to some early signs of worsening conditions whose scope and depth are not yet clear. Once the recession period actually starts, ʊ decreases sharply presumably because now the market direction is obvious, i.e. market is less ambiguous. On the other hand, the sharpest recession-related increase in ʊ is observed in October 2008 which coincides with the middle of the recession caused by global financial crisis. The huge crash in the markets in October 2008 could increase market ambiguity as the financial crisis proved to be considerably more serious than expected. Apart from recessionary periods, ʊ is also able to reflect the perception in the financial markets on important political and economic events as Brenner and Izhakian (2018) argues. For instance, ʊ is high during European credit crisis (2010m12), the UK Brexit decision (2016m7), and the US elections (2016m11).

The association between recession periods and the other ambiguity measures seems more contemporaneous, being especially obvious in UNC and VIX series. As for GDP_{AMB}, the peak is during 2020m7, which can be seen a post-recessionary increase considering it is three months later than the recession period. This point also stands as a outlier in the GDP_{AMB} trend and helps explaining its large skewness and kurtosis.

Figure 4 plots cross correlation of market ambiguity with GDP_{AMB} (Panel A), UNC (Panel B), VIX (Panel C) in order to explore the any possible lead and lag association. Panel D depicts the correlation coefficients across lags; blue dotted line,

red dash line, and green solid line show the correlation coefficient for GDP_{AMB}, UNC, VIX respectively. The negative and low correlation between ʊ and GDP_{AMB} persists at up to minus and plus five lags, being larger for the positive lags. The coefficient becomes almost zero for the larger negative lags and positive for the larger positive lags. This implies that higher GDP_{AMB} leads to lower ambiguity in the short run and higher market ambiguity in the long run. There is almost zero correlation between UNC and market ambiguity at lag zero and preceding and following lags. At the larger plus and minus lags, however, there is a stronger positive correlation. The strongest correlation between ʊ and VIX occurs at around lag zero and persistent up to 30 lags. High negative contemporaneous shows that both ʊ and VIX respond to some other exogenous factors simultaneously but in a different direction. The persistency of negative correlation at both positive and negative lags implies that higher (lower) than average market ambiguity leads to lower (higher) than average VIX level and vice versa.

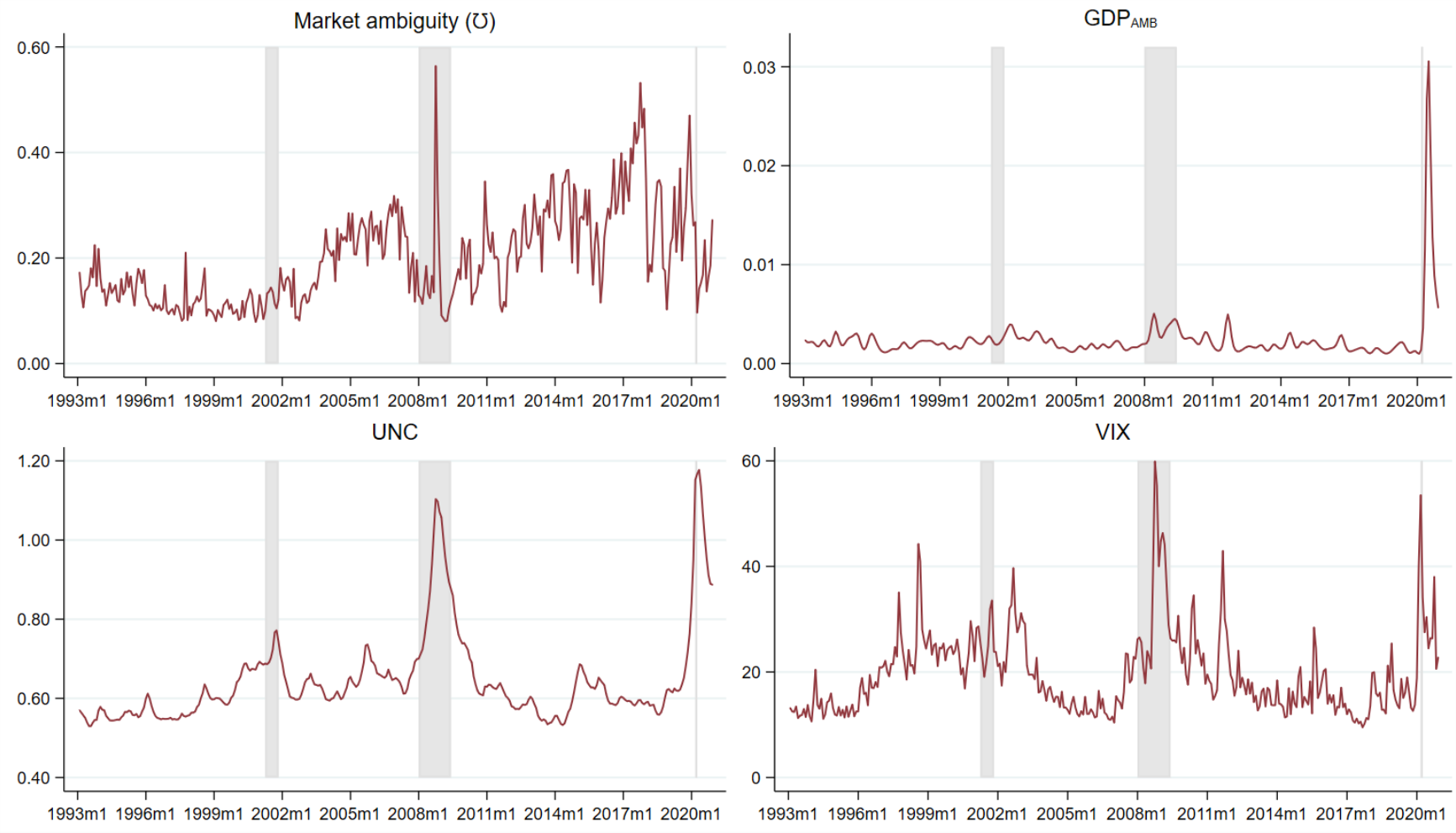


Figure 3. Market ambiguity (ʊ), GDP_{AMB}, UNC, and VIX

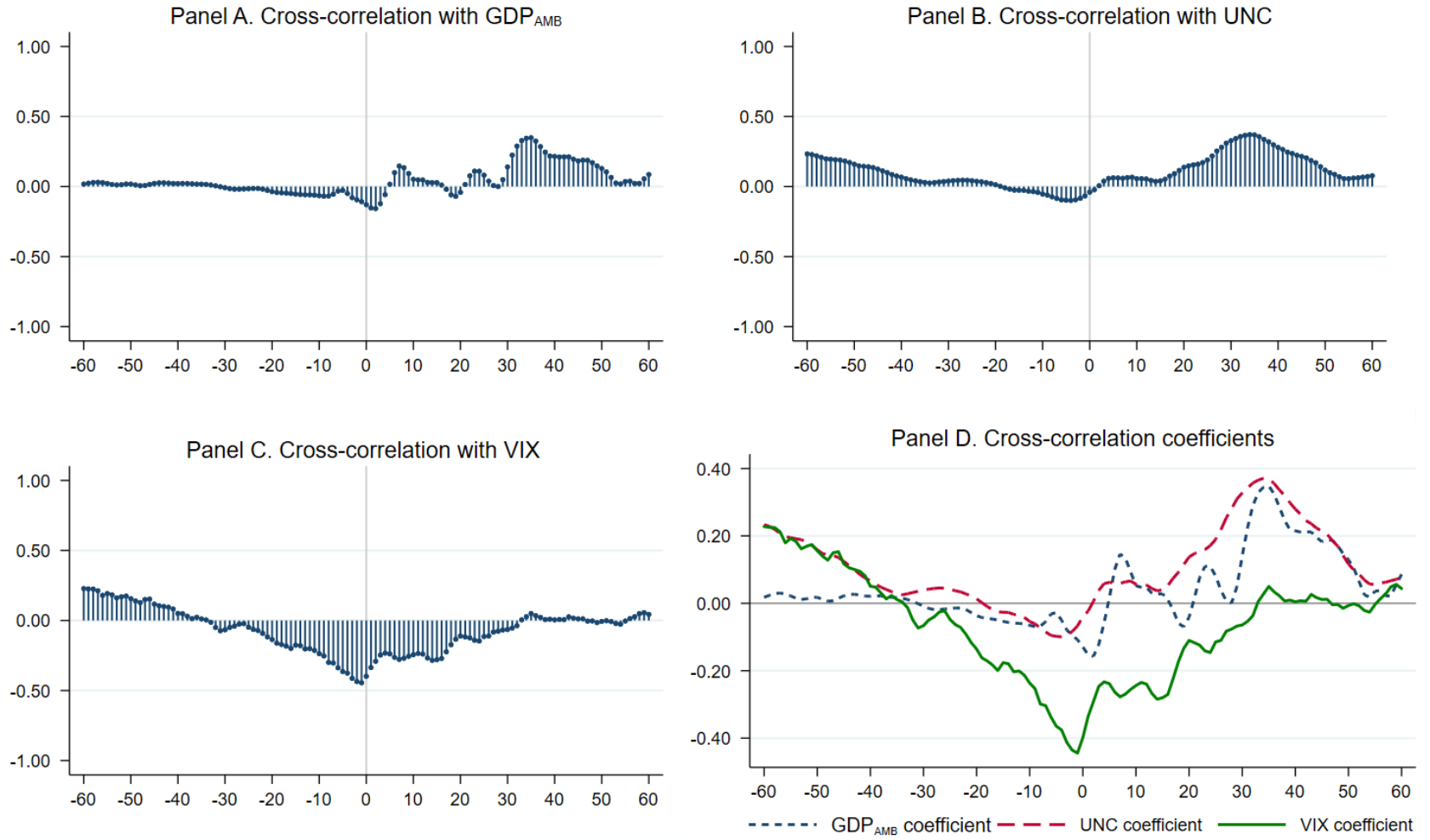


Figure 4. Cross correlation

4.4.2 Univariate portfolio-level analysis

In each month t, we sort stock into quintiles based on their ambiguity beta ($\beta^{\mho}$) in previous month. Decile 1 consists of stocks with the lowest $\beta^{\mho}$ during the past month, and decile 10 consists of stocks with the highest $\beta^{\mho}$ during the previous month. Table 8 shows the descriptive statistics of each portfolio. Panel A indicates that the ambiguity beta is ranges from -102.34 to 95.84, meaning that there is a significant cross-sectional variation in average of $\beta^{\mho}$. The percentage of significant $\beta^{\mho}$ is highest for decile 1 and decile 10. All stocks included in the first (last) three deciles have negative (positive) ambiguity beta. Stocks are evenly distributed for each portfolio.

Panel B of Table 8 shows average stock characteristics. There is no clear trend and dramatic difference across portfolios in terms of SIZE, SYNC, and ANALYST. There is a gradual increase in BM value from lowest ambiguity beta portfolio to highest ambiguity beta portfolio, except the decile 10 having the second lowest BM after decile 1. Similarly, β^{LIQ} increases as moving toward decile 10 except the decile 1 having the second highest β^{LIQ}after decile 10. However, decile 10 has a considerably higher average β^{LIQ} than decile 1. MOM, REV, β^{MKT} IVOL, are highest for two extreme portfolios and gradually decreases as we move to the medium portfolio. There is no straightforward trend in average ambiguity of market beta (SE) across decile portfolios. COSKEW has a clear inverse association with ambiguity beta. Overall, we can conclude that stocks with high absolute ambiguity exposure (decile 1 and decile 10) are on average growth stocks with high momentum, high short term-return reversion, high market beta, and high idiosyncratic volatility. However, decile 1 and decile 10 are different from each other

in such a way that former has positive co-skewness and comparably lower exposure to market liquidity.

Panel C of Table 8 presents the average exposure to other ambiguity measures. β^{UNC} and β^{VIX} have comparably lower average values and β^{GDP} has comparably larger average values because of the scale differences. Decile 10 has the highest average β^{GDP} but there is no straightforward trend in β^{GDP} across deciles. $\beta^{VIX}$and β^{UNC} are inversely related with $\beta^{\mho}$. There seems no variation in the average dispersion in the analysts' recommendation (STDEV) except there is a very weak inverse trend as moving from decile 1 to decile 10.

We check the persistency of $\beta^{\mho}$ to explore whether estimated ambiguity betas are good proxies for the true conditional betas. The first column of Table 9 shows the average slope coefficients on $\beta^{\mho}$ from the univariate Fama-MacBeth regressions of one-month- to five-year- ahead $\beta^{\mho}$ on lagged $\beta^{\mho}$. The second column reports the average slope coefficients on $\beta^{\mho}$ from multivariate Fama-MacBeth regressions after controlling for lagged cross-sectional predictors: SIZE, BM, MOM, REV, β^{MKT}, IVOL, COSKEW, SYNC, ANALYST, β^{LIQ}, and SE. The results show that, in both cases, $\beta^{\mho}$ is persistent up to five years with large t values, meaning that high (low) $\beta^{\mho}$ stocks are likely to be remain so in next five years. Therefore, estimated historical ambiguity betas is able to successfully predict future ambiguity betas.

Table 8. Descriptive Statistics of Decile Portfolios

Panel A. Average exposure to market ambiguity										
	(lowest)				(medium)					(highest)
	1	2	3	4	5	6	7	8	9	10
$\beta^{\mho}$	-102.34	-36.15	-21.46	-12.22	-5.16	0.91	7.09	14.84	26.95	95.84
Absolute standard error	72.79	36.57	30.08	26.31	23.98	22.63	23.56	26.93	31.93	72.08
% significant at 0.10 level	0.45	0.23	0.12	0.05	0.01	0.01	0.01	0.02	0.09	0.35
% positive $\beta^{\mho}$	0.00	0.00	0.00	0.01	0.13	0.39	0.92	1.00	1.00	1.00
Average stock number	388.13	387.57	387.68	387.51	387.48	387.76	387.69	387.60	387.69	387.26
Panel B. Average stock characteristics										
SIZE	6.47	6.79	6.73	6.67	6.63	6.58	6.55	6.60	6.59	6.32
BM	-0.92	-0.81	-0.78	-0.77	-0.77	-0.74	-0.74	-0.72	-0.73	-0.87
MOM	22.65	15.40	13.19	12.35	11.64	11.23	11.83	12.75	14.52	21.54
REV	2.71	1.59	1.36	1.27	1.12	1.16	1.14	1.24	1.41	2.36
β^{MKT}	1.56	1.10	0.91	0.80	0.76	0.72	0.75	0.81	0.88	1.33
SE (ambiguity of β^{MKT})	1.96	1.51	1.70	1.83	2.32	2.16	3.02	1.95	1.78	2.05
IVOL	13.53	9.10	7.80	6.74	6.22	5.97	6.24	7.02	8.20	11.32
COSKEW	0.03	0.01	0.00	0.00	0.00	-0.01	-0.01	-0.01	-0.02	-0.07
SYNC	0.31	0.32	0.31	0.31	0.32	0.32	0.32	0.30	0.29	0.27
ANALYST	6.86	7.12	6.41	5.79	5.46	5.16	5.36	5.86	6.05	5.85
β^{LIQ}	3.73	-2.12	-0.89	-0.32	0.10	0.11	0.32	1.28	3.15	26.33
Panel C. Average exposure to other ambiguity measures										
β^{GDP}	326.65	424.31	415.61	397.48	360.01	332.38	298.80	268.86	238.97	1222.23
β^{UNC}	53.52	10.54	7.37	5.19	3.79	2.71	1.93	2.07	1.06	-15.27
β^{VIX}	0.69	0.14	0.07	0.03	0.00	-0.02	-0.04	-0.07	-0.12	-1.57
STDEV	0.70	0.71	0.70	0.69	0.69	0.69	0.68	0.68	0.67	0.67

Table 9. Persistency of Ambiguity Beta

Months	Univariate Predictive Regression	Controlling for stock characteristics
n = 1	0.96	0.94
	(175.60)	(117.15)
n = 2	0.92	0.87
	(94.27)	(62.28)
n = 3	0.88	0.82
	(67.10)	(44.18)
n = 6	0.78	0.69
	(36.90)	(27.44)
n = 9	0.70	0.60
	(28.13)	(21.76)
n = 12	0.63	0.53
	(24.42)	(18.71)
n = 24	0.42	0.30
	(13.94)	(11.94)
n = 36	0.25	0.17
	(9.07)	(8.15)
n = 48	0.12	0.08
	(5.67)	(5.66)
n = 60	0.01	0.01
	(1.69)	(3.02)

Newey-west t values in parentheses

Table 10 shows average excess returns, five-factor loadings, and alphas for equal-weighted portfolios. Newey-West t –statistics are displayed in parentheses. All returns are displayed in percentages. Since we require at least 24 monthly observations for our rolling regression, our observation number reduces to 311 covering the period between February 1995 and December 2020. Table 10 addresses a non-linear relationship with the returns and ambiguity beta. Equal-weighted excess portfolio returns and risk adjusted returns (alpha) are highest for both highest and lowest deciles while they gradually decrease as we move to medium portfolio from two extreme portfolios. Hence, rather than an upward or downward sloping line, the relationship between return and ambiguity beta can be described as U-shaped curve. Decile 1 and decile 10 are not significantly different from each other while both of them have statistically higher excess return than medium portfolio (decile-5). Alpha on decile 10 remains significantly higher than alpha of decile 5 with a t-statistics

usually equal or greater than 3.00 which is the cut-off t-statistics offered by Harvey, Liu, and Zhu (2016) for the newly discovered factors. Factor loadings imply that two extreme portfolios behave like small and growth stocks as they have higher loadings on SMB and lower loadings on HML. WML loadings are negative for all decile, being the highest for decile 1 and lowest for decile 10. LIQ loading is negative for decile 1, positive for decile 10, and positive and higher for decile 4. However, as Diether et. al. (2002) argues these factor loadings tend to portrait a misleading picture of the risk characteristics of these stocks, because they are not stable over time. Therefore, our main interest is on these models' alpha, which represents the abnormal risk adjusted return.

Table 11 presents the results for the value-weighted portfolios. The average value-weighted excess returns on portfolios from decile 1 to decile 10 follow a U-shaped trend, but a weaker one compared to equal-weighted portfolios. More specifically, excess return on decile 10 is statistically higher from decile 5 at 90% confidence interval (i.e. t-value is 1.71) while the excess return on decile 1 is not statistically different form neither decile 10 not decile 5. Trend in alphas is not U-shaped but upward sloping linear line, meaning that risk-adjusted return increases as moving from decile 1 to decile 10. More specifically, alphas on extreme portfolios (decile 1 and decile 10) is not statistically higher than medium portfolio (decile 5). On the other hand, except alphas on CAPM, alpha on decile 10 is higher than decile 1 with statistically significant but marginally low t-values ranging from 1.67 to 2.25. Hence, when portfolio returns are calculated value-weighted, then risk-adjusted abnormal return only exists for portfolios with higher ambiguity beta (from decile 7 to decile 10). Figure 5 plots the excess returns and five factor alphas of equal and

value-weighted portfolios. X -axis stands for the label for decile portfolios and y-axis shows the return in percentage.

Table 10. Equal-Weighted Return on Decile Portfolios

	(lowest)				(medium)					(highest)			
	1	2	3	4	5	6	7	8	9	10	10-1	1-5	10-5
Excess Return	1.50	1.18	1.04	0.94	0.96	0.91	1.02	1.08	1.22	1.55	0.05	0.54	0.59
	(4.13)	(4.18)	(4.13)	(4.09)	(4.38)	(4.14)	(4.36)	(4.15)	(4.43)	(4.63)	(0.35)	(2.42)	(3.55)
MARKET	1.06	0.86	0.77	0.70	0.66	0.64	0.68	0.76	0.83	0.94	-0.12	0.39	0.27
	(33.88)	(45.64)	(35.42)	(35.64)	(24.17)	(20.82)	(20.54)	(20.48)	(24.31)	(30.42)	(-2.34)	(8.29)	(12.59)
SMB	0.91	0.55	0.39	0.31	0.29	0.26	0.29	0.35	0.42	0.67	-0.24	0.62	0.38
	(17.08)	(21.08)	(15.63)	(9.25)	(8.22)	(7.08)	(6.72)	(6.75)	(6.64)	(9.35)	(-2.21)	(7.79)	(8.03)
HML	0.05	0.26	0.27	0.30	0.30	0.31	0.33	0.38	0.34	0.16	0.11	-0.25	-0.13
	(1.42)	(7.72)	(6.07)	(7.58)	(6.86)	(6.10)	(6.15)	(5.59)	(5.51)	(2.80)	(1.45)	(-3.89)	(-3.86)
WML	-0.06	-0.04	-0.06	-0.05	-0.07	-0.06	-0.07	-0.08	-0.10	-0.15	-0.09	0.02	-0.07
	(-1.79)	(-2.23)	(-3.51)	(-3.45)	(-3.54)	(-2.25)	(-2.33)	(-2.56)	(-3.63)	(-5.07)	(-1.68)	(0.35)	(-3.55)
LIQ	-0.46	0.80	0.44	1.71	0.95	0.95	0.19	0.91	0.46	0.79	1.25	-1.41	-0.16
	(-0.21)	(0.77)	(0.51)	(2.22)	(1.13)	(0.96)	(0.19)	(0.76)	(0.36)	(0.47)	(0.41)	(-0.56)	(-0.10)
Observations	311	311	311	311	311	311	311	311	311	311	311	311	311
Adjusted R^2	0.92	0.94	0.93	0.93	0.91	0.89	0.87	0.90	0.92	0.91	0.19	0.68	0.65
CAPM alpha	0.55	0.43	0.38	0.34	0.40	0.37	0.46	0.44	0.51	0.70	0.16	0.15	0.31
	(2.88)	(2.88)	(2.64)	(2.43)	(2.80)	(2.58)	(2.97)	(2.52)	(2.85)	(3.52)	(0.96)	(0.75)	(2.21)
FF3 alpha	0.54	0.42	0.37	0.33	0.38	0.35	0.44	0.42	0.49	0.69	0.15	0.17	0.31
	(5.10)	(5.46)	(4.46)	(4.08)	(4.47)	(3.92)	(4.37)	(3.89)	(4.58)	(5.38)	(0.97)	(1.14)	(2.99)
FF4 alpha	0.58	0.45	0.41	0.36	0.43	0.39	0.49	0.47	0.56	0.79	0.21	0.16	0.36
	(5.64)	(5.85)	(5.18)	(4.63)	(5.01)	(4.31)	(4.82)	(4.36)	(5.09)	(5.92)	(1.28)	(1.08)	(3.49)
FF5 alpha	0.57	0.47	0.42	0.41	0.45	0.41	0.49	0.49	0.57	0.81	0.24	0.12	0.36
	(5.54)	(6.21)	(5.32)	(5.09)	(5.33)	(4.51)	(5.13)	(4.90)	(5.20)	(6.33)	(1.48)	(0.82)	(3.53)

Newey-west t values in parentheses

Table 11. Value-Weighted Return on Decile Portfolios

	(lowest)				(medium)					(highest)			
	1	2	3	4	5	6	7	8	9	10	10-1	1-5	10-5
Excess Return	0.76	0.78	0.83	0.61	0.70	0.70	0.85	0.91	0.84	1.00	0.24	0.07	0.30
	(1.91)	(2.53)	(2.87)	(2.31)	(2.67)	(2.56)	(3.32)	(3.64)	(3.07)	(2.84)	(1.14)	(0.30)	(1.71)
MARKET	1.27	1.04	1.02	0.92	0.94	0.92	0.90	0.92	0.94	1.11	-0.16	0.33	0.17
	(29.29)	(29.56)	(34.28)	(37.59)	(31.14)	(26.00)	(32.06)	(31.16)	(25.69)	(26.47)	(-2.32)	(6.23)	(4.26)
SMB	0.50	0.15	0.06	-0.01	-0.13	-0.14	-0.17	-0.16	-0.12	0.19	-0.31	0.63	0.32
	(5.67)	(1.94)	(0.89)	(-0.35)	(-3.98)	(-3.11)	(-3.19)	(-3.13)	(-2.60)	(3.45)	(-2.92)	(6.75)	(6.72)
HML	-0.18	-0.13	0.04	0.09	0.04	0.13	0.11	0.06	-0.01	-0.15	0.03	-0.22	-0.19
	(-2.47)	(-2.35)	(0.80)	(2.98)	(1.39)	(2.54)	(1.98)	(1.29)	(-0.21)	(-1.72)	(0.21)	(-2.71)	(-2.60)
WML	0.02	0.06	0.08	0.01	-0.02	0.01	-0.06	-0.03	-0.13	-0.15	-0.16	0.03	-0.13
	(0.32)	(1.29)	(1.76)	(0.43)	(-0.70)	(0.52)	(-1.52)	(-1.14)	(-2.68)	(-3.30)	(-2.29)	(0.68)	(-2.55)
LIQ	0.90	-1.06	-1.65	1.50	-1.10	2.66	-1.23	0.14	2.05	-0.68	-1.59	2.00	0.42
	(0.36)	(-0.54)	(-1.23)	(0.89)	(-0.73)	(1.58)	(-0.62)	(0.09)	(1.03)	(-0.22)	(-0.42)	(0.67)	(0.12)
Observations	311	311	311	311	311	311	311	311	311	311	311	311	311
Adjusted R^2	0.840	0.828	0.863	0.875	0.901	0.867	0.847	0.854	0.846	0.818	0.133	0.509	0.284
CAPM alpha	-0.29	-0.03	0.06	-0.10	-0.01	0.01	0.18	0.22	0.10	0.07	0.36	-0.28	0.08
	(-1.63)	(-0.22)	(0.58)	(-0.98)	(-0.09)	(0.06)	(1.58)	(2.39)	(0.83)	(0.41)	(1.61)	(-1.39)	(0.43)
FF3 alpha	-0.28	-0.02	0.06	-0.10	-0.01	0.00	0.17	0.21	0.09	0.08	0.36	-0.27	0.09
	(-1.84)	(-0.17)	(0.58)	(-1.06)	(-0.14)	(0.00)	(1.68)	(2.35)	(0.84)	(0.45)	(1.67)	(-1.70)	(0.51)
FF4 alpha	-0.29	-0.06	0.01	-0.11	0.00	-0.01	0.21	0.24	0.18	0.18	0.47	-0.30	0.18
	(-1.96)	(-0.47)	(0.09)	(-1.15)	(0.03)	(-0.10)	(2.06)	(2.66)	(1.50)	(1.14)	(2.25)	(-1.84)	(1.08)
FF5 alpha	-0.27	-0.09	-0.03	-0.07	-0.03	0.06	0.18	0.24	0.24	0.16	0.43	-0.24	0.19
	(-1.60)	(-0.71)	(-0.32)	(-0.71)	(-0.29)	(0.56)	(1.97)	(2.46)	(1.82)	(1.06)	(1.93)	(-1.30)	(1.09)

Newey-West t-values in parentheses



Figure 5. Excess returns and five factor alphas

Fama (1998) reports that returns on equal-weighted portfolios are affected by microcaps and unlikely to be exploitable fully. Since we exclude the stocks with a price per share less than $5, the difference in results between equal and value weighted portfolios is not likely to be driven by the microcap stocks. Besides, as Table 8 shows, there is no considerable size variation among decile portfolios. Therefore, we conjecture that the zero value weighted alphas for the low ambiguity portfolios is not simply size effect.

In order to explore the different interpretation between equal weighted and value weighted portfolios, we control the size variable by forming decile portfolios

conditional on the SIZE. More specifically, for each month, we first sort stocks into quintiles based on their SIZE. Then, within each SIZE quintile, we sort firms into decile portfolios based on their market ambiguity beta. Ultimately, we have five sets of decile portfolios based on ambiguity beta, each set having the similar SIZE. Appendix B (Table B1 and Table B2) shows both equal weighted and value weighted average excess returns and five factor alphas for each one of 50 portfolios. Figure 6 depicts the excess return and five factor alpha on decile portfolios in each size group. X -axis stands for the decile portfolios sorted within each size group, decile 1 (decile 10) being the portfolio including stocks with lowest (highest) ambiguity beta. Y-axis shows the return in percentage. Figure 6 shows that the relationship between the ambiguity beta and expected return differs across size groups. If the premium observed for the low ambiguity beta stocks was driven by only size effect, then we should not see any U-shaped pattern for the any of the five size neutral portfolios. However, for the small stocks (size group 1 and size group 2), there is a clear U-shaped pattern in returns across decile. This means that small stocks with higher absolute exposure to market ambiguity deliver higher expected return. This pattern is robust to both equal-weighted and value-weighted portfolio returns as well as both excess return and five factor alphas. Starting with the medium size stocks (size group 3), U-shaped pattern becomes subtler. For the larger than average stocks (size group 4), returns are almost uniform across different deciles. Return on largest stocks (size group 5), on the other hand, increases as the ambiguity beta increases which is even more obvious in the value weighted portfolios.



Figure 6. Excess returns and five factor alphas for size neutral portfolios

As Figure 6, Table B1, and Table B2 indicate size has a sort of moderating effect on the relationship between expected return and ambiguity beta. This explains why decile portfolios have different return patterns for equal and value weighted portfolios. More specifically, value-weighted portfolios overweight the pattern for large stocks (linear relationship) while equal-weighted portfolios overweight the pattern for small stocks (U-shaped relationship). These findings also imply that small stocks tend to be affected by market ambiguity more since both lower and higher than average ambiguity exposure increase their required rate of returns. In cases when the factor tested on cross-section of returns is more influential on the small stocks, it is more convenient to use equal-weighted portfolios. For example, Baker and Wurgler (2006) test sentiment on the equal-weighted decile portfolios by arguing that value weighting tends to obscure relevant patterns because large companies are less sensitive to sentiment. Because of the similar arguments, equal-weighted portfolios are reported by Fang and Peress (2009) while testing media coverage, by Chan (2003) while testing news, by Kumar and Lee (2006) while testing retail investor sentiment, by Diether et al. (2002) while testing dispersion in analysts' forecasts. Following these studies, we report equal-weighted portfolio returns for the rest of our analysis as value-weighting may underestimate the impact of market ambiguity exposure on the expected returns.

4.4.3 Bivariate portfolio-level analysis

U-shaped relationship between market ambiguity exposure and cross-section of returns may be driven from the well-known return predictors in the cross-section. Market ambiguity beta is obviously distinct from size effect, however, it may be related with other cross-sectional predictors such as BM, MOM, REV, β^{MKT}, IVOL,

COSKEW, SYNC, ANALYST, β^{LIQ}. Or the return variation may come from not variation in market ambiguity beta but variation in the ambiguity of β^{MKT} (SE). On the other hand, given that market ambiguity has serious lead and lag association with other ambiguity proxies and even high contemporaneous correlation with VIX, the role of $\beta^{\mho}$ may be statistically indistinguishable from other ambiguity proxies.

In this section, we form conditional bivariate portfolios in order to control the well-known cross-sectional return predictors (defined in Table A2) and other ambiguity proxies (defined in Table A1). In each month, we first sort stocks into quintiles based on their values on the relevant variable. Then, within each quintile, we sort firms into decile portfolios based on their market ambiguity beta ($\beta^{\mho}$). We also perform unconditional bivariate portfolios which are formed by crossing the stocks sorted independently by the relevant variables. The results are the same. However, we report only the conditional bivariate portfolios because as Lee et.al. (2019) discuss, unconditional bivariate portfolio may fall short of controlling variables that are highly correlated with $\beta^{\mho}$. Following Bali et.al (2017) and Lee et. al. (2019), we report average portfolio returns across the five control portfolios within each decile of $\beta^{\mho}$.

Table 12 presents average equal-weighted returns across quintiles sorted by cross-sectional predictors. Similar with the univariate analysis, excess return on decile 1 and decile 10 are not significantly different from each other while the excess returns on decile 5 is statistically lower than two extreme portfolios (decile 1 and decile 10). As for risk-adjusted returns, decile 10 remains significantly higher than alpha of decile 5 with high t-values except MOM portfolios which has a lower t-value (t = 1.66). Figure C1 depicts the average five-factor alphas for bivariate portfolios displayed in Table 12 to ease to follow return patterns. U-shaped return

pattern in excess return and five-factor alphas persists after controlling for the widely known cross-sectional predictors. It is worth noting, however, that the trend becomes subtler after MOM and IVOL is controlled, but does not subside completely. Table 13 shows the average equal-weighted returns across quintiles sorted by other ambiguity proxies. Figure C2 depicts the average five-factor alphas. Similar with the previous results excess return on two extreme portfolios beat the excess return on decile 5 and the alpha on decile 10 statistically higher than decile 5. The high t-statistics and the obvious U-shaped trend shows that the impact of market ambiguity beta to cross-section of expected returns is distinguishable from the exposure to other ambiguity proxies.

Table 12. Bivariate Portfolio Returns Sorted by β^{υ} and Cross-Sectional Predictors

.Panel A. Excess Return													
	(lowest)				(medium)					(highest)			
	1	2	3	4	5	6	7	8	9	10	10-1	1-5	10-5
SIZE	1.49	1.22	1.08	0.94	0.95	0.94	0.98	1.09	1.23	1.51	0.02	0.54	0.56
	(4.19)	(4.45)	(4.36)	(4.18)	(4.33)	(4.16)	(4.10)	(4.17)	(4.38)	(4.48)	(0.13)	(2.50)	(3.29)
BM	1.60	1.27	1.13	1.10	1.17	1.08	1.05	1.16	1.25	1.60	-0.00	0.43	0.43
	(4.28)	(4.11)	(4.00)	(4.14)	(4.45)	(4.12)	(3.86)	(4.08)	(4.23)	(4.47)	(-0.01)	(2.13)	(2.73)
MOM	1.25	1.04	1.00	0.96	0.98	0.98	1.00	1.06	1.17	1.27	0.02	0.26	0.28
	(3.93)	(4.03)	(4.06)	(4.08)	(4.28)	(4.28)	(4.24)	(4.34)	(4.31)	(3.96)	(0.14)	(1.73)	(2.17)
REV	1.45	1.12	1.06	1.00	0.97	1.03	1.01	1.07	1.22	1.50	0.04	0.49	0.53
	(4.23)	(3.99)	(4.20)	(4.32)	(4.32)	(4.50)	(4.28)	(4.16)	(4.39)	(4.61)	(0.32)	(2.55)	(3.52)
β^{MKT}	1.33	1.09	1.03	1.03	0.97	0.99	1.07	1.15	1.25	1.51	0.18	0.36	0.54
	(4.53)	(4.18)	(4.22)	(4.25)	(4.02)	(4.07)	(4.28)	(4.39)	(4.53)	(4.77)	(1.48)	(2.80)	(4.46)
IVOL	1.10	0.97	1.08	1.06	1.04	1.09	1.08	1.15	1.15	1.28	0.18	0.07	0.24
	(4.12)	(3.85)	(4.36)	(4.38)	(4.11)	(4.41)	(4.29)	(4.47)	(4.45)	(4.45)	(1.25)	(0.71)	(3.06)
COSKEW	1.39	1.19	1.06	1.01	0.98	0.98	0.99	1.11	1.18	1.52	0.13	0.41	0.53
	(4.05)	(4.43)	(4.29)	(4.44)	(4.25)	(4.30)	(4.12)	(4.32)	(4.22)	(4.63)	(0.91)	(2.14)	(3.51)
SYNC	1.38	1.20	0.98	0.93	0.91	0.90	1.00	1.02	1.19	1.46	0.08	0.47	0.55
	(4.10)	(4.51)	(4.00)	(4.19)	(4.19)	(4.15)	(4.25)	(3.97)	(4.42)	(4.37)	(0.55)	(2.43)	(3.44)
ANALYST	1.55	1.24	1.08	1.04	1.01	1.00	1.11	1.12	1.27	1.53	-0.02	0.54	0.52
	(4.25)	(4.46)	(4.22)	(4.36)	(4.25)	(4.14)	(4.38)	(4.29)	(4.52)	(4.51)	(-0.14)	(2.53)	(3.23)
β^{LIQ}	1.41	1.15	1.06	0.99	1.01	0.95	1.01	1.11	1.24	1.48	0.07	0.40	0.47
	(4.00)	(4.14)	(4.34)	(4.27)	(4.39)	(4.08)	(4.30)	(4.43)	(4.59)	(4.62)	(0.50)	(2.06)	(3.38)
SE	1.46	1.21	1.03	0.92	0.93	0.93	1.04	1.08	1.26	1.52	0.07	0.53	0.59
	(4.07)	(4.42)	(4.14)	(4.07)	(4.36)	(4.26)	(4.42)	(4.23)	(4.56)	(4.38)	(0.48)	(2.46)	(3.36)
Panel B. Five-factor alpha													
	(lowest)				(medium)					(highest)			
	1	2	3	4	5	6	7	8	9	10	10-1	1-5	10-5
SIZE	0.59	0.52	0.47	0.40	0.45	0.42	0.44	0.50	0.57	0.77	0.18	0.14	0.32
	(6.34)	(6.55)	(5.54)	(5.17)	(5.57)	(4.70)	(4.81)	(5.30)	(5.31)	(6.19)	(1.27)	(1.15)	(3.03)
BM	0.65	0.44	0.37	0.40	0.55	0.45	0.41	0.51	0.55	0.85	0.20	0.11	0.31
	(5.75)	(5.58)	(4.95)	(5.45)	(6.21)	(6.07)	(5.36)	(5.39)	(5.39)	(6.30)	(1.16)	(0.70)	(2.62)

MOM	0.45	0.38	0.38	0.40	0.43	0.44	0.43	0.48	0.55	0.55	0.11	0.02	0.12
	(4.86)	(5.85)	(4.72)	(5.47)	(5.37)	(5.44)	(4.69)	(5.21)	(5.29)	(4.65)	(0.74)	(0.13)	(1.66)
REV	0.62	0.43	0.41	0.41	0.43	0.49	0.46	0.47	0.54	0.79	0.17	0.19	0.36
	(6.27)	(5.80)	(5.32)	(5.50)	(5.71)	(5.28)	(4.90)	(4.96)	(5.15)	(6.58)	(1.14)	(1.53)	(3.54)
β^{MKT}	0.57	0.46	0.42	0.44	0.39	0.41	0.50	0.52	0.58	0.82	0.25	0.19	0.44
	(7.01)	(5.71)	(5.77)	(5.23)	(5.29)	(4.85)	(5.72)	(5.74)	(5.70)	(6.65)	(1.67)	(1.76)	(4.81)
IVOL	0.43	0.34	0.45	0.43	0.44	0.49	0.50	0.54	0.55	0.64	0.20	-0.01	0.20
	(4.65)	(4.51)	(6.08)	(5.64)	(5.84)	(6.05)	(5.52)	(6.34)	(5.51)	(5.24)	(1.26)	(-0.05)	(2.59)
COSKEW	0.52	0.48	0.44	0.46	0.45	0.45	0.45	0.54	0.54	0.76	0.24	0.07	0.31
	(5.74)	(6.69)	(6.03)	(6.29)	(5.28)	(5.17)	(4.70)	(5.23)	(4.97)	(6.07)	(1.46)	(0.56)	(3.07)
SYNC	0.50	0.48	0.39	0.42	0.43	0.40	0.47	0.45	0.55	0.71	0.22	0.06	0.28
	(4.84)	(6.56)	(4.60)	(4.84)	(5.21)	(4.44)	(5.61)	(4.62)	(4.96)	(5.33)	(1.32)	(0.45)	(2.55)
ANALYST	0.63	0.51	0.42	0.44	0.45	0.45	0.55	0.49	0.62	0.80	0.16	0.19	0.35
	(6.86)	(6.43)	(5.35)	(5.59)	(5.27)	(5.45)	(6.16)	(5.11)	(5.86)	(6.40)	(1.09)	(1.43)	(3.51)
β^{LIQ}	0.54	0.44	0.47	0.46	0.48	0.40	0.47	0.51	0.59	0.75	0.21	0.06	0.27
	(5.51)	(5.78)	(5.90)	(6.45)	(5.43)	(4.87)	(5.49)	(5.48)	(5.38)	(6.34)	(1.38)	(0.49)	(2.96)
SE	0.54	0.50	0.42	0.41	0.43	0.44	0.52	0.50	0.60	0.74	0.19	0.12	0.31
	(5.67)	(6.69)	(5.44)	(4.94)	(5.63)	(5.31)	(5.42)	(5.51)	(5.76)	(5.44)	(1.20)	(0.95)	(2.76)

Newey-West t-values in parentheses

Table 13. Bivariate Portfolio Returns Sorted by β^{υ} and Other Ambiguity Proxies

Panel A. Excess Return													
	(lowest)				(medium)					(highest)			
	1	2	3	4	5	6	7	8	9	10	10-1	1-5	10-5
β^{GDP}	1.47	1.15	1.07	1.04	0.97	0.96	1.02	1.11	1.16	1.46	-0.02	0.50	0.49
	(4.51)	(4.20)	(4.23)	(4.34)	(4.19)	(4.16)	(4.30)	(4.35)	(4.32)	(4.48)	(-0.13)	(2.86)	(3.63)
β^{UNC}	1.38	1.16	1.04	1.01	1.02	0.99	1.05	1.09	1.22	1.44	0.07	0.36	0.43
	(4.09)	(4.29)	(4.15)	(4.37)	(4.43)	(4.20)	(4.39)	(4.33)	(4.56)	(4.50)	(0.55)	(2.08)	(3.14)
β^{VIX}	1.39	1.10	1.08	1.02	1.03	1.04	1.04	1.12	1.11	1.47	0.08	0.37	0.45
	(4.16)	(4.17)	(4.36)	(4.43)	(4.52)	(4.24)	(4.23)	(4.45)	(4.06)	(4.63)	(0.63)	(2.20)	(3.28)
STDEV	1.50	1.31	1.19	1.12	1.08	1.12	1.10	1.16	1.34	1.50	0.00	0.42	0.42
	(3.89)	(4.24)	(4.19)	(4.20)	(4.13)	(4.20)	(4.08)	(4.10)	(4.45)	(4.20)	(0.03)	(1.92)	(2.61)
Panel B. Five-factor alpha													
	(lowest)				(medium)					(highest)			
	1	2	3	4	5	6	7	8	9	10	10-1	1-5	10-5
β^{GDP}	0.64	0.43	0.44	0.47	0.44	0.42	0.48	0.54	0.51	0.73	0.08	0.20	0.29
	(6.73)	(6.92)	(5.96)	(6.02)	(5.15)	(5.18)	(5.40)	(5.43)	(4.80)	(6.15)	(0.54)	(1.59)	(3.35)
β^{UNC}	0.59	0.47	0.43	0.46	0.47	0.45	0.50	0.47	0.57	0.69	0.11	0.12	0.23
	(6.33)	(6.72)	(5.54)	(5.83)	(6.04)	(4.79)	(5.90)	(5.28)	(5.57)	(5.97)	(0.75)	(1.01)	(2.54)
β^{VIX}	0.56	0.44	0.47	0.46	0.49	0.47	0.47	0.53	0.49	0.72	0.16	0.07	0.23
	(6.17)	(5.83)	(6.17)	(5.57)	(6.36)	(5.30)	(4.95)	(5.83)	(4.88)	(5.73)	(1.04)	(0.63)	(2.41)
STDEV	0.49	0.45	0.41	0.40	0.41	0.49	0.48	0.46	0.59	0.69	0.20	0.08	0.28
	(4.72)	(5.46)	(5.44)	(5.17)	(4.89)	(5.38)	(6.05)	(4.52)	(5.18)	(5.10)	(1.16)	(0.57)	(2.51)

Newey-West t-values in parentheses

4.4.4 Stock-level fama macbeth regression

Bivariate portfolio construction process is very widely-used and clever way to examine the effect of test variable while controlling some well-known cross-sectional predictors. However, this process enables to control only a limited number of variables simultaneously. Two-step Fama and MacBeth (1973) regression, on the other hand, is able to control various factors simultaneously and help to examine the explanatory power of test variables in a more controlled framework. Therefore, this section applies stock level Fama-MacBeth regression to test role of the market ambiguity exposure on cross-section of returns.

It should be noted that Fama-MacBeth regression is designed to catch the linear relationship between return and the test (and the control) variables. Our results so far show a U-shaped pattern between return and market ambiguity beta. As expected, in the unreported results, ambiguity beta is found to be insignificant in all of the Fama-MacBeth regression models with different control variables. In order to examine whether the non-linear U-shaped is significant, we perform Fama-MacBeth regression by putting square of the market ambiguity beta. Following Fama and MacBeth (1973), Table 14 reports time-series averages coefficient of the variables in the models. As the range of (β^{υ}) square is very large compared to stock return range, the coefficient retrieved is extremely small. Therefore, coefficients for (β^{υ}) square in Table 14 are rescaled and reported in basis point. The coefficients of the other ambiguity betas are also rescaled accordingly for a more robust comparison. Because of multicollinearity among SIZE, COSKEW, ANALYST, and SYNC, they are not included in the models simultaneously.[12] For example, if the model includes

[12] Previous studies also confirm the high correlation between SIZE and ANALYST (Arbel & Strebel, 1982), SIZE and COSKEW (Barone Adesi, Gagliardini, & Urga, 2004; Harvey & Siddique, 2000) as well as between SYNC and SIZE, and SYNC and ANALYST (Devos, Hao, Prevost, & Wongchoti, 2015).

ln(SIZE), we exclude COSKEW, and ln(ANALYST) and SYNC. Or if the model includes ln(ANALYST), we exclude COSKEW, ln(SIZE and SYNC. The models that control other ambiguity betas (β^{GDP}, β^{UNC}, β^{VIX}) we only report the model with ln(SIZE) rather than the versions including ln(ANALYST) or SYNC. The results of other versions are essentially the same.

Table 14 shows a positive and significant slope for ($\beta^{\cup}$) square confirming a statistically significant U-shaped trend exists between market ambiguity beta and expected stock return. The t-values are not very large though. However, the performance of ($\beta^{\cup}$) square are comparable with market beta given that β^{MKT} has also t-values ranging from 1.46 to 2.13. The most significant and the robust return predictors are size (SIZE) and short-term return reversal (REV). Surprisingly, momentum is insignificant in all models. This is, in fact, consistent with the findings of Bhattacharya Li and Sonaer (2017), who report that momentum profits have become negligible since the late 1990s, the initial period of our sample. Persistency of ($\beta^{\cup}$) square in terms of both sing and magnitude as well as the acceptable level of t values across different shows that U-shaped pattern between expected return and market ambiguity exposure is robust and significant in US market.

Table 14. Stock Level Fama MacBeth Regression

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
(β^{U}) square	0.43	0.30	0.20	0.25	0.29	0.32	0.31	0.25	0.22	0.23
	(2.40)	(1.77)	(1.66)	(1.85)	(2.09)	(1.67)	(2.21)	(1.83)	(1.69)	(1.73)
β^{MKT}		0.38	0.23	0.22	0.29	0.38	0.27	0.22	0.22	0.23
		(2.13)	(1.61)	(1.53)	(1.92)	(2.16)	(1.79)	(1.54)	(1.46)	(1.56)
ln(SIZE)			-0.14	-0.17				-0.16	-0.17	-0.16
			(-4.60)	(-4.45)				(-4.41)	(-4.42)	(-4.40)
ln(BM)			-0.03	-0.07	0.02	0.04	-0.01	-0.07	-0.07	-0.07
			(-0.59)	(-1.05)	(0.35)	(0.71)	(-0.14)	(-1.18)	(-1.08)	(-1.10)
MOM			0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
			(0.85)	(0.53)	(0.35)	(0.08)	(0.23)	(0.13)	(0.61)	(0.45)
REV			-0.04	-0.03	-0.03	-0.04	-0.03	-0.03	-0.03	-0.03
			(-7.35)	(-6.72)	(-6.64)	(-7.10)	(-6.83)	(-6.97)	(-6.76)	(-6.77)
β^{LIQ}				0.00	0.00	0.00	0.00	0.00	0.00	0.00
				(0.98)	(1.06)	(0.54)	(0.91)	(1.01)	(1.13)	(1.05)
SE				0.01	0.02	0.05	0.02	0.01	0.01	0.02
				(1.26)	(2.00)	(2.51)	(1.81)	(1.23)	(1.19)	(1.35)
STDEV				0.04	-0.16	-0.14	-0.06	0.03	0.04	0.04
				(0.77)	(-2.55)	(-2.27)	(-1.32)	(0.65)	(0.68)	(0.71)
COSKEW					-0.50					
					(-1.13)					
SYNC						-0.96				
						(-3.01)				
ANALYST							-0.02			
							(-2.40)			
β^{GDP}								0.35		
								(1.62)		
β^{UNC}									-0.30	
									(-0.16)	
β^{VIX}										-0.38
										(-0.36)
Constant	1.10	0.73	1.72	1.87	0.82	1.01	0.90	1.85	1.84	1.84
	(4.48)	(4.92)	(6.35)	(5.51)	(4.24)	(5.04)	(4.51)	(5.46)	(5.44)	(5.48)
Adj R square	0.66%	3.98%	5.49%	6.29%	5.68%	6.19%	5.88%	6.46%	6.44%	6.44%
Number of groups	311	311	311	311	311	311	311	311	311	311

Newey-West t-values in parentheses

4.4.5 Different formation and holding periods

In this section, we try different formation and holding periods up to one year in order to ensure that U-shaped pattern is not just a statistical phenomenon. We first examine effect of waiting several months before assigning stocks to portfolios. Next, we examine holding the stocks longer than one month. At the end of each month, stocks are sorted into deciles based on their contemporaneous ambiguity beta, and then hold in the relevant portfolio for T >1 months. Similar to Jegadeesh and Titman (1993) and Diether (2002), 1/Tth of each portfolio reinvested monthly. More specifically, if T is three months, then portfolio return in month t equals to the average return of the decile portfolios formed in month t-3, t-2, and t-1. Table 15 and Table 16 show the excess return and five factors alphas for decile portfolios with different formation and holding periods respectively. Besides, Figure C3 plots the five factor alphas provided in Table 15 and Table 16 for selected periods (n = 3, n = 6, n = 9, n = 12) to follow the return trends more easily. Our results show that the higher return on decile 1 and decile 10 is robust to different holding and formation periods.

Table 15. Different Formation Periods

Panel A. Excess Return													
	(low)				(medium)					(high)			
	1	2	3	4	5	6	7	8	9	10	10-1	1-5	10-5
n = 2	1.53	1.19	1.03	0.98	0.95	0.91	1.03	1.11	1.20	1.58	0.05	0.58	0.62
	(4.33)	(4.23)	(4.12)	(4.26)	(4.41)	(4.21)	(4.38)	(4.37)	(4.37)	(4.80)	(0.36)	(2.70)	(3.66)
n = 3	1.62	1.20	1.03	0.99	0.94	0.95	1.01	1.12	1.20	1.59	-0.02	0.68	0.65
	(4.39)	(4.35)	(4.07)	(4.30)	(4.31)	(4.31)	(4.33)	(4.53)	(4.48)	(4.89)	(-0.20)	(3.15)	(3.93)
n = 4	1.56	1.22	1.06	0.94	0.99	0.96	1.01	1.12	1.23	1.58	0.02	0.57	0.59
	(4.33)	(4.23)	(4.14)	(4.00)	(4.49)	(4.37)	(4.48)	(4.53)	(4.55)	(4.82)	(0.18)	(2.72)	(3.46)
n = 5	1.49	1.29	1.03	0.99	0.98	0.95	1.05	1.09	1.23	1.56	0.07	0.51	0.58
	(4.18)	(4.50)	(4.06)	(4.21)	(4.42)	(4.39)	(4.71)	(4.36)	(4.49)	(4.78)	(0.62)	(2.49)	(3.46)
n = 6	1.53	1.18	1.08	0.99	0.96	0.96	1.01	1.11	1.24	1.61	0.08	0.56	0.65
	(4.29)	(4.16)	(4.24)	(4.27)	(4.45)	(4.28)	(4.44)	(4.41)	(4.45)	(4.83)	(0.73)	(2.75)	(3.77)
n = 7	1.50	1.17	1.04	1.01	0.95	0.95	1.00	1.16	1.24	1.63	0.13	0.55	0.68
	(4.28)	(4.21)	(4.16)	(4.17)	(4.24)	(4.42)	(4.27)	(4.39)	(4.37)	(4.72)	(0.94)	(2.74)	(3.74)
n = 8	1.46	1.17	1.08	0.95	0.93	0.97	1.03	1.15	1.24	1.64	0.19	0.52	0.71
	(4.20)	(4.20)	(4.18)	(4.03)	(4.02)	(4.40)	(4.51)	(4.38)	(4.31)	(4.69)	(1.27)	(2.68)	(3.95)
n = 9	1.46	1.13	1.07	0.98	1.00	0.92	1.01	1.15	1.26	1.67	0.21	0.46	0.67
	(4.21)	(4.08)	(4.08)	(4.13)	(4.39)	(4.09)	(4.27)	(4.43)	(4.30)	(4.72)	(1.53)	(2.35)	(3.52)
n = 10	1.49	1.13	1.11	0.97	0.97	0.94	1.01	1.20	1.30	1.69	0.20	0.52	0.72
	(4.28)	(4.08)	(4.22)	(4.01)	(4.20)	(4.16)	(4.37)	(4.63)	(4.39)	(4.78)	(1.57)	(2.69)	(3.76)
n = 11	1.52	1.13	1.07	0.98	1.00	0.94	1.04	1.16	1.29	1.70	0.18	0.52	0.70
	(4.34)	(4.11)	(4.09)	(3.93)	(4.22)	(4.15)	(4.56)	(4.44)	(4.29)	(4.81)	(1.51)	(2.71)	(3.71)
n = 12	1.51	1.16	1.01	1.02	1.02	0.98	1.01	1.14	1.28	1.75	0.23	0.49	0.72
	(4.33)	(4.15)	(3.89)	(4.30)	(4.18)	(4.49)	(4.27)	(4.29)	(4.24)	(4.60)	(1.50)	(2.48)	(3.01)
Panel B. Five-factor alpha													
	(low)				(medium)					(high)			
	1	2	3	4	5	6	7	8	9	10	10-1	1-5	10-5
n = 2	0.60	0.45	0.42	0.44	0.47	0.40	0.51	0.53	0.56	0.82	0.22	0.13	0.35
	(5.63)	(5.61)	(5.55)	(5.40)	(5.73)	(4.16)	(5.28)	(5.30)	(4.61)	(6.59)	(1.34)	(0.95)	(3.34)
n = 3	0.71	0.46	0.41	0.47	0.44	0.45	0.47	0.55	0.58	0.83	0.12	0.27	0.39
	(6.12)	(5.71)	(5.18)	(5.57)	(5.50)	(4.82)	(4.78)	(5.33)	(4.86)	(6.76)	(0.73)	(1.98)	(3.74)
n = 4	0.64	0.47	0.43	0.44	0.51	0.47	0.47	0.55	0.61	0.81	0.17	0.12	0.30

	(6.09)	(5.39)	(5.32)	(5.34)	(5.75)	(4.71)	(4.46)	(5.60)	(5.51)	(6.61)	(1.21)	(0.91)	(3.06)
n = 5	0.59	0.56	0.42	0.49	0.50	0.46	0.53	0.52	0.63	0.74	0.15	0.09	0.25
	(6.15)	(5.78)	(5.11)	(5.88)	(5.54)	(4.64)	(4.88)	(5.31)	(5.83)	(6.26)	(1.14)	(0.77)	(2.53)
n = 6	0.67	0.45	0.49	0.50	0.49	0.47	0.50	0.56	0.63	0.81	0.14	0.18	0.31
	(7.04)	(5.33)	(5.55)	(6.09)	(5.70)	(4.39)	(4.77)	(5.59)	(5.71)	(6.75)	(1.14)	(1.54)	(3.28)
n = 7	0.68	0.46	0.45	0.52	0.46	0.48	0.49	0.60	0.61	0.85	0.16	0.22	0.39
	(7.06)	(5.69)	(5.60)	(5.73)	(5.72)	(4.78)	(4.45)	(5.83)	(5.59)	(6.49)	(1.22)	(1.94)	(3.58)
n = 8	0.65	0.50	0.52	0.45	0.44	0.47	0.55	0.62	0.60	0.87	0.22	0.21	0.43
	(6.50)	(6.33)	(5.68)	(5.41)	(5.20)	(5.06)	(4.94)	(5.27)	(5.35)	(6.41)	(1.61)	(1.87)	(3.92)
n = 9	0.66	0.44	0.53	0.47	0.53	0.43	0.54	0.60	0.63	0.91	0.25	0.13	0.38
	(6.27)	(5.51)	(5.29)	(5.66)	(6.59)	(4.33)	(4.55)	(5.55)	(5.52)	(6.90)	(1.85)	(1.15)	(3.42)
n = 10	0.65	0.44	0.52	0.45	0.50	0.45	0.52	0.64	0.68	0.90	0.25	0.16	0.40
	(6.40)	(4.86)	(5.65)	(5.44)	(6.19)	(4.47)	(4.64)	(5.85)	(5.93)	(6.76)	(1.93)	(1.38)	(3.46)
n = 11	0.68	0.48	0.50	0.47	0.51	0.46	0.55	0.61	0.71	0.90	0.22	0.17	0.38
	(6.18)	(5.62)	(5.11)	(5.12)	(6.49)	(4.72)	(4.95)	(5.53)	(6.23)	(6.89)	(1.75)	(1.46)	(3.39)
n = 12	0.72	0.51	0.45	0.53	0.56	0.51	0.52	0.58	0.67	0.98	0.26	0.16	0.41
	(6.49)	(5.48)	(5.53)	(6.10)	(5.41)	(5.47)	(4.70)	(5.53)	(6.06)	(5.46)	(1.60)	(1.12)	(2.35)

Newey-West t-values in parentheses

Table 16. Different Holding Periods

Panel A. Excess Return													
	(low)				(medium)					(high)			
	1	2	3	4	5	6	7	8	9	10	10-1	1-5	10-5
n = 2	1.51	1.18	1.03	0.95	0.95	0.91	1.02	1.09	1.21	1.56	0.05	0.56	0.61
	(4.22)	(4.19)	(4.12)	(4.16)	(4.38)	(4.16)	(4.37)	(4.25)	(4.39)	(4.70)	(0.36)	(2.56)	(3.62)
n = 3	1.54	1.19	1.03	0.97	0.95	0.92	1.02	1.10	1.21	1.57	0.03	0.60	0.62
	(4.27)	(4.24)	(4.09)	(4.21)	(4.35)	(4.20)	(4.35)	(4.33)	(4.41)	(4.76)	(0.22)	(2.76)	(3.74)
n = 4	1.54	1.19	1.04	0.96	0.96	0.93	1.02	1.10	1.21	1.57	0.03	0.58	0.61
	(4.27)	(4.23)	(4.10)	(4.15)	(4.38)	(4.23)	(4.38)	(4.37)	(4.44)	(4.76)	(0.26)	(2.75)	(3.68)
n = 5	1.53	1.21	1.03	0.96	0.96	0.93	1.02	1.10	1.21	1.57	0.04	0.57	0.61
	(4.25)	(4.28)	(4.07)	(4.15)	(4.38)	(4.25)	(4.43)	(4.35)	(4.43)	(4.75)	(0.33)	(2.71)	(3.64)
n = 6	1.52	1.20	1.04	0.96	0.96	0.93	1.02	1.09	1.21	1.57	0.05	0.56	0.62
	(4.22)	(4.24)	(4.09)	(4.15)	(4.36)	(4.23)	(4.41)	(4.33)	(4.41)	(4.74)	(0.46)	(2.69)	(3.67)
n = 7	1.50	1.19	1.03	0.97	0.95	0.93	1.00	1.10	1.21	1.57	0.07	0.55	0.62
	(4.18)	(4.20)	(4.07)	(4.13)	(4.31)	(4.22)	(4.35)	(4.30)	(4.37)	(4.71)	(0.62)	(2.66)	(3.67)
n = 8	1.49	1.18	1.04	0.96	0.95	0.93	1.01	1.10	1.21	1.58	0.08	0.55	0.63
	(4.17)	(4.18)	(4.07)	(4.11)	(4.25)	(4.22)	(4.35)	(4.30)	(4.34)	(4.69)	(0.74)	(2.67)	(3.73)
n = 9	1.49	1.18	1.04	0.96	0.95	0.93	1.00	1.10	1.21	1.59	0.10	0.54	0.64
	(4.16)	(4.16)	(4.06)	(4.09)	(4.26)	(4.19)	(4.32)	(4.29)	(4.33)	(4.69)	(0.89)	(2.63)	(3.73)
n = 10	1.51	1.18	1.06	0.97	0.96	0.94	1.01	1.12	1.24	1.62	0.11	0.55	0.66
	(4.22)	(4.18)	(4.11)	(4.10)	(4.27)	(4.21)	(4.35)	(4.36)	(4.37)	(4.75)	(0.99)	(2.70)	(3.82)
n = 11	1.50	1.17	1.05	0.97	0.96	0.93	1.01	1.12	1.23	1.62	0.12	0.54	0.66
	(4.21)	(4.15)	(4.08)	(4.06)	(4.23)	(4.17)	(4.33)	(4.33)	(4.33)	(4.73)	(1.05)	(2.71)	(3.82)
n = 12	1.51	1.17	1.05	0.97	0.96	0.93	1.01	1.12	1.23	1.63	0.13	0.55	0.67
	(4.23)	(4.14)	(4.05)	(4.06)	(4.22)	(4.19)	(4.31)	(4.31)	(4.30)	(4.72)	(1.12)	(2.74)	(3.78)
Panel B. Five-factor alpha													
	(low)				(medium)					(high)			
	1	2	3	4	5	6	7	8	9	10	10-1	1-5	10-5
n = 2	0.59	0.46	0.42	0.42	0.46	0.40	0.50	0.51	0.57	0.82	0.23	0.13	0.36
	(5.67)	(6.03)	(5.56)	(5.31)	(5.59)	(4.37)	(5.34)	(5.16)	(4.93)	(6.50)	(1.43)	(0.89)	(3.51)
n = 3	0.63	0.46	0.42	0.44	0.45	0.42	0.49	0.53	0.57	0.82	0.20	0.17	0.37
	(5.93)	(6.06)	(5.51)	(5.50)	(5.60)	(4.54)	(5.18)	(5.25)	(4.94)	(6.62)	(1.24)	(1.26)	(3.64)
n = 4	0.62	0.46	0.42	0.44	0.47	0.43	0.49	0.53	0.58	0.82	0.20	0.15	0.35

	(6.13)	(6.24)	(5.56)	(5.52)	(5.71)	(4.63)	(5.05)	(5.40)	(5.13)	(6.65)	(1.29)	(1.15)	(3.53)
n = 5	0.62	0.48	0.42	0.45	0.48	0.44	0.50	0.53	0.59	0.81	0.19	0.14	0.33
	(6.27)	(6.31)	(5.52)	(5.66)	(5.76)	(4.68)	(5.07)	(5.42)	(5.31)	(6.60)	(1.29)	(1.07)	(3.35)
n = 6	0.63	0.48	0.43	0.46	0.48	0.44	0.50	0.53	0.60	0.81	0.18	0.15	0.33
	(6.52)	(6.26)	(5.74)	(5.79)	(5.77)	(4.64)	(5.03)	(5.47)	(5.40)	(6.71)	(1.32)	(1.17)	(3.44)
n = 7	0.63	0.48	0.44	0.47	0.48	0.45	0.49	0.54	0.60	0.82	0.18	0.16	0.34
	(6.72)	(6.27)	(5.82)	(5.98)	(5.79)	(4.67)	(4.93)	(5.54)	(5.46)	(6.77)	(1.38)	(1.28)	(3.56)
n = 8	0.63	0.48	0.45	0.47	0.47	0.45	0.50	0.55	0.60	0.82	0.19	0.16	0.35
	(6.88)	(6.33)	(5.91)	(5.96)	(5.77)	(4.74)	(4.96)	(5.57)	(5.49)	(6.78)	(1.46)	(1.38)	(3.69)
n = 9	0.64	0.48	0.46	0.47	0.48	0.45	0.51	0.56	0.61	0.84	0.20	0.16	0.36
	(6.96)	(6.33)	(5.95)	(5.96)	(5.91)	(4.71)	(4.93)	(5.60)	(5.55)	(6.87)	(1.57)	(1.37)	(3.74)
n = 10	0.64	0.47	0.46	0.47	0.48	0.45	0.51	0.57	0.61	0.85	0.21	0.16	0.37
	(7.00)	(6.18)	(5.98)	(5.92)	(5.94)	(4.67)	(4.89)	(5.64)	(5.63)	(6.91)	(1.66)	(1.40)	(3.82)
n = 11	0.65	0.47	0.47	0.47	0.48	0.45	0.51	0.57	0.62	0.85	0.21	0.16	0.37
	(7.08)	(6.20)	(5.96)	(5.89)	(6.03)	(4.69)	(4.92)	(5.67)	(5.73)	(6.96)	(1.70)	(1.46)	(3.83)
n = 12	0.66	0.48	0.47	0.47	0.49	0.45	0.51	0.57	0.63	0.87	0.21	0.17	0.38
	(7.19)	(6.19)	(5.99)	(5.94)	(6.07)	(4.78)	(4.91)	(5.67)	(5.77)	(6.94)	(1.75)	(1.51)	(3.79)

Newey-West t-values in parentheses

4.4.6 Time-varying market ambiguity exposure

Our results show that stocks heavily exposed to absolute market ambiguity (both positive and negative) have higher expected returns compared to others. This pattern is more pronounced for smaller stocks. However, our findings appear to be quite puzzling under a standard asset pricing framework which assumes a linear relationship between expected returns and ambiguity (Epstein & Schneider, 2010; Izhakian, 2020; Ju & Miao, 2012; Kogan & Wang, 2003; Mukerji et al., 2022; Ui, 2011). The previous empirical studies find either positive (Hsiao, 2022; Izhakian, 2012; Jiang & Sun, 2014) or negative (Bali et al., 2017; Diether et al., 2002; Lee et al., 2019) association.

Our previous analyses exhibit only an average association between expected return and market ambiguity exposure over the period 1993 – 2020. However, cross-sectional association of β^{υ} and stock return may be state dependent and may change over time, being one of the strongest candidates to explain U-shaped return pattern across decile portfolios. For example, while in good states one of the two extreme portfolios may bring higher return than average, in bad state the other one may deliver higher than average return. A related approach takes part in model of Ju and Miao (2012). Their model predicts that stock with low ambiguity beta should have higher average required rate of return because they deliver lower return during high ambiguity periods and investors demand a premium to hold them, which is empirically documented by Bali et. al. (2017) and Lee et. al. (2019). Adapting this to our state-dependent hypothesis, we may expect low β^{υ} stocks (decile 1) bring lower return during bad states when ambiguity is presumably higher. Similarly, high β^{υ} stocks (decile 10) may deliver higher return during bad states when ambiguity is higher.

We use different variables to determine state of economy. First we define good (bad) states as periods with low (high) ambiguity. Therefore, our first set of state variables consists of market ambiguity ($\mho$) and other ambiguity measures (GDP_{AMB}, UNC, and VIX). More specifically, we calculate the median of relevant ambiguity proxy during our sample period and define a month as being in the good state if relevant ambiguity measure is lower than or equal to its median value during previous month. Similarly, a month is defined as being in the bad state if relevant ambiguity measure in the previous month is higher than its median value. Secondly, we use NBER based and OECD based recession dummies to define good and bad states by defining recession (expansionary) periods as bad (good). Finally, we define states based on SENT^ and SENT indices from Baker and Wurgler (2006). Accordingly, a month is defined as being in the bad (good) state if sentiment index in the previous month is below (more than equal to) the median value.

Table 17 presents and Figure C4 plots the five-factor alphas for each state defined by relevant state variables. Findings show that hypothesis that U-shaped pattern is driven by the time-varying state-dependent behavior of the $\beta^{\mho}$ is not valid. For both good and bad states defined by various state variables, return on stocks with higher absolute exposure to market ambiguity is higher than those with lower absolute exposure. However, the U-shaped pattern is more clear for the high uncertainty periods except when state variable is market ambiguity where the low uncertainty periods is associated with higher sloped U-shaped curve. This is expected considering that market ambiguity has a negative correlation with all of the other ambiguity proxies. Two extreme portfolios (decile 1 and decile 10) delivers even larger return than the rest of the portfolios during the NBER recession periods, with decile 10 being even higher. There are no important return differences during good

and bad states when state variable is OECD recession dummy. As for sentiment-based state variables, the U-shaped pattern does not completely disappear yet becomes asymmetric for good states (high sentiment) and bad states (low sentiment). While the stocks with most positive ambiguity beta (decile 10) brings larger returns during good states, stocks with most negative ambiguity beta (decile 1) brings larger returns during bad states, which is more obvious when state variable is SENT^. This is the closest observation that fits with hypothesis that U-shaped pattern may be driven by – at least inflated by – time varying behavior of market ambiguity beta offered by Ju and Miao (2012), Bali et. al. (2017), Lee et.al. (2019). However, the results favor just the reverse of the arguments by Ju and Miao (2012), Bali et. al. (2017), Lee et.al. (2019). More particularly, these studies argue that high ambiguity beta stocks have lower expected returns because they bring high return when ambiguity is high and thus they serve as natural hedging tool. Therefore, in a state dependent framework, we would expect high ambiguity beta stocks deliver high return during bad states and vice versa. However, Table 15 and Figure C3 shows that high ambiguity beta stocks (decile 10) brings high return during good states (high sentiment) and that low ambiguity beta stocks (decile 1) brings high return during bad states (low sentiment).

Table 17. Five-factor Alphas on Decile Portfolios During Different States

Panel A. Good States														
	(low)				(medium)					(high)				Number of
	1	2	3	4	5	6	7	8	9	10	10-1	1-5	10-5	months
$\beta^{\mho}$	0.80	0.61	0.57	0.52	0.58	0.52	0.64	0.58	0.82	1.05	0.25	0.22	0.46	156
	(7.05)	(5.61)	(4.60)	(4.17)	(4.12)	(3.20)	(3.85)	(3.43)	(4.38)	(4.77)	(1.07)	(1.11)	(2.62)	
β^{GDP}	0.50	0.40	0.31	0.31	0.30	0.24	0.31	0.35	0.32	0.51	0.01	0.20	0.21	156
	(2.81)	(3.65)	(3.30)	(3.70)	(3.87)	(3.18)	(4.19)	(4.69)	(3.90)	(4.66)	(0.06)	(0.97)	(1.65)	
β^{UNC}	0.55	0.41	0.39	0.31	0.36	0.31	0.48	0.43	0.47	0.61	0.06	0.18	0.25	156
	(2.78)	(4.26)	(4.71)	(3.64)	(3.98)	(3.06)	(3.79)	(4.76)	(4.94)	(5.09)	(0.28)	(0.79)	(2.14)	
β^{VIX}	0.33	0.23	0.26	0.26	0.34	0.24	0.30	0.37	0.38	0.55	0.23	-0.01	0.22	156
	(1.51)	(2.21)	(2.89)	(2.71)	(4.11)	(2.67)	(3.47)	(4.88)	(6.01)	(5.26)	(0.88)	(-0.04)	(1.74)	
NBER	0.55	0.46	0.41	0.39	0.44	0.37	0.46	0.46	0.50	0.72	0.18	0.11	0.29	283
	(4.91)	(5.82)	(5.32)	(5.27)	(5.64)	(4.95)	(5.47)	(5.84)	(5.32)	(6.20)	(1.13)	(0.78)	(2.73)	
OECD	0.55	0.51	0.40	0.40	0.37	0.29	0.36	0.36	0.42	0.68	0.13	0.19	0.32	192
	(4.28)	(5.59)	(5.78)	(5.45)	(4.95)	(3.64)	(3.92)	(3.81)	(4.10)	(4.53)	(0.74)	(1.15)	(2.33)	
SENT^	0.42	0.37	0.31	0.40	0.51	0.44	0.49	0.50	0.69	0.95	0.54	-0.10	0.44	142
	(2.96)	(3.05)	(2.54)	(3.36)	(4.70)	(3.77)	(3.54)	(4.50)	(4.73)	(4.89)	(2.64)	(-0.55)	(2.44)	
SENT	0.50	0.44	0.36	0.37	0.53	0.49	0.56	0.52	0.72	1.00	0.49	-0.03	0.46	143
	(3.65)	(3.85)	(3.10)	(3.32)	(4.68)	(4.15)	(4.05)	(4.10)	(4.18)	(4.67)	(2.39)	(-0.19)	(2.54)	
Panel B. Bad States														
	(low)				(medium)					(high)				Number of
	1	2	3	4	5	6	7	8	9	10	10-1	1-5	10-5	months
$\beta^{\mho}$	0.51	0.28	0.20	0.22	0.24	0.20	0.25	0.26	0.23	0.50	-0.01	0.27	0.26	155
	(2.60)	(2.68)	(1.94)	(2.25)	(2.81)	(2.70)	(3.66)	(3.74)	(3.10)	(4.45)	(-0.04)	(1.24)	(1.99)	
β^{GDP}	0.65	0.53	0.53	0.48	0.59	0.59	0.69	0.63	0.82	1.11	0.45	0.06	0.51	155
	(5.50)	(4.97)	(4.10)	(3.65)	(4.21)	(3.80)	(4.14)	(3.64)	(4.32)	(5.26)	(1.88)	(0.30)	(3.47)	
β^{UNC}	0.62	0.60	0.49	0.56	0.59	0.53	0.53	0.59	0.73	1.07	0.45	0.03	0.48	155
	(5.44)	(5.52)	(4.09)	(4.86)	(4.95)	(3.97)	(3.80)	(3.88)	(4.20)	(4.91)	(2.01)	(0.17)	(2.70)	
β^{VIX}	0.77	0.66	0.62	0.53	0.54	0.46	0.61	0.55	0.71	1.03	0.26	0.24	0.50	155
	(6.06)	(6.52)	(5.44)	(4.55)	(4.21)	(3.09)	(3.90)	(3.49)	(4.08)	(4.74)	(1.10)	(1.21)	(2.75)	
NBER	1.04	0.77	0.52	0.66	0.53	0.64	0.65	0.62	1.21	1.88	0.84	0.51	1.35	28
	(2.85)	(2.69)	(1.60)	(1.67)	(1.19)	(1.00)	(1.07)	(1.20)	(2.20)	(3.50)	(1.30)	(0.88)	(5.28)	
OECD	0.67	0.43	0.43	0.41	0.61	0.60	0.67	0.67	0.75	1.00	0.33	0.07	0.39	119
	(3.59)	(3.15)	(2.74)	(2.67)	(3.72)	(3.50)	(3.91)	(3.63)	(3.80)	(4.86)	(1.13)	(0.26)	(2.60)	
SENT^	0.93	0.54	0.44	0.30	0.29	0.22	0.30	0.29	0.33	0.56	-0.37	0.64	0.27	145
	(5.88)	(6.39)	(4.66)	(3.14)	(2.39)	(1.67)	(2.22)	(2.16)	(2.59)	(3.85)	(-1.81)	(3.04)	(2.44)	
SENT	0.75	0.45	0.40	0.34	0.32	0.24	0.31	0.34	0.37	0.61	-0.14	0.42	0.29	144
	(4.38)	(4.83)	(4.09)	(3.55)	(3.08)	(2.12)	(2.72)	(3.06)	(3.33)	(4.33)	(-0.58)	(1.93)	(2.39)	

Newey-West t-values in parentheses

4.4.7 Test portfolios

This section estimates market ambiguity beta of various test portfolios after controlling for the market. The value weighted decile portfolio returns are retrieved from data library of Chen and Zimmermann[13] who provides data and code for a large set of cross-sectional predictors. For each test variable, decile 1 corresponds to the portfolio with lowest test variables and decile 10 corresponds to the portfolio with highest test variables. We find that ambiguity betas of 15 out of 172 test variables changed monotonically across deciles and the ambiguity beta of highest decile and lowest decile are significantly different from each other. Among these 15 test variables, the ambiguity beta is increasing across deciles for seven variables, three being strictly increasing and four being moderately increasing. Among the rest of eight variables, ambiguity beta is strictly decreasing for three variables and moderately decreasing for five variables. We do not report test variables that show an obvious trend in ambiguity betas but are not statistically significant. Similarly, we do not report test variables that are significant but do not show any sign of trend across deciles. Appendix A3 provides the summary table for the selected test variables in alphabetic order[14].

Table 18 reports increasing ambiguity betas across deciles. In general, the significant betas with larger absolute size are clustered around the lower deciles. The betas estimated for the larger deciles are usually close to zero and statistically insignificant. For example, LRreversal stands for long-run reversal reported to be negatively associated with the expected return by De Bondt and Thaler (1985). Stocks with lowest long-run reversal (past losers) have a large, negative, and statistically significant ambiguity beta while stocks with highest long-run reversal

[13] https://www.openassetpricing.com/
[14] We use the same variable denotation with Chen and Zimmermann (2021).

(past winners) have a smaller, positive, and insignificant ambiguity beta. Similarly, stocks with lowest Price – calculated as log(Price)- have the most negative market ambiguity exposure with the highest t-value while ambiguity beta of highest price portfolio is lower than unity with insignificant t-value.

A striking feature of Table 18 is that it includes various growth variables, each with a large and significant negative beta for the decile 1 (lowest growth stocks). Among them, MeanRankRevGrowth stands for the revenue growth rank which is calculated as weighted average of past five-year annual revenue growth rank and argued to be positively related with stock return (Lakonishok, Shleifer, & Vishny, 1994). While AssetGrowth is calculated by change in total assets (Cooper, Gulen, & Schill, 2008), InvGrowth corresponds to inventory growth deflated by GNP (Thomas & Zhang, 2002), both reported to be negative predictors of cross-section of expected returns. The results indicate that stocks with lowest growth in terms of revenue, asset, or even inventory have a statistically and economically more negative market ambiguity exposure compared to highest growth stocks.

A final variable set in Table 18 is intangible returns. According to Daniel and Titman (2006), information can be decomposed as tangible and intangible information. If the total return is taken a good measure for overall information; the portion of total return attributable to summary accounting measures can be defined tangible information (return) and the residuals can be regarded as intangible information (return). By decomposing various price-scaled accounting variables which are found to be related with expected into their tangible and intangible return component, Daniel and Titman (2006) conclude that mainly the intangible component of these variables can explain the variation in stock returns. Our results show that stocks with lower intangible returns have the most negative ambiguity

exposure, which is more obvious when we measure intangible return based on sale-to-price ratio (IntanSP). As moving to highest decile, the ambiguity beta fairly increases and becomes closer to zero. Ambiguity beta becomes even positive (but insignificant) for the portfolio with highest intangible returns based on BM ratio (IntanBM). Actually, neither of deciles sorted by IntanBM has significant ambiguity beta, but the difference between the highest and lowest decile is significant.

Table 19 reports decreasing ambiguity betas across deciles. Unlike Table 18, this time the significant betas with larger absolute size are clustered around the higher deciles and betas for lowest deciles are mostly statistically insignificant. First test variable having a strict negative trend with ambiguity beta is Frontier standing for the distance to the efficient frontier index; or more specifically, residual of regression equation regressing firm value to various firm characteristics and fundamentals. Nguyen and Swanson (2009) report that stocks closer to efficient market frontier index (lower Frontier) underperforms compared to stocks more distant to index (higher Frontier). Our results show that there is a strict decrease in ambiguity beta as moving to higher Frontier portfolio and that highest Frontier is associated with the most negative ambiguity beta.

Table 19 includes four variables related to return persistency, three of them being different versions of momentum provided by Heston and Sadka (2008). Heston and Sadka (2008) examine the momentum behavior in the stock returns in a very comprehensive manner by covering horizon up to 20 years and by decomposing overall momentum into off season and on season for each horizon setting. In simplified terms, while on season refers to sorting stocks based on their average return on the annual lags such as past 12-month, past 24-month, past 36-month etc., off season refers to sorting stocks based on their average return on other months than

the annual lags. Among various versions of momentum variables, ambiguity beta is significantly reducing from lowest to highest decile sorted by three momentum variables, namely on season momentum between 6-year and 11-year horizon (MomSeason06YrPlus), off season momentum between 6-year and 11-year horizon (MomOffSeason06YrPlus), and on season momentum between 2-year and 5-year horizon (MomSeason). This implies that stocks with higher average return during longer horizon (6 to 11 years) have the most negative ambiguity beta in both on and off season. Over medium-to-long horizon (2 to 5 years), on the other hand, only stocks with higher on season momentum have largest negative coefficient beta.

The other variable related with the return persistency is trend factor which is designed to catch simultaneously the short, medium, and long-term trend in the price by using information in the moving average prices at different lags (Han, Zhou, & Zhu, 2016). As our results so far show that ambiguity exposure is strongly related with the return persistency and reversal, it is not surprising that portfolios sorted decile by trend factor has statistically different ambiguity betas. The ambiguity beta across decile sorted by trend factor is not only strictly negative and linear, it also has significant coefficients with different sign for highest decile and lowest decile. While lowest trend factor portfolio has significant and positive ambiguity beta, highest trend factor portfolio has a significant and negative ambiguity beta. Finally, ambiguity beta is the most negative for the stocks with higher growth over inventory growth (Sales growth over inventory growth), higher advertising expense (AdExp), and highest cash flow-to-price volatility (VarCF).

Overall, none of the test variables in Table 18 and Table 19 is able to provide a fair ambiguity beta spectrum from most negative to most positive. Rather, they help distinguishing the most negative ambiguity beta stocks from the zero ambiguity beta

stocks. Only trend factor (Trend) and the Revenue Growth Rank (MeanRankRevGrowth) may be an exception because they have significant coefficient with different signs for the highest and lowest decile, the t values for at least one of the portfolios are not quite large though.

Appendix D1 and Appendix D2 plots the ambiguity betas across deciles and label the significant and insignificant betas with differing colors to portrait the related trends. Although it is necessary to approach the results with caution and avoid making too ambitious conclusions, we can infer that negative ambiguity beta stocks tend to be losers over long-term, more speculative (low price), desperate in terms of not only fundamental growth (low revenue growth, low asset growth, low inventory growth) but also intangible growth (low intangible returns). Besides, they tend to be more inefficient (higher distance to efficient frontier index), have higher return persistency (high off and on season momentum, have higher slope on trend factor), higher revenue growth compared to inventory growth, higher cash-flow volatility, and higher advertising. expense. On the other hand, positive ambiguity beta stocks tend to have lower slope on the trend factor and have higher revenue growth.

Table 18. Increasing Ambiguity Beta

Panel A. Strictly increasing trend in ambiguity beta												
		(low)				(medium)					(high)	
Denoted by	Variable name	1	2	3	4	5	6	7	8	9	10	10-1
LRreversal	Long-run reversal	-12.10	-7.22	-5.81	-2.50	-2.45	-1.66	-0.02	-0.03	0.35	1.10	13.20
		(-3.44)	(-2.72)	(-3.73)	(-2.21)	(-1.77)	(-1.50)	(-0.02)	(-0.03)	(0.30)	(0.68)	(3.35)
Price	Price	-12.06	-5.44	-6.89	-4.07	-1.61	-4.02	-1.47	-1.57	-0.46	0.67	12.73
		(-2.02)	(-1.43)	(-1.81)	(-1.38)	(-0.80)	(-2.10)	(-1.18)	(-1.42)	(-0.57)	(1.14)	(2.04)
MeanRankRevGrowth	Revenue Growth Ranking	-5.16	-1.88	-0.60	-0.60	-0.93	-0.96	-0.33	-1.09	-0.75	2.56	7.72
		(-3.37)	(-1.53)	(-0.73)	(-0.56)	(-0.78)	(-0.89)	(-0.28)	(-0.87)	(-0.51)	(1.88)	(3.68)
Panel B. Moderately increasing trend in ambiguity beta												
		(low)									(high)	
Denoted by	Variable name	1	2	3	4	5	6	7	8	9	10	10-1
AssetGrowth	Asset Growth	-6.03	-2.58	-0.77	0.07	-1.91	-0.58	-2.45	-1.04	-0.79	-0.06	5.97
		(-3.22)	(-2.19)	(-0.69)	(0.06)	(-2.14)	(-0.61)	(-3.74)	(-1.12)	(-0.63)	(-0.04)	(2.68)
InvGrowth	Inventory Growth	-4.77	-2.11	-3.59	-0.31	-0.22	-1.14	-2.07	0.51	-2.00	0.77	5.54
		(-3.30)	(-1.41)	(-2.51)	(-0.31)	(-0.19)	(-0.99)	(-1.97)	(0.45)	(-1.19)	(0.35)	(2.40)
IntanSP	Intangible return using sale-to-price	-8.75	-5.58	-2.29	-2.29	-1.6	-0.68	-0.19	-0.57	1.48	-1.55	7.20
		(-2.47)	(-2.39)	(-1.23)	(-1.31)	(-1.14)	(-0.55)	(-0.19)	(-0.44)	(1.27)	(-1.06)	(1.74)
IntanBM	Intangible return using BM	-3.82	-2.17	-3.43	-1.91	-3.00	-0.54	-1.05	0.03	0.7	1.59	5.41
		(-1.40)	(-1.16)	(-1.47)	(-1.19)	(-1.87)	(-0.42)	(-0.75)	(0.04)	(0.66)	(1.48)	(1.72)

Newey-West t-values in parentheses

Table 19. Decreasing Ambiguity Beta

Panel A. Strictly decreasing trend in ambiguity beta												
		(low)				(medium)					(high)	
Denoted by	Variable name	1	2	3	4	5	6	7	8	9	10	10-1
Frontier	Distance to efficient frontier index	1.86 (1.10)	0.01 (0.01)	1.05 (1.02)	-0.01 (-0.01)	-2.25 (-1.66)	-1.42 (-1.07)	-5.34 (-2.41)	-3.70 (-1.86)	-4.58 (-1.73)	-7.73 (-2.43)	-9.58 (-2.35)
TrendFactor	Trend Factor	3.26 (1.78)	1.26 (0.90)	0.57 (0.49)	0.91 (0.77)	1.31 (1.31)	-0.56 (-0.74)	0.91 (1.12)	-1.97 (-1.69)	-4.53 (-1.86)	-5.66 (-1.67)	-8.92 (-2.05)
MomSeason06YrPlus	Return seasonality years 6 to 10	3.11 (1.33)	1.12 (0.66)	0.39 (0.32)	-1.01 (-0.80)	0.22 (0.16)	0.01 (0.01)	-1.93 (-1.54)	-2.47 (-1.82)	-4.47 (-3.17)	-5.54 (-3.32)	-8.65 (-2.84)
Panel B. Moderately decreasing trend in ambiguity beta												
		(low)				(medium)					(high)	
Denoted by	Variable name	1	2	3	4	5	6	7	8	9	10	10-1
MomOffSeason06YrPlus	Off season reversal years 6 to 10	1.35 (0.55)	-0.7 (-0.46)	-0.5 (-0.37)	-1.52 (-1.15)	0.39 (0.31)	-0.57 (-0.39)	-1.54 (-0.98)	-3.24 (-1.93)	-3.06 (-2.07)	-3.91 (-2.37)	-5.26 (-1.71)
MomSeason	Return seasonality years 2 to 5	0.6 (0.31)	2.3 (1.54)	1.4 (1.08)	-1.19 (-0.93)	-0.69 (-0.47)	-0.63 (-0.53)	-0.4 (-0.49)	-1.09 (-0.90)	-4.42 (-2.73)	-4.78 (-2.45)	-5.39 (-2.15)
GrSaleToGrInv	Sales growth over inventory growth	-0.09 (-0.05)	-1.17 (-0.80)	0.01 (0.01)	-0.68 (-0.62)	0.31 (0.33)	-0.87 (-0.82)	-1.01 (-1.31)	-1.53 (-1.21)	-2.60 (-1.98)	-3.65 (-2.80)	-3.57 (-1.75)
VarCF	Cash flow-to-price variance	-0.95 (-1.20)	-0.25 (-0.35)	0.56 (0.75)	1.08 (1.09)	-1.23 (-1.38)	-3.63 (-2.99)	-1.13 (-0.71)	-2.08 (-1.36)	-2.84 (-1.50)	-7.46 (-2.61)	-6.51 (2.01)
AdExp	Advertising expense	0.57 (0.31)	0.70 (0.40)	-0.76 (-0.50)	-0.31 (-0.21)	-1.09 (-0.67)	-0.04 (-0.03)	1.39 (0.85)	0.06 (0.04)	-2.7 (-1.39)	-6.49 (-2.42)	-7.06 (-1.94)

Newey-West t-values in parentheses

4.5 Conclusion

Theoretical studies have long accepted that uncertainty can be separated into risk and uncertainty and propose models for how these two affect financial markets (Epstein & Schneider, 2010; Izhakian, 2020; Ju & Miao, 2012; Kogan & Wang, 2003; Mukerji et al., 2022; Ui, 2011). However, empirical studies measuring the cross-sectional impact of ambiguity are very limited because, unlike risk, there is no straightforward and agreed-upon measure of ambiguity. Previously used ambiguity proxies are disagreement in future stock performance (Anderson et al., 2005, 2009; Diether et al., 2002; Hsiao, 2022; Jiang & Sun, 2014) or exposure to aggregate ambiguity such as exposure to uncertainty index (Bali et al., 2017), exposure to VIX (Bekaert et al., 2013; Bloom, 2009), or exposure to disagreement on expected GDP-growth (Lee et al., 2019).

Aside from these ambiguity measures give conflicting results, they may be insufficient at some points to accurately measure ambiguity. For example, the measure of disagreement may have an overstatement or understatement bias when the forecaster has an incentive to do so (Kim & Zapatero, 2022). The exposure to aggregate ambiguity, on the other hand, usually derived from conditional volatility or dispersion in forecast of macroeconomic variables. And thus it underweights either ambiguity hidden at the higher moments or the ambiguity driven by market itself.

In this study, we aim to test whether the market ambiguity is priced in the cross-section of returns by quantifying the market ambiguity with the monthly standard deviation of daily market return distribution following Brenner and Izhakian (2018). Our results indicate that market uncertainty ($\mho$) is non-linearly priced in the cross-section of return for US market, as there is a U-shaped relationship between them, being more pronounced for smaller stocks. Besides, U-shaped relationship is

usually left-skewed, meaning that absolute exposure to market ambiguity (absolute value of β^{υ}) is a positive predictor of expected returns, and the absolute effect of positive exposure being the larger compared to the absolute effect of negative exposure. Results are essentially robust to widely-known cross-sectional return predictors (size, value, momentum, reversal, market beta, idiosyncratic volatility, synchronicity, market liquidity exposure, analyst coverage etc.), other ambiguity proxies (exposure to dispersion in forecast of GDP growth, exposure to uncertainty index, exposure to VIX, disagreement among analyst recommendation), different portfolio holding and formation periods, and different states (high and low ambiguity periods, recessions, and sentiment). It is worth noting, though, that there are two exceptional cross-sectional predictors – namely momentum and idiosyncratic volatility- that reduce, but fail to fully explain, the U-shaped pattern representation. Besides, we find that sentiment partly explains overall U-shaped trend as the trend becomes asymmetric during high and high and low sentiment periods.

We also estimate ambiguity betas of the portfolios sorted decile by various test variables. Our analysis on test portfolios mainly help having further insights on the features of negative ambiguity beta stocks. Although driving too general conclusion is not proper as these were not bivariate portfolios that can strongly control for tested variables, we may still infer that stocks with high negative market ambiguity beta are tend to be long-term loser and more speculative (low price) stocks, having desperate fundamental and intangible growth. They are also more likely to be distant from efficient market frontier, have higher return persistency, higher revenue growth compared to inventory growth, higher cash-flow volatility, and higher advertising expense. On the other hand, stocks having positive ambiguity exposure tend to be lower slope on the trend factor and have higher revenue growth.

Overall, U-shaped association between ambiguity exposure and expected stock return is quite puzzling because a standard asset pricing framework predicts a linear relationship between expected returns and ambiguity. None of the robustness tests we employed could fully explain this trend. Resolving the reasons for the U-shaped trend, therefore, requires further analysis that is the beyond of this chapter. However, we conjecture that this puzzling observation may be resolved with one or more of the following further tests. First, our analysis shows that ambiguity beta is essentially associated with return persistency (or reversal). Further testing can be done to detect any causal and other form of relevance between momentum and ambiguity beta. Second, our study does not account for idiosyncratic ambiguity because we do not have intraday returns for each stocks. Incorporating idiosyncratic ambiguity may differentiate the returns on highest and lowest deciles and portrait a rather linear relationship as hypothesized by many theoretical studies. Finally, option-theoretic argument may be relevant for our results. According to Merton (1974), equity can be regarded as a call option on the firm's asset with a strike value equal to value of debt. Thus asset volatility and expected stock returns are negatively associated, and this association is more pronounced for firms with higher leverage. Based on option-theoretic argument, for example, Johnson (2004) argues analyst dispersion is a good proxy for unpriced information risk when asset value is not observable and show this as the reason why Diether et. al. (2002) found a counterintuitive negative association between stock return and disagreement. From a similar perspective, further testing can diagnose how ambiguity beta, financial distress and asset volatility are related.

CHAPTER 5

CONCLUDING REMARKS

Starting from Knight (1921), decision making theory has recognized that there are two different forms of uncertainty: risk and ambiguity. While risk corresponds to the situations that a unique probability can be assigned to each outcomes, ambiguity refers to the situations in which a unique probability distribution does not exit. Many theoretical financial models perceive ambiguity as an important missing piece that largely explains the systematic difference between prices predicted by fundamental models and those actually observed. However, until recently, the hypothesized theoretical effect of ambiguity on stock prices was not empirically documented since ambiguity is not directly observable and is difficult be quantified with an agreed proxy.

While number of empirical evidence is still very limited, the existing ones have some conflicting results as well. In this respect, the effect of ambiguity in financial markets, which is claimed to be very strong in theory, has been empirically revealed in a very limited and fuzzy way. Therefore, the main aim of this study is to contribute to the literature on the quantifying ambiguity and empirical testing its effect on asset prices. For quantifying the ambiguity, we use a recent methodology offered by Brenner and Izhakian (2018) who measure ambiguity by probability perturbations (uncertain probabilities) and aversion to ambiguity by aversion to mean-preserving spreads in these probabilities. Beyond being simple and intuitive by definition, this measure is also risk-independent, making it highly applicable for the empirical testing.

Since empirical studies focused mainly on the US market, our initial empirical analysis measures the ambiguity and ambiguity attitude in Turkey which is one of the major emerging markets. Although two empirical studies using the analogous ambiguity measure with Brenner and Izhakian (2018) find that ambiguity is associated negatively with stock prices in Turkish market (Altay, 2015; Şahin & Danışoğlu, 2022), none of them tracts the attitude toward ambiguity. We find that between January 1998 and February 2021, there is an ambiguity premium that is distinct from risk premium in Turkish market. Besides, our results show that ambiguity attitude in Turkish market is different from reported attitude in US markets by Brenner and Izhakian (2018). While invertors trading in US market are ambiguity averse (lover) when they expect a gain (loss), invertors trading in Turkish market are ambiguity averse under positive and negative market expectations, but they remain ambiguity neutral for lateral markets, where direction is not clear. Results essentially same when we control for higher moments return, sentiment, and recessions. Our results point that attitude towards risk and ambiguity may differ across markets with different national and cultural characteristics.

Our second empirical analysis examines cross-sectional effect of ambiguity exposure on stock return in US from February 1993 through December 2020. The results show that market ambiguity, is non-linearly related with expected return and exhibits a left skewed U-shaped pattern, meaning that absolute effect of positive ambiguity exposure is larger than absolute effect of negative ambiguity exposure. Besides, U-shaped association is more pronounced for smaller firms. Results are essentially robust to cross-sectional return predictors, other ambiguity proxies, different holding or formation periods, and different states. However, momentum and idiosyncratic volatility are found to reduce the slope of U-shaped pattern but they fail

to totally explain non-linear association. We also find that sentiment partially explains the overall U-shaped trend because the trend is asymmetric during periods of high and low sentiment. Finally, we estimate ambiguity beta for the deciles sorted by 172 different test portfolios find that 15 of them helps distinguishing the features of negative ambiguity beta stocks. More specifically, the stocks with most negative market ambiguity exposure tend to be long-term losers, speculative (low-priced) stocks with more desperate fundamental and intangible growth. Moreover, they have higher distance from efficient market frontier, higher return persistency, higher revenue growth compared to inventory growth, higher cash-flow volatility, and higher advertising expense.

Overall, our study portraits how ambiguity priced in financial markets by covering one major emerging market (Turkey) and one major developed market (US). The findings for both emerging and developed markets contain some interesting insights. However, investigating the determinants of the relationships depicted requires further analysis and is beyond the scope of this thesis. For example, ambiguity attitudes of investors traded in Turkish market is completely different than the US counterparts. Differences in cultural attitudes towards ambiguity may result from variations in cultural and national dynamics such as exposure to different types of socio-political and/or economic uncertainty. We also find a left skewed U-shaped association between ambiguity exposure and cross-sectional expected stock return in US market, which is puzzling under a standard asset pricing model which predicts ambiguity linearly tied to expected stock return. Non-linear relationship may be explained by relevance of momentum with ambiguity, idiosyncratic ambiguity, or option-theoretic models. However, these conjectures for the possible causes are purely hypothetical and should be investigated by further analysis. Future research

may reveal the underlying dynamics by conducting detailed tests to identify the causes of the observed effects of ambiguity on the financial market.

APPENDIX A

VARIABLE DEFINITION

Table A1. Ambiguity Proxies and Betas

Denoted by	Variable Name	Explanation	Relevant regression equation
$\beta^{\mho}$	Market ambiguity beta	Estimate of the loading on the ambiguity after controlling for the market	$r_{i,t} - r_f = \alpha_{i,t} + \beta_{i,t}^{\mho} . \mho_t + \beta_{i,t} . MKT_t + \varepsilon_{i,t}$
β^{GDP}	GDP-based ambiguity beta	Estimate of the loading on the GDP-based ambiguity after controlling for the market	$r_{i,t} - r_f = \alpha_{i,t} + \beta_{i,t}^{GDP} . GDP_{AMB_t} + \beta_{i,t} . MKT_t + \varepsilon_{i,t}$
β^{UNC}	Economic uncertainty index beta	Estimate of the loading on the economic uncertainty index after controlling for the market	$r_{i,t} - r_f = \alpha_{i,t} + \beta_{i,t}^{UNC} . UNC_t + \beta_{i,t} . MKT_t + \varepsilon_{i,t}$
β^{VIX}	VIX beta	Estimate of the loading on the VIX index after controlling for the market	$r_{i,t} - r_f = \alpha_{i,t} + \beta_{i,t}^{VIX} . VIX_t + \beta_{i,t} . MKT_t + \varepsilon_{i,t}$
$\mho$	Market ambiguity	The monthly standard deviation of daily market return distribution	-
GDP_{AMB}	GDP-based ambiguity	The cross-sectional standard deviation of the forecasted GDP growth	-
STDEV	Dispersion in the analysts' recommendation	Cross-sectional standard deviation of five-tier recommendation retrieved from I/B/E/S	
UNC	Economic uncertainty index	Conditional volatility of the unforecastable component of a large number of economic indicators	-
VIX	VIX	Chicago board options exchange volatility index	-

Table A2. Cross-Sectional Return Predictors

Denoted by	Variable Name	Explanation	Relevant regression equation
β^{LIQ}	Liquidity beta	Estimate of the loading on the liquidity in five factor model	$r_{i,t} - r_f = \alpha_{i,t} + \beta_{i,t}^{1}.MKT_t + \beta_{i,t}^{2}.SMB_t + \beta_{i,t}^{3}.HML_t + \beta_{i,t}^{4}.WML_t + \beta_{i,t}^{LIQ}.LIQ_t + \varepsilon_{i,t}$
β^{MKT}	Market beta	Estimate of the loading on market premium in CAPM	$r_{i,t} - r_f = \alpha_{i,t} + \beta_{i,t}^{MKT}.MKT_t + \varepsilon_{i,t}$
ANALYST	Analyst coverage	Number of analyst following the relevant stock	-
BM	Value	Book value of stockholder equity plus deferred taxes and investment tax credit minus the book value of preferred stock at the end of the last fiscal year t −1, divided by the market value of equity at the end of December of year t −1	-
COSKEW	Co-skewness	Estimate of the loading on market excess return square in regression equation regressing excess stock return on market excess return and square of market excess return	$r_{i,t} - r_f = \alpha_{i,t} + \beta_{i,t}.MKT_t + \gamma_{i,t}.MKT_t^2 + \varepsilon_{i,t}$
IVOL	Idiosyncratic volatility	Standard deviation of residuals of the five-factor model	$r_{i,t} - r_f = \alpha_{i,t} + \beta_{i,t}^{1}.MKT_t + \beta_{i,t}^{2}.SMB_t + \beta_{i,t}^{3}.HML_t + \beta_{i,t}^{4}.WML_t + \beta_{i,t}^{LIQ}.LIQ_t + \varepsilon_{i,t}$
MOM	Momentum	Stock return in the past year skipping the most recent month (11 months total)	-
REV	Short-term reversal	Stock return over prior month	-
SE	Ambiguity of market beta	Absolute value of standard error of the market beta scaled by coefficient of market beta	-
SIZE	Size	Market capitalization which is the product of price per share and shares outstanding.	-
SYNC	Synchronicity	R-square of five factor model	$r_{i,t} - r_f = \alpha_{i,t} + \beta_{i,t}^{1}.MKT_t + \beta_{i,t}^{2}.SMB_t + \beta_{i,t}^{3}.HML_t + \beta_{i,t}^{4}.WML_t + \beta_{i,t}^{LIQ}.LIQ_t + \varepsilon_{i,t}$

Table A3. Selected Test Variables

Denoted by	Variable Name	Details	Related Study	Reported Relationship with Return	Estimated Relationship with Ambiguity Beta
AdExp	Advertising Expense	Advertising expense scaled by market value of equity	Chan, Lakonishok and Sougiannis (2001)	(+)	Moderate (-)
AssetGrowth	Asset Growth	Growth of total asset (annual basis)	Cooper, Gulen and Schill (2008)	(-)	Moderate (+)
Frontier	Distance to efficient frontier index	Residual of monthly rolling regression equation regressing log(BM) on firm characteristics and fundamentals	Nguyen and Swanson (2009)	(+)	Strict (-)
GrSaleToGrInv	Sales growth over inventory growth	Sales growth during previous month minus inventory growth during previous month	Abarbanell and Bushee (1998)	(+)	Moderate (-)
IntanBM	Intangible return using BM	Residual of monthly rolling regression equation regressing stock return on 5-year lagged log(BM) and a constructed regressor that is the change in BM from 5 years ago to today plus the five-year stock return.	Daniel and Titman (2006)	(-)	Moderate (+)
IntanSP	Intangible return using Sale to Price	Residual of monthly rolling regression equation regressing stock return on 5-year lagged sales to price ratio (SP) and a constructed regressor that is the change in SP from 5 years ago to today plus the five-year stock return.	Daniel and Titman (2006)	(-)	Moderate (+)
InvGrowth	Inventory Growth	Inventory growth deflated by GNP	Thomas and Zhang (2002)	(-)	Moderate (+)
LRreversal	Long-run reversal	Return between t-36 and t-13	De Bondt and Thaler (1985)	(-)	Strict (+)
MeanRankRevGrowth	Revenue Growth Ranking	Weighted average of past five-year ranking based on annual revenue growth	Lakonishok, Shleifer, and Vishny (1994)	(+)	Strict (+)

MomOffSeason06YrPlus	Off season reversal years 6 to 10	Average return in the other months during the past 6 to 10 years.	Heston and Sadka (2008)	(-)	Moderate (-)
MomSeason	Return seasonality years 2 to 5	Average return in the same month during the past 2 to 5 years.	Heston and Sadka (2008)	(+)	Moderate (-)
MomSeason06YrPlus	Return seasonality years 6 to 10	Average return in the same month during the past 2 to 5 years.	Heston and Sadka (2008)	(+)	Strict (-)
Price	Price	Logarithm of price	Blume and Husic (1972)	(-)	Strict (+)
TrendFactor	Trend Factor	For each month and each stock, a trend signal is calculated as normalized moving average stock prices, and slope of trend signal in month t is estimated by cross-section regression of return over aggregated signal. Expected return for each stock during month t is calculated as multiple sums of trend signal in t-1 and expected trend signal in month t. The return difference between highest expected return quintile and lowest expected return quintile is the trend factor.	Han, Zhou, and Zhu (2016)	(+)	Strict (-)
VarCF	Cash-flow to price variance	Rolling variance of cash flow. Cash flow is calculated as (net income + depreciation)/ market equity	Haugen and Baker (1996)	(-)	Moderate (-)

APPENDIX B

CONDITIONAL BIVARIATE PORTFOLIO RETURNS

Table B1. Equal-Weighted Returns on Decile Portfolios Conditional on Size (SIZE)

Panel A. Excess return

		Ambiguity beta (β^{U})												
		(lowest)				(medium)					(highest)			
		1	2	3	4	5	6	7	8	9	10	10-1	1-5	10-5
SIZE	1 (smallest)	2.24	1.29	1.18	0.98	1.07	1.10	1.16	1.30	1.47	2.08	-0.16	1.17	1.01
		(7.14)	(5.48)	(5.57)	(4.88)	(5.68)	(5.43)	(5.41)	(5.56)	(5.62)	(6.92)	(-1.10)	(5.39)	(5.60)
	2	2.01	1.66	1.15	0.95	0.90	0.94	1.08	1.16	1.31	1.92	-0.09	1.11	1.02
		(5.11)	(5.39)	(4.37)	(4.38)	(4.50)	(4.44)	(4.72)	(4.19)	(4.41)	(5.31)	(-0.47)	(3.85)	(4.30)
	3	1.45	1.29	1.23	0.94	0.98	0.95	0.91	1.11	1.31	1.46	0.01	0.47	0.49
		(3.61)	(4.06)	(4.45)	(3.68)	(3.81)	(3.94)	(3.33)	(3.84)	(3.98)	(3.84)	(0.08)	(1.77)	(2.20)
	4	0.95	1.08	1.05	1.01	0.95	0.89	0.86	0.98	1.16	1.09	0.14	-0.00	0.13
		(2.42)	(3.40)	(3.39)	(3.70)	(3.60)	(3.26)	(3.25)	(3.23)	(3.76)	(2.84)	(0.75)	(-0.02)	(0.73)
	5 (largest)	0.82	0.80	0.83	0.83	0.87	0.81	0.90	0.90	0.87	0.99	0.17	-0.05	0.12
		(2.17)	(2.78)	(3.12)	(3.33)	(3.45)	(3.12)	(3.28)	(3.44)	(3.07)	(2.88)	(0.88)	(-0.23)	(0.68)

Panel B. Five factor alpha

		Ambiguity beta (β^{U})												
		(lowest)				(medium)					(highest)			
		1	2	3	4	5	6	7	8	9	10	10-1	1-5	10-5
SIZE	1 (smallest)	1.70	1.06	1.00	0.74	0.82	0.77	0.84	0.94	1.08	1.65	-0.05	0.87	0.82
		(11.07)	(6.98)	(6.87)	(5.27)	(6.61)	(5.96)	(6.77)	(7.46)	(7.35)	(11.35)	(-0.39)	(5.75)	(6.16)
	2	1.01	0.96	0.53	0.55	0.47	0.49	0.56	0.55	0.66	1.19	0.19	0.53	0.72
		(6.93)	(7.65)	(4.36)	(4.25)	(3.87)	(4.45)	(4.41)	(4.42)	(5.19)	(6.06)	(0.89)	(2.76)	(3.70)
	3	0.45	0.40	0.43	0.25	0.40	0.37	0.28	0.40	0.51	0.65	0.20	0.04	0.24
		(3.01)	(3.23)	(3.96)	(2.68)	(3.75)	(3.19)	(2.26)	(3.27)	(3.49)	(3.99)	(0.97)	(0.23)	(1.40)
	4	-0.11	0.17	0.28	0.31	0.31	0.26	0.22	0.30	0.40	0.20	0.32	-0.43	-0.11
		(-0.80)	(1.61)	(2.58)	(3.36)	(3.26)	(2.28)	(2.02)	(2.44)	(2.58)	(1.35)	(1.54)	(-2.24)	(-0.78)
	5 (largest)	-0.07	0.01	0.09	0.16	0.24	0.19	0.29	0.33	0.20	0.18	0.24	-0.30	-0.06
		(-0.44)	(0.09)	(0.96)	(1.72)	(2.41)	(2.27)	(2.91)	(3.07)	(1.67)	(1.04)	(1.08)	(-1.67)	(-0.37)

Newey-West t-values in parentheses

Table B2. Value-Weighted Returns on Decile Portfolios Conditional on Size (SIZE)

Panel A. Excess return

		Ambiguity beta (β^{U})												
		(lowest)				(medium)					(highest)			
		1	2	3	4	5	6	7	8	9	10	10-1	1-5	10-5
SIZE	1 (smallest)	2.22	1.27	1.17	0.95	1.07	1.07	1.16	1.27	1.46	2.08	-0.14	1.15	1.01
		(6.60)	(5.30)	(5.34)	(4.76)	(5.73)	(5.24)	(5.22)	(5.29)	(5.34)	(6.73)	(-0.93)	(4.95)	(5.15)
	2	1.96	1.68	1.13	0.93	0.94	0.96	1.07	1.17	1.30	1.89	-0.07	1.02	0.95
		(5.00)	(5.43)	(4.19)	(4.16)	(4.52)	(4.39)	(4.64)	(4.17)	(4.32)	(5.13)	(-0.39)	(3.52)	(3.96)
	3	1.43	1.29	1.24	0.95	0.96	0.95	0.92	1.08	1.30	1.44	0.01	0.47	0.48
		(3.57)	(4.04)	(4.52)	(3.64)	(3.69)	(3.85)	(3.29)	(3.71)	(3.92)	(3.79)	(0.08)	(1.76)	(2.13)
	4	0.90	1.07	1.04	1.01	0.93	0.88	0.86	0.96	1.14	1.04	0.15	-0.03	0.12
		(2.32)	(3.38)	(3.38)	(3.62)	(3.53)	(3.24)	(3.22)	(3.15)	(3.71)	(2.69)	(0.78)	(-0.12)	(0.61)
	5 (largest)	0.77	0.61	0.67	0.62	0.68	0.62	0.88	0.83	0.84	0.91	0.14	0.08	0.22
		(1.93)	(2.01)	(2.29)	(2.41)	(2.68)	(2.22)	(3.36)	(3.38)	(3.03)	(2.87)	(0.69)	(0.35)	(1.40)

Panel B. Five factor alpha

		Ambiguity beta (β^{U})												
		(lowest)				medium)					(highest)			
		1	2	3	4	5	6	7	8	9	10	10-1	1-5	10-5
SIZE	1 (smallest)	1.62	0.99	0.97	0.71	0.81	0.74	0.82	0.85	1.00	1.60	-0.02	0.81	0.80
		(10.76)	(6.56)	(6.30)	(5.06)	(6.57)	(5.57)	(6.42)	(6.70)	(6.58)	(10.78)	(-0.11)	(5.32)	(5.39)
	2	0.96	0.97	0.49	0.50	0.50	0.49	0.55	0.54	0.62	1.14	0.18	0.46	0.64
		(6.43)	(7.35)	(3.92)	(3.79)	(3.82)	(4.40)	(4.32)	(4.27)	(4.79)	(5.63)	(0.86)	(2.24)	(3.23)
	3	0.42	0.40	0.43	0.24	0.38	0.35	0.30	0.35	0.47	0.61	0.19	0.04	0.23
		(2.87)	(3.04)	(4.02)	(2.56)	(3.52)	(2.93)	(2.38)	(2.95)	(3.23)	(3.77)	(0.90)	(0.20)	(1.31)
	4	-0.16	0.15	0.27	0.31	0.29	0.26	0.20	0.31	0.41	0.18	0.34	-0.46	-0.12
		(-1.10)	(1.41)	(2.41)	(3.18)	(3.05)	(2.32)	(1.72)	(2.40)	(2.53)	(1.15)	(1.58)	(-2.33)	(-0.82)
	5 (largest)	-0.27	-0.21	-0.18	-0.06	0.01	-0.09	0.23	0.19	0.21	0.13	0.40	-0.28	0.12
		(-1.39)	(-1.67)	(-1.70)	(-0.55)	(0.14)	(-0.79)	(2.42)	(1.87)	(1.82)	(0.90)	(1.81)	(-1.38)	(0.70)

Newey-West t-values in parentheses

APPENDIX C

FIVE-FACTOR ALPHAS

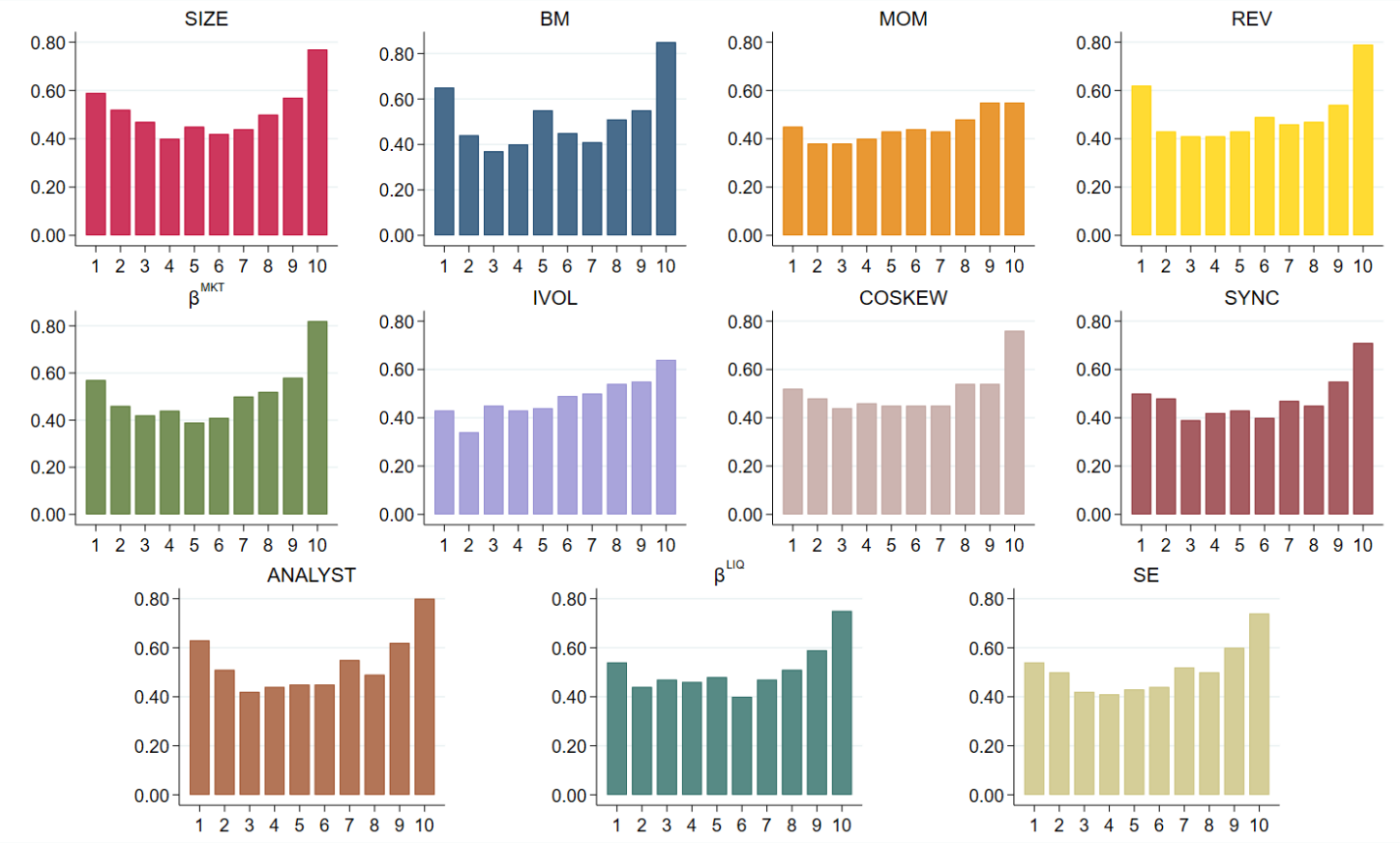


Figure C1. Average five-factor alphas on decile portfolios conditional on cross-sectional return predictors

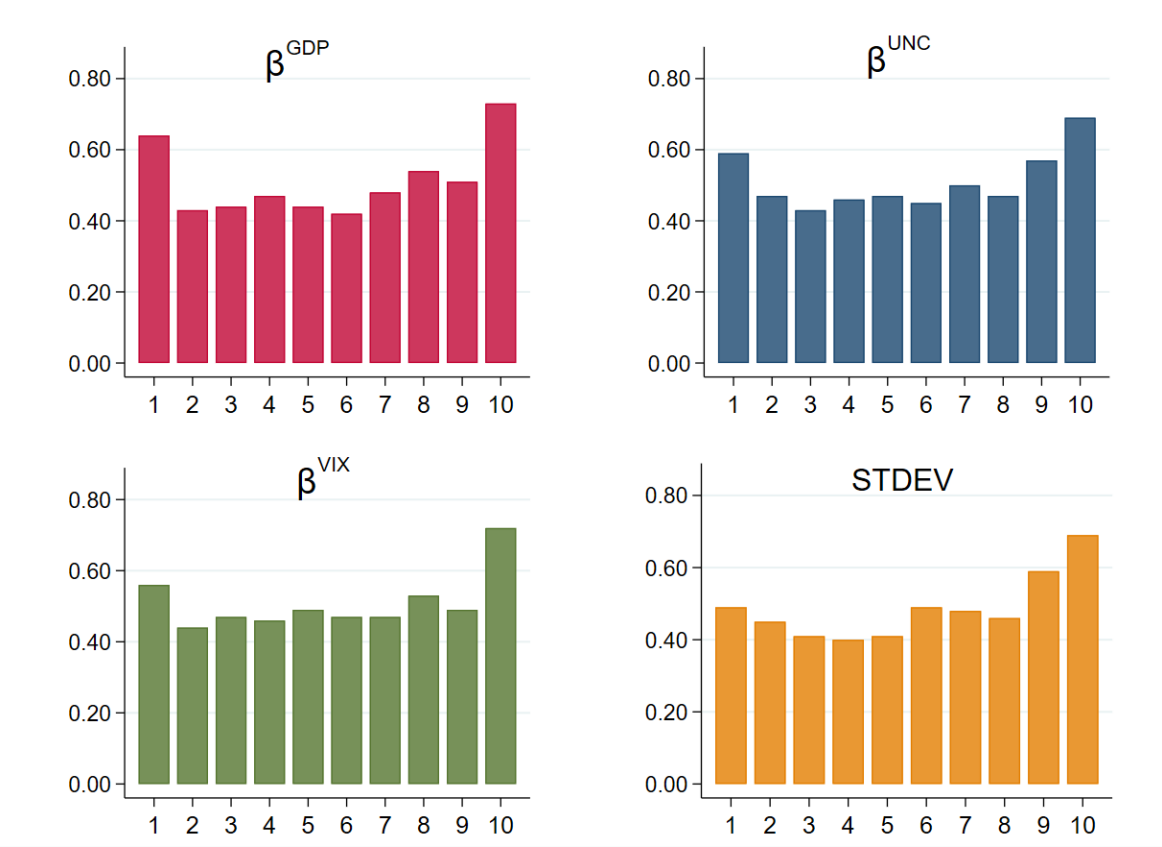


Figure C2. Average five-factor alphas on decile portfolios conditional on other ambiguity measures

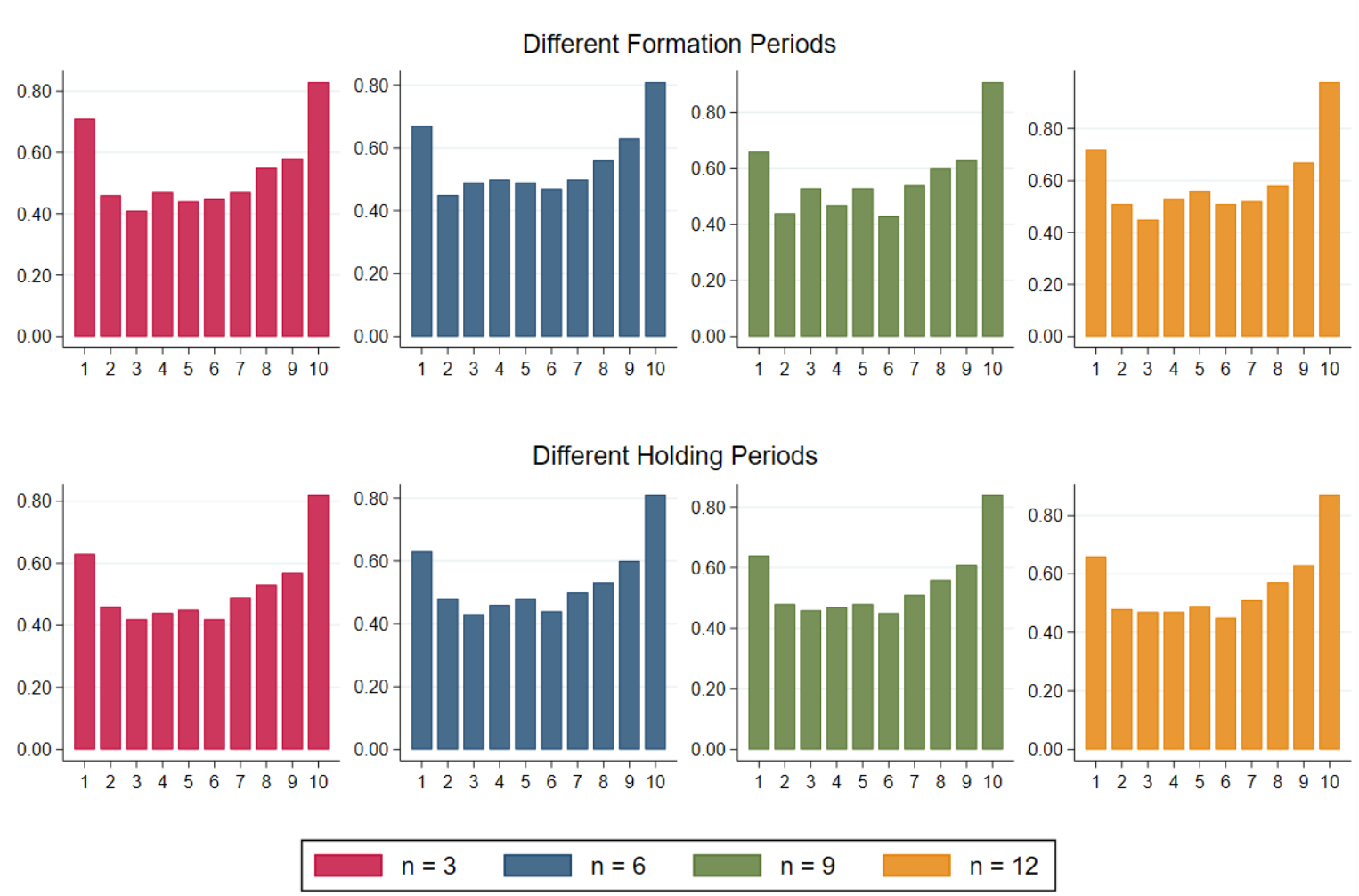


Figure C3. Five-factor alphas on decile portfolios with different formation and holding periods

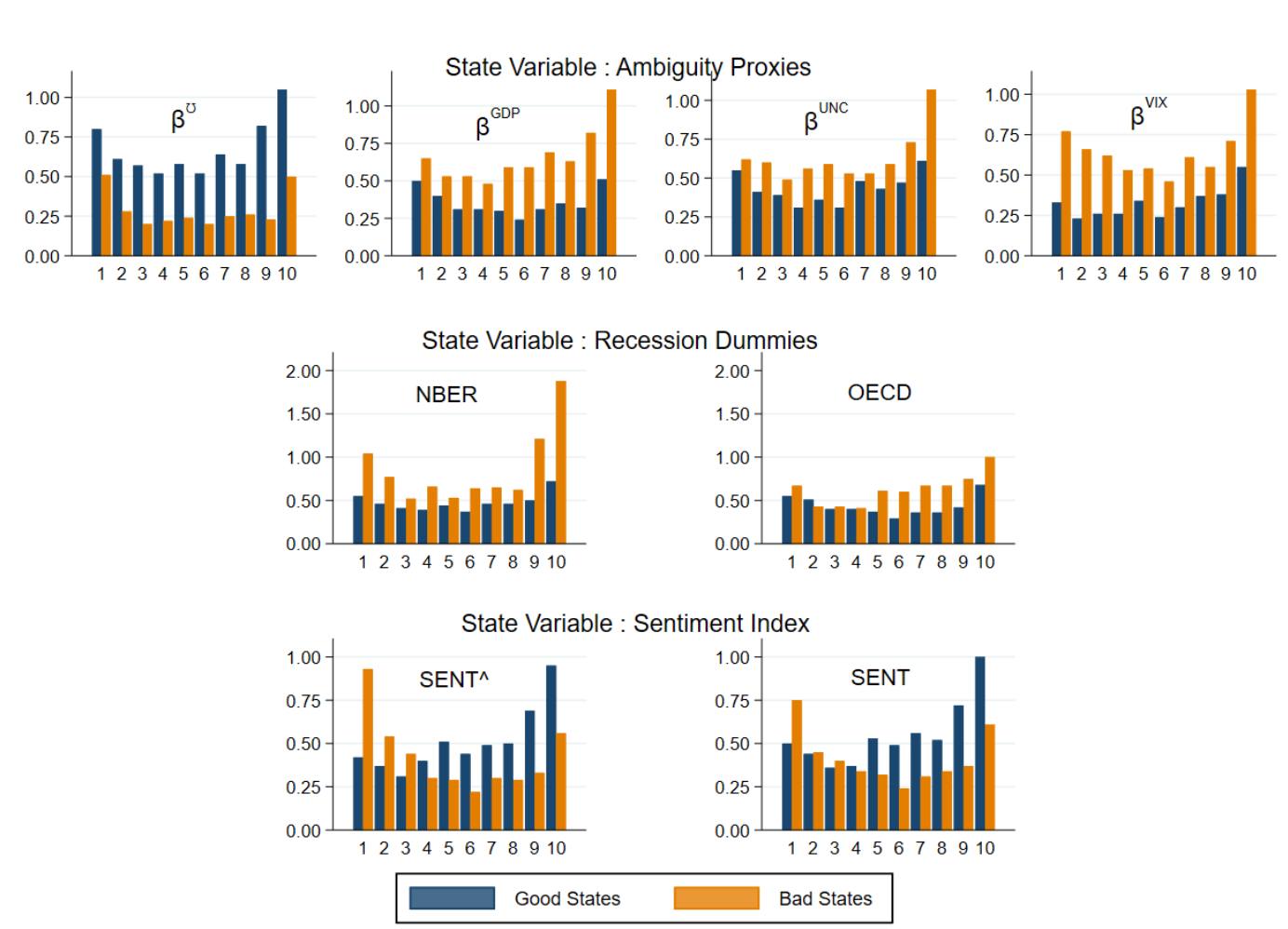


Figure C4. Five-factor alphas on decile portfolios during different states

APPENDIX D

AMBIGUITY BETA ACROSS TEST PORTFOLIOS

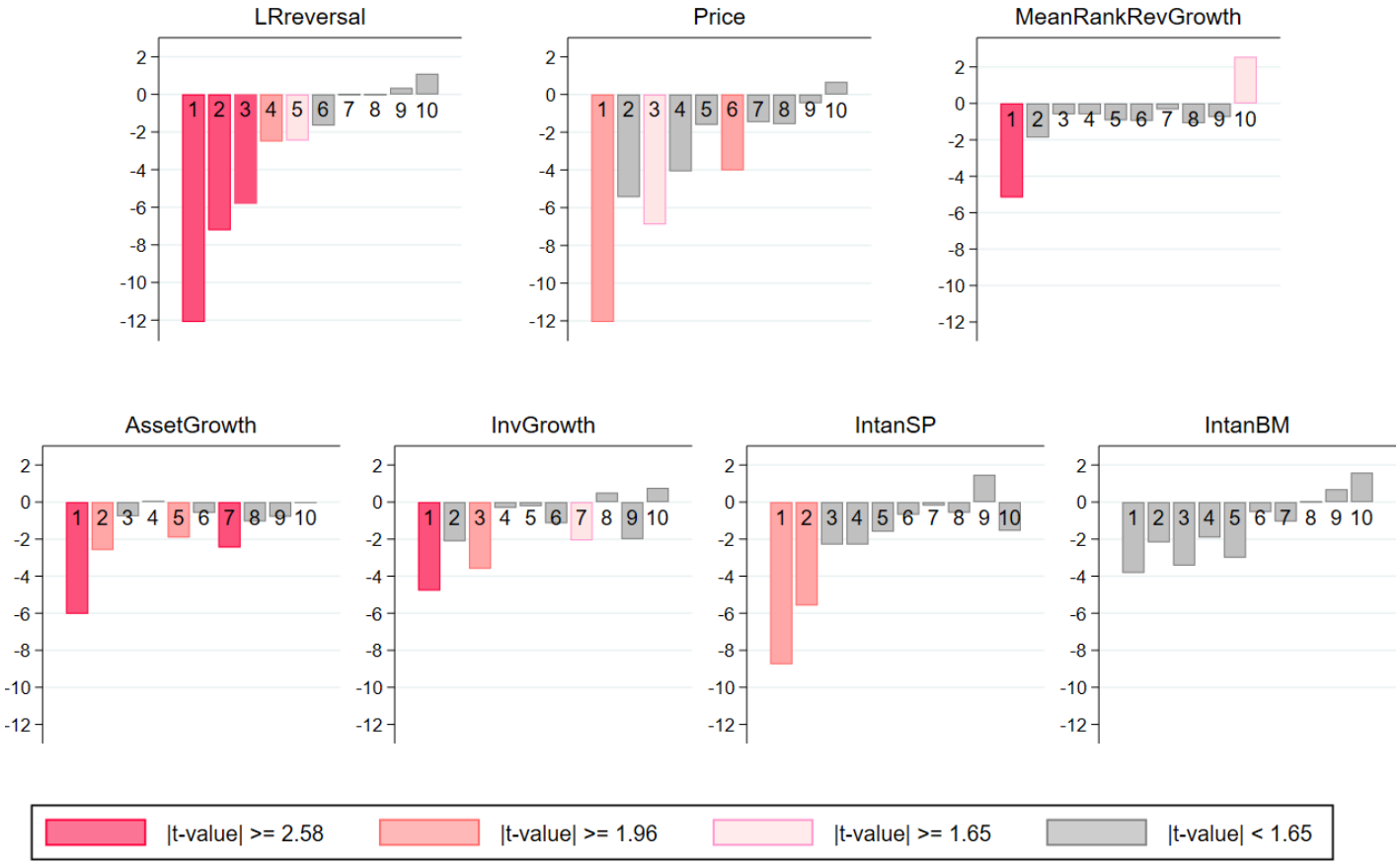


Figure D1. Increasing ambiguity beta coefficient

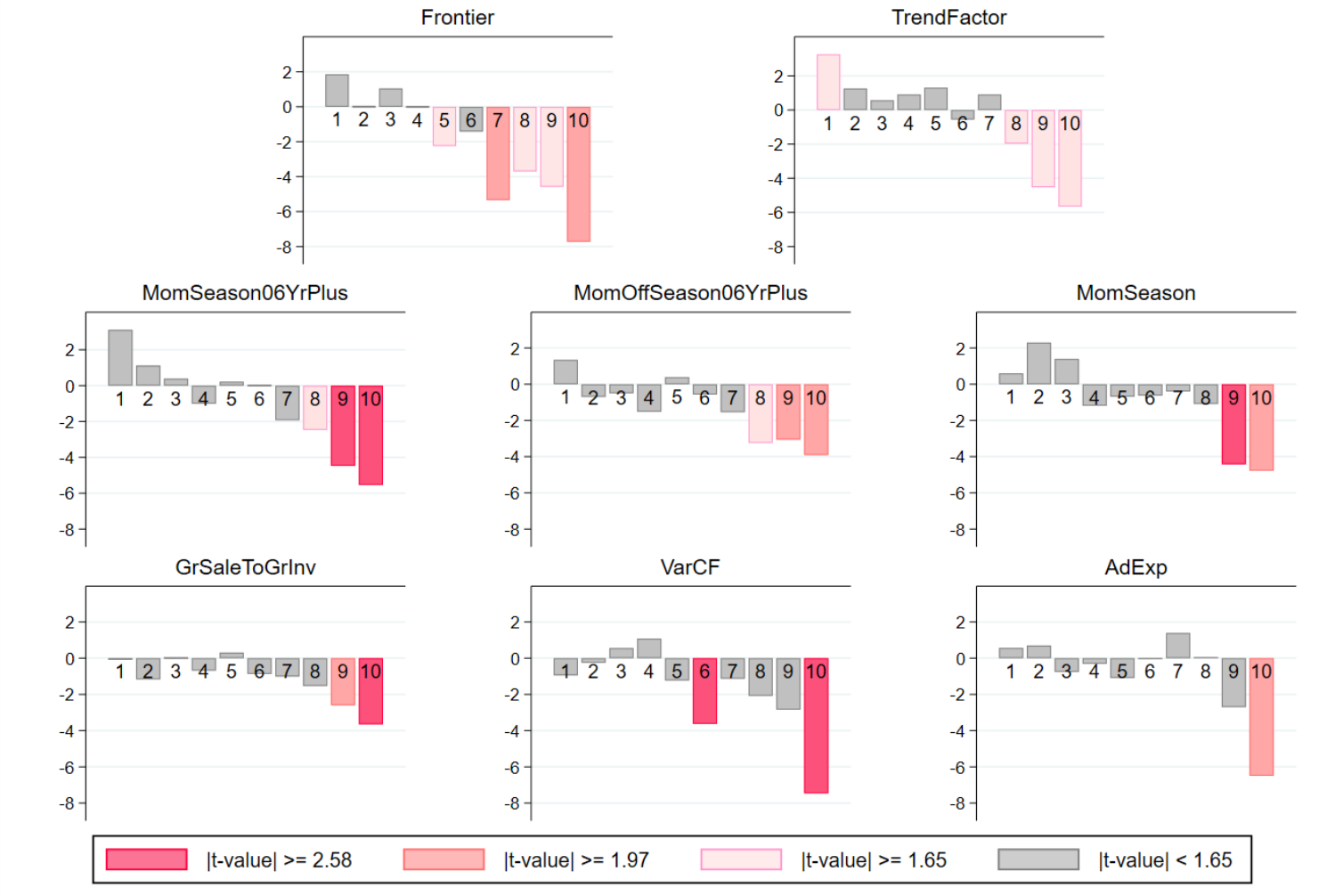


Figure D2. Decreasing ambiguity beta coefficient

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