EXAMINING THE ROLE OF NUMBER SENSE ON SEVENTH GRADE STUDENTS' PROPORTIONAL REASONING

by

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ABSTRACT

EXAMINING THE ROLE OF NUMBER SENSE ON SEVENTH GRADE STUDENTS' PROPORTIONAL REASONING

Number sense is the general understanding of a person about the numbers, and related operations, and this person's tendency to use this understanding in variable contexts in order to create effective plans and sound mathematical conclusions (Reys et al., 1997). In this context, considering teaching learning process of mathematics, number sense provides both students and teachers with in-depth understanding for mathematics subjects (Cain et al., 2007). This study examined number sense in the context of proportional reasoning, which is an essential mathematics subject related to higher mathematics (Lesh, Post, Behr, 1988). In this study which was conducted with the participation of 328 7th grade students, the aim was to determine number sense levels of the students, to investigate whether proportional reasoning of the students differentiate based on their number sense levels, and to display how these students use their number sense on the proportional reasoning problems. Explanatory sequential design, a mixed method design, was conducted. Quantitative data were collected through Number Sense Scale and Proportional Reasoning Test, while qualitative data were collected through audio-recorded interviews that include various proportional reasoning problems. The results of the study revealed that the students have a low-level number sense in total, and proportional reasoning statistically and significantly differentiates based on number sense levels of students. Also, the findings of the qualitative data revealed that students with both good level number sense and proportional reasoning were at forefront in applying different strategies for certain proportional situations.

ÖZET

7. SINIF ÖĞRENCİLERİNİN ORANTISAL AKIL YÜRÜTME BECERİLERİNDE SAYI HİSSİ ROLÜNÜN İNCELENMESİ

Sayı hissi, kişinin sayılar ve ilgili işlemler hakkındaki genel anlayışı ve kişinin bu anlayışı etkili planlar ve sağlam matematiksel sonuçlar oluşturmak için değişken bağlamlarda kullanabilme eğilimidir (Reys *et al.*, 1997). Bu bağlamda matematiğin öğrenme öğretme süreci göz önünde bulundurulduğunda, sayı hissi hem öğretmenlere hem de öğrencilere matematik konuları için derinde bir anlayış kazandırır (Cain *et al.*, 2007). Bu çalışmada ise sayı hissi, yüksekmatematikle bağdaştırılabilen önemli bir matematik konusu olan (Lesh, Post, Behr, 1988) orantısal akıl yürütme bağlamında incelenmiştir. 328 7. sınıf öğrencisinin katılımıyla gerçekleştirilen çalışmada öğrencilerin sayı hissi düzeylerinin belirlenmesi, öğrencilerin orantısal akıl yürütmelerinin sayı hissi düzeylerine göre farklılaşıp farklılaşmadığının araştırılması ve bu öğrencilerin sayı hislerini orantısal akıl yürütme problemlerinde nasıl kullandıklarının ortaya konulması amaçlanmıştır. Karma yöntem desenlerinden açıklayıcı sıralı desenden faydalanılan çalışmada, nicel veriler Sayı Hissi Ölçeği ve Orantısal Muhakeme Testi aracılığıyla, nitel veriler ise, çeşitli orantısal akıl yürütme sorularının yöneltildiği, ses kaydı alınan görüşmeler yoluyla toplanmıştır. Araştırmanın bulguları, öğrencilerin genel olarak düşük düzeyde sayı hissine sahip olduklarını, orantısal akıl yürütmenin öğrencilerin sayı hissi düzeylerine göre istatistiksel ve anlamlı düzeyde farklılaştığını ortaya koymuştur. Ayrıca nitel verilerin bulgularına göre hem sayı hissi hem de orantısal muhakemesi iyi düzeyde olan öğrenciler, belirli orantı durumlarında farklı stratejiler uygulayabilme konusunda ön plandadır.

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LIST OF ACRONYMS/ABBREVIATIONS

NSS	Number Sense Scale
PRT	Proportional Reasoning Test
MoNE	Ministry of National Education
NCTM	National Council of Teachers of Mathematics

1. INTRODUCTION

It is an undeniable fact that math is not just algorithms, formulas, and procedural operations with numbers, but an essential part of our daily lives. Therefore, even prior to instructional school process, children can use math in their everyday life as a thinking way.

Sorting the sizes, being able to distinguish most from few, recognizing numbers symbolically and visually through a certain number of objects the prominent examples of this idea. Considering these, the literature reveals number sense as a significant concept for both primary school mathematics and daily-life situations that requires mathematical thinking (National Research Council [NRC], 2001). Number sense can be described as a person's general understanding of numbers and mathematical operations as well as their aptitude and propensity to use this understanding in a variety of contexts to form sound mathematical conclusions and create effective management plans for numerical situations (Reys *et al.*, 1997). In this regard, Purnomo, Alyani, and Assiti, (2014) indicates that understanding of the concept of numbers and any sense related to them takes a great place in mathematics learning, which also includes the topics that are taught in mathematics classes at schools. Besides, considering the importance of number sense in school mathematics, Yang, Reys and Reys (2007) states that students start to understand the value and meaning of numbers when teachers believe that developing number sense is more important than mastering the rules governing written computation.

At very this point, Cain, Doggett, Faulkner and Hale (2007) examined number sense, as different from other researchers, with instructional aims by paying attention to teaching and learning mathematics. They constructed a classification including mathematics subjects such as algebraic thinking, proportional reasoning, form of a number (decimal, percentage, fraction), equality, base ten, and quantity and magnitude with the idea of developing teacher's content knowledge and instructional skills

through number sense. By taking the study of Yang *et al.* (2007) into consideration, it is very important for teachers to make sense of number sense and reflect it in their teachings, instead of transferring algorithmic processes, formulas and procedures to students, for the development of students' number sense and further mathematics learning. Based on the studies of Cain, Doggett, Faulkner and Hale (2007), the studies on number forms, that is, decimals, fractions, percentages, and algebraic thinking as a mathematical subject, to be examined in line with number sense are in the foreground. On the other hand, proportional reasoning is another mathematics subject emphasized by Cain *et al.*, (2007). Besides, proportional reasoning is a crucial part of mathematics teaching learning process since it is strongly related to further subjects such as functions, algebra, probability, fractions and statistics (Akatugba and Wallace, 2009). To define it, proportional reasoning is the capability to comprehend the multiplicative relationship inherent in comparison situations (Behr *et al.*, 1992).

However, proportional reasoning is a mathematics subject both students and teachers have difficulty in (Weiland *et al.*, 2019) with so many reasons including inability for many of the number concepts and relationships. Fernández *et al.*, (2012) states that one of these reasons could be the failure in distinguishing proportional and non-proportional situations, and the literature suggests that having a good understanding of numbers help students to do it. Besides, Lamon(2007) indicates that multiplicative reasoning is a requisite part of proportional reasoning, and Sowder (2007) states that difficulty could stem from this transition additive thinking to multiplicative thinking. Moreover, insufficient content knowledge of teachers (Hilton and Hilton, 2016), and instructional methods that are limited with algorithmic processes (Lamon 2006, 2007) could be displayed the other sources of these difficulties. Regarding these difficulties, and the importance of proportional reasoning, Bently and Yates (2017) advocates that it is necessary to look into more efficient methods of teaching proportional reasoning techniques. Moreover, researchers indicate that reasoning could be developed and improved through some interventions such as PBL (problem-based learning) (Misnasanti *et al.*, 2017), and contextual-real life- connected problems (Ortiz *et al.*, 2022).

However, there is a limited number of studies that pay attention to remove both student and teachers from algorithmic processes as Lamon (2006, 2007) suggests, in order to improve proportional reasoning considering school mathematics.

At very this point, it could be quite contributive to regard the perspective of Cain, Doggett, Faulkner and Hale (2007), which is to improve instructional strategies in the context of school mathematics for some mathematical subjects including proportional reasoning through number sense. Hence, this study specifically aims to examine the role of number sense on proportional reasoning of 7th grade students.

1.1. Purpose of the Study

The main purpose of this study is to investigate whether there is a significant difference among proportional reasoning scores of students who have low, medium, and high level of number sense according to the Number Sense Scale, and if so, to see how, where, and to what extent these students use their number sense based on their number sense levels in their proportional reasoning.

1.2. Significance of the Study

As mentioned in the introduction part, number sense has an important place in mathematics learning, since it enables student to be flexible for numerical operations (Carpenter, 1989), and since students can find much more effective, practical, and unusual solutions for any type of problems. In addition, with the help of number sense, students do not need be attached to rule-based processes and specific algorithms to cope with problems (Pilmer, 2008). Hence, number sense started to gain importance in terms of curriculum to some extent (MEB,2009). On the other hand, number sense in teaching programs is indeed constricted to estimation skills in some mathematics topics such as fractions, decimals, and area and length measurement. Whereas number sense is beyond estimation considering its relations with subjects in mathematics. Along with teaching programs and the curriculum, in the literature, number sense has generally

been examined in terms of its development, the factors that affects it, and particularly some mathematics subjects such as form of a number and algebraic thinking in the context of school mathematics. However, number sense could directly affect mathematics achievement (Yang, Li, and Lin, 2008), including other subjects of mathematics for students. In this regard, Cain, Doggett, Faulkner and Hale (2007) constructed seven main components (algebraic and geometric thinking, quantity/magnitude, numeration, equality, base ten, form of a number, and proportional reasoning) for number sense as different from previous examinations by considering school mathematics subjects and instructional aims that can benefit from number sense. Cain, *et al.*, (2007) advocates that their classifications were constructed so that teachers can have an understanding of mathematics in depth, which indicates that number sense enables teachers and therefore students to indeed have understanding of mathematical topics deeply. In other words, teachers could improve their mathematical instructions through number sense.

As mentioned before, the other components such as quantity/magnitude, form of a number, algebraic thinking, constructed by Cain, *et al.*, (2007) has been often investigated in the literature. Yet, when it comes to proportional reasoning there is indeed limited study that relates with number sense. However, Lamon (2006, 2007) emphasizes that instructions are restricted with algorithmic processes for proportional reasoning in schools, and Deizmann and English (2001) states that students are supposed to be provided with diagram literacy instead of algorithmic and symbolic universe of proportion. Faulkner (2009) suggests these ideas by defending that actual proportional reasoning requires keeping away from the capabilities that are dependent to formulas and symbolic displays. Considering all this, number sense can be an effective way for both teachers and students to achieve understanding of proportional reasoning in-depth without relying on symbols and algorithms.

Unlike other subjects, number sense has not been studied much together with the subject of proportional reasoning, number sense has not been given enough attention in research for proportional reasoning's development although proportional reasoning

directly concerns higher mathematics, and it is very important because it is connected to daily life, and is generally a subject that both teachers and students have difficulty in.

To sum up, considering that;

- instructions for proportional reasoning is generally constricted with algorithmic process in schools (Lamon, 2006, 2007)
- both students and teacher have difficulty in proportional reasoning (Weiland *et al.*, 2019)
- the numerical, concrete mathematics of arithmetic and the abstraction that follows in many higher-level mathematical topics are linked by proportional reasoning (Lesh, Post, and Behr, 1988)
- it is imperative to investigate more effective ways to instruct proportional reasoning skills (Bentley and Yates, 2017)
- actual proportional reasoning is achieved by staying away from formulas and symbolics (Faulkner, 2009)
- number sense enables both teachers and therefore students to have in depth understanding in essential mathematics subjects such as algebraic thinking and proportional reasoning (Cain *et al.*, 2007)
- despite importance of the development of proportional reasoning, there is limited study about number sense and proportional reasoning together

examining the role of number sense for proportional reasoning could be meaningful for the literature and teaching learning process. Moreover, if number sense can be used for proportional reasoning skills, content knowledge and instructional methods of teachers could be improved, and therefore students' proportional reasoning can be improved, and for teaching programs, it may be possible that number sense is not limited to the estimation dimension. Besides, since proportional reasoning is also related to many other subjects such as algebra, geometric thinking and fractions, future experimental studies can be suggested considering number sense.

2. LITERATURE REVIEW

There is no doubt that mathematics is not just a discipline, but it is a way of thinking. At this point, it would not be wrong to think that engaging with mathematics starts at very early years and it is not all about school mathematics and numbers. Exploring the real world could be a pathway for the beginning of mathematical thinking. To give an example, a child, who has not started going to school yet and who does not take any education in terms of school mathematics, still can divide a rectangular cake into equal three parts for his or her three friends. Yet, of course, the child does it by utilizing different ways of thinking. Besides, a child, who does not know numbers yet, can determine the equal numbers of the things by observing. Therefore, mathematics is not just a lesson and not the accumulation of procedural knowledge with numbers, and it is about thinking. At very this point, number sense, which has many different definitions, stands out.

2.1. Number Sense

Numbers have been always considered essential for mathematics, especially for primary school mathematics which leads to handling daily-life situations and problems, as a basement for following general mathematics curriculum, and improving sense of the numbers (National Research Council [NRC], 2001). Moreover, NRC (2001) emphasized the importance and the utilization of numbers' concept in algebra and geometry. Therefore, having an understanding of the concept of numbers and any sense related to them plays a great role in mathematics learning (Purnomo, Alyani, and Assiti, 2014). Hence, number sense has been a crucial concept that researchers and mathematics educators have emphasized, been interested in, and studied recently, and it has started to take place in mathematics curriculum and textbooks to some extent. (Acar and Peker, 2022).

Çekirdekci and Yorulmaz (2021) stated that number sense skill took place as a concept in mathematics teaching programs in both Turkey (MEB, 2009; 2015; 2018) and other countries such as Hong Kong and Japan by giving importance operations and numbers. MEB (2009) has determined the effective use of prediction and mental calculation skills as one of the general aims of mathematics education. Examples of problems in which this aim is included in the textbook published by the Ministry of National Education can be given (MoNE, 2019). In the problem below, an estimation problem, which is directly related to daily life, with its solution is presented about decimal numbers in 6th grade.

Pınar measured the duration of her one-lap run with a stopwatch and saw that she came from the starting point to the destination in 60.23 seconds. Let's estimate how many seconds it takes for Pınar to run 4 laps. (We can estimate the results of operations by rounding the given numbers to numbers that are easier to use in calculations).

The time it takes to run one lap, 60.23 seconds, is approximately 60.25 seconds. Estimated time for 4 laps of running: 60.25. 4 = 241 seconds

After this example problem with solution, two different estimation problems about decimals are asked in textbook for students in "*your turn*" part as a follow-up exercise.

30 kilograms of lentils will be placed in 0.28-kilogram bags. Estimate how many bags are needed for this job.

Ms. Ekin bought 3 kilos of apples, which cost 2.9 liras per kilo, and 2.26 kilos of oranges, which cost 2 liras per kilo, from the greengrocer. Estimate how much Ms. Ekin will pay to the grocery store.

Additionally, similar examples can be seen in area measurement subject, in 5th grade.

The appearance of a room with a square base is as shown below. Estimate the area of the surface covered by the carpet in the room, in square metres.

When it comes to what number sense exactly is, there are many different definitions and explanations made by different researcher for number sense. According to Yang (2003), number sense is the basic comprehension of numbers and their operations, as well as the capacity and skill to create effective, adaptable, flexible and practical solutions to situations involving numbers. Besides, the ability to understand numbers and their relationships intuitively can be defined as number sense (Nickerson and Whitacre, 2008). As different from Yang's definition, intuition is emphasized as having a great place when it comes to number sense. As for another and more holistic definition, the term "number sense" describes general number understanding of a person and mathematical operations as well as their aptitude and propensity to use this understanding in a variety of contexts to form sound mathematical conclusions and create effective management plans for numerical situations. Besides, there is the perception of numbers as significant objects and the expectation that mathematical operations and their results ought to make sense (McIntosh, Reys, Reys, and Farrell, 1997).

As the learner engages in mathematical thinking, number sense appears in multitude of forms, such as sensitivity to the rationality of calculations and understanding of varying accuracy levels and trying to understand numerical situations is one of its defining traits (McIntosh, Reys, Reys, and Farrell, 1997). Besides, NCTM (1989) stated that number sense is a fundamental idea in mathematics, and determined prominent indicators of number sense like :

- good comprehension of the meaning of numbers
- multiple numerical relationships' existence and use
- understanding the relative size of numbers
- understanding how operations on numbers affect them relative to other operations
- using environment-relevant referents to measure common objects and situations

As compatible with these ideas, Mohamed and Johnny (2010) states that the following traits are frequently displayed by individuals who have strong number sense when performing mental calculations; "sense making approach, planning and control, flexibility and appropriateness sense of reasonableness" (Mohamed and Johnny, 2010, p.317). A good level of calculation in the mind, estimation of numbers, and quantitative judgment are some other characteristics of the individuals with good number sense (Greeno, 1991). Moreover, these individuals can move with ease between numerical statements' mathematical framework and quantitative reality (Sengul, and Gulbagci, 2012). As for another attribute of people who have a better level of number sense, Case (1998) emphasizes that these people are capable of creating their own methods for performing mathematical operations and representing the same number in various ways considering the context and aim of the representation. At this point regarding all these advantages of number sense, we can easily say that it is expected to take a great place in teaching learning process of mathematics. When it comes to school mathematics, according to Yang, Reys and Reys (2007), when teachers think that cultivating number sense is more significant than mastering the rules governing written computation, students begin to see the value and meaning of numbers.

2.2. The Features of Number Sense Considering School Mathematics

Yang and Wu (2010) evaluated number sense in the framework of teaching and learning of it in primary school, and determined four main reasons for its importance:

(i) Number sense represents being creative, flexible, and reasonable.
(ii) Number sense comprise numbers, operations, and relationship between them, which are expected to be implemented smoothly in daily life.
(iii) Number sense has an impact on adults' representations of numbers and mathematical thinking.
(iv) The excessive importance given to written calculations both restrict children's mathematical thinking and prevent their improvement of number sense.

When the reasons are examined, it can be easily stated that keeping the number sense away, especially in primary school education, and directing students to algorithmic calculations can negatively affect students' mathematical thinking, and can directly affect the relationship with numbers in later ages and in daily life adversely.

Along with that, considering number sense at school atmosphere, it can be mostly regarded as students' move between different representations of numbers (e.g., percent, decimals, fractions), determining size of numbers, and relating numbers, operations, and symbols. From this perspective, number sense has some features to be cared about. First of all, number sense is quite specific to each student, and number sense varies depending on different students (Pilmer, 2008), and it stems from the ideas about numbers and the way of obtaining these ideas (McIntosh, Reys, Reys, and Hope, 1997). According to Pilmer (2008), instruction in the classroom, tasks and activities play a great role for number sense development on students. This approach actually emphasizes that number sense can develop or be improved from time to time, and Howden (1989) suggests this idea by stating that number sense development is gradual, and number sense is obtained at various levels from student to student. At this point Reys (1989) emphasizes that these levels expand as long as students have new experiences and understandings. Hence, considering Pilmer's (2008) attention on the importance of task and instruction that students engaged for the development of number sense, Reys suggest this idea by bringing the importance of experience students face with. Here, the point that is meant from experience is not just working and operating with numbers as an algorithmic process, but also having a chance to explore numbers by indeed experiencing them through insight.

Along with these, Howden (1989) also indicates that students should be given opportunity for the exploration of numbers covering both formal and informal education of them in order to foster number sense. Moreover, according to Gülbağci Dede (2015), using different methods for operations, investigating how students make reasoning numerically and the reasonableness of the results, relating mathematics to real life and seeing problems as experiences for students, creating mathematical discussion

atmosphere at classrooms so that students can express themselves for their reasoning process, and giving an opportunity to making guess and comparing actual results with guesses could be the main strategies to develop number sense . Therefore, there is a consensus among researchers about the development of number sense regarding some factors such as experience students have and improvement of understanding through exploration of the world of numbers. Yet, at very this point, it would be very beneficial to indicate that all students cannot develop their number sense with only time and experience (McIntosh, Reys, Reys, and Hope, 1997).

As for another feature, number sense enables student to be flexible for numerical operations. (Carpenter, 1989). With this feature, students do not necessarily have to be attached to rule-based procedures and certain algorithms to cope with problems, and they can obtain much more efficient, useful, and unusual ways for any kind of problems. (Pilmer, 2008).

Along with them, by going back and benefiting from the definitions, number sense has also an intuitive extent. Pilmer (2008) explains this approach by stating that some students are capable of gaining different perspective and understanding without any effort to have an idea about which strategy is better to use and to handle with problems. When considering all these features, we can easily say that number sense is directly related to the interaction among the skills of an individual, knowledge of an individual and the problem's nature. Besides, it should not be forgotten that number sense is a concept that can be learned and taught (Griffin, 2004).

2.3. Different Classifications and Components of Number Sense

Along with many different definitions and features, number sense has been also examined under different components and classifications for deeper understanding. According to Şengül and Gülbağcı Dede (2013), researchers could not construct any common terminology for the components of number sense, and they used different components for different age groups.

First of all, Greeone (1991) determined the number sense components such as flexible thinking, estimation ability for calculation, and quantitative judgment. Flexible thinking could be considered as the ability to see numbers' and different operations' equivalence when it comes to mental computation. Estimation ability in calculation is to be aware of the approximate values of the numbers and to make reasoning through these values to solve the problems with an effective way. As for quantitative judgement, it means making judgment and inferences for different numerical values. A year after Greeone (1991), McIntosh, Reys and Reys (1992) determined three different components for number sense such as knowledge and facility with numbers, knowledge and facility with operations, and applying knowledge of and facility with numbers and operations to computational settings. Moreover, there are some other subcomponents for each main three components. Therefore, it is a classification that shows the numbers, operations and their application in cases requiring calculation, and what this whole process depends on in detail, with sub-components.

When other classifications about number sense were considered, it can obviously be said that McIntosh, Reys and Reys (1992) constructed the most detailed and systematic number sense components considering mathematical properties, mathematical problem contexts, and applying knowledge on computational settings, which could be considered as the reason for selecting this classification for the analysis in this specific study that cares about proportional reasoning problems. According to this classification, number sense components (McIntosh, Reys, Reys, 1992, p.4) are as follows:

(i) Knowledge and facility with numbers
sense of orderliness of numbers
place value
relationships between number types
ordering numbers within and among number types

multiple representations for numbers
graphical/symbolic

equivalent numerical forms
comparison to benchmarks

sense of relative and absolute magnitude of numbers
comparing to physical referent
comparing to mathematical referent

system of benchmarks
mathematical
personal

(ii) Knowledge and facility with operations
understanding the effects of operations
operating on whole numbers
operating on fractions/decimals

understanding mathematical properties
commutativity
associativity
distributivity
identities
inverses

understanding the relationships between operations
addition/multiplication
subtraction/division
addition/subtraction
multiplication/division

(iii) Applying knowledge and facility with operations
understanding the relationships between problem context and the necessary com-

putation
recognize data as exact or approximate
awareness that solutions may be exact or approximate

awareness that multiple strategies exist
ability to create and/or invent strategies
ability to apply different strategies
ability to select an efficient strategy

inclination to utilize an efficient representation and/or method
facility with various methods (mental, calculator, paper/pencil)
facility choosing efficient number(s)

inclination to review data and result for sensibility
recognize reasonableness of data
recognize reasonableness of calculation

Along with these two classifications, Markovits and Sowder (1994) divided number sense into three main components such as magnitude of numbers, mental computation, and estimation in calculation. Magnitude of numbers explains ordering numbers and finding numbers between two numbers. To give an example, understanding that there are infinite numbers between 0.5 and 0.6 is related to the skills of this component. Mental computation can be described as the methods that are different from the usual and standardized computation methods. As for another component which is estimation in calculation is a skill that could be difficult to students. When adding large numbers with many digits, not including a two-digit number between them in the total and adding it later can be given as an example. Yet, ignoring a number as a first step, and considering it as 0 could be really challenging for student.

As a following step and considering the study of McIntosh *et al.* (1992), Reys *et al.* (1999) constructed five main components for number sense, and as similar to

McIntosh *et al.* (1992), benchmarks, meaning and size of numbers, and operational effects are the prominent components.. Although the sentences and titles are different, we cannot say that the content has changed much. According to this classification, number sense components (Reys *et al.*, 1999, p.62) are as follows:

(i) Understanding of the meaning and size of numbers
defining relative size of numbers
comparing two given fractions
determining the number between two numbers

(ii) Understanding and use of equivalent representations of numbers
understanding the equivalents of numbers and their equivalent representations

(iii) Understanding the meaning and effect of operations
the ability to notice how the result will change based on the change of the value of a number or operation in computational situations. (realizing that multiplication does not always make numbers bigger)

(iv) Measurement benchmarks
identifying and using reference/benchmarks that may be suitable for different situations.

(v) Flexible computing and counting strategies for mental computation, written computation, and calculator use
being able to solve problems and predict the results without calculating using paper and pen, and calculating mentally.

Meanwhile, in 1995, Yang constructed six main components for number sense. Deep comprehension of numbers and their magnitudes, constructing benchmarks and efficient use of them, operations, and their relationships with numbers , and flexibility for both numbers and operations are the main characteristics of the component classification created by Yang (1995). As different from previous classifications, composing

and decomposing numbers are present in this classification that consists of six different components:

(i) Understanding the basic meaning of numbers
(ii) Being able to compose and decompose numbers
(iii) Recognizing the magnitude of numbers
(iv) Being able to use benchmarks appropriately
(v) Recognizing the relative effect of operations on numbers
(vi) Flexibility in applying numerical and operational knowledge to computational

As seen in tables above, the components for number sense are indeed similar to each other. Yet, there is no one single framework that covers these similar components constructed by McIntosh, Reys and Reys (1992), Markovits and Sowder (1994), Reys *et al.* (1999), and Yang (1995). However, Cain, Doggett, Faulkner and Hale (2007) constructed seven main com- ponents for number sense, and these components are slightly different from previous ones. For, Cain *et al.* (2007) focus on school mathematics and teaching learning process of mathematics, and they determine the components of number sense considering students' learning of mathematics and its subjects. Moreover Faulkner (2009) states that schoolteachers have not understood number sense clearly and sufficiently, and these components could be constructed to develop teachers' content knowledge as an instructional model through number sense.

Table 2.1. Number Sense Components suggested by Cain *et al.* (2007).

Components	**Explanations**
Quantity/Magnitude	Mathematical topics could be taught and modeled through quantity.
Numeration	Mathematical expression requires the skill of numeration, and students should comprehend the numeration system which is at the rate of 10.
Equality	A=B means, A and B are equal in value, but they are not exactly the same, so teachers should be careful about teaching the meaning of equality.

Table 2.1 Number Sense Components suggested by Cain *et al.* (2007). (cont.)

Components	**Explanations**
Forms of a Number	Mathematical topics such as trigonometry, subtraction, addition, and fractions are connected to forms of a number as a common, which improves the understanding of students for equality.
Base Ten	In order to be prepared to integrate larger numbers as they work with them, students must learn to think in powers of 10.
Proportional Reasoning	Actual proportional reasoning is achieved when students know about diagram literacy instead of fitting numbers to symbolic formulas.
Algebraic and Geometric Thinking	When teaching algebra and geometric thinking, teachers should create a process in which students can use their mathematical knowledge, rather than going directly to symbols and a clear result with an end goal.

Along with that, as different from other researchers and their classifications, Lago and DiPerna (2010) constructed a task list in order to determine the factors that affect pre-school (kindergarten) children's number sense.

Table 2.2. The List for Number Sense by Lago and DiPerna (2010).

Components	**Explanations**
Counting aloud	Counting out loud in order, starting from 1
Recognizing quantity	Being able to distinguish which of the two numbers selected from two boxes with numbers between 0 and 20 is larger
Identification of Numbers	Naming numbers which are between 1-30 when they are shown
Nonverbal Calculation	Performing addition and subtraction operations whose result is not more than 10 with a stick

Table 2.2 The List for Number Sense by Lago and DiPerna (2010). (cont.)

Measurement Concepts	Understanding and learning of basic measurement concepts such as shorter or longer while comparing things

2.4. Studies on Number Sense

Studies on number sense are becoming more prevalent in our country day after day, and researchers and mathematics educators emphasize its importance (Acar and Peker, 2022).

As a general perspective, Acar and Peker (2022) state that most of the studies uses quantitative method and descriptive survey design, samples mostly consist of 5-8 grade students from primary school, and the interests of studies are mostly about the deeper understanding of number sense, and the measurement of number sense level, identifying number sense components, examining the factors that affect number sense, and other concepts related to number sense. Acar and Peker (2022) also emphasize that for the studies related to number sense conducted in Turkey, despite the rising number of studies, it was determined that a sufficient level of diversity could not be achieved, and more number sense studies should be conducted considering its importance.

When it comes to general results of studies, it was determined that students have a low level of number sense (Facun and Naul, 2002; Kayhan Altay, 2010), and they are adhering to rule based, standard, algorithmic solutions instead of number sense (Kayhan Altay, 2010; Şengul and Gulbağcı Dede, 2014). Moreover, students' reference use and estimation skills as components of number sense are insufficient (Facun and Nool, 2012; Kartal, 2016; Yang, 2007). Yang, Li and Lin (2008) suggested these results by stating that it was determined that the worst performances of the students were in making logical estimations in computational processes. Besides, Zanzali and Ghazali (1999) states that multiple representation of numbers and relating between number are the two main components that students have difficulty in.

There are also some studies related to grade level and gender on number sense. Menon (2004) indicated that girls' performances on number sense are better than boys' performances. On the other hand, Singh (2009) stated that in terms of gender, it was seen that the mean score of boys was higher than that of girls, but there was no significant difference between boys and girls. Harç (2010) suggests Singh (2009) and emphasizes that number sense is not dependent on gender. As a similar studies, Kayhan Altay (2010) asserts that the use of number sense in male students was higher than that of female students, but there was no significant difference in terms of gender.

Grade level has also been examined for number sense. Menon (2004) explains that it was observed that as the grade level increased, the use of number sense decreased, but the performance of applying standard operations increased. As a similar result, there is inverse proportion between grade level and the use of number sense (Kayhan Altay, 2010). On the other hand, Singh (2009) and Takır (2016) indicated that it was observed that the mean score for number sense increased as the grade levels increased. Hence, in both grade level and gender, results indicating two different and opposite opinions were found.

As another perspective, Yang, Li and Lin (2008) states that there is a significant relationship between students' number sense and their mathematics achievement. Besides, Bütüner (2017) suggests that mathematical thinking develops with the development of number sense. Researchers also has been interested in fostering and developing number sense considering its importance on school mathematics, and they studied the factors that foster number sense. Rich teaching activities and process-oriented activities (Markovits and Sowder, 1994; Yang, 2002), real life problems and realistic activities in math lessons (Yang, 2003; Yang and Wu, 2010), and problem-solving based mathematics education (Tsao, 2004a) are considered as the factors that foster number sense for students.

2.5. Proportional Reasoning

There is no doubt reasoning indeed takes a great place in mathematics learning. In other words, mathematics is one of the areas where reasoning is used most intensively, and mathematical reasoning is a base for mathematics learning (Kahraman *et al.*, 2019). Thanks to reasoning, students can solve the problems more successfully, and they can have more achievement level of mathematics because according to Bal-İncebacak and Ersoy (2016) the reasoning skill provides the opportunity to evaluate the events with different perspectives and then transfer them to new situations. Besides, mathematical reasoning enables students to form a permanent and open-ended mathematics (Poçan, Yaşaroğlu, and İlhan, 2017), and mathematical reasoning is an indispensable part of the mathematics learning and teaching process (Duatepe *et al.*, 2005). Moreover Umay (2003) divides mathematical reasoning into four categories according to the subject such as algebraic, proportional, statistical and geometrical reasoning, and proportional reasoning is an essential way to reach mathematical reasoning skills for students.

When it comes to proportional reasoning, we need to have a look at the meanings of ratio and proportion which are essential terms for the understanding of proportional reasoning. These two terms (ratio and proportion) are usually used together, but they have different and a kind of related meanings. Dole, Clarke, Wright, and Hilton (2012) defined the ratio as the situation in terms of comparison (a/b), and the proportion as the use of comparison to identify an associated situation in the same comparison (a/b=c/d). As for the definitions of proportional reasoning, actually there are not so many different views, and there is a consensus about the meanings of it. Proportional reasoning is the capability to comprehend the multiplicative relationship inherent in comparison situations (Behr *et al.*, 1992). Besides, according to Lesh, Post, and Behr (1988), the capacity to translate ideas from students' concrete operational level into formal thinking is known as proportional reasoning. Being able to recognize a proportion, describe it symbolically, and solve problems that require proportional reasoning could be also considered as proportional reasoning (Cramer and Post, 1993). Owens (1993), on the other hand, defined proportional reasoning as the ability to be aware

of the distinction of proportional and non-proportional situations, which is a different and quite general, comprehensive perspective for a definition. According to Baxter and Junker (2001), proportional reasoning includes not only examining the relationship between two concrete objects, but also defining, predicting and evaluating the relationship between two relationships. As for predicting extent of it, according to Lamon (2020), proportional reasoning entails the intentional use of multiplicative relationships to compare quantities and to predict the value of one quantity based on the values of another. Sowder *et al.* (1998) brings a different perspective by stating that proportional reasoning is a type of complex mathematical and psychological reasoning based on intuition learned in preschool and school and developed through experience. In this regard, intuition could be considered as the collection of attitudes which are shaped by personal and societal experience. Also in intuitive approach, verbal expressions take precedence over numerical expressions. Here, both intuition and experience are emphasized as crucial for the proportional reasoning as different from other approaches. As a comprehensive view, Misnasanti, Utami, and Suwanto (2017) indicates that the capacity for proportional reasoning involves the connection between the idea of multiplication and formal thinking in problem-solving and can be developed by drawing on students' experience.

2.6. The Importance of Proportional Reasoning

Lesh, Post, and Behr (1988) consider proportional reasoning as an essential link between the numerical, tangible mathematics of arithmetic and the abstraction that follows in many higher-level mathematical topics. Akatugba and Wallace (2009) suggest this idea by stating that proportional reasoning development is quite significant for the understanding of functions, algebra, probability, fractions and statistics. Besides, understanding of some fundamental concepts such as multiplication, division, and decimals are related to proportional reasoning (Boyer and Levin, 2015). Moreover, in everyday circumstances such as grocery shopping, proportional reasoning is crucial for determining the best deals based on a food item's unit pricing and for doing measurement and such as currency conversions like converting inches to centimeters

or *eure* to *dolar* (Im and Jitendra, 2020). Along with all these, considering the importance of proportional reasoning, Cramer, Post and Currier (1993) determined three kind of tasks such as missing value-problems (the task is to locate the fourth or missing piece of information when three pieces of information are given), numerical comparison problems (a situation in which two complete rates or ratios are presented, a numerical response is not necessary, but the rates or ratios must be compared), and qualitative prediction and comparison problems (comparisons that are not reliant on particular numerical values) to assess and evaluate the proportional reasoning.

2.7. Difficulties with Proportional Reasoning

In most of the countries of the world, ratio and proportion as a mathematics topic is presented in schools' curriculum in 6th and 7th grade. However, In Grade 3, students begin formally learning about ratios by multiplying and dividing whole numbers; this knowledge serves as the foundation for learning about fractions, ratios, and proportional reasoning (Im and Jitendra, 2020). On the other hand, NCTM (2000) indicates that proportional reasoning skills can be developed in students through 5-8 grades, which also shows us proportional reasoning can be developed and improved. According to Lobato, Ellis and Zbiek (2010), the development of proportional reasoning is a change that can be used to assess students' thinking, which also emphasizes the developable side of proportional thinking. On the other hand, despite the importance of proportions in daily life math and the time spent teaching them in medium school, both students and teachers have difficulty with proportions (Post *et al.*, 1988; Weiland *et al.*, 2019). Teachers' inadequate content knowledge for utilizing suitable representations to gain a thorough understanding of multiplicative concepts could be one reason of this approach about difficulties (Hilton and Hilton, 2016). Besides, as for students, the absence of a solid conceptual foundation for multiplicative or relative thinking; an excessive reliance on algorithmic methods like cross-multiplication, which may yield the desired result but obscure the multiplicative structures in a proportionate way (Berk *et al.*, 2009). These two ideas emphasize that the importance of multiplicative thinking on proportional reasoning.

In this way, another reason to have difficulty in proportional reasoning could be considered as the resistance to transition from additive thinking to multiplicative thinking because early instruction places a lot of emphasis on multiplication as just repeated addition and creating ratios between two quantities, using multiple comparisons, and changing the conceptual framework from additive comparisons to ratios between two quantities present challenges (Im and Jitendra, 2020). Singh (2000) evaluates this transition from additive to multiplicative by stating that there are changes both in what the numbers are and what the numbers are about.

As a different perspective, failing to distinguish proportional and nonproportional situations could be another reason for these difficulties (Fernández *et al.*, 2012). Also, Im and Jitendra (2020) address that medium school students with MLD (mathematics learning disability) have some challenges for proportional reasoning since they have not yet mastered many of the number concepts and relationships. This means that number concepts and relationships take a great place when it comes to proportional reasoning. Hence, there is a consensus among researchers about both students and teachers have difficulty in proportional reasoning with different possible reasons. Yet, this lack of comprehension does not preclude students from inventing a proportional relationship or from figuring out problems. Understanding proportionality involves more than just using mathematical equations like a/b=c/d to solve problems. Proper reasoning for proportionality is necessary for further understanding (Park, Park and Kwon, 2010). To make the sentence simple, students can solve proportion problems and equations, and can reach the correct answer through algorithmic ways and manipulating numbers, but this does not mean that they have a real understating of it. It is widely acknowledged that students' computational abilities do not accurately reflect how well they comprehend the ideas that underlie the calculations (Dole, Clarke, Wright and Hilton, 2012). In spite of these difficulties, it should not be forgotten that proportional reasoning can be developed, and problem solving through contextual problems and schema-based instruction are the frequently used methods to develop proportional reasoning in the literature.

2.8. Fostering Proportional Reasoning

When it comes to fostering proportional reasoning, since it is difficult for both teachers and students in school mathematics, studies have mentioned the importance of encouraging both students and teachers. As for student, Cragg and Chevalier (2012) states that when a person is able to choose from a variety of representations or tactics to adapt to the needs of a circumstance, they are demonstrating cognitive flexibility. In order to trigger cognitive flexibility, multiple categorizations, which is a process that enables someone to examine a certain thing from several perspectives, could be benefited from, and this method, which is based on the development of cognitive flexibility through multiple categorizations, also played an improving role in the proportional reasoning of primary school students (Scheibling-Sève *et al.*, 2022). Besides with cognitive flexibility through multiple categorizations, comprehension of multiplicative constructions are essentials in proportional reasoning (Behr, Harel, Post and Lesh, 1992), and Hart (1991) indicates that children intuitively tend to make additive constructions while solving proportional problems. Therefore, teachers' guidance for students in constructing multiplicative structures could be an essential move for the students' development of proportional reasoning (Dole, 2010). These two approach (Cragg and Chevalier (2012) ; Dole (2010)) appears to attempt to get to the heart of proportional reasoning to foster it for students. Considering the major impact of technology on education field, use of digital technology on teaching proportional reasoning to students is quite beneficial when it comes to removing misconceptions and simulation of real world, daily-life scenarios that can be problematized (Ortiz *et al.*, 2022).

Another important aspect in terms of in-class teaching strategies for triggering proportional reasoning of students could be flipped classroom model as different from traditional model in terms of choosing more accurate strategies for solutions and being more successful in proportional reasoning problems (Ergene and Karabogaz, 2024).

Moreover, problem-based learning (PBL), which is a model that benefits from real-life, contextualized problems so that students can think critically (Kunandar,

2010), have a role in improving students' skills about proportional reasoning (Misnasanti *et al.*, 2017).

Inquiry pedagogy, which is an approach that pays attention to classroom-group discussions and engagement of students to the lessons by allowing them to express, defend, and elaborate their opinions (Cobb and McClain 2006) to manage their inquiry (Fielding-Wells and Makar, 2012). According to Fielding-Wells *et al.*, (2014) inquiry model helps students transfer their additive thinking to multiplicative thinking through their mathematical inquiry process.

Along with fostering students' proportional reasoning, the development of teachers' proportional reasoning has been crucial in the literature. According to Lamon (2006, 2007), it seems that the acknowledged techniques for teaching proportional reasoning are mainly limited to a variety of algorithms and mechanical processes, which that teachers in general are not sufficiently equipped in proportional reasoning. Research findings indicate that incorporating practice-based materials into a teacher education program can enhance the content knowledge of aspiring educators in mathematics concepts (Steele, 2006). Similarly, Pişkin Tunç and Çakıroğlu (2022) advocates that the implementation of practice-based instructions through proportional reasoning tasks as a teacher education has a positive impact in the development of teachers' proportional reasoning especially in transition from algebraic techniques to reasoning deeply. Besides, Arican (2023) emphasizes that it is crucial to have an understanding where the teachers are exactly in terms of proportional reasoning and how to develop their proportional reasoning. In other words, Arican (2023) suggests that in this regard, it would be better to first identify the basic problems through interviews and in-depth questions for teachers and then implement training programs to remove lacks and develop their proportional reasoning.

Van de Walle (2001) looked at this situation from a broader perspective and made a few recommendations for teachers to improve their students' proportional reasoning:

- provide students with proportional problem situations in different contexts and relevant to their real lives.
- use qualitative-type problems as well as quantitative-type problems because qualitative-type problems encourage students to use proportional reasoning without having to use numbers.
- help student distinguish proportional situations from non-proportional situations
- encourage students to discuss the estimation and comparisons of ratios.

2.9. Types of Proportional Reasoning Problems

In the literature, one of the studies on proportional reasoning was to determine the types of proportional reasoning problems (Cramer and Post, 1993; Akkuş and Duatepe-Paksu, 2006). In this sense, the prominent proportional reasoning problem types can be listed as follows:

- Missing Value: Two variables have a proportionality relationship and four quantities form a proportion.
  If 3 sandwiches are 30 liras, how much is 8 sandwiches?
- Quantitative Comparison: In these problems, two ratios are given, and these ratios are asked to be compared without the need for a numerical answer.
  In a sandwich eating contest, Ayşe ate 20 sandwiches in 4 minutes and Ahmet ate 10 sandwiches in 5 minutes. Who won the competition which is based on the speed?
- Qualitative Comparison: Regardless of certain numerical values, ratios are asked to be compared.
  In a sandwich eating contest, Ayşe ate more sandwiches in a shorter time than Ahmet. Who won the competition?
- Problem Involving a Non-proportional Type of Relationship: Although the data of such questions are not proportionally related, their form may give the impression that they require proportional reasoning.
  In a sandwich eating contest, Ahmet and Ayşe's eating speeds are equal. Ayşe

started eating the sandwiches first. If Ayşe ate 5 sandwiches when Ahmet ate 10 sandwiches, how many sandwiches will Ahmet eat when Ayşe ate 15 sandwiches?

- Inverse Proportion: There is an inverse proportion between the ratios, and while one of the given that is constituent for ratio increases, the other given of ratio decreases.

  12 sandwiches are enough for 3 people in a group for 4 days. If there were 3 more people in the group, how many days would 12 sandwiches be enough?

2.10. Strategies for Solving of Proportional Reasoning Problems

Cramer and Post (1993) determined that students have four different solution strategies for proportional reasoning problems:

- Unit-rate: It makes a comparison between a specific number of units of one quantity and units of another.

  In a sandwich eating contest, Ayşe ate 20 sandwiches in 4 minutes, and Ahmet ate 10 sandwiches in 5 minutes. Who won the competition? In this kind problem, students try to find out find out how many sandwiches Ayşe and Ahmet eat in a minute *(Ayşe ate 5 sandwiches in a minute, and Ahmet ate 2 sandwiches in a minute).*
- Change of multiplier: Students who benefits from this strategy thinks that *if Ayşe ate 20 sandwiches in 4 minutes, she would eat half of 20 sandwiches (10=20/2) in 2 minutes as a half of 4 minutes.*
- Cross Multiplication Algorithm: Students construct a proportion through algorithm and solve the equations.

  20 sandwiches 4 minutes 10 sandwiches x minutes (x= 10x4/20=2 minutes)
- • Equivalent Fractions Strategy: Ratios are considered as equivalent fractions. The purpose is to find a fraction that is equivalent to the given fraction.

  4 minutes/ 20 sandwiches= x/10 sandwiches, and x= 2 minutes

2.11. Studies on Proportional Reasoning

Looking at international studies, the focus is generally on finding the essence of proportional reasoning, identifying the factors affecting its development, relationship with other mathematical concepts and strategies mostly used by students.

Studies investigating not only the school period but also pre-school period tried to gain information about the development of proportional reasoning. Vanluydt *et al.*, (2022) conducted a study whose participants are 5-8 age and concluded that proportional reasoning starts to develop in pre-school ages, and its development still goes on even if students are in 3rd grade. Also this study brought an issue that development of children's proportional reasoning is directly related to spatial ability and numerical skills.

In another interesting study that examines the development of proportional reasoning, 7th grade students are participants, and the task that are completed with LEGO robotics has a positive impact on the development of 7th graders' proportional reasoning with the help of collaborative works through using a new technology (Casler-Failing and Swann, 2023).

In another study conducted with teachers, based on the idea that algebraic thinking could be benefited from while solving proportional reasoning problems, and some real life connected proportional reasoning problems were directed to K-8 teachers. As a finding, teachers had difficulty in both solving these problems and transferring variables to algebraic forms, and use of algebraic thinking for the solutions of proportional reasoning problems were quite challenging for the teachers (Lamber *et al.*, 2021).

Apart from algebraic concept, other contexts such as probability was studied together with proportional reasoning. In the study, the participants were 3rd graders, and it was conducted to see the role of contexts (fair-sharing, probability, and mixture) on students' proportional reasoning. As result, students were best in fair sharing

context, and they mostly tended to additive thinking in mixture context (Supply *et al.* 2023).

Along with specific mathematical contexts and concepts, Özdemir (2011) studied on the relationship between proportional reasoning and constructing problems related to ratio and proportion. The participants were 7th and 8th graders, and results showed that the students, who are insufficient in proportional reasoning skills, could not construct ratio and proportion problems to a large extent. On the other hand, the students, who are good at proportional reasoning, were more successful in constructing problems.

As for another study that examines proportional reasoning problems, the researchers investigated the students' solution strategies for proportional reasoning types. 8th graders were the participants, and fifteen open-ended problems were asked.

As a result, the students used mostly cross multiplication in missing value problems, equivalent form of fractions in quantitative comparison problems, and they chose wrong and incorrect strategies to type of problem that distinguishes proportional from non-proportional (Keleş and Cinar, 2023).

As related to students' solution strategies, Pakmak (2014) investigated the strategies that 6th grade students use in quantitative and qualitative proportional reasoning problems. In the result of the study, inverse proportion algorithm is the most used strategy in qualitative proportional reasoning problems while unit rate is the most used strategy in quantitative proportional reasoning problems.

Also, while students turned to numerical operations in quantitative problems, they turned to methods of digitizing, symbolizing or drawing the given data in qualitative problems.

In the development of students' solution strategies and attitudes towards proportional reasoning problems, Ben-Chaim *et al.*, (1998) suggested a new curricular

perspective through contextual problems including rational numbers and proportional reasoning, and collaborative process of solutions of these problems for students. As a result of using this suggestion instead of traditional, teacher-centered education, 7th grade students created their own proportional reasoning and gave more creative answers to problems.

2.12. Proportional Reasoning and Number Sense

First of all, there is indeed very limited study and research in the literature when it comes to proportional reasoning and number sense together. On the other hand, as different from other studies about number sense, Cain, Doggett, Faulkner and Hale (2007) constructed number sense components considering school mathematics and an instructional model for teachers. Based on this classification, proportional reasoning is a subcomponent of number sense. Faulkner (2009) states that he and his colleagues think that there is a need to a number sense model that support firstly the teachers' understanding of mathematics in depth. In other words, this classification was developed with an instructional purpose for teachers so that they can improve their mathematical instructions through number sense model with its components. Moreover, Faulkner (2009), who consider proportional reasoning as a compound skill and inherent to higher mathematics, emphasizes the importance of estimation, deduction, and both qualitative and quantitative comparisons for it. In this regard, Deizmann and English (2001) indicates that students should be given a chance to develop the diagram literacy instead of symbolic and algorithmic world of proportions. Faulkner (2009) suggests this idea by stating that the instruction that obtains diagram literacy trigger actual proportional reasoning instead of the instruction and ability that obtains formulas and symbolic processes. Also, Faulkner (2009) gives an example about Pi number as a crucial proportion since it is circumference divided by diameter, and emphasizes that it is important for the teacher to ask students questions such as giving the circumference of the circle and asking for its diameter, or giving the diameter and asking for its circumference. For, engaging students with this kind of questions, develop both proportional reasoning and number sense of students according to Faulkner

(2009). Moreover, numbers have an impact on students' reasoning proportionally or not. In other words, students can distinguish proportional and nonproportional situations by having a good understanding of numbers (Van Dooren *et al.*, 2005), which is essential when it comes to proportional reasoning. Also, if students have a strong number sense, they could be more comfortable in applying algorithms and operations such as multiplication, and they can use different strategies flexibly during reasoning proportionally. Therefore, the effect of numbers and number sense on proportional reasoning should not be ignored.

3. METHODOLOGY

3.1. Research Questions

- Research Question 1:
  What are the number sense levels of 7th graders as measured by "Number Sense Scale towards Medium Grade Students"?

- Research Question 2:
  Is there a statistically significant difference between proportional reasoning scores of seventh grade students with low, medium, and high number sense levels?

- Null Hypothesis:
  There is no a statistically significant difference between of 7th graders who have lower number sense score , medium number sense score , and high number sense score on their proportional reasoning test scores.

- Research Question 3:
  How do 7th graders' number sense competency take a role in solving problems related to proportional reasoning?

3.2. Operational Definitions

Number Sense: A person's general understanding of numbers and mathematical operations as well as their aptitude and propensity to use this understanding in a variety of contexts to form sound mathematical conclusions and create effective management plans for numerical situations (Reys *et al.*, 1997).

In this study, students' Number Sense competiences were measured by the total score gained by Number Sense Scale that is consisted of seventeen items, with minimum

total score 0, and maximum total score 17. Scores were then categorized as low level (0-5), medium level (6-11), and high level (12-17).

Proportional Reasoning: Proportional reasoning is the capability to comprehend the multiplicative relationship inherent in comparison situations (Behr *et al.*, 1992). In this study, proportional reasoning capability of students were measured by the total score gained by Proportional Reasoning Test that includes 15 items, with minimum total score 0, and maximum total score 53.

3.3. Design of the Study

In this study, the main purpose was to examine the role of number sense on the proportional reasoning skills of 7th grade students. Based on this, the study obtained a mixed method that blended quantitative data and qualitative data, and explanatory sequential design in which qualitative data helped explain quantitative data (Creswell *et al.*, 2016) The number sense levels of 7th grade students were determined as a first step, and students were divided into three groups as lower, medium, and higher according to these levels. Then, it was investigated whether there were statistically significant differences between the proportional reasoning skills of the students in these three groups. As for quantitative data, Number Sense Scale towards Medium Grade Students (Kayhan and Umay, 2013) and Proportional Reasoning (Akkuş and Duatepe Paksu, 2006) test were used to determine the number sense levels and proportional reasoning skills of 7th grade students, and these quantitative data were evaluated through casual-comparative research design that aimed to assign the consequence of difference that already exist between three groups such as lower, medium, and high number sense level without any manipulation or intervention for proportional reasoning skills. Besides, in order to follow and support quantitative data, an audio-recorded interview was conducted to see how students use their number sense in problems related to proportional reasoning. Thus, through quantitative data, the difference in proportional reasoning skills between number sense level groups was determined, and through qualitative data, as an effort to explain these differences. How students with different

levels of number sense used their number sense in proportional reasoning problems was investigated.

3.4. Sampling and Participants

The target population in this study is 7th graders in Adana, Turkey. As a sampling method, convenience sampling was used to select participants considering both attainability and presence. Hence, 7th grade students from three different state schools of Kozan/Adana district of Turkey were selected as participants through convenience sampling. To elaborate for these three state school, each class consists of approximately 30 students, and all three schools have classes based on success levels. When the student profiles in these schools are examined, it can be said that they belong to the same socio-economic class on average. As for the quantitative part of the research, the total number of participants was 328, and they were 7th graders from three different state schools in Kozan/Adana. These 328 7th grade students attended both Number Sense Scale and Proportional Reasoning Test, and before the students were subjected to measurement tools, parental consent forms were obtained with parent signatures. In cases where students answered the scale questions but did not answer the test questions or vice versa, the answers of these students were not accepted as data and were destroyed.

As for qualitative part of the research, 10 students from the same three state schools out of 328 students were selected for the audio-recorded interview that consisted of five proportional reasoning problems. While selecting these 10 students, care was taken to ensure that the voice recording permission option was approved in the parental consent forms. Of these 10 students, 3 of them were successful in both the proportional reasoning test and the number sense scale, 3 of them had low success levels in both measurement tools, 2 of them were successful in the proportional reasoning test but inadequate in the number sense scale, and 2 of them were successful in the number sense scale but inadequate in the proportional reasoning test. Those whose number sense scale score was over 12 out of 17 and whose proportional reasoning test was over

35 out of 53 were considered as good levels for the interview. Those whose number sense was lower than 12 points were selected between 0-5 points, as in the first grouping for low levels. Besides, in determining the students to be selected for the interview, the opinions of the teachers of the students at the schools were also taken.

3.5. Instruments

3.5.1. Instruments for Quantitative Data

Quantitative data was collected to determine whether there is a statistically significant difference between proportional reasoning skills based on number sense levels, using a scale called as "Number Sense Scale towards Medium Grade Students" which is developed by Kayhan and Umay (2013), and a test called as "Proportional Reasoning Skill Test" which is developed by Akkuş and Duatepe Paksu (2006). "Number Sense Scale towards Medium Grade Students" was constructed based on six components that are described by Yang (1995). The scale was implemented to 584 medium grade students from four different schools of the two different districts of Ankara. The Cronbach's alpha coefficient was calculated as .86 for the scale that includes seventeen items. Exploratory factor analysis was conducted, and it revealed that the scale is composed of three dimensions. The first dimension includes eight items, and it is defined as flexibility in calculation. The second dimension involves four items, and it is defined as conceptual thinking in fraction. The third dimension includes five items, and it is defined as using benchmark (reference points). The score that can be taken from the scale varies between 0 and 17. Students who solve the question using the sense of number are given 1 point, those who solve the question by making calculations in a standard-routine way and cannot reach the correct result are given 0 points. Moreover, for this specific study, the Cronbach's alpha coefficient for Number Sense Scale through 328 participants was calculated as .85, which could be considered as reliable (Pallant, 2016).

As for Proportional Reasoning Skill Test, it includes "15 open ended proportional reasoning items of different types (seven of them are missing value, three of them are quantitative comparison, four of them are qualitative comparison and the remaining one is inverse relation type)" (Akkuş and Duatepe Paksu, 2006, pp.1).

One mathematician and one mathematics educator checked the items to test their appropriateness for the purpose. In order to evaluate the data gathered from this instrument, three-holistic rubrics were constructed. The test was applied to 304 7th and 8th grade students, and the Cronbach's alpha coefficient was calculated as .86. In this study, the test was applied to 328 7th grade students, and the Cronbach's alpha coefficient was calculated as .88, which is evidence for an instrument to be reliable (Pallant, 2016).

3.5.2. Interviews

To examine the role of number sense competency of students on how they use it in proportional reasoning problems, qualitative data was collected through an audio-recorded interviews with 10 students. The interview questions consisted of 5 main problems developed by the researcher considering five different problem types related to proportional reasoning. These problem types are: *"finding missing value, quantitative comparison, qualitative comparison, problem involving a non-proportional type of relationship, and inverse proportion"* (Duatepe, Çıkla, and Kayhan, 2005, pp76). Moreover, there were also sub-problems in options a and b of the 1st, 3rd and 4th problems. Therefore, the students solved and answered with their opinions 8 problems related to proportional reasoning in total. After creating the problems, the researcher shared the interview problems with thesis advisor, a mathematics teacher working in a state school, and a graduate student in mathematics education. During the sharing, it was especially emphasized to check the understandability of the problems and their compatibility with the five different proportional reasoning problem types indicated above. Based on the feedback, the problems created were found to be compatible and understandable, but with the suggestion from the mathematics teacher, it was decided

that it would be useful to state that 1 liter is equal to 1000 milliliters in the option a of 3rd problem. When the necessary changes were made, the researcher conducted interviews with 6 students who were 7th graders, accessible and not among the research participants, in order to conduct a pilot study, based on the problems created. At the end of this entire process, it was deemed appropriate to use the created problems to collect qualitative data.

3.6. Data Collection

In this study, data was collected by means of a scale and a test that were applied to students face to face with paper pen method, and through an audio-recorded interview. Before the implementation of data collection, necessary permissions were obtained from the Ministry of National Education and Boğaziçi University ethics committee. Moreover, parental consent forms were distributed to the 7th grade students in three schools where data would be collected .Necessary explanations for the data collection process were made, a deadline was presented, and the consent forms were asked to be signed and returned by the parents if deemed appropriate. The students who were allowed to participate were determined after parental consent documents were collected by the school deputy principals. When the signing of parental consent forms was completed in all three schools, two data collection tools were applied on separate days for each school, on one day and on the same day. Since the Proportional Reasoning Test was required to be administered to the students to whom the Number Sense scale was applied, as required by the research question, care was taken to apply both data collection tools in two consecutive lesson hours on the same day for each school.

Additionally, the data was collected in the third week of May 2023, after the topic of ratio and proportion was taught in mathematics lessons. Thus, both data collection tools were applied to a total of 328 students from three different state schools located in Adana. Then, by looking at the results of the data collection tools applied, taking the opinions of the teachers in these three schools, and paying attention to the voice recording permissions in the parent consent forms, the schools were informed that

interviews would be held with 10 students from three different schools. In the last week of June 2023 interviews were held with these 10 students on three separate days and the data collection process was completed.

3.7. Data Analysis

When it came to the data analysis part, evaluating quantitative data was the first step of data analysis phase. First, for the answers of 328 students on the number sense scale, 1 point was given to students who reached the solution using number sense, 0 points were given to students who did not use number sense and gave incorrect answers, and a score was determined for each student out of 17 points. Student answers to some problems in the scale that were solved using number sense and those that were not solved are shown in the table. Then, the Proportional Reasoning test answers were analyzed for 328 students through detailed, three-holistic rubrics created by Akkuş and Duatepe Paksu (2006), and each student was given a score out of 53 points. The scoring results of two data collection tools for each student were transferred to SPPS. In response to research question 1, the number sense average was determined based on the results of all participating students. In response to the research question 2, number sense scale scores ranging from 0 to 17 points were divided into three main level groups: those between 0 and 5 points were lower, those between 6 and 11 were medium, and those between 12 and 17 were higher.

Normality analyzes were performed according to proportional reasoning test scores for all three groups. Since the distributions in the groups were normal and one independent variable (number sense levels) had more than two groups for one dependent variable (proportional reasoning test scores), One-way ANOVA, a parametric method, was used. According to the one-way ANOVA results, since there was a statistically significant difference between the groups, a Post hoc test was conducted to fully compare the groups on the basis of these differences. In this way, quantitative data was analyzed.

Then, in order to evaluate the qualitative data, the 10 students interviewed were divided into four groups according to their number sense levels and proportional reasoning skills. Three students have both good level number sense and proportional reasoning skills, three students have both low-level number sense and proportional reasoning skills, two students have good level number sense but low-level proportional reasoning skills, and other two students have low level number sense and good level proportional reasoning skills.

The answers of the students in these four groups were transcribed separately for each question according to the audio recordings. Frequencies and percentages of number sense components as themes, categories, and codes were determined using the classification developed by McIntosh, Reys and Reys, (1992). In this classification, there are three main components of number sense, but these three subcomponents are divided into subcomponents, and a total of eleven newly formed subcomponents are divided into thirty subcomponents in total. While coding, first three components were considered as the themes, eleven different subcomponents were considered as categories, and thirty different subcomponents that came from categories were considered as codes of the analysis. Coding was done by the researcher , but then expert opinion was taken for what was done afterwards.

To give information, the expert is an associate professor studying at a state university in the field of mathematics education. Although there was disagreement on the two codes, a consensus was reached as a result of discussions, and no codes were observed other than subcomponents in the classification.

For each group, the themes, categories and codes that stand out according to student answers are explained in detail, giving examples from student answers and providing photographic evidence. Thus, qualitative analysis was completed with descriptive content analysis.

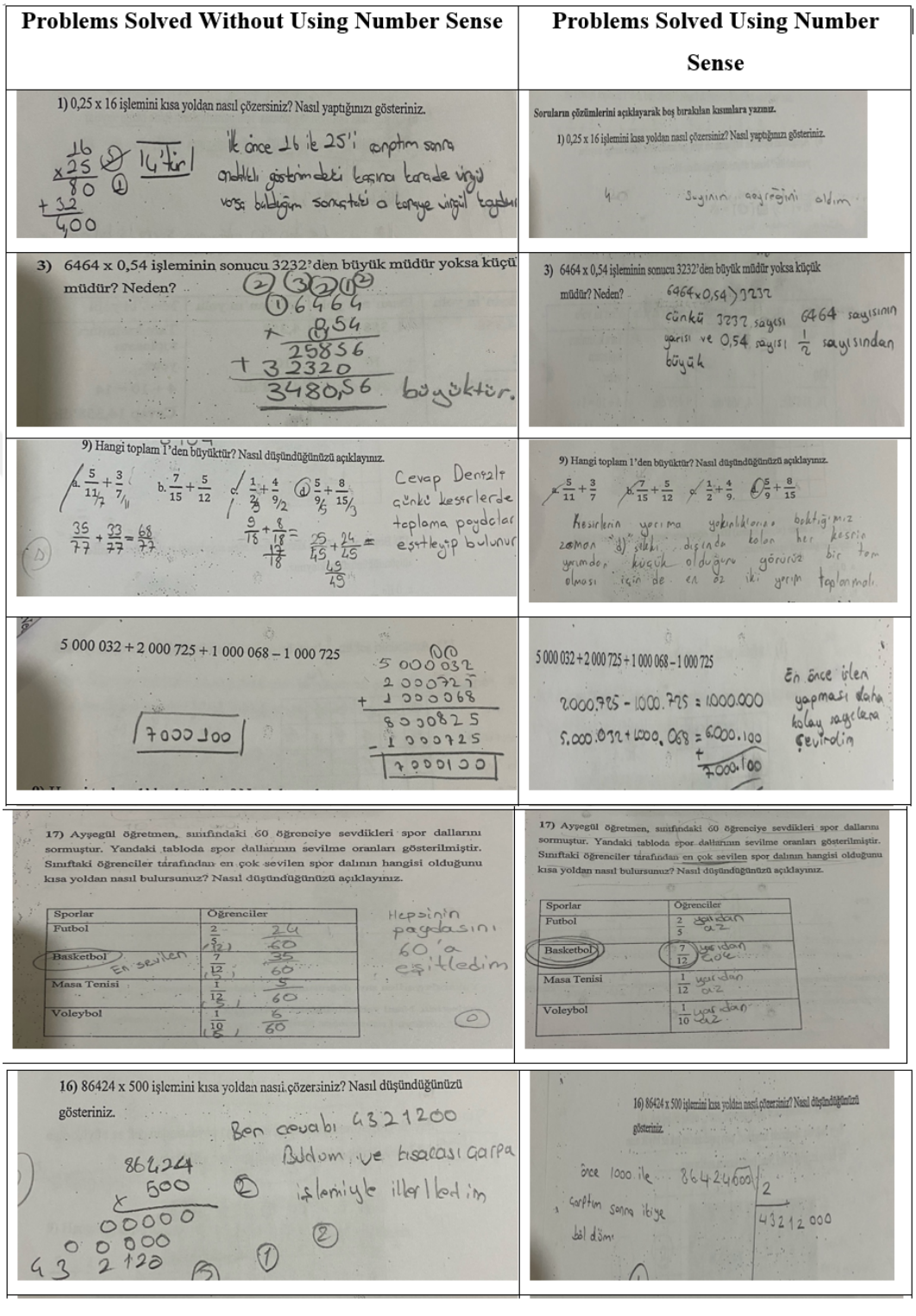


Figure 3.1. Example solutions for number sense 1.

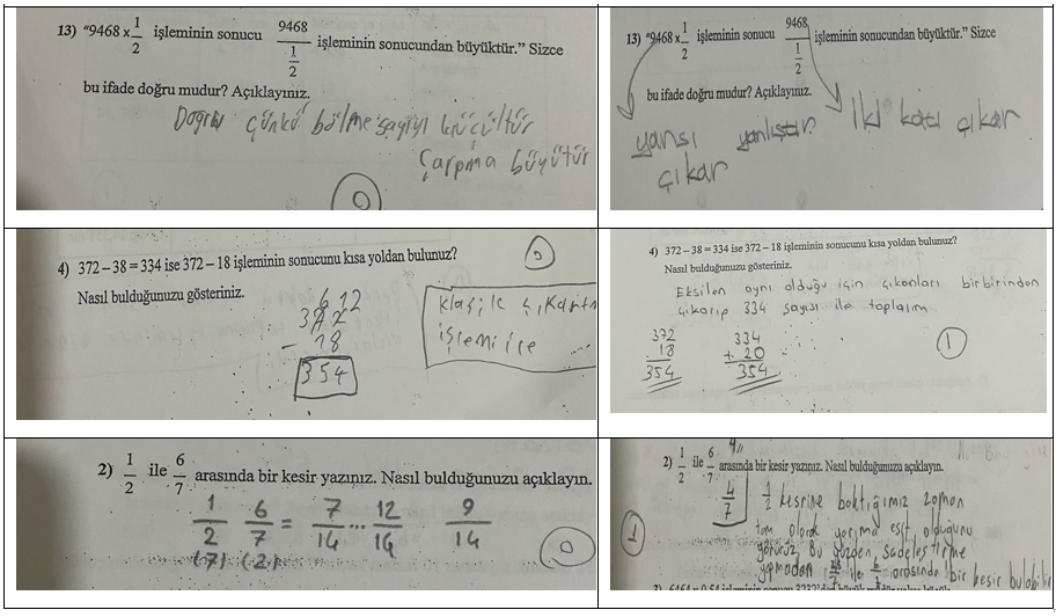


Figure 3.2. Example solutions for number sense 2.

In the first problem in the table, the student who solved problem without using number sense, wrote decimal numbers under each other, multiplied them and placed comma as following the rules learned. On the other hand, the student who solved the same problem by using number sense, S(he) defined 0.25 as a quarter and directly divided 16 by 4 in order to reach solution.

Similarly, in the second problem in the table, first student did a long multiplication operation to get the result, but second student realized that 3232 was half of 6464 and stated that multiplying by a number greater than half would increase the result, which is evidence for number sense.

As for third problem in the table, while the first student tried to reach the result by equalizing the denominators of the fractions in the options and performing operations, the other student stated that at least two halves were needed to obtain one whole, without doing any operations, and reached the result by examining the closeness of the fractions to halves.

In problem 4 in the table, the student added and subtracted the questions she saw in the same order. However, another student grouped the given numbers in order to reach the result more easily. In other words, s(he) chose efficient numbers in his/her own way.

In the next problem, as similar to problem 3, first student tried to equalize the denominators to compare the given fractions, but second student examined the closeness of fractions to halves, and avoided from calculations.

In problem 6 in the table, although the problem was asked to be solved in a shorter way, the first student wrote the numbers one under the other and multiplied them for a long time. The other student came up with the solution because s(he) knew the shortcut to multiplying by 1000 and that multiplying by 500 was actually multiplying by 1000 and dividing by two.

In problem 7 in the table, the first student argued that division always reduces, and multiplication always increases by ignoring that one of the factors is rational number. On the other hand, the other student, without adhering to a general rule or generalization, realized that multiplying a number by 1/2 means getting half of that number, and dividing by 1/2 means getting twice that number.

In problem 8 in the table, the first student could not find a shortcut and did the subtraction directly. Yet, second student looked at the difference between the subtrahends since the minuends were the same, and added the difference to the first result, which was 334. This provides both a good use of number sense and also a good understanding of subtraction.

In the last problem in the table, knowing that the fraction 1/2 is half, the second student stated that s(he) could also express half as 3.5 / 7 and easily found a fraction between 6/7 and 3.5 / 7. Yet, the first student equalized the denominators through a rule based way as s(he) learned.

4. RESULTS

In the very first part of this chapter, based on Research Question 1, the descriptive statistics for Number Sense Scale and Proportional Reasoning Test Scores gained by the participants was presented as a first step.

4.1. Number Sense levels of students according to "Number Sense Scale towards Medium Grade Students"?

Table 4.1. Descriptive Statistics for the Scores of the Participants Gained through P.R.T and N.S.S.

	N	**Mean**	**Std. Deviation**
P.R.T	328	19.58	12.23
N.S.S	328	4.61	3.79

The data was collected from three different state schools with 328 participants for both P.R.T (Proportional Reasoning Test) and N.S.S (Number Sense Scale). The mean of the participants for the scores of Proportional Reasoning Test is 19.58 out of 53, and it is possible to say that it is less than half of the maximum score. Besides, the mean of the participants for the scores of Number Sense Scale is 4.61 out of 17, which is a quite low level considering the maximum score that could be gained from the N.S.S. Hence, it is possible to state that the level of number sense is very low, but it is also lower than the level of proportional reasoning based on the participants' scores.

In the second part of this chapter, One Way ANOVA test was used to analyze Research Question 2 since there were three (more than two) groups (lower (0-5), medium (6-11), higher (12-17)) of one independent variable (number sense levels based on scores) on one dependent variable (proportional reasoning test scores).

Yet, before analyzing through One Way ANOVA, the three assumptions such as normal distribution of dependent variable through groups, homogeneity of variance for

dependent variable, and independency of scores gaining (Morgan, Barrett, Leech, and Gloeckner, 2019) were checked first since One Way ANOVA is used for parametric statistics.

4.2. Comparison of Proportional Reasoning Test Scores of Students with Respect to Their Number Sense Levels

Table 4.2. Descriptive Statistics for P.R.T scores based on N.S.S Score Level Groups.

Groups	N	Mean	Std. Deviation	Std. Error	95% Confidence Interval for Mean		Minimum	Maximum
					Lower Bound	Upper Bound		
Lower	217	15.28	10.40	0.706	13.89	16.67	0.00	41.00
Medium	91	26.13	10.37	1.087	23.97	28.29	5.00	47.00
Higher	20	36.50	10.82	2.419	31.44	41.56	7.00	53.00
Total	328	19.58	12.23	0.675	18.25	20.91	0.00	53.00

As seen in the table, the data was collected from 328 participants including 217 lower-level N.S.S score (0-5), 91 medium level N.S.S score (6-11), and 20 higher level N.S.S score (12-17). The mean of lower-level group for P.R.T scores is 15.28, while the mean of medium group is 26.13 and the mean of higher group 36.50 out of 53. As a result of the descriptive statistics, it was determined that the number of participants was concentrated in the lower-level group from N.S.S score level groups.

4.2.1. Assumption 1 (Normality) for One Way ANOVA of

Skewness values for each group were checked for normality assumption as seen in the table below. The skewness value of lower-level group is 0.630, while medium level group is equal to -0.780, and higher-level group is -.834, which indicates that

the skewness values of all three groups is between -1 and 1. Hence, the normality assumption is met, and it is not violated.

Table 4.3. Skewness Values.

ACHV Groups	**N**	**Skewness**	**Std. Error**
Lower	217	0.630	0.165
Medium	91	-0.780	0.500
Higher	20	-0.834	0.512

Q-Q PLOTS

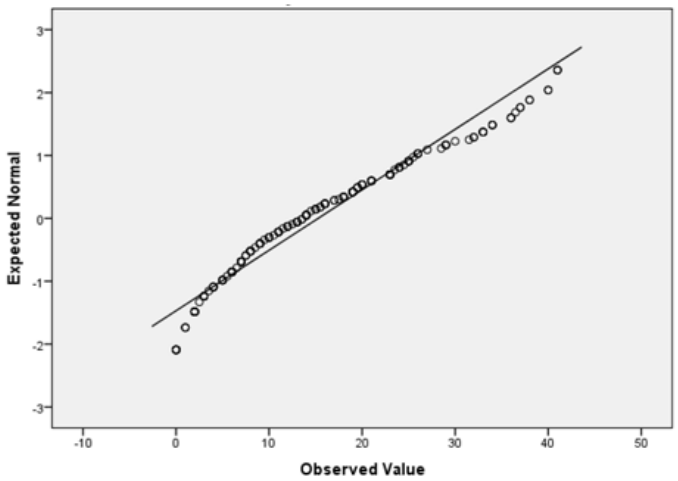


Figure 4.1. Q-Q Plot for the data (scores) of lower-level group.

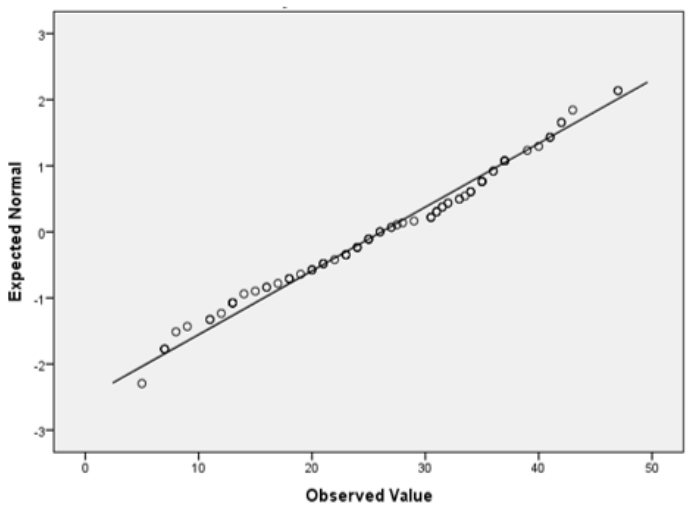


Figure 4.2. Q-Q Plot for the data (scores) of medium-level group.

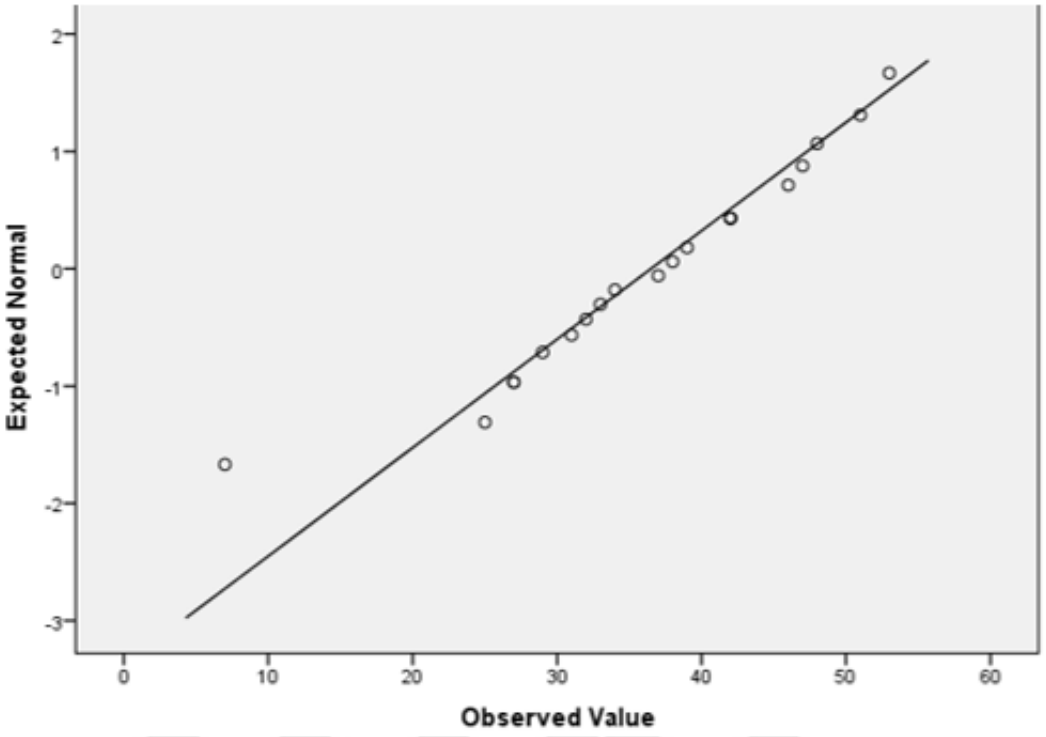


Figure 4.3. Q-Q Plot for the data (scores) of higher-level group.

Skewness values were checked for normality assumption of Research Question2, and histograms and Q-Q plots were checked to support skewness values. As seen in the figures, data are very close to the straight line for each group, which indicates that assumption is not violated, and data are normally distributed in each group.

Table 4.4. Test of homogeneity of variances.

	Levene Statistic	**df1**	**df2**	**Sig.**
P.R.T Scores	0.017	2	325	0.984

Based on the table gathered from SPSS, Levene's test indicates that F (2, 325) = 0.017, p=0.984, which states that F is not significant, and the assumption 2 (homogeneity of variance) is not violated.

4.2.2. Assumption 3 (Independent Data) for One Way ANOVA of Research Question 2

This assumption is not violated because one participant's score is not a pathway for another participant's score. In other words, the data collected from 328 students from three different schools, and one participant's score is not affected by another participant's score. Hence, data and scores of participants are independent from each other.

Thus, all three assumptions were met to conduct One Way ANOVA as a parametric test.

ANOVA

Table 4.5. Proportional reasoning test scores.

	Sum of Squares	**df**	**Mean Square**	**F**	**Sig.**
Between Groups	13644.40	2	6822.20	62.88	0.000
Within Groups	35261.21	325	108.50		
Total	48905.61	327			

Considering the table from SPSS, $F(2, 325) = 62.88$, $p = 0.000$, which states that F is significant, and there is a statistically significant difference between lower, medium, and higher-level groups of N.S.S on the proportional reasoning test scores of the 328 participants. In other words, there is a statistically significant difference by number sense level on proportional reasoning of the students, and the null hypothesis is rejected. In this regard, a post hoc test was conducted to see exactly which groups and to what extent the difference was, in other words, to compare the average between pairs.

Table 4.6. Multiple Comparisons, Post Hoc Test Tukey HSD.

Dependent Variable	**Groups**	**Between Groups**	**Mean Difference**	**Std. Error**	**Sig.**	**95% Confidence Interval**	
						Lower Bound	**Upper Bound**
Proportional Reasoning Test Scores	**Lower-Level N.S**	**Medium**	-10.850*	1.301	0.000	-13.91	-7.79
		Higher	-21.224*	2.434	0.000	-26.95	-15.49
	Medium Level N.S	**Lower**	10.850*	1.301	0.000	7.79	13.91
		Higher	-10.374*	2.572	0.000	-16.43	-4.32
	Higher Level N.S	**Lower**	21.224*	2.434	0.000	15.49	26.95
		Medium	10.374*	2.572	0.000	4.32	16.43

Post hoc Tukey HSD test revealed that lower-level number sense group and medium level number sense group significantly differed from each other based on their proportional reasoning scores ($p < 0.05$), and considering mean difference (Md= 10.850) students who have medium level number sense have higher mean of proportional reasoning test score compared to lower level group. Besides, lower-level number sense group and higher-level number sense group significantly differed from each other on their proportional reasoning scores (p=0.000). Mean difference is 21.224 between lower-level and higher-level groups, and higher-level group has greater mean of proportional reasoning test scores. Lastly, there is a statistically significant difference between the proportional reasoning test scores of medium level number sense group and higher-level number sense group ((p=0.000). Based on mean difference, Md= 10.374, the mean of higher-level number sense group for proportional reasoning test score is greater than the mean of medium level number sense group.

4.3. Qualitative Findings

In this part of the study, the interview questions were analyzed for four different groups which were constructed based on Number Sense Scale and Proportional Reasoning Test scores. In these four groups, the first group has a good number sense level and a low proportional reasoning level; the second group has a low number sense level and a good proportional reasoning level; the third group has low levels of both number sense and proportional reasoning; the fourth group consists of students with high levels of both number sense and proportional reasoning.

Table 4.7. Students with Good Level N.S and Low Level P.R.

Theme	**Category**	**Code**	**f**	**%**
1. Knowledge and facility with numbers	1.1 Sense of relative and absolute magnitude of numbers	1.1.1 Comparing to mathematical referent	1	**2.8**
	1.2 System of benchmarks	1.2.1 Mathematical	8	**22.2**
2. Knowledge and facility with operations	2.1Understanding the effects of operations	2.1.1Operating on whole numbers	1	**2.8**
		2.1.2 Operating on fractions/decimals	3	**8.3**
	2.2 Understanding the relationship between operations	2.2.1 Addition/ Subtraction	2	**5.6**
		2.2.2 Multiplication/ Division	4	**11.1**
3. Applying knowledge of and facility with numbers	3.1 Awareness that multiple strategies exist	3.1.1 Ability to apply different strategies	3	**8.3**

Table 4.7 Students with Good Level N.S and Low Level P.R. (cont.)

Theme	**Category**	**Code**	**f**	**%**
and operations to computational settings.		3.1.2 Ability to select an efficient strategy	2	**5.6**
	3.2 Inclination to utilize an efficient representation and/or method	3.2.1 Facility with various methods (mental, calculator, paper/pencil)	2	**5.6**
	3.3 Inclination to review data and result for sensibility	3.3.1 Recognize reasonableness of data	4	**11.1**
		3.3.2 Recognize reasonableness of calculation	6	**16.7**

4.3.1. Students with Good Level Number Sense and Low-Level Proportional Reasoning

The first group whose members are the two 7th grade students with good level number sense and low-level proportional thinking, focused on 3 themes, 7 categories related to them, and 11 different codes. As seen at the table, “applying knowledge of and facility with numbers and operations to computational settings” is the prominent theme (f=17, %= 47.3). The featured category of the theme is “inclination to review data and result for sensibility” (f=10, %= 27.8). Besides, students mostly utilized from the code, “recognize reasonableness of calculation” (f=6, %= 16.7) related to this theme. On the other hand, as different from other groups, “recognize reasonableness of data” code was emphasized by students with good level number sense and low level proportional reasoning.

To elaborate, students' answers for the "recognize reasonableness of calculation" code are like in below:

> AHMET: 2 in 6 will be one third, so we will divide by 3. We will divide 27.30 by 3. (Problem 1.a)
> BUSE: We need to divide 6 by 3 to make it 2 kg, so we divide 27.30, that is, the price, by 3. (Problem 1.a)
> AHMET: Here it will be 3/4, so this is 3/4 of 60, which is 45. If there are 150 centimeters of shadow in 200 centimeters, I simplify it to 3/4. I also simplify 60 centimeters to 3/4, 45. (Problem 1.b)
> AHMET: We need to look at the difference. Mahmut started late. Burak fills 60 glasses from 30 to 90 glasses. Mahmut should also fill 60 glasses in this duration because they don't waste any time filling in up to 90. Both fill in equal time. (Problem 2)
> AHMET: 6 taps fill it in 72 hours. We will multiply 6 and 72 to find the total and divide it by four. (Problem 5)
> Researcher: What do you mean by total?
> AHMET: What I mean by total is the amount of work that just one tap has its own, that is, how many hours does it take to fill with one tap alone? If we multiply 6 and 72, we get 432. One tap alone fills it in 432 hours. Yet, there are 4 taps, and we will share the filling work between 4 identical taps equally, which will be 108 hours. (Problem 5)
> Researcher: Where and how did you learn this method? What is the source of your method?
> AHMET: Actually, I knew it myself, but the more my teachers explained it, the more I improved.

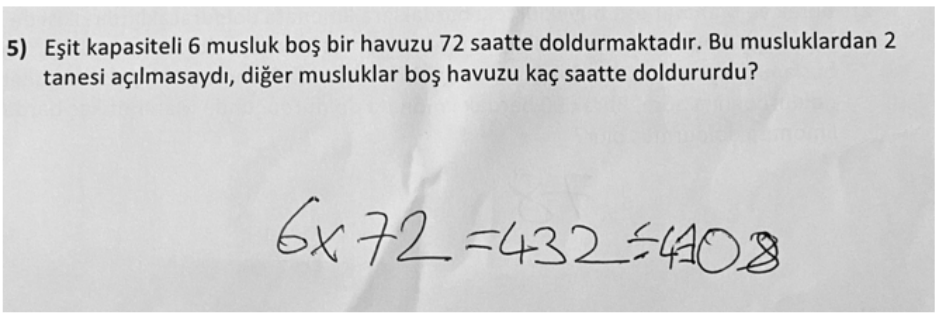


Figure 4.4. Example solution for problem 5.

When the first three answers which are related to problem 1, are examined, it can be stated that the students avoided from the algorithmic process such as cross multiplication and division for unit price, and that they made reasoning for their calculations. Besides, in problem 2, AHMET realized that he was supposed to make additive reasoning instead of multiplicative reasoning for the calculations s(he) did.

In problem 5, which is an inverse proportion problem, AHMET used a method unlike all other students interviewed in different groups. The other 9 students used the algorithmic calculation method used in the inverse proportion questions without actually having an understanding of the numbers and operations and could not reach the solution with any other method. Yet, in this group, AHMET was able to explain that if 6 taps fill it in 72 hours, the result of 6 times 72 operations will give how many hours it will take for one tap to fill it on its own, which means AHMET is quite aware of the plausibility of her/his calculation. Moreover, when the researcher asked about the source of method, AHMET stated that despite he had knowledge, the teachers helped improvement.

Related to this category, inclination to review data and result for sensibility, there is another code that stands out and is not seen in other groups, named as "recognize reasonableness of data" (f=4, %=11.1). The students' answers for this code were seen in problem 2 and 5, and they were like in below:

> AHMET: Some people may fall into a trap here. Instead of looking at the difference, they can think of a threefold relationship between 30 and 90 and apply this to 18 as well. (Problem 2)
> BUSE: They fill them into equal sized glasses and they fill them at the same speed. If we find the difference, we need to add it to the other one. I think we have nothing to do with the multiple here. (Problem 2).
> AHMET: 6 taps fill it in 72 hours. So, I think someone who doesn't know this can divide 72 by 6, and I think some could have fallen into a trap. I mean there is an inverse proportion here, but they may think it is a direct proportion (Problem 5).
> BUSE: There is an inverse proportion. If 2 of them were not opened, there would be 4 taps. For, while one increases at the same rate, the other decreases at the same rate. (Problem 5)

The students who can recognize reasonableness of data also can be indeed aware of what to do and what not to do with this data or givens in the problems. When the answers were examined specifically for this code, AHMET realized additive reasoning in problem 2 and did the calculation based on that, but he also emphasized that some other students could consider it multiplicatively, and hence that they would make a mistake. In other words, AHMET was able to reason to reach the correct conclusion

based on what was given, but he also identified possible mistakes as traps that could be fallen into. Similarly, in problem 5, AHMET again indicated that a student who could not distinguish inverse proportion from direct proportion would do a different and wrong calculation. When it comes to BUSE, she could also differentiate additive and multiplicative reasoning in problem 2, and direct proportion and inverse proportion in problem 5 with justifications.

"Knowledge and facility with operations" is the second prominent theme (f=10, %= 27.8) in this group. Related to the theme, "understanding the effects of operations" is one of the featured categories (f=4, %=11.1), and operations on fractions/decimals (f=3, %=8.3) is a crucial code that should be examined. As in other groups, students in this group also resorted to operating with fractions and decimals in problem 1. Yet, compared to answers from other groups, AHMET stated a different deduction in problem 1.a:

> AHMET: *Two in six will be one third, so we will divide by 3. We will divide 27 by 3, 9, and we divide 30 by 3, 10. So 9 liras, 10 cents* (27, 30:3).
> Researcher: *Could you do this operation in a different way?*
> AHMET: *27.30 is a decimal number, but since both the whole part and the decimal part are multiples of 3, I was able to divide directly. I couldn't have done it this way if it wasn't completely multiple of each other, which would be difficult for me.*
> AHMET clearly indicated that s(he) cannot always perform operations with decimal numbers in this way, but if the numbers are multiples of each other, this method would be much easier and preferable.
> In this group, as different from other answers, "comparing to mathematical referent" (f=1, %=2.8) code as related to "sense of relative and absolute magnitude of numbers" (f=1, %=2.8) category and "knowledge and facility with numbers" (f=9, %=24) theme, emerged.
> BUSE: *Classroom 7-A, consisting of 15 students, is given 10 pizzas, and class 7-B, consisting of 20 students, is given 10 pizzas. While each student in 7-B will receive half a pizza, each student in 7-A will receive more than half a pizza. So the answer would be class 7-A* (Problem 3.b).

Here, the student realized that 10 divided by 20 is half, s(he) took half as basis, and described the other operation (10/15) in terms of half.

In other words, to find out which class bought the most pizza, instead of writing it as a fraction and equating the denominator, she noticed that one of the classes bought half, and took the half as a reference and compared them accordingly.

Table 4.8. Students with Low Level N.S and Good Level P.R.

Theme	**Category**	**Code**	**f**	**%**
1. Knowledge and facility with numbers	1.1 System of benchmarks	1.1.1 Mathematical	8	34.8
2. Knowledge and facility with operations	2.1 Understanding the effects of operations	2.1.1 Operating on whole numbers	1	4.3
	2.2 Understanding the relationship between operations	2.1.2 Operating on fractions/decimals	3	13
		2.2.1 Addition/ Multiplication	1	4.3
		2.2.2 Addition/ Subtraction	1	4.3
		2.2.3 Multiplication/ Division	1	4.3
3. Applying knowledge of and facility with numbers and operations to computational settings.	3.1 Awareness that Multiple strategies exist	3.1.1 Ability to apply different strategies	1	4.3
	3.2 Inclination to utilize an efficient representation and/or method	3.2.1 Facility with various methods (mental, calculator, paper/ pencil)	2	8.7
	3.3 Inclination to review data and result for sensibility	3.3.1 Recognize reasonableness of calculation	5	21.7

4.3.2. Students with Low Level Number Sense and Good Level Proportional Reasoning

As for as the findings of analysis based on the answers of two 7th graders who have low level of number sense and good level of proportional reasoning, "applying knowledge of and facility with numbers and operations to computational settings" (f= 8, %=34.7) is one of the prominent themes as well as previous group. Depending on the theme, "inclination to review data and result for sensibility" category was highlighted by students (f=5, %=21.7), and the featured code for this category is "recognize reasonableness of calculation" (f=5, %= 21.7). Besides, the code was emphasized in the 1st and 5th ("finding unknown value" and "inverse proportion" problem types of proportional reasoning) problems. Related to this code, answers of students with low level number sense and good level proportional reasoning is like below:

> ZEYNEP: *Sir, since it is six kg, we will actually divide it into three. Because two is one third of six* (Problem 1.a).
> ZEYNEP: *3/4 of 200 is 150. If the height is 60 cm, 3/4 of 60 is 45. Because it should be the same ratio* (Problem 1.b).
> ZEYNEP: *If 6 taps fill it in 72 hours, 4 taps will fill it later because there are fewer taps* (Problem 5).
> AYŞE: *There is a proportion here, but I'm trying to understand whether it's inverse or direct. If 2 of them were not turned on, there would be 4 taps. All 4 taps will fill it in more than 72 hours because it is not 6 taps. This is the inverse proportion* (Problem 5).
> AYŞE: *We need to multiply side by side in inverse proportion. We will multiply 6 and 72 and divide by 4 to find x. We multiply side by side because it is an inverse proportion, not a direct proportion. All 4 taps fill it in 108 hours. If it was lower than 72, I would have made a mistake* (Problem 5).

As for the 1.a problem, instead of finding the unit price and moving forward, ZEYNEP realized that she could determine the numerical relationship between kilograms and apply this directly to prices. Hence, it is easy to say that she was able to recognize the reasonableness of her calculation from the very beginning. Also, ZEYNEP decided that she could determine the ratio between the tree and its shadow length and apply the same ratio to the sapling and its shadow, which explains that she noticed the reasonableness of the calculation she will make, and reached the result. Problem 5

is another problem in which the students utilize this code. Both students were able to state that the filling time would be longer than the given 72 hours due to the decrease in the number of taps, even before making the calculation. Moreover, AYŞE specifically indicated that she would have made a mistake if the result of her calculation had been a number smaller than 72 instead of 108. In short, students reasoned about the result and tested the reasonableness of their calculations before implementing the calculation for the problem.

The other prominent theme for this group is "knowledge and facility with numbers". (f= 8, %=34.8). As related to the theme, the category "system of benchmarks" (f= 8, %=34.8) was emphasized by 7th grade students. The featured code of this category is "mathematical"with the same frequency and percentage level. Along with that, when all interview problems were considered, students mostly utilized from their mathematical benchmarks in qualitative and quantitative comparison problems (3rd and 4th) from the types of proportional reasoning problems. Related to this code, the student answers for problems are like below:

> ZEYNEP: *Kemal filled 12 glasses in 1 minute, and Nisa filled 4 glasses in 1 minute. If we look at the number of glasses filled per minute, it is Kemal* (Problem 3.a).
> AYŞE: *If I divide 20 by 5, it becomes 4. Nisa fills a glass in 4 minutes, and if I divide 36 by 3, Kemal fills a glass in 12 minutes. So, Nisa fills the jug firs* (Problem 3.a).

For this specific problem, ZEYNEP considers the number of glasses filled per unit time as a benchmark for the filling duration, without using any other data such as jug and glass size given in the problem. However, apparently, AYŞE had difficulty in making reasoning about given data. As a result of division operation, she was expected to find the number of glasses filled per unit time, but she evaluated the result as the duration to fill a glass, which caused student to construct a wrong benchmark. "Interestingly enough, although the fact that these two students had low level of number sense based on the "number sense scale", ZEYNEP was good at constructing his/her mathematical benchmark, which is an important subcomponent of number sense. Also,

both students avoided from unnecessary operation and paper-pen calculation by eliminating givens in the problem, which could be another trigger point for number sense.

As for another problem, 3.b for this "mathematical benchmark code" the student answers are like in below:

> ZEYNEP: *For class 7-A, we must divide 15 by 20, and for class 7-B, we must divide 20 by 20. The answer will be 7-A. Fewer students will get more pizza.*
> AYŞE: *Here, I can use my logic without making any calculation. 7-A and 7/B, each takes 10 pizzas. If we divide 10 pizzas into 20, class 7-B will eat them in smaller slices. That's why I think 7-a students have much more pizza. I mean, I will divide 10 pizzas among 7-A and there are 15 students. I will distribute it to the 7-B in the same way, but there will be 20 students here. Since there are fewer students in class 7-A and more students in class 7-B, 7-A students will take more.*

As seen in the answers, ZEYNEP, evaluated given numbers' meanings in a wrong way, and she divided the number of students by the number of pizzas in order to find amount of pizza per student, instead of dividing number of pizzas by number of students. Yet, then she actually ignored the given numbers and their meanings, and considered the amount of pizza as related to number of students, which means the number of students was a benchmark for comparing the amount of pizzas per student. On the other hand, AYŞE not only correctly interpreted what was given and proceeded with the correct calculation, but also used the number of students as a benchmark for the amount of pizza per student.

As for 4.a problem about the prominent theme, category, and code, the students' answers are like in below:

> ZEYNEP: *Buğra ran fewer laps in a longer period of time than Ece. Both fewer laps and longer periods of time mean slowness. Then the answer is Ece.*
> AYŞE: *Ece ran more laps in less time than Buğra. The answer will be Ece.*
> Researcher: *What do you attribute for being fast?*
> AYŞE: *More laps should be run in less time.*

As for the answers of problem 4.a, ZEYNEP attributed that slowness to fewer laps and more time, and AYŞE attributed that being fast to more laps and fewer time after researchers' question, which demonstrates us how they construct their benchmarks.

As for 4.b problem about the prominent theme, category, and code, the students' answers are like in below:

> ZEYNEP: *If the parquet flooring he used in room A was larger, he installed room B faster. Since the parquet flooring used in room A is larger and the rooms are of equal size, B should also work faster.*
> AYŞE: *As far as I understand, he used larger parquets in room A and smaller parquets in room B. He finishes A room faster because small parquets require more effort.*
> Researcher: *But he finished the work for both rooms at the same time. The problem states that finishing time is the same for both rooms.*
> AYŞE: *If he finished it in equal time, I think it will have nothing to do with the size of the parquet.*
> Researcher: *If the finishing times are equal, isn't it possible to work faster in one room and slower in the other room, even if the parquet floors are large?*
> *Let's say you and I are doing a job. We both finished it in 5 minutes. I had more work to do than you. But I still worked very fast, I caught up with you and finished the job. It could be like that, right?*
> AYŞE: *Parquet was different and A was bigger. But now it could be B, because it worked faster so they finished it in the same time. Yes, I understand now. The answer will be B. I don't need to do any operation on this question.*

ZEYNEP had a deep understanding of givens, and she decided that the worker worked fast in room B considering the equal size of rooms. Yet, AYŞE obviously had difficulty in having understanding of the problems and givens by ignoring finishing at the same time for both rooms. Also ZEYNEP stated that if the finishing times are equal for both rooms, parquet sizes are trivial. Then, when the researcher explained problem with a real-life situation example, student can easily reach the answer. Here students again construct their own mathematical benchmarks to reach the result without any operation.

Another theme that remains in the background compared to the other two themes in this group is "knowledge and facility with operations" (f=7, %=30.2), and the prominent category of the theme is "understanding the effects of operations" (f=4,

%= 17.3). "Operating on fractions- decimals" (f=3, %= 13) and "operating on whole numbers" (f=1, %= 4.3) are the two codes of this category, and these codes were used by students mostly in problem 1. As for "operating on fractions-decimals" code, the students' answers are like in below:

> ZEYNEP: *If we divide 27 by 3, it equals 9. If we divide thirty by 3, it equals 10. Then, the answer is 9 lira 10 cents.*
> AYŞE: *If 6 kilograms is 27.30, 2 kilograms is x. If we do the cross product; we multiply x by 6 and multiply 2 by 27.30. This is a direct proportion, we do cross multiplication. Instead of multiplying by 2, I write it twice (27, 30+27, 30) and add them together because it is difficult to multiply numbers with commas. It makes 54.60. We will divide by 6. First of all, I think we will do it without seeing the comma. (wrote 5460). Then I'll shift the comma. 910 this is 9 lira 10 cents.*
> ZEYNEP: *3/4 of 200 is 150. If the height is 60 cm, 3/4 of 60 is 45.*

When student answers were examined, it was seen that they did not that much difficulty in operating with decimals and fractions, even though they used different methods. On the other hand, AYŞE stated that she preferred addition operation instead of multiplication when it came to decimal numbers by indicating that later is difficult.

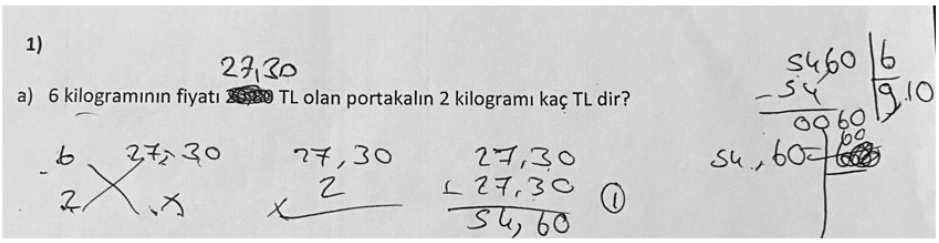


Figure 4.5. Example for the solution of problem 1.a.

Moreover, as for "operating on whole numbers" code, the student's answer is like in below:

> AYŞE: *There is a proportion here again. I'm doing cross multiplication again.*
> Researcher: *You wrote 150 opposite 200 and x opposite 60. When you need to multiply 150 and 60, you write them down and multiply them. Can we multiply 150 by 60 in a different, shorter way?*
> AYŞE: *If we delete the zeros and add them later, it is possible.*

Researcher: *So which one do you prefer more?*
AYŞE: *I do the same as in the first method, using zeros and writing them one under the other.*
Researcher: *Why?*
AYŞE: *It's easier and because I've been used to it this way since primary school.*

The student did cross multiplication to find unknown value in direct proportion problem. Yet, s(he) did not use shortcuts when it came to multiply by zeros despite the fact that s(he) is 7th grader. On the other hand, it is obvious the student has knowledge about shortcuts because when the researcher asked about any different and shorter way of multiplication, she could answer it easily. Yet, she does not prefer the shortcut due to the idea of its difficultness and previous knowledge from primary school.

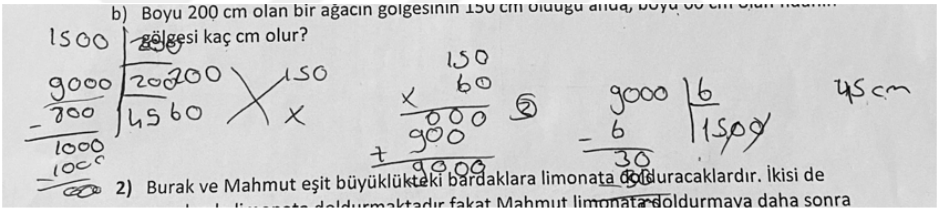


Figure 4.6. Example for the solution of problem 1.b.

Table 4.9. Students with Both Low Level N.S and P.R.

Theme	**Category**	**Code**	**f**	**%**
1. Knowledge and facility with numbers	1.1 System of benchmarks	1.4.1 Mathematical	**11**	**30.6**
2. Knowledge and facility with operations	2.1Understanding the effects of operations	2.1.1 Operating on whole numbers	**7** **3**	**19.4** **8.3**
	2.2Understanding the relationship between operations	2.1.2 Operating on fractions/decimals	**1**	**2.8**
		2.2.1 Addition/ Subtraction	**6**	**16.7**

Table 4.9 Students with Both Low Level N.S and P.R. (cont.)

Theme	**Category**	**Code**	**f**	**%**
		2.2.2 Multiplication/ Division		
3. Applying knowledge of and facility with numbers and operations to computational settings.	3.1 Awareness that multiple strategies exist	3.1.1 Ability to apply different strategies	**2**	**5.6**
		3.1.2 Ability to select an efficient strategy	**1**	**2.8**
	3.2 Inclination to utilize an efficient representation and/or method	3.2.1 Facility with various methods (mental, calculator, paper/pencil)	**2**	**5.6**
	3.3 Inclination to review data and result for sensibility	3.3.1 Recognize reasonableness of calculation	**3**	**8.3**

4.3.3. Students with Low-Level Number Sense and Low-Level Proportional Reasoning

The group which consists of three seventh grade students with low level number sense and low-level proportional thinking, focused on 3 themes, 6 categories, and 9 different codes as related to each other. "Knowledge and facility with operations" (f=17 %= 47.2) is the most emphasized theme for this group, and compared to other three groups, third theme is quite in the background with its categories and codes in this group.

Besides, "understanding the effects of operations" (f=10, %=27.7) is the featured category of the most emphasized theme with its prominent code, "operation on whole numbers" (f=7, %=19.4). This specific code was mostly utilized by the three stu-

dents in problem 1 (finding unknown value), problem 3 (quantitative comparison), and problem 5 (inverse proportion). Some of the students' answers for this code was like below:

> MEHMET: *I write the height information under the other height information and the shade information under the other. I apply direct proportion, and I multiply 150 and 60* (Problem 1.b)
> Researcher: *Here you write 150 and 60 one under the other and multiply them. Is there a shorter way?*
> MEHMET: *We'll ignore the zeros, multiply the numbers, and then add those zeros back in.*
> Researcher: *So, why did you prefer the first method?*
> *MEHMET: Writing one under the other and multiplying with the long way gives me a feeling of certainty.*
> Researcher: *Can you do it another way?*
> MEHMET: *Yes, I could do it mentally, but visually placing numbers on paper for cross multiplication make me feel comfortable.*

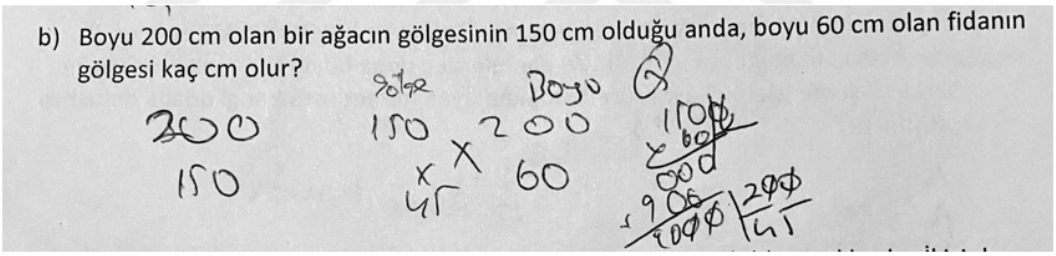


Figure 4.7. Example solution for problem 1.b.

As seen in the conversation between MEHMET and the researcher above, although the student knew how to multiply the short way, he chose the long way to feel certainty. Also, the researcher's question also helped the student express the shortcut. In other words, the student did not apply the shorter way in the first step.

> SİNAN: *First, by adding three zeros, I convert 3 liters to 3000 milliliters. Then I divide 3000 ml by 50 to find out how many glasses 3000 ml is. (the student divided them in shorter way). For Nisa, we will divide 60 by 20, and find 3. We multiply 3 by 5, which is 15 minutes to fill the jug. For Kemal, I used the multiples rule because 60 is not divisible by 36 glasses in three minutes, then 12 glasses in one minute. So I divide 60 by 12, 5 minutes. Hence, Kemal is faster* (Problem 3).
> Researcher: *Can we solve this in a way other than using 3000 ml etc.*
> SİNAN: *No, we cannot.*

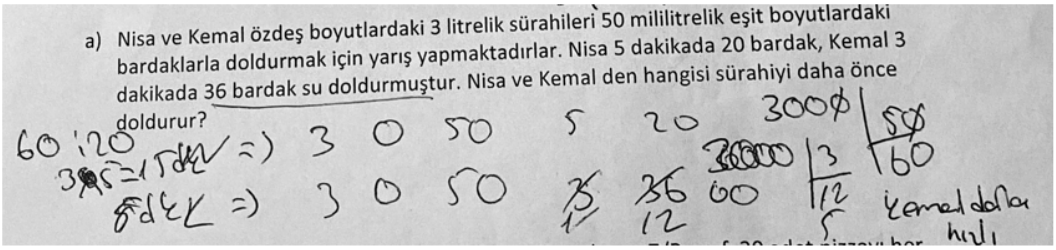


Figure 4.8. Example solution for problem 3.a.

First, the student converted 3 liters to 3000 milliliters by adding directly three zeros in as the short form of multiplying with 1000. Similarly, when dividing 3000 by 60, she took the shortcut and reached the result by deleting the zeros with simplification. These two steps are the crucial parts of operation on whole numbers. On the other hand, as different from other students in the groups, instead of looking at the number of glasses filled per unit time and establishing a mathematical benchmark, s(he) used all the numbers given, made lengthy calculations, and rejected any other solution.

In problem 5, which is inverse proportion problem, students also benefited from this code. MEHMET and SİNAN used the algorithm for inverse proportion directly by multiplication and division operations as seen below. Yet, MEHMET confused the places where he should place the numbers in the algorithm as seen below.

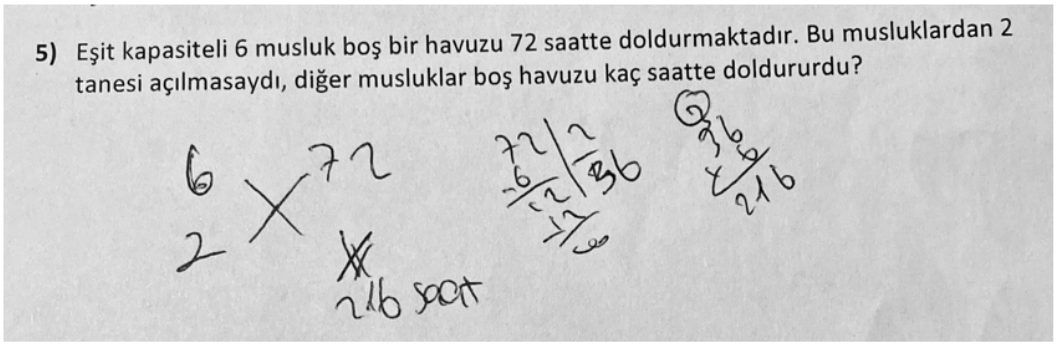


Figure 4.9. Example solution for problem 5.

As for SİNAN, he preferred the algorithm directly as well, but instead of inverse proportion, he did cross multiplication as like direct proportion. When the result was 48, SİNAN stated that ‘‘*while the duration was 72, it became 48, it decreased, but*

it should have increased due to the number of taps". Then, the student changed the algorithm and reached the correct result.

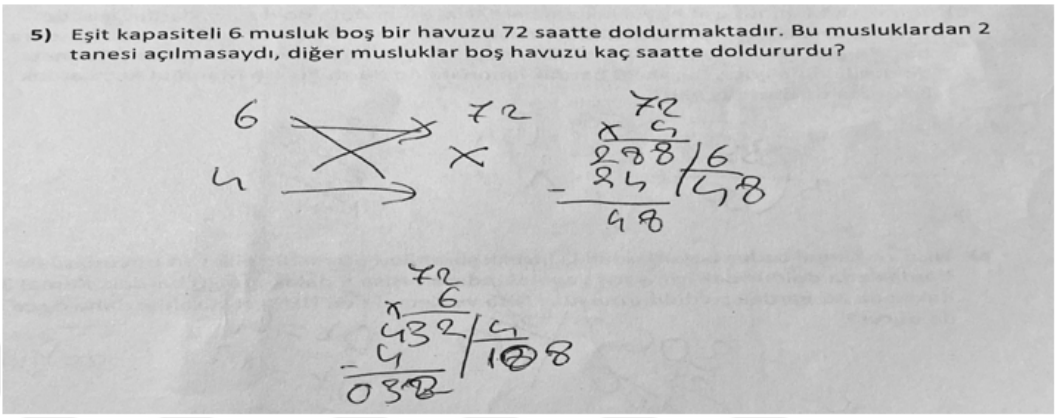


Figure 4.10. Example solution for problem 5.

Moreover, in the problems mentioned above, these students tended to calculate directly by doing procedures in the first step instead of making reasoning compared to most of the students in the other groups.

In this group, "knowledge and facility with numbers" (f=11 %= 30.6) is the second prominent theme with one category "system of benchmarks" and one code "mathematical". As in previous groups, students benefited from this code most in the 3rd (quantitative comparison) 4th (qualitative comparison) and 5th (inverse proportion) problems. Some of the related student answers are below:

> MERVE: *Nisa filled 20 glasses of water in 5 minutes, and Kemal filled 36 glasses of water in 3 minutes. Kemal filled more water in a shorter time. Nisa, on the other hand, filled fewer glasses than him in a long time. I think Kemal fills the glass first here because he filled more glasses than Nisa in a very short time* (Problem 3.a).
> MERVE: *If the pizza quantities were not equal, for example, if they gave 7-B a larger pizza, B would also took more. But since they are equal size of pizza here, since class 7-B consists of 20 students, they receive less share. There is less pizza for each student in 7-B because Class 7-A has 15 students and their number of people is less, there is more pizza per person* (Problem 3.b).
> SİNAN: "*Running more laps in less time shows that you are fast*" (Problem 4.a).
> SİNAN: "*Here he worked faster in room B. Sir, because the parquets in room A are larger and can take up space quickly, but since the parquets in room B are small, you need to work a little faster than in room A*" (Problem 4.b).

MERVE: *Since the parquet size is smaller in room B, if the worker worked faster in this room, he could equalize the finishing times* (Problem 4.b).
MERVE: *If 4 taps work and it took 72 hours to fill it with 6 taps... It will definitely take more than 72 hours to filli.* (Problem 5).

In the first answer by MERVE, the student attributed fast filling to filling more glasses in a shorter time, established her mathematical benchmark and reached the result without using other numerical expressions or performing any operations.

In his answer to problem 3.b, the student emphasized that the amount of pizza given and the number of students are criteria for the amount of pizza per student, but since the amounts of pizza are equal, it will be sufficient to look at the number of students.

As well as, SİNAN in problem 4.a, attributed fast running to less time and more laps, and in problem 4.b, both SİNAN and MERVE stated that the worker had to work faster in room B in order to finish in the same time, as it would take a lot of time to place small parquets.

In problem 5, even if the student could not reach the numerical result, she stated that the number of taps is a criterion in the filling time of the pool, and emphasized that the result will definitely be more than 72 hours as the number of taps decreases.

Table 4.10. Students with Both Good Level N.S and P.R.

Theme	**Category**	**Code**	**F**	**%**
1. Knowledge and facility with numbers	1.1 Sense of orderliness of numbers	1.1.1 Relationships between number types	**1**	**1.7**
		1.1.2 Ordering numbers within and among number types	**2**	**3.3**

Table 4.10 Students with Both Good Level N.S and P.R. (cont.)

Theme	**Category**	**Code**	**F**	**%**
	1.2 Sense of relative and absolute magnitude of numbers	1.2.1 Comparing to mathematical referent	**1**	**1.7**
	1.3 System of benchmarks	1.3.1 Mathematical	**13**	**21.7**
2. Knowledge and facility with operations	2.1 Understanding the effects of operations	2.1.1 Operating on whole numbers	**3**	**5**
	2.2 Understanding the relationship between operations	2.1.2 Operating on fractions/decimals	**6**	**10**
		2.2.1 Addition/ Subtraction	**2**	**3.3**
		2.2.2 Multiplication/ Division	**6**	**10**
3. Applying knowledge of and facility with numbers and operations to computational settings.	3.1 Understanding the relationships between problem context and the necessary computation	3.1.1 Recognize data as exact or approximate	**1**	**1.7**
	3.2 Awareness that multiple strategies exist	3.2.1 Ability to apply different strategies	**7**	**11.7**
	3.3 Inclination to utilize an efficient representation and/or method	3.2.2 Ability to select an efficient strategy	**4** **6**	**6.7** **10**
	3.4 Inclination to review data and result for sensibility	3.1.1 Facility with various methods (mental, calculator, paper/pencil) 3.4.1 Recognize reasonableness of calculation	**8**	**13.3**

4.3.4. Students With Good-Level Number Sense and Good-Level Proportional Reasoning

Fourth and last group members, who were three seventh graders with good level number sense and good level proportional reasoning, focused on three main themes, nine different categories, and 14 different codes. Among four groups, the highest number and variety of codes were examined in this group.

To begin with the prominent theme for this group, "applying knowledge of and facility with numbers and operations to computational settings" (f=26, % = 43.4), "awareness that multiple strategies exist" category was mostly emphasized by the students in this theme. Moreover, considering of the theme, members of this group were more successful in applying their knowledge of numbers and operations on the problems.

The category that consists of two different codes, "ability to apply different strategies" (f=7, %= 11.7) and "ability to select an efficient strategy" (f=4, %=6.7), displays how students realize different strategies and how they choose those which are most efficient and appropriate for them.

Some of the students' answers for the featured code of the leading category belonging to the most highlighted theme were like below:

> DOĞA: *Since 2 is one third of 6, we divide 27.30 by 3. We divide 27 by 9 to get 3, we divide 30 by 3 to get 10. The answer is 9.10* (Problem 1.a) .
> Researcher: Since they are divisible here, you could divide it very easily. Let's say it is not divisible, let it be 5 kg instead of 6 kg, or 20.80 instead of 27.30 here. How would you do it then?
> DOĞA: *Then I would find the price of one kilogram.*
> ALİ: I *will do it through cross multiplication. considering that if a 200 cm tree has a 150 cm shadow, how many cm (x) of shadow will a 60 cm sapling have. I can find x by simplification. Also, there is a 75% relationship between 200 and 150, I can get 75 percent of 60 to get 45*(Problem 1.b).

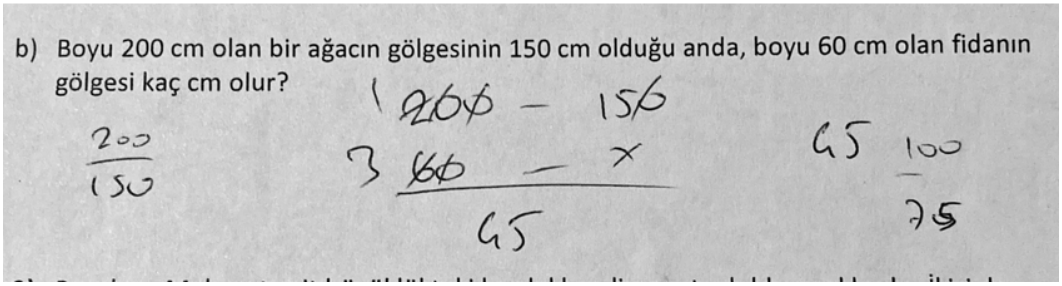


Figure 4.11. Example solution for problem 1.b.

DOĞA: *To run faster, she needs to run more in less time, so the answer is Ece. When I want to verify this, I can verify it by giving a numerical value. (like Ece ran 10 laps in 60 seconds, Buğra ran 5 laps in 120 seconds)* (Problem 4.a)
ALİ: *The parquets in B are smaller, and since it will be difficult to place small ones, the one in B is faster. For example, we can do a 10 square meter area in 2 times with 5 square meters of parquet, but with 10 square meters of parquet, we can finish it faster in one time* (Problem 4.b).

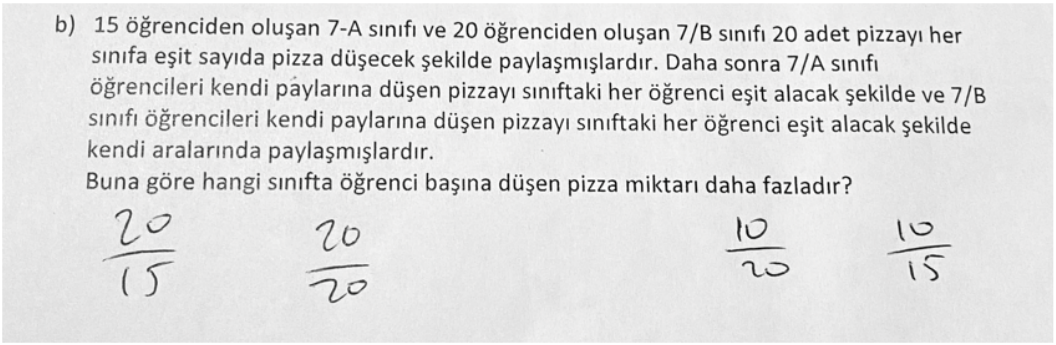
Considering the first dialogue, when the researcher offered different numbers which were not easily divisible for the solution, DOĞA explained a new, different solution strategy to apply instead of persistently thinking on existing solution. In other words, in the given proportional reasoning problem, the student coped with a possible difference and challenge by using a certain component of number sense.

As for ALİ, in problem 1.b, he stated that along with cross multiplication, the ratio could be expressed as a fraction, and then it could be evaluated as a percentage, which also could be an indicator of code 1.1.1 since the student could realize the relationship between fractions and percentages. Hence, even if this student does not know cross multiplication or has trouble knowing the algorithm or procedure and where to place the numbers correctly, he will be able to develop her own methods and use the number relations she knows to reach the correct result.

As related to this code, in problem 5, which is a qualitative comparison problem from the kinds of proportional reasoning problems, DOĞA and ALİ stated that even if they initially reached conclusions by reasoning, they could also reach conclusions or strengthen their ideas by giving numerical values.

As similar to code 3.2.1, another code of the same category, "ability to select an efficient strategy" was utilized by these students, and some of the answers were like below:

> MELİSA: *Since the shadow of the tree is three-fourths of its height, we find three-fourths of 60, and it will be 45. Actually, we could do this with cross multiplication, but it would be a slow method that took time* (Problem 1.b).
> DOĞA: *If Nisa fills 20 glasses in 5 minutes, she can fill 4 glasses in 1 minute. If Kemal fills 36 glasses in 3 minutes, he can fill 12 glasses in 1 minute. So here Kemal fills faster. I decide by looking at the glass they fill per unit time* (Problem 3.a).
> Researcher: *I understand. So, could you solve this in another way?*
> DOĞA: *Considering 3 liters, that is, the volume of the jug and the volume of the glass, we would find out exactly how many minutes they would take to fill a jug, but this would be more difficult and would take longer.*
> DOĞA: *One class consists of 15 students and the other class consists of 20 students, and there are 10 pizzas for each class. Here, of course, you know, the question asks who gets more pizza for fewer students. We can also look at it as a fraction. 10/15 is for one, and 10/20 is for another one. Here too, it asks us the larger fraction as a fraction. It becomes 7-A. Yet, it can be understood by looking directly at the number of people.* (Problem 3.b).


b) 15 öğrenciden oluşan 7-A sınıfı ve 20 öğrenciden oluşan 7/B sınıfı 20 adet pizzayı her sınıfa eşit sayıda pizza düşecek şekilde paylaşmışlardır. Daha sonra 7/A sınıfı öğrencileri kendi paylarına düşen pizzayı sınıftaki her öğrenci eşit alacak şekilde ve 7/B sınıfı öğrencileri kendi paylarına düşen pizzayı sınıftaki her öğrenci eşit alacak şekilde kendi aralarında paylaşmışlardır.
Buna göre hangi sınıfta öğrenci başına düşen pizza miktarı daha fazladır?

$\frac{20}{15}$ $\frac{20}{20}$ $\frac{10}{20}$ $\frac{10}{15}$

Figure 4.12. Example solution for problem 3.b.

In the first answer, it can be seen that MELİSA is aware that there is more than one solution, and cross multiplication, which is maybe the most common one among students, will take more time compared to the method she preferred. Also, in Problem 3.a (quantitative comparison problem), DOĞA, who quickly found a solution by taking into account the number of glasses filled per unit time, argued that she could actually reach exact numerical values by using other information such as jug and glass size, but this would be a difficult and time-consuming method.

In problem 3.b, DOĞA first focused on the number of pizzas and people, then tried to find the number of pizzas per person as a fraction, but finally specifically stated that the problem could be solved by looking directly at the number of people. Here, the student divided the number of pizzas by the number of people, established a ratio, and in his own words, ordered the fractions and determined that the bigger one would take more pizza, which is also compatible with code 1.1.2. On the other hand, she emphasized that there should be fewer students before and after implementing this procedure as an efficient strategy, which is also compatible with code 1.3.1 since the number of students was taken as a benchmark.

When considered generally based on students' answers for this specific code 3.2.2, it was generally encountered in the 3rd problem (the quantitative comparison problem), and although the students were aware of various methods, they were able to choose the one that was more efficient for them. What they meant by efficiency here was generally on a duration/time basis.

As different from other three groups and related to third theme and its first category 3.1 "understanding the relationships between problem context and the necessary computation", (f=1, %=1.7) code 3.1.1 "recognize data as exact or approximate" was used by DOĞA in problem 2 (problem involving a non-proportional type of relationship) and emerged in this group.

> DOĞA: *When Burak filled 30 glasses, Mahmut filled 18 glasses. The difference between them is 12. Since they fill it at the same speed, when Burak fills 90, the difference should be 12 again. Then it becomes 78.*
> Researcher: *In the previous problem (finding unknown value), you looked at the difference as a multiple. You thought of it as multiplication or division. Here you think like addition and subtraction. You looked directly at the difference. What is the reason?*
> DOĞA: *If I had done it like the previous problems, the difference would have become much greater and this time they would not have filled it in equal time.*

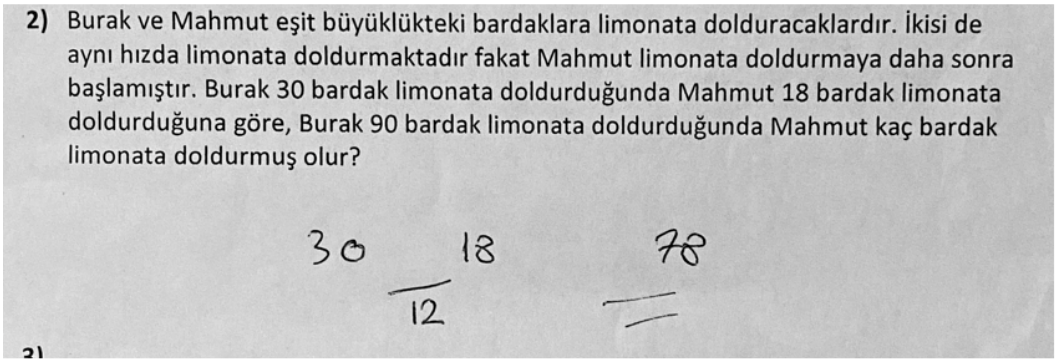


Figure 4.13. Example solution for problem 2.

Here, the student understood what was given in the problem, she was able to distinguish what was given, and reached a conclusion. Although at first glance there seems to be a proportional difference and there is a possibility of making the mistake of progressing through multiplicative reasoning, the student has realized that she needs to think additively. In response to the researcher's question, the student stated that if she had thought multiplicatively, the difference would have become much larger, and this would have been reflected in the filling times. In other words, as stated in the related category, the student comprehended the relationship between the problem context and the calculative process that must be followed to achieve the result.

Generally, the three students in this group reached their conclusions by comprehending the problems and givens as a first step, and even if they were aware of other strategies, by choosing the most efficient strategy with the help of different codes of number sense specific to that situation, rather than performing operations.

5. DISCUSSION AND CONCULISION

5.1. Summary of the Study

To remind, the main aim of this study, which examines whether 7th grade students' number sense makes a significant difference in their proportional reasoning, is to reveal how 7th grade students use their number sense in proportional reasoning problems. In order to reach the answers of research questions, explanatory sequential design, which is a mixed method design, was conducted. According to the descriptive statistical results, the level of number sense for 328, 7th grade students is quite low. Also, there is a statistically significant difference among proportional reasoning test scores of the students based on their number sense levels. The students who have high level number sense have also higher level of proportional reasoning based on Proportional Reasoning Test, while the students who have low level number sense have also lower level of proportional reasoning. Besides, the students who have medium level number sense are better at proportional reasoning than students who have low number sense. As a follow-up to quantitative results, in order to explain how students use their number sense in the proportional reasoning problem types (finding missing value, quantitative comparison, qualitative comparison, problem involving a non-proportional type of relationship, and inverse proportion), four different group was constructed:

(i) Students With Good Level Number Sense and Low Level Proportional Reasoning
(ii) Students With Low Level Number Sense and Good Level Proportional Reasoning
(iii) Students With Low Level Number Sense and Low Level Proportional Reasoning
(iv) Students With Good Level Number Sense and Good Level Proportional Reasoning

Two students were included for each group in first and second group, and three students were included for each group in third and fourth group.

As for the general findings of the groups, all three themes were seen in all groups. 11 different codes were observed in first group, 9 different codes were observed in second group, while 13 different codes were observed in fourth group, and 9 different codes were observed in third group. While the number and diversity of codes is highest in the fourth group, the least is in the third group.

As for the first group, whose members have good level number sense and low level proportional reasoning, they focused on third theme (applying knowledge of and facility with numbers and operations to computational settings), and "reasonableness of calculation" code in especially problem 1 (missing value), problem 2 (quantitative comparison), and problem 5 (inverse proportion). The students in this group were away from algorithmic processes, and they could make reasoning in their calculation methods. Besides, they were good at distinguishing multiplicative reasoning from additive reasoning although they had low level proportional reasoning. As different from other groups, "reasonableness of data" code was encountered in this group. While interpreting the data, they were able to take into account every possible situation and make comments accordingly. Moreover, "comparing to mathematical referent" code was only seen in this group. In the problem five, which is inverse proportion problem, one of the students in this group reached the solution as different from other nine students in all groups because the student solved the problem by making reasoning rather than inverse proportion algorithm.

As for second group, whose members have low level number sense and good level proportional reasoning, they focused on the first theme (knowledge and facility with numbers), and "mathematical benchmark" code in problem 3 (quantitative comparison) and problem 4 (qualitative comparison). In these two qualitative and quantitative comparison problems, students tended to solve the problem by establishing a mathematical benchmark without doing operation. However, one of the students had difficulty setting up the benchmark correctly and achieving results. Yet, the researcher's questions helped the student think and reach the correct conclusion. Moreover, when it comes to operations with numbers, the student who had difficulty in constructing

benchmarks correctly, also had difficulty in shortcuts for the multiplication of numbers, and operation on decimals. Lastly, the students in this group were left behind in applying different strategies for problems.

As for third group, whose members have both low-level number sense and proportional reasoning, they focused on the second theme (knowledge and facility with operations), and "operation on whole numbers" code in especially problem 1 (missing value), problem 3 (quantitative comparison), and problem 5 (inverse proportion). In these problems, the students generally established direct proportion and inverse proportion algorithms, and long calculations, which sometimes leaded confusion on them. On the other hand, of the students reached the solutions in problem 3 and problem 4 by establishing mathematical benchmark instead of long calculations. Besides, third theme (applying knowledge of and facility with numbers and operations to computational settings) were quite in the background in this group compared to other groups. Also, the number and variety of codes are at least in this group.

As for last and fourth group, whose members have both good level number sense and good level proportional reasoning, they focused on third theme and, "awareness that multiple strategies exist" code. Regarding of this, the students in this group, made reasoning about given data, they showed that they knew different strategies and tended to choose the most effective and easiest strategy instead of doing long calculations and operations. Besides, the number and variety of codes were highest in this group, and "recognize data as exact or approximate" code was seen in only there.

5.2. The Implications for the Study

First of all, as Facun and Naul (2002), and Kayhan and Umay (2010) stated in their studies, this study also revealed that 7th grade students' number sense levels are quite low according to the mean of their scores gained by Number Sense Scale. At this point, regarding the importance of number sense on mathematics learning (Cain *et al.*, 2007), searching for methods to improve number sense and perhaps incorporating

number sense into the education process may contribute to the development of students' number sense. Along with that, the study also revealed that number sense takes an active role in students proportional reasoning skills. Students with good number sense were more successful in proportional reasoning problems than students with medium or low number sense, and students with medium number sense were more successful than students with low number sense based on proportional reasoning problems. Regarding this statistical result, it could be obviously and easily said that number sense could be benefited from to foster proportional reasoning, which may mean that teaching proportional reasoning in schools can be carried out a little more independently of algorithmic processes as compatible with the idea of Lamon (2006; 2007) about the restricted instruction in schools.

After the role of number sense on proportional reasoning was determined statistically, the study also examined how students used their number sense in proportional reasoning problem types. Five different proportional reasoning problems were used, and interviews were held with four different groups that consisted of ten students in total.

When the first group that consisted of two students with good level number sense and low level proportional reasoning was examined, it was observed that students reached the correct conclusion in proportional reasoning questions such as finding the missing value and quantitative comparison, without depending on any operation or algorithmic process.

Although it is so common to use direct proportion algorithms in missing value problems, these two students reached the conclusions without them. These students also were good at what to do with given data as different from other groups (compatible with "reasonableness of data" code). They were good at distinguishing inverse and direct proportion, and additive and multiplicative reasoning despite that they have low level on proportional reasoning test. This may actually show that students with a good sense of numbers can reach the correct conclusion in proportional reasoning

problems with their own reasoning, but presenting the problems to the student in a test or exam style may lead the student to the style or algorithmic processes taught at school. Moreover, even if their proportional reasoning test results were not good, they were very good at reasoning mathematically because they can use their number sense appropriately, and as Faulkner (2009) stated, proportional reasoning is actually achieved when moving away from formulas and procedures. Lastly, in this group, a student as different from other nine students in the interview regardless of their groups, solved the inverse proportion problem by making reasoning. On the other hand, other nine students tried to solve it by establishing an inverse proportion algorithm, which shows that there is indeed a problem in the understanding of inverse proportion, which may stem from the instructions and inadequate content knowledge of teachers.

As for the examination of second group which consisted of two students with low level number sense and good level proportional reasoning, even though these students did not make much use of different strategies or other components of number sense, they achieved results by using mathematical benchmarks, especially in quantitative and qualitative comparison questions. In this group, where the variety and number of codes for number sense were low, one of two students preferred long paths when operating with decimals and integers, and explained that the reason was that the long path felt more easy and comfortable since s(he) has been used to it since primary school. This can be considered as an example of how great the impact of teaching in schools has on students.

When it comes to third group that consists of three students with low level number sense and low level proportional reasoning, these students tended to establish direct proportion and inverse proportion algorithms as a first step especially in missing value, quantitative comparison, and inverse proportion problems. Regarding this step, one of three students stated that s(he) receives a certainty from these algorithms, and the other rejected reaching the solution through another way except long operations. Therefore, the students in this group were focused on performing calculations and operations, and doing procedures based on what was given in the problems. In this

group, where the number and diversity of codes was least, students were good at using mathematical benchmarks in operations with integers and especially in qualitative comparison questions. Considering that they focus on algorithmic processes and their initial inclination is always to perform operations, their reason for executing operations with integers and knowing shortcuts may be to follow what is taught and comply exactly.

In this group, where there were three students who have both good number sense and proportional reasoning, the number and variety of codes related to number sense are highest. The most important feature of the students in this group is that although they know more than one method to solve the problem, they have the will to choose the most efficient and applicable one. These students stated that even if both methods lead to the correct result, making calculations and using algorithms such as cross multiplication and direct proportion algorithms instead of reasoning is long, tiring and time-consuming. Likewise, they were very good at establishing mathematical benchmarks.

In general, all groups were good at establishing mathematical benchmarks, especially in quantitative and qualitative comparison problems. Moreover, the students used their number sense the most in problem 1 (missing value), problem 3 (quantitative comparison) and problem 4 (qualitative comparison). Additionally, students with good number sense tended to make reason using number sense components rather than long operations and procedures. Students with low number sense tended to create direct or inverse proportion algorithms using the given numbers especially in problem 1, 3 and 5. Besides, in groups with students with a good level number sense, using different and effective strategies came to the fore instead of sticking tightly to known or learned methods and applying them. Considering Yang and Wu (2010), it is an undeniable fact that strict dependency to written calculations instead of the development of number sense limits students' mathematical thinking.

Regarding all these, it is an undeniable fact that number sense takes an important place when it comes to proportional reasoning of 7th grade students. Hence, considering that both students and teachers have difficulty in proportional reasoning (Weiland *et al.*, 2019) , that proportional reasoning remains only algorithmic teaching in schools (Lamon, 2006, 2007), that different methods are needed for the development of proportional reasoning (Bentley and Yates, 2017) and that number sense gives deep meaning to both teachers and students in mathematics subjects (Cain *et al.*, 2007), the possible positive effects of number sense on proportional reasoning should not be ignored.

5.3. Limitations and Suggestions

Even though the study was completed successfully and there were important results, there are some limitations that should be cared about in the study to regard for future studies. Based on the feedback of some of the participants (7th grade students) one lesson hour was inadequate to express their ideas for the problems in Number Sense Scale. Since one lesson hour was insufficient for some students, and two lesson hours will be too much for this scale consisting of 17 questions, attention can be paid to allowing time between these two time periods in terms of timing.

Along with that, since those who took the number sense scale also had to take the proportional reasoning test, the two measurement tools were administered in two consecutive lesson periods on the same day, before the lunch break. Yet, this was also considered as tiring by some students. Although the future researcher may collect data on the same day to ensure that the same participants receive both measurement tools, it may be more beneficial to leave a time period between the application of the two measurement tools.

Besides with timing, following the 7th grade curriculum, data was collected after the proportional reasoning topic was covered, but since it was close to summer vacation, it was very difficult to find students in the classrooms and make them fo-

cused. The school administration and deputy principals were very supportive in this regard. Therefore, it would be beneficial to work collaboratively with administrators and teachers, and perhaps not collect data close to holidays.

Another limitation about the study was interviewing with only ten students. Therefore, there were 2-3 students in each group, which may have been limiting in getting more diverse answers. Hence, , t may be useful to increase the number of students in groups for future studies.

Moreover, as a little suggestion, in these two measurement tools, attention is paid to students' explanations rather than their correct or incorrect answers. Therefore, it should be emphasized that students should be kept away from subjects such as exams, grades, and scoring from the very beginning, and that explaining their ways of thinking is the most important criterion in the evaluation.

Lastly, as a contributive study to this research, experimental studies that examine number sense as an intervention on proportional reasoning, and other topics related to the indirect or direct inclusion of number sense in curriculum and, teachers' number sense development and their ability to reflect this in their instruction, could be studied

REFERENCES

Acar, S. and Peker, B. 2022, "Türkiye'de Matematik Eğitimi Alanında Yayımlanan Sayı Hissi ile İlgili Makalelerin İçerik Analizi", *International Journal of Educational Studies in Mathematics*, Vol. 9, No. 1, pp. 14-32.

Akatugba, A. H. and J. Wallace, 2009, "An Integrative Perspective on Students' Proportional Reasoning in High School Physics in a west African Context", *International Journal of Science Education*, Vol. 31, No. 11, pp. 1473-1493.

Akkuş, O. and A. Duatepe Paksu, 2006, *Orantısal Akıl Yürütme Becerisi Testi ve Teste Yönelik Dereceli Puanlama Anahtarı Geliştirilmesi*, Eurasian Journal of Educational Research, Ankara.

Anghileri, J., 2006, *Teaching Number Sense* (2nd Ed.), Continuum International Publishing Group, London.

Arican, M., 2023, "The Development and Application of an Interview Structure on Determining Preservice Mathematics Teachers' Competence in Proportional Reasoning", *Mathematics Education Research Journal*, Vol. 35, No. 1, pp. 55-79.

Bal-İncebacak, B. ve E. Ersoy, 2016, "7. Sınıf Öğrencilerinin Matematiksel Muhakemelerinin Timms'e Göre Analizi", *Uluslararası Sosyal Araştırmalar Dergisi*, Vol. 9, No, 46, pp. 474-481.

Baykul, Y., 2009, *İlköğretimde Matematik Öğretimi 6-8. Sınıflar*, Pegem Akademi, Ankara.

Baxter, G.P. and B.A. Junker, 2001, *Designing Cognitive-Developmental Assessments: A Case Study in Proportional Reasoning*, Paper Presented at the Annual Meeting

of the National Council for Measurement in Education Seattle, Washington.

Behr, M.J., G. Harel, T.R. Post, R. Lesh, 1992, "Rational *Number, Ratio, and Proportion*, *Handbook of Research on Mathematics Teaching and Learning: A Project of the National Council of Teachers of Mathematics*, Vol. 1, No. 1, Pp. 296–333.

Ben-Chaim, D., J.T. Fey, W.M. Fitzgerald, C. Benedetto and J. Miller, 1998, "Proportional Reasoning Among 7th Grade Students with Different Curricular Experiences", *Educational Studies in Mathematics*, Vol. 36, No. 1, pp. 247-273.

Bentley, B. and G.C. Yates, 2017, "Facilitating Proportional Reasoning Through Worked Examples: Two Classroom-Based Experiments", *Cogent Education*, Vol. 4, No. 1, pp. 1297213-1297265.

Berk, D., S.B. Taber, C.C. Gorowara and C. Poetzl, 2009, "Developing Prospective Elementary Teachers' Flexibility in the Domain of Proportional Reasoning", *Mathematical Thinking and Learning*, Vol. 11, No. 3, pp. 113-135.

Boyer, T.W. and S.C. Levine, 2015, "Prompting Children to Reason Proportionally:, Processing Discrete Units as Continuous Amounts", *Developmental Psychology*, Vol. 51, No. 5, pp. 615-645.

Bütüner, S.Ö., 2017, "Comparing the Use of Number Sense Strategies Based on Student Achievement Levels", *International Journal of Mathematical Education in Science and Technology*, Vol. 1, NO. 1, PP. 1-32.

Cain, C., M. Doggett, V. Faulkner and C. Hale, 2007, *The Components of Number Sense, Raleigh, NC: NC Math Foundations Training*, Exceptional Children's Division of the North Carolina Department of Public Instruction, New York.

Carpenter, T.P., 1989, "Number Sense and Other Nonsense", *Establishing Foundations*

for Research on Number Sense and Related Topics: Report of A Conference San Diego: San Diego State University Center for Research in Mathematics and Science Education, Vol. 1, No. 1, pp. 89-91, San Diego.

Case, R., 1998, *A Psychological Model of Number Sense*, Paper Presented at the Annual Meeting of the American Educational Research Association, San Diego.

Casler-Failing, S.L. and L.C. Swann, 2023, "Developing Proportional Reasoning Via Lego Robotics: Experiences of A 7th Grade Mathematics Class", *Medium Grades Review*, Vol. 9, No. 3, pp. 5-16.

Cobb, P. and K. McClain, 2006, "Guiding inquiry-based math learning", in K. Sawyer (Ed.), *The Cambridge handbook of the learning sciences New York: Cambridge University Press*, Vol. 1, No. 1, pp. 171–186.

Cragg, L. and N. Chevalier, 2012, "The Processes Underlying Flexibility in Childhood", *Quarterly Journal of Experimental Psychology*, Vol. 65, No. 2, pp. 209-232.

Cramer, K. ve T. Post, 1993, "Connecting Research to Teaching Proportional Reasoning", *Mathematics Teacher*, Vol. 86, No. 5, pp. 404- 407.

Cramer, K., T. Post and S. Currier, 1993, "Learning and Teaching Ratio and Proportions: Research Implications", In D.T. Owens (Ed), *Research Ideas for the Classroom, Medium Grades Mathematics*, New York: Macmillan Publishing Company, Vol. 1, No. 1, pp. 159-178.

Creswell, J.W. and C.N. Poth, 2016, *Qualitative Inquiry and Research Design: Choosing Among Five Approaches*, Sage Publications, New York.

Çekirdekci, S. and A. Yorulmaz, 2021, "İlokul Matematik Dersi Öğretim Programlarının Sayı Hissine Göre Incelenmesi", *Atatürk Üniversitesi Kazım Karabekir Eğitim*

Fakültesi Dergisi, Vol. 1, No. 43, pp. 254-278.

Deizmann, C.M. and L.D. English, 2001, "Promoting the Use of Dioramas as Tools for Thinking", in A.A. Cuoco (Ed.), *The Roles of Representation in School Mathematics Reston, VA: National Council of Teachers Of Mathematics*, Vol. 1, No. 1, pp. 77-89.

Dole, S., 2010, *Making Connections to the Big Ideas in Mathematic*, Promoting Proportional Reasoning, London.

Dole, S., D. Clarke, T. Wright and G. Hilton, 2012, "Students' Proportional Reasoning in Mathematics and Science", *In Proceedings of the 36th Conference of the International Group for the Psychology of Mathematics Education International Group for the Psychology of Mathematic*, Vol. 1, No. 1, pp. 195-202, Canada.

Duatepe, A., O.A. Çıkla and M. Kayhan, 2005, "Orantısal Akıl Yürütme Gerektiren Sorularda Öğrencilerin Kullandıkları Çözüm Stratejilerinin Soru Türlerine Göre Değişiminin Incelenmesi", *Hacettepe Üniversitesi Eğitim Fakültesi Dergisi*, Vol. 28, No. 28, pp. 73-81.

Ergene, Ö. and Y. Karabogaz, 2024, "The Effect of the Flipped Classroom Model on Students' Proportional Reasoning", *Journal of Pedagogical Research*, Vol. 8, No. 1, pp. 294-311.

Facun, Remedios D. and Nelvin R. Nool, 2012, "Assessing the Number Sense of Grade 6 Pupils", *International Proceedings of Economics Development and Research*, Vol. 30, No. 1, pp. 297-301.

Fernández, C., S. Llinares, W. Van Dooren, D. De Bock and L. Verschaffel, 2012, "The Development of Students' Use of Additive and Proportional Methods Along Primary and Secondary School", *European Journal of Psychology of Education*, Vol. 27, No. 1, pp. 421-438.

Fielding-Wells, J. and K. Makar, 2012, "Developing primary students' argumentation skills in inquiry-based mathematics classrooms", In: J. van Aalst, K. Thompson, M.J. Jacobson and P. Reimann, (Eds.) *The future of learning: Proceedings of the 10th International Conference of the Learning Sciences International Society of the Learning Sciences*, Vol. 2, No. 1, pp. 149-153, Australia.

Fielding-Wells, J., S. Dole and K. Makar, 2014, "Inquiry Pedagogy to Promote Emerging Proportional Reasoning in Primary Students", *Mathematics Education Research Journal*, Vol. 26, No. 1, pp. 47-77.

Greeno, J.G., 1991, "Number Sense as Situated Knowing in A Conceptual Domain", *Journal for Research in Mathematics Education*, Vol. 22. No. 3, pp. 170-218.

Griffin, S., 2004, "Building Number Sense with Number Worlds: A Mathematics Program for Young Children", *Early Childhood Research Quarterly*, Vol. 19, No. 1, pp. 173-180.

Gülbağci Dede, H., 2015, "İlköğretim Ve Ortaöğretim Matematik Öğretmen Adaylarının Sayı Hissinin Incelenmesi", *Turkish Journal of Computer and Mathematics Education*, Vol. 7, No. 2, pp. 285-303.

Harç, Sevinç., 2010, *6. Sınıf Öğrencilerinin Sayı Duygusu Kavramı Açısından Mevcut Durumlarının Analizi*, YuKsek Lisans Tezi, Marmara Üniversitesi, İstanbul.

Hart. K., 1981, "Children's Understanding of Mathematics", *John Murray*, Vol. 1, No. 1, pp. 11-16.

Hilton, A. and G. Hilton, 2016, "Proportional Reasoning: An Essential Component of Scientific Understanding", *Teaching Science*, Vol. 62, No. 4, pp. 32-42.

Howden, H., 1989, "Teaching Number Sense", *The Arithmetic Teacher*, Vol. 36, No.

6, pp. 6-11.

Im S.H. and A.K Jitendra, 2020, "Analysis of Proportional Reasoning and Misconceptions Among Students with Mathematical Learning Disabilities", *The Journal of Mathematical Behavior*, Vol. 57, No. 1, pp. 100753-100798.

İymen, E. and A. Duatepe-Paksu, 2015, Analysis of 8th Grade Students' Number Sense Related to the Exponents in Terms of Number Sense, Researchgate, Vol. 40, No. 177, pp.3456-3678.

İymen, Esra., 2012, *8. Sınıf Öğrencilerinin USlu Ifadeler Ile Ilgili Sayı Duyularının Sayı Duyusu Bileşenleri Bakımından Incelenmesi, YuKsek Lisans Tezi*, Pamukkale Üniversitesi.

Kahraman, H., K.U.L. Elif and T.A. İskenderoglu, 2019, "7. Ve 8. Sınıf Öğrencilerinin Nicel Karşılaştırma Içeren Orantısal Akıl Yürütme Problemlerinde Kullandıkları Stratejiler", *Turkish Journal of Computer and Mathematics Education*, Vol. 10, No. 1, pp. 195-216.

Kartal, A., 2016, *8. Sınıf Öğrencilerinin Kesirlerde Sayı Duyularının Incelenmesi*, Doktora Tezi, Recep Tayyip Erdoğan Üniversitesi.

Kayhan Altay, M. and A. Umay, 2013, "The Development of Number Sense Scale Towards Medium Grade Students", *Egitim ve Bilim-Education and Science*, Vol. 38, No. 167, pp. 4567-4599.

Kayhan A.M., 2010, *İlköğretim Ikinci Kademe Öğrencilerinin Sayı Duyularının; Sınıf DuZeyine, Cinsiyete ve Sayı Duyusu Bileşenlerine Göre Incelenmesi*, Doktora Tezi, Hacettepe Üniversitesi.

Keleş, A.T. and C. Cinar, 2023, "Sekizinci Sınıf Öğrencilerinin Orantısal Akıl Yürütme

Gerektiren Problemlerinin Çözümünde Kullandıkları Stratejiler", *Online Journal of Mathematics, Science and Technology Education*, Vol. 4, No. 1, pp. 1-44.

Kunandar. Langkah Mudah Penelitian Tindakan Kelas sebagai Pengembangan Profesi Guru, 2010, "PT RajaGrafindo Persada", *Jakarta*, Vol. 1, No. 1, pp. 354-358,

Lago, Rachel M. and DiPerna, C. James, 2010, "Number Sense In Kindergarten: A Factor-Analytic Study of the Construct", *School Psychology Review*, Vol. 39, No. 2, pp. 164-180.

Lamberg, T., T. Grewall, A. Goyer and S. Gil, 2021, *Representing Proportional Reasoning Algebraically to Problem Solve*, North American Chapter of the International Group for the Psychology of Mathematics Education, North.

Lamon, S.J., 2006, "Teaching fractions and ratios for understanding: Essential content knowledge and instructional strategies for teachers Mahwah", *Erlbaum*, Vol. 2, No. 1, pp. 123-145

Lamon, S.J., 2007, "Rational Numbers and Proportional Reasoning: Towards a Theoretical Framework for Research", F.K. In Lester (Ed.), *Second Handbook of Research on Mathematics Teaching and Learning Charlotte, NC: Information Age*, Vol. 1, No. 1, pp. 629-667.

Lamon, S.J., 2020, *Teaching Fractions and Ratios for Understanding: Essential Content 83 and Instructional Strategies for Teachers, Routledge*, London.

Lesh, R., T.R. Post and M. Behr, 1988, "Proportional Reasoning, in Number Concepts and Operations in the Medium Grades National Council of Teachers of Mathematics", *Lawrence Erlbaum Associates*, Vol. 1, No. 1, pp. 93-118.

Lobato, J., A. Ellis and R.M. Zbiek, 2010, "Developing Essential Understanding of Ra-

tios", Proportions, and Proportional Reasoning for Teaching Mathematics: Grades 6-8, *National Council of Teachers of Mathematics, 1906 Association Drive*, Reston, VA Vol. 1, No. 1, pp. 20191-1502.

Markovits, Z. and J. Sowder, 1994, *Developing Number Sense: an Intervention Study in Grade 7*, Jou, New York.

McIntosh, A., B. Reys, R. Reys, J., Bana and B. Farrell, 1997, *Number Sense in School Mathematics: Student Performance in four Countries National Council of Teachers of Mathematics, 1989*, Curriculum and Evaluation Standards for School Mathematics, Reston.

McIntosh, A., R.E. Reys, B.J. Reys and J. Hope, 1997, "Number Sense", *Grades*, Vol. 1, No. 1, pp. 6-8.

McIntosh, A., B.J. Reys and R.E. Reys, 1992, "A Proposed Framework for Examining Basic Number Sense", *For the Learning of Mathematics*, Vol. 12, No. 3, pp. 2-44.

Menon, Ramakrishnan., 2004, "Elementary School Children's Number Sense, International Journal for Mathematics Teaching and Learning, Retrieved From", http:// www. cimt. org. uk/ journal/ ramamenon. pdf, accessed on July 21, 2024.

Milli Eğitim Bakanlığı (MEB), 2009, "İlköğretim Matematik Dersi 6-8. Sınıflar Öğretim Programı", *Ankara: Milli Eğitim Basımevi,Rnal for Research in Mathematics Education*, Vol. 25, No. 1, pp. 4-29.

Misnasanti, R.W. Utami and F.R. Suwanto, 2017, "Problem Based Learning to Improve Proportional Reasoning of Students in Mathematics Learning", *In AIP Conference Proceedings, AIP Publishing*, Vol. 1868, No. 1, pp. 050002-050022, İstanbul.

Model, A.I., 2009, *The Components of Number Sense Teaching Exceptional*, Children

25, Germany.

Mohamed, M. and J. Johnny, 2010, "Investigating Number Sense Among Students", *Procedia-Social and Behavioral Sciences*, Vol. 8, No. 1, pp. 317-324.

Morgan, G.A., K.C. Barrett, N.L. Leech and G.W. Gloeckner, 2019, *IBM SPSS for Introductory Statistics: Use and Interpretation: Use and Interpretation*, Routledge, New York.

National Council of Teachers of Mathematics, 2000, "Principles and Standards for School Mathematics", *National Council of Teachers of Mathematics*, Vol. 1, No. 1, pp. ?

National Research Council., 2001, *Adding it Up: Helping Children Learn Mathematics*, National Academy Press, Washington.

Nickerson, S.D. and I. Whitacre, 2010, "A Local Instruction Theory for the Development of Number Sense", *Mathematical Thinking and Learning*, Vol. 12, No. 3, pp. 227-252.

Ortiz, E.I., C.A. Cuevas-Vallejo, J. Orozco-Santiago and de A. Puebla, 2003, *Promoting Proportional* Reasoning with the Support of Digital Technology, Washington.

Özdemir, E.Y., 2011, *İlköğretim Öğrencilerinin Orantısal Akıl Yürütme Becerileri ile Oran-Orantı Problemi Kurma Becerileri Arasındaki İlişki*, Pamukkale University Journal of Education,

Pakmak, G.S., 2014, *6. Sınıf Öğrencilerinin Niceliksel ve Niteliksel Orantısal Akıl Yürütme Problemlerinin Çözümündeki Anlayışlarının Incelenmesi*, M.S. Thesis, Pamukkale Üniversitesi.

Pallant, J., 2016, *SPSS Survival Manual: A Step by Step Guide to Data Analysis Using SPSS Program* (6th Ed.), Mcgraw-Hill Education, London.

Park, J.S., J.H. Park and O.N. Kwon, 2010, "Characterizing the Proportional Reasoning of Medium School Students the SNU", *Journal of Education Research*, Vol. 1, No. 1, pp. 119-144.

Pilmer, C.D., 2008, *Number Sense, Canada: Nova Scotia School for Adult Learning*, Department of Labour and Workforce Development, Canada.

Pişkin Tunç, M. and E. Çakıroğlu, 2022, "Fostering Prospective Mathematics Teachers' Proportional Reasoning Through a Practice-Based Instruction", *International Journal of Mathematical Education in Science and Technology*, Vol. 53, No. 2, pp. 269-288.

Poçan, S., C. Yaşaroğlu and A. İlhan, 2017, "Ortaokul 7. Ve 8. Sınıf Öğrencilerinin Matematiksel Akıl Yürütme Beceri Düzeylerinin Bazı Değişkenler Açısından İncelenmesi", *Journal of International Social Research*, Vol. 10, No. 52, pp.?

Purnomo, Y.W., F. Alyani and S.S. Assiti, 2014, "Assessing Number Sense Performance of Indonesian Elementary School Students'", *International Education Studies*, Vol. 7, No. 8, pp. 74-84.

Reys, R., B. Reys, G. Emanuelsson, B. Johansson, A. McIntosh and D.C. Yang, 1999, "Assessing Number Sense of Students in Australia, Sweden, Taiwan, and the United States", *School Science and Mathematics*, Vol. 99, No. 2, pp. 61-70.

Reys, R., 1989, "Some Personal Reflections on the Conference on Number Sense, Mental Computation", *And Estimatio, in Establishing Foundations for Research on Number Sense and Related Topics: Report of A Conference San Diego: San Diego State University Center for Research in Mathematics and Science Education*, Vol.

1, No. 1, pp. 65-66, San Diego.

Scheibling-Sève, C., K. Gvozdic, E. Pasquinelli and E. Sander, 2022, "Enhancing Cognitive Flexibility Through a Training Based on Multiple Categorization: Developing Proportional Reasoning in Primary School", *Journal of Numerical Cognition*, Vol. 8, No. 3, pp. 443-472.

Sengul, S. and H. Gulbagci, 2012, "An Investigation of 5th Grade Turkish Students' Performance in Number Sense on the Topic of Decimal Numbers", *Procedia-Social and Behavioral Sciences*, Vol. 46, No. 1, pp. 2289-2293.

Singh, P., 2000, "Understanding the Concepts of Proportion and Ratio Constructed by two Grade six Students", *Educational Studies in Mathematics*, Vol. 43, No. 3, pp. 271-292.

Singh, Parmjit., 2009, *An Assessment of Number Sense Among Secondary School Students*, International Journal for Mathematics Teaching and Learning, London.

Sowder, J., B. Armstrong, S. Lamon, M. Simon, L. bentlşey and A. Thompson, 1998, "Educating Teachers to Teach Multiplicative Structures in the Medium Grades", *Journal of Mathematics Teacher Education*, Vol. 1, No. 2, pp. 127-155.

Sowder, J.T., 2007, "The Mathematical Education and Development of Teachers", F.K. In Lester, Second Handbook of Research on *Mathematics Teaching and Learning*, Vol. 1, No. 1, pp. 157-223. Charlotte, NC: Information Age.

Steele,M.D., 2006, *Medium Grades Geometry And Measurement: Examining Change in Knowledge Needed for Teaching Through A Practice-Based Teacher Education Experience*, Ph.D. Thesis, University of Pittsburgh.

Supply, A.S., E. Vanluydt, W. Van Dooren and P. Onghena, 2023, "Out of Proportion

or Out of Context, Comparing 8-To 9-Year-Olds' Proportional Reasoning Abilities Across Fair-Sharing, Mixtures, and Probability Context", *Educational Studies in Mathematics*, Vol. 113, No. 3, pp. 371-388.

Şengül, S. and H.G. Dede, 2013, "Sayı Hissi Bileşenlerine Ait Sınıflandırmaların İncelenmesi", *The Journal of Academic Social Science Studies*, Vol. 6, No. 8, pp. 645-664.

Şengul, Sare ve Gulbağcı Dede, Hande, 2014, "Matematik Öğretmenlerinin Sayı Hissi Problemlerini Çözerken Kullandıkları Stratejiler", *Turkish Journal of Computer and Mathematics Education*, Vol. 5, No. 1, pp. 73-88.

Takır, Aygil., 2016, 6., 7. Ve 8. Sınıf Öğrencilerinin Sayı Duyusu Becerilerinin Bazı Değişkenler Açısından İncelenmesi'", *Dicle Üniversitesi Ziya Gökalp Eğitim FakuLtesi Dergisi*, Vol.29, No. 1, pp. 309-323.

Tsao, Yea-Ling., 2004, "Effects of A Problem-Solving-Based Mathematics Course on Number Sense of Preservice Teachers", *Journal of College Teaching and Learning*, Vol. 1, No. 2, pp. 33-49.

Umay, A., 2003, "Matematiksel Muhakeme Yeteneği", *Hacettepe Üniversitesi Eğitim Fakültesi Dergisi*, Vol. 24, No. 1, pp. 234-243.

Van Dooren, W., D. De Bock, A. Hessels, D. Janssens and L. Verschaffel, 2005, "Not Everything is Proportional: Effects of Age and Problem Type on Propensities for Overgeneralization", *Cognition and Instruction*, Vol. 23, No. 1, pp. 57-86.

Van de Walle, J., 2001, *Elementary and Medium School Mathematics: Teaching Developmentally*, Addison Wesley Longman, Germany.

Yang, D.C., C.J. Hsu and M.C. Huang, 2004, "A Study of Teaching and Learning Number Sense for Sixth Grade Students in Taiwan", *International Journal of Science*

and Mathematics Education, Vol. 2, No. 3, pp. 407-430.

Yang, D.C., R.E. Reys and B.J. Reys, 2007, N*umber Sense Strategies Used by Pre-Service Teachers in Taiwan*, International Journal of Science and Mathematics Education, Taiwan.

Yang, D.C., 1995, *Number Sense Performance and Strategies Possessed by Sixth and Eighth Grade Students in Taiwan*, Ph.D: Thesis, University of Missouri, Columbia.

Yang, Der-Ching., 2003, "Developing Number Sense Through Realistic Setting", *Australian Primary Mathematics Classroom*, Vol. 8, No. 3, pp. 12-17.

Yang, Der-Ching., Mao-Neng Li and Lin, I. Chih, 2008, "A Study of the Performance of 5th Graders in Number Sense and Its Relationship to Achievement in Mathematics", *International Journal of Science and Mathematics Education*, Vol.6, No. 4, pp. 789-807.

Yang, Der-Ching., 2002, "Teaching and Learning Number Sense: one Successful Process-Oriented Activity with Sixth Grade Students in Taiwan", *School Science And Mathematics*, Vol. 102, No. 3, pp. 1-6.

Yang, Der-Ching and Wan-Ru Wu, 2010, "The Study of Number Sense: Realistic Activities Integrated into Third-Grade Math Classes in Taiwan", *The Journal of Educational Research*, Vol. 103, No. 6, pp. 379-392.

Yang, D.C., M.F. Li and W.J. Li, 2008, "Development of A Computerised Number Sense Scale for 3rd Graders: Reliabilty and Validity Analysis", *International Electronic Journal of Mathematics Education*, Vol. 3, No. 1, pp. 110-124.

Yang, D.C., 2003, "Teaching and Learning Number Sense - An Intervention Study of Fifth Grade Students in Taiwan", *International Journal of Science and Mathematics*

Education, Vol. 1, No. 1, pp. 115-134.

Zanzali, Noor A.A. and Ghazali, Munirah, 1999, "Assessment of School Children's Number Sensei in Proceedings of the International Conference on Mathematics Education", *The 21st Century: Societal Changes: Issues and Approaches*, Vol. 1, No. 1, pp. 345-478, Indian.

Weiland, T., C.H. Orrill, R.E. Brown and G.G. Nagar, 2019, "Mathematics Teachers' Ability to Identify Situations Appropriate for Proportional Reasoning", *Research in Mathematics Education*, Vol. 21, No. 3, pp. 233-250.

APPENDIX A: THE APPROVAL OF BOGAZICI UNIVERSITY INSTITUTIONAL REVIEW BOARD FOR RESEARCH WITH HUMAN SUBJECTS

Appendix A includes ethics committee approval from Boğaziçi University Institutional Review Board for Research with Human Subjects.

Evrak Tarih ve Sayısı: 14.03.2023-117666

T.C.
BOĞAZİÇİ ÜNİVERSİTESİ REKTÖRLÜĞÜ
Fen Bilimleri ve Mühendislik Alanları İnsan Araştırmaları Etik Kurulu
(FMİNAREK)

Sayı : E-84391427-050.01.04-117666 14.03.2023
Konu : 2023/10 Kayıt no'lu başvurunuz hakkında

Sayın Dr. Öğr. Üyesi Yeşim İMAMOĞLU
Matematik ve Fen Bilimleri Eğitimi Bölüm Başkanlığı - Öğretim Üyesi

"7. sınıf Öğrencilerinin Orantısal Akıl Yürütmelerinde Sayı Hissi Rolünün İncelenmesi" başlıklı projeniz ile Boğaziçi Üniversitesi Fen Bilimleri ve Mühendislik Alanları İnsan Araştırmaları Etik Kurulu (FMİNAREK)'e yaptığınız 2023/10 kayıt numaralı başvuru 06.03.2023 tarihli ve 2023/03 No.lu kurul toplantısında incelenerek etik onay verilmesi uygun bulunmuştur.

Bu karar tüm üyelerin toplantıya on-line olarak katılımıyla ve oybirliği ile alınmıştır. Onay mektubu tüm üyeler adına Komisyon Başkanı tarafından e-imzalanmıştır.

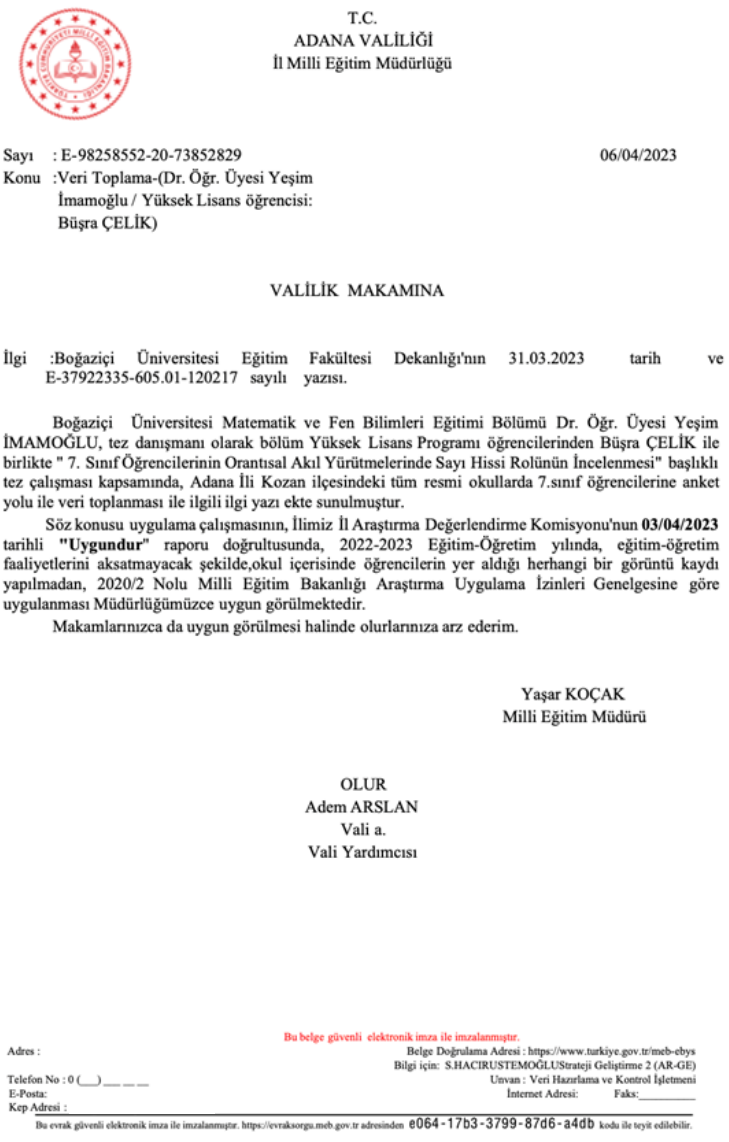
Saygılarımızla bilginize sunarız.

Prof. Dr. Tınaz EKİM AŞICI
Başkan

Figure A.1. Ethics committee.

APPENDIX B: DATA COLLECTION APPROVAL FROM MEB

Appendix B includes data collection approval from Ministry of National Education.



T.C.
ADANA VALİLİĞİ
İl Milli Eğitim Müdürlüğü

Sayı : E-98258552-20-73852829 06/04/2023
Konu :Veri Toplama-(Dr. Öğr. Üyesi Yeşim İmamoğlu / Yüksek Lisans öğrencisi: Büşra ÇELİK)

VALİLİK MAKAMINA

İlgi :Boğaziçi Üniversitesi Eğitim Fakültesi Dekanlığı'nın 31.03.2023 tarih ve E-37922335-605.01-120217 sayılı yazısı.

Boğaziçi Üniversitesi Matematik ve Fen Bilimleri Eğitimi Bölümü Dr. Öğr. Üyesi Yeşim İMAMOĞLU, tez danışmanı olarak bölüm Yüksek Lisans Programı öğrencilerinden Büşra ÇELİK ile birlikte " 7. Sınıf Öğrencilerinin Orantısal Akıl Yürütmelerinde Sayı Hissi Rolünün İncelenmesi" başlıklı tez çalışması kapsamında, Adana İli Kozan ilçesindeki tüm resmi okullarda 7.sınıf öğrencilerine anket yolu ile veri toplanması ile ilgili ilgi yazı ekte sunulmuştur.

Söz konusu uygulama çalışmasının, İlimiz İl Araştırma Değerlendirme Komisyonu'nun **03/04/2023** tarihli **"Uygundur"** raporu doğrultusunda, 2022-2023 Eğitim-Öğretim yılında, eğitim-öğretim faaliyetlerini aksatmayacak şekilde,okul içerisinde öğrencilerin yer aldığı herhangi bir görüntü kaydı yapılmadan, 2020/2 Nolu Milli Eğitim Bakanlığı Araştırma Uygulama İzinleri Genelgesine göre uygulanması Müdürlüğümüzce uygun görülmektedir.

Makamlarınızca da uygun görülmesi halinde olurlarınıza arz ederim.

Yaşar KOÇAK
Milli Eğitim Müdürü

OLUR
Adem ARSLAN
Vali a.
Vali Yardımcısı

Bu belge güvenli elektronik imza ile imzalanmıştır.
Adres :
Belge Doğrulama Adresi : https://www.turkiye.gov.tr/meb-ebys
Bilgi için: S.HACIRUSTEMOĞLUStrateji Geliştirme 2 (AR-GE)
Unvan : Veri Hazırlama ve Kontrol İşletmeni
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Kep Adresi :
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Figure B.1. Approval from MoNE.

APPENDIX C: PARENTAL CONSENT FORM

Appendix C includes the consent form presented to parents in order to work with students under the age of 18.

T.C.
BOĞAZİÇİ ÜNİVERSİTESİ
FEN BİLİMLERİ VE MÜHENDİSLİK ALANLARI
İNSAN ARAŞTIRMALARI ETİK KURULU
KATILIMCI BİLGİ ve ONAM FORMU

Araştırmayı destekleyen kurum: Boğaziçi Üniversitesi

Araştırmanın adı: 7. sınıf Öğrencilerinin Orantısal Akıl Yürütmelerinde Sayı Hissi Rolünün İncelenmesi

Araştırmacı: Büşra ÇELİK
E-mail adresi:

Proje Yürütücüsü: Dr. Öğretim Üyesi Yeşim İmamoğlu
Adresi: ˙

E-mail adresi:
Telefonu:

Proje konusu: Sayı hissi ve orantısal akıl yürütme literatürde de belirtildiği gibi matematiksel öğretim öğrenim sürecinin iki önemli kavramıdır. Bununla birlikte orantısal ilişkiler ve buna bağlı akıl yürütme ile günlük hayatta sıkça karşılaşılmakta olup, orantısal akıl yürütme matematiğin diğer birçok önemli konusu ile de direkt olarak bağlantılıdır. Orantısal akıl yürütme bu denli önemli iken, hem öğretmenler hem de öğrenciler orantısal akıl yürütmede zorlanmaktadırlar. Bu sebeple orantısal akıl yürütmeye etki edebilecek diğer kavramlar da önemsenmektedir. Bu çalışma ise 7. Sınıf öğrencilerinin sayı hissi düzeylerini belirleyerek, sayı hissi düzeyi düşük ve sayı hissi düzeyi yüksek olan öğrencilerin orantısal akıl yürütme becerileri arasında istatiksel olarak anlamlı bir fark olup olmadığını inceler ve temel amaç sayı hissinin

7. Sınıf öğrencilerinin orantısal akıl yürütme becerileri üzerindeki rolü hakkında bilgi sahibi olmaktır.

Onam: Sayın veliler,

Ben, Büşra ÇELİK, Boğaziçi Üniversitesi, Fen Bilimleri Enstitüsü, Matematik ve Fen Bilimleri Eğitimi Bölümü'nde yüksek lisans öğrencisiyim. Yürüttüğüm tez çalışmam kapsamında, 7. sınıf öğrencilerinin orantısal akıl yürütmelerinde sayı hissi rolünü incelemeyi amaçlıyorum. Bu amaçla Adana ilinin Kozan ilçesinin iki farklı devlet okulundaki 7. Sınıf öğrencilerinden veri toplayacağım. Verileri biri 17 diğeri 15 açık uçlu sorudan oluşan iki farklı test aracılığı ile elde edeceğim. İki test aynı gün içerisinde veya yalnızca bir iki gün aralıkla 2023 yılının Nisan ayında ara tatil sonrası uygulanacaktır. Ayrıca bu iki testin sonucuna göre, seçilen 10 gönüllü öğrenci ile yine matematik sorularından oluşan bir mülakat yapılacaktır Mülakata seçilen öğrencilerden sayı hislerini orantısal akıl yürütmeyle ilişkili problemlerde nasıl kullandığını görmek için sesli düşünmeleri istenecek ve ses kaydı alınacaktır. Araştırma uygulamasına katılım tamamıyla gönüllülük esasına dayalı olmaktadır. Çocuğunuz çalışmaya katılıp katılmamakta özgürdür, katılımı tamamen onun ve sizin isteğinize bağlıdır, reddedebilir ya da herhangi bir aşamasında ayrılabilirsiniz. Araştırmaya katılma, katılmamama veya katıldıktan sonra vazgeçme durumunda öğrencilerin akademik başarıları, okul ve öğretmenleriyle olan ilişkileri etkilemeyecektir. Çalışmaya katıldıktan sonra vazgeçme durumunda, öğrencilerin isimlerinin de bulunduğu test yanıt kağıtları tamamen gizli tutulacak ve hemen ardından araştırmacı tarafından imha edilecektir. Araştırma çocuğunuz için herhangi bir istenmeyen etki ya da risk taşımamaktadır. T.C. Milli Eğitim Bakanlığı'nın ve okul yönetiminin de izni ile gerçekleşmektedir. Çalışmada kişisel veri olarak yalnızca isim soy isimler alınacaktır fakat çalışmanın yazılı ya da sözlü hiçbir aşamasında isimler paylaşılmayacaktır. İsimler yalnızca verilerin eşleştirilmesinde kullanılacaktır. Cevaplar ve isimler tamamen gizli tutulacak ve sadece araştırmacılar tarafından değerlendirilecektir. Uygulanacak olan testler matematik sorularından oluşmakta olup kişisel rahatsızlık verecek durumlar ve sorular içermemektedir. Fakat

katılım sırasında herhangi bir sebepten çocuğunuz kendisini rahatsız hissederse, testi yarıda bırakıp çıkabilir. Böyle bir durumda, rahatsızlığın giderilmesi için yardımcı olunacak ve testleri tamamlamamak çocuğunuza herhangi bir sorumluluk getirmeyecektir. Onay vermeden önce araştırma hakkında daha detaylı bilgi sahibi olmak isterseniz sizlere sağlanılan iletişim adreslerinden bize ulaşabilirsiniz. Çalışma sonrasında da aynı şekilde herhangi bir sorunuz için bizimle iletişim kurabilirsiniz. Saygılarımızla,

Ses kaydı alınmasına izin veriyorum/vermiyorum.

Araştırma hakkında soru sormak ya da ek bilgi almak için araştırmacı Büşra ÇELİK ile iletişime geçebilirsiniz Araştırmayla ilgili haklarınız konusunda Boğaziçi Üniversitesi Fen Bilimleri ve Mühendislik Alanları İnsan Araştırmaları Etik Kurulu'na danışabilirsiniz.

Öğrencinin velisi ..... olarak yukarıdaki metni okudum, anladım ve çalışma hakkında soru sorma imkanı buldum. Velisi bulunduğum öğrencinin bu çalışmayı istediği zaman ve herhangi bir neden belirtmek zorunda kalmadan bırakabileceğini ve bıraktığı takdirde herhangi olumsuzluk ile karşılaşmayacağımı anladım.

Bu koşullarda söz konusu araştırmaya velisi olduğum öğrencinin katılmasına, hiçbir baskı ve zorlama olmaksızın, izin veriyorum.

Formun bir örneğini aldım / almak istemiyorum.

Öğrencinin Velisinin Adı-Soyadı: .....

İmzası: .....

Tarih (gün/ay/yıl): ...../...../.....

APPENDIX D: INTERVIEW PROBLEMS

Appendix D includes the problems asked to students in the interviews.

Soruları dikkatle okuyup, düşüncelerimizi açıklayarak çözelim.

1)

a) 6 kilogramının fiyatı 27,30 TL olan portakalın 2 kilogramı kaç TL dir?

b) Boyu 200 cm olan bir ağacın gölgesinin 150 cm olduğu anda, boyu 60 cm olan fidanın gölgesi kaç cm olur?

2) Burak ve Mahmut eşit büyüklükteki bardaklara limonata dolduracaklardır. İkisi de aynı hızda limonata doldurmaktadır fakat Mahmut limonata doldurmaya daha sonra başlamıştır. Burak 30 bardak limonata doldurduğunda Mahmut 18 bardak limonata doldurduğuna göre, Burak 90 bardak limonata doldurduğunda Mahmut kaç bardak limonata doldurmuş olur?

3)

a) Nisa ve Kemal özdeş boyutlardaki 3 litrelik sürahileri 50 mililitrelik eşit boyutlardaki bardaklarla doldurmak için yarış yapmaktadırlar. Nisa 5 dakikada 20 bardak, Kemal 3 dakikada 36 bardak su doldurmuştur. Nisa ve Kemal den hangisi sürahiyi daha önce doldurur? (1 litre= 1000 ml)

b) 15 öğrenciden oluşan 7-A sınıfı ve 20 öğrenciden oluşan 7/B sınıfı 20 adet pizzayı her sınıfa eşit sayıda pizza düşecek şekilde paylaşmışlardır. Daha sonra 7/A sınıfı öğrencileri kendi paylarına düşen pizzayı sınıftaki her öğrenci eşit alacak şekilde ve 7/B sınıfı öğrencileri kendi paylarına düşen pizzayı sınıftaki her öğrenci eşit alacak şekilde kendi aralarında paylaşmışlardır.

Buna göre hangi sınıfta öğrenci başına düşen pizza miktarı daha fazladır?

4)

a) Bir koşu parkurunda Buğra Ece den daha uzun zamanda daha az tur koşmuştur. Buğra mı Ece mi daha hızlı koşucudur?

b) Ahmet usta metrekare bakımından aynı büyüklükteki A ve B odalarına parke döşeyecektir. Ahmet ustanın A odasında kullandığı eş büyüklükteki parkeler, B odasında kullandığı eş büyüklükteki parkelerden daha büyüktür. A ve B odalarının parke döşeme işlemini aynı sürede tamamlayan Ahmet usta hangi odada daha hızlı çalışmıştır?

5) Eşit kapasiteli 6 musluk boş bir havuzu 72 saatte doldurmaktadır. Bu musluklardan 2 tanesi açılmasaydı, diğer musluklar boş havuzu kaç saatte doldururdu?

APPENDIX E: SCALE PERMISSION

Appendix E incudes permission to use the Number Sense Scale from the creators of the scale.



MESTURE KAYHAN ALTAY
Alıcı: ben

9 Mart Per 15:49 (11 gün önce)

Merhaba Büşra,

Öncelikle tez çalışmanızda başarılar dilerim.

Değerli hocam Prof. Dr. Aysun Umay ile birlikte geliştirmiş olduğumuz sayı duyusu ölçeğini tez çalışmanızda kullanabilirsiniz.

İyi çalışmalar dilerim.

Mesture Kayhan Altay

Gönderen: Büşra Çelik <
Gönderildi: 7 Mart 2023 Salı 15:32:17
Kime: Aysun UMAY; MESTURE KAYHAN ALTAY
Konu: Büşra ÇELİK-Tez için Ölçek İzni

Figure E.1. Scale permıssıon.

APPENDIX F: PROPORTIONAL REASONING TEST PERMISSION

Appendix F incudes permission to use the Proportional Reasoning Test from the creators of the scale.

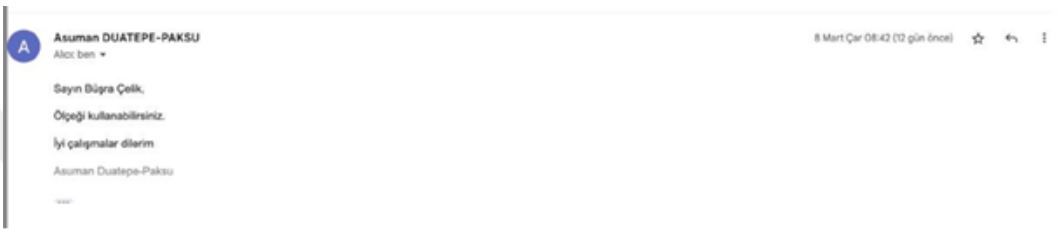

Figure F.1. Proportıonal reasonıng test permıssıon.