THE EFFECT OF ORIGAMI-BASED INSTRUCTION ON SPATIAL VISUALIZATION, GEOMETRY ACHIEVEMENT AND GEOMETRIC REASONING OF TENTH-GRADE STUDENTS

by

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B.S., Secondary School Science and Mathematics Education, Boğaziçi University, 2009

Submitted to the Institute for Graduate Studies in
Science and Engineering in partial fulfillment of
the requirements for the degree of
Master of Science

Graduate Program in Secondary School Science and Mathematics Education
Boğaziçi University
2012

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DATE OF APPROVAL: 30.01.2012

ACKNOWLEDGEMENTS

First of all, I would like to thank to my thesis supervisor Fatma Aslan-Tutak for all her support and patience during my study. Her encouragement contributed so much to me.

I would also like to thank to my thesis committee members. I am grateful to Prof. Emine Erktin and Assist. Prof. Hülya Kılıç for their assistance.

Furthermore, I would like to thank to all academic members of Secondary School Science and Mathematics Education Program. I have learned so much from their courses and knowledge. I thank to Prof. Ali Baykal, Prof. Füsun Akarsu, Prof. Dilek Ardaç, Assoc. Prof. Ayşenur Yontar-Toğrol, Assist. Prof. Buket Yakmacı-Guzel and Gülseren Karagöz-Akar for their contributions.

Finally, I thank to my mother Safiye, father Yüksel and sister Sevim for their encouragement and faith in me from the beginning of my study.

ABSTRACT

THE EFFECT OF ORIGAMI-BASED INSTRUCTION ON SPATIAL VISUALIZATION, GEOMETRY ACHIEVEMENT AND GEOMETRIC REASONING OF TENTH-GRADE STUDENTS

The purpose of this study was to investigate the effect of instruction type (origami-based and traditional instruction) on tenth-grade students' spatial visualization, geometry achievement and geometric reasoning concerning certain key topics in triangles. The study was conducted in a public high school in Tekirdağ with 184 tenth-grade students by using a quasi-experimental pretest-posttest design. One group (n = 94) received traditional instruction while the other group (n = 90) received origami-based instruction. In the origami-based instruction, there were twelve class hours in total. The origami-based instructional materials were designed by the researcher based on the literature. The van Hiele Test (vHT) was used to compare group differences in terms of geometric reasoning just prior to the treatment. The van Hiele Test results indicated that students in experimental and control groups had similar van Hiele levels, thereby similar geometric thinking levels before treatment. The Card Rotation Test (CRT), Cube Comparison Test (CCT) and Paper Folding Test (PFT) were used to measure students' spatial visualization ability. Furthermore, Geometry Achievement Test (GAT) and Geometric Reasoning Test (GRT) were developed by the researcher to evaluate students' geometry achievement and geometric reasoning ability, respectively. A Repeated Measures Analysis of Variance (ANOVA) was used to analyze data collected from each pretest and posttest. The results of the study revealed that there was a statistically significant change in spatial visualization, geometry achievement and geometric reasoning abilities of students who received origami-based instruction across two time periods (from pre-treatment to post-treatment). The results of this study suggested that origami-based geometry instruction could be effective to enhance students' spatial visualization, geometry achievement and geometric reasoning abilities.

ÖZET

ORİGAMİ TEMELLİ ÖĞRETİMİN 10. SINIF ÖĞRENCİLERİNİN UZAMSAL GÖRSELLEŞTİRME, GEOMETRİ BAŞARISI VE GEOMETRİK AKIL YÜRÜTMELERİ ÜZERİNE ETKİSİ

Bu çalışmanın amacı, öğretim türünün (origami temelli ve geleneksel öğretimin) 10. sınıf öğrencilerinin üçgenlerle ilgili bazı temel konularda uzamsal görselleştirme, geometri başarısı ve geometrik akıl yürütmeleri üzerine etkisinin incelenmesidir. Bu çalışma, yarı-deneysel öntest-sontest tasarımı kullanılarak Tekirdağ'da bir genel lisedeki 184 onuncu sınıf öğrencisiyle yürütülmüştür. Bir grup öğrenci (94 kişi) geleneksel geometri öğretimi görürken, diğer grup öğrenci (90 kişi) origami temelli geometri öğretimi görmüştür. Origami temelli öğretim toplamda on iki ders saatidir. Origami temelli öğretim materyalleri literatüre dayanarak araştırmacı tarafından hazırlanmıştır. Öğretimden önce geometrik düşünme becerileri açısından grup farklılıklarını karşılaştırmak amacıyla van Hiele testi kullanılmıştır. Van Hiele test sonuçlarına göre, öğretimin uygulanmasından önce deney ve kontrol gruplarındaki öğrencilerin benzer van Hiele seviyeleri, dolayısıyla benzer geometrik düşünme seviyeleri vardır. Öğrencilerin uzamsal görselleştirme yeteneği, Kart Çevirme, Küp Karşılaştırma ve Kâğıt Katlama testleri kullanılarak ölçülmüştür. Ayrıca, öğrencilerin geometri başarıları ve geometrik akıl yürütme yeteneklerini değerlendirmek için sırasıyla, araştırmacı tarafından geliştirilen Geometrik Başarı Testi ve Geometrik Akıl Yürütme Testi kullanılmıştır. Ayrıca, öntest ve sontestlerin her birinden elde edilen veriler, Tekrarlı Varyans Analiz Yöntemi kullanılarak analiz edilmiştir. Çalışmanın sonuçları, origami temelli öğretim gören öğrencilerin uzamsal görselleştirme, geometri başarıları ve geometrik akıl yürütme yeteneklerinde zamana dayalı (öntesten sonteste) istatistiksel açıdan anlamlı bir değişiklik olduğunu göstermiştir. Bu çalışmanın sonuçları, origami temelli öğretimin öğrencilerin uzamsal görselleştirme, geometri başarıları ve geometrik akıl yürütme yeteneklerini geliştirmede etkili olabileceğini işaret etmektedir.

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LIST OF SYMBOLS

df	Degree of Freedom
M	Mean
n	Number

LIST OF ACRONYMS / ABBREVIATIONS

ANOVA	Analysis of Variance
SD	Standard Deviation
Std. Error	Standard Error
CCT	Cube Comparison Test
CRT	Card Rotation Test
ctr	Control Group
exp	Experimental Group
GAT	Geometry Achievement Test
GRT	Geometric Reasoning Test
MEB	Milli Eğitim Bakanlığı
NCTM	National Council of Teachers of Mathematics
PISA	Programme for International Student Assessment
PFT	Paper Folding Test
PostCCT	Cube Comparison Posttest Scores
PostCCTctr	Cube Comparison Posttest Scores in the Control Group
PostCCTexp	Cube Comparison Posttest Scores in the Experimental Group
PostCRT	Card Rotation Posttest Scores
PostCRTctr	Card Rotation Posttest Scores in the Control Group
PostCRTexp	Card Rotation Posttest Scores in the Experimental Group
PostGAT	Geometry Achievement Posttest Scores
PostGATctr	Geometry Achievement Posttest Scores in the Control Group
PostGATexp	Geometry Achievement Posttest Scores in the Experimental Group
PostGRT	Geometric Reasoning Posttest Scores
PostGRTctr	Geometric Reasoning Posttest Scores in the Control Group
PostGRTexp	Geometric Reasoning Posttest Scores in the Experimental Group
PostPFT	Paper Folding Posttest Scores

PostPFTctr	Paper Folding Posttest Scores in the Control Group
PostPFTexp	Paper Folding Posttest Scores in the Experimental Group
PreCCT	Cube Comparison Pretest Scores
PreCCTctr	Cube Comparison Pretest Scores in the Control Group
PreCCTexp	Cube Comparison Pretest Scores in the Experimental Group
PreCRT	Card Rotation Pretest Scores
PreCRTctr	Card Rotation Pretest Scores in the Control Group
PreCRTexp	Card Rotation Pretest Scores in the Experimental Group
PreGAT	Geometry Achievement Pretest Scores
PreGATctr	Geometry Achievement Pretest Scores in the Control Group
PreGATexp	Geometry Achievement Pretest Scores in the Experimental Group
PreGRT	Geometric Reasoning Pretest Scores
PreGRTctr	Geometric Reasoning Pretest Scores in the Control Group
PreGRTexp	Geometric Reasoning Pretest Scores in the Experimental Group
PrePFT	Paper Folding Pretest Scores
PrePFTctr	Paper Folding Pretest Scores in the Control Group
PrePFTexp	Paper Folding Pretest Scores in the Experimental Group
SM	Science-Mathematics Orientation
SPSS	Statistical Package for the Social Sciences
TIMSS	Trends in International Mathematics and Science Study
TM	Turkish-Mathematics Orientation
TS	Turkish-Social Orientation
vHT	van Hiele Test

1. INTRODUCTION

"*I hear, and I forget. I see, and I remember. I do, and I understand*."

Chinese saying

Geometry is one of the important and oldest branches of mathematics that helps people to understand space, shape, and dimension. Geometry has different application areas in life. Architecture, art, navigation, engineering, and industrial design of certain objects are some areas in which geometry is used. In addition to daily-life applications of geometry, it has also links to other areas of mathematics such as algebra and measurement. Considering its applications and benefits of learning, geometry is included in school curriculum. The importance of geometry learning is also emphasized in the recent Turkish high school geometry curriculum. Geometry may develop some skills needed in science and other aspects of mathematics (Milli Eğitim Bakanlığı [MEB], 2010). The new Turkish Ministry of National Education geometry curriculum (MEB, 2010a) suggests that geometry education should be designed so that the fundamental skills such as reasoning, proving, problem solving, communication, making connections, creative and innovative thinking, and inquiry should be developed.

Considering the benefits of geometry, an effective geometry instruction in schools is needed. It is essential for educators the learning process in geometry in order to design an effective instruction of geometry and to develop new instructional strategies for such effective geometry lessons. Three theoretical frameworks may be used in order to understand geometry learning. Duval's (1998) model of geometric thinking may give insight in understanding the cognitive dimensions in geometry learning. Duval (1998) proposes that geometric thinking involves three kinds of cognitive processes which are visualization, construction, and reasoning. Smith (2010) suggests that geometric thinking consists of argumentation, justifying, and proving. Another framework is developed by van Hiele in order to explain student difficulties in geometry learning (Fuys, Geddes and Tischler, 1984). The van Hiele model of geometry thinking proposes that geometric thinking develops in stages.

Spatial thinking ability is also important in geometry learning. Geometry and spatial abilities are claimed to be interconnected (Clements and Battista, 1992; Goos, Stillman and Vale, 2007). Studies have showed that spatial ability and geometry achievement are positively related (e.g. Bayrak, 2008; Çakmak, 2009; Delialioğlu, 1996; Ünal, 2005). Thus, geometry lessons may provide a context to improve spatial ability. The new high school curriculum in Turkey also points out the importance of geometry lessons in developing spatial ability (MEB, 2010a). Studies show that origami may be used to increase spatial ability, particularly, spatial visualization (Bayrak, 2008; Boakes, 2006; Çakmak, 2009). Spatial visualization is considered an important component of spatial ability. Ekstrom, French and Harman (1976) have defined spatial visualization as "the ability to manipulate or transform the image of spatial patterns into other arrangements" (p. 173). One part of this study investigates the effect of using origami in geometry lessons on spatial visualization of tenth-grade students.

Although geometry is taught beginning from earlier grades in Turkish curriculum, Turkish students still have difficulty in learning geometry when they come to high school. The international comparative studies also support this observation. For instance, the results of the Programme for International Student Assessment (PISA) and the Trends in International Mathematics and Science Study (TIMSS) show that middle school students in Turkey have performed below the international average for mathematics, in particular geometry. According to the results reported by Mullis, Martin and Foy (2008), Turkey is so below the TIMSS 2007 average scores for all domains in eight-grade mathematics. The lowest content domain score in math is geometry for Turkey.

In order to increase geometry achievement of students, new instructional methods may be designed in addition to traditional instruction or as an alternative to traditional instruction. Hands-on materials may play an important role in enhancing geometry achievement. Studies show that the use of manipulatives in geometry lessons may help students learn geometry better (e.g. Clements and Battista, 1992; Jones, 2010). Manipulatives can provide concrete experiences that may enhance geometric reasoning. In this context, origami, the art of paper folding, may be a useful tool to teach geometry.

Origami, the art of paper folding, has been claimed to be as an effective instructional tool in many references (Boakes, 2006, 2010; Brumbaugh and Rock, 2001; Coad, 2006; Çakmak, 2009; Higginson and Colgan, 2001; Kavici, 2005; Pope, 2002; Robichaux and Rodrigue, 2003; Sağsöz, 2008; Tuğrul and Kavici, 2002). Origami offers many educational benefits such as behavioral gains, socio-emotional gains, psychomotor developmental gains, language development, multicultural awareness, cooperative learning, cognitive development, and mathematical skills (Tuğrul and Kavici, 2002; Valentini, 2005). For instance, as one of behavioral gains of origami, a person learns to be patient in folding origami models (MEB, 2011). Besides, as one of socio-emotional gains, a student can get an opportunity to be accepted by others since the student has created a product out of origami (MEB, 2011).

Although there are studies to support the benefits of using origami in geometry education, more research studies are needed. In general, there is a limited number of research studies in high school level even if practical references support the use of origami for high school geometry instruction. Moreover, research studies about the use of origami in geometry education in Turkey are mostly about elementary or middle school level. The purpose of this study is to investigate the effect of origami-based lessons on tenth grade students' geometry achievement, spatial visualization and geometric reasoning abilities.

2. REVIEW OF RELATED LITERATURE

Due to the rapid changes in accessing and evaluating information in our era, the need to understand and be able to use mathematics in everyday life and in the workplace has been more evident than before. For instance, mathematics is necessary for everyday routines such as making purchasing decisions, choosing insurance, voting and interpreting graphs in media since they require the evaluation of the quantitative information (National Council of Teachers of Mathematics [NCTM], 2000). Therefore, understanding and doing mathematics is fundamental for adapting to change in our era and opportunities of pursuing a good career in future.

Geometry, which literally means "earth measuring", is one of the important and oldest branches of mathematics that helps people to understand space, shape, and dimension. Like many aspects of mathematics, geometry has been evolving through the contributions of different cultures and civilizations. Today, the geometry of Euclid is not the only type of geometry but new geometries (such as hyperbolic and elliptic geometries) have emerged. Thus, geometry is a dynamic and broad field that has evolved throughout the historical contributions.

Felix Klein is one of such contributors to the development of geometry. Klein's characterization of a geometry, which is based on the invariants of a transformation group in his "Erlanger Programm" in 1872 has affected the later development of research in geometry (Kilpatrick, 1992). Klein's such definition has enabled the classification of geometries into related strands ranging from topology as the most general, through projective and affine geometries, to Euclidean geometry as the most specific (Jones, 2000). Therefore, this new way of seeing geometry has opened the gate to generate many more geometries.

Considering its variety, geometry has many applications in life. Robotics, medical imaging, computer animation and visual presentations, computational chemistry, shapes of molecules, material physics, biology, Geographic Information Systems (GIS), many fields of engineering (Jones, 2000), architecture, art, navigation and designing objects are just a few examples of applications of geometry. With its rich applications, geometry takes its

place in school curricula and Goos, Stillman and Vale (2007) emphasize the necessity of geometry by comparing the relative importance of shape and number in our physical environment: “Objects in our world cannot exist without shape, but can be described without number” (p. 200).

Besides the real-world applications of geometry, it has connections to other areas of mathematics such as measurement, proportional reasoning, and algebra. For example, the Pythagorean relationship is an algebraic, a geometric, and a metric relationship, and similar objects can visually represent proportionality (Van de Walle, 2007). As Goos and her colleagues (2007) have suggested, the study of geometry can provide opportunities for teachers and learners to integrate thinking and learning because of rich connections between geometry and other mathematical concepts. As another important link between geometry and other fields of mathematics, visual thinking and reasoning, which are used in all fields of mathematics, can be developed in the context of geometry (Goos *et al*., 2007).

2.1. Geometric Reasoning

For this literature review, three main theoretical frameworks were used. They were developed by Duval (1998), Smith (2010) and van Hiele (1957) (as cited in Fuys, Geddes and Tischler, 1984). Duval’s and Smith’s frameworks approached geometry from a cognitive point of view whereas van Hiele theory mainly focused on the developmental aspect of geometry.

Considering its rich applications and benefits of learning geometry, geometry takes its place in school curriculum. For effective teaching of geometry, it is crucial to understand how geometry learning occurs. Jones (1998) presents three theoretical frameworks for the learning of geometrical reasoning that have implications for teaching of geometry. These frameworks discussed by Jones (1998) are Fischbein’s theory of figural concepts, Duval’s cognitive model of geometrical reasoning, and the van Hiele model of thinking in geometry. In this study, initially, Fischbein’s framework is briefly discussed. Then, frameworks of Duval, Smith and van Hiele are discussed in more detail.

In *The Theory of Figural Concepts*, Fischbein (1993) argues that a geometrical figure such as a triangle or rectangle is not just a concept but also an image. According to Fischbein (1993), an image (in fact, a mental image) is "a *sensorial* representation of an object or phenomenon" (p.139) whereas a concept is a symbolic (usually verbal) or ideal representation of a class of objects based on their common properties. Fischbein (1993) says that "all geometrical figures represent mental constructs which possess simultaneously, conceptual and figural properties" (p. 142). Fischbein views *figural concepts* as the things of investigation and manipulation in geometrical reasoning and these mental entities involve two components as spatial properties (shape, position, magnitude), and conceptual qualities (such as ideality, abstractness, generality, perception). Thus, as Jones (1998) has also emphasized, the interaction between the two aspects, the figural and the conceptual properties characterizes geometrical reasoning. As Fischbein (1993) points out, there may be tensions between the two aspects of figural concepts (concepts and images) and this may be one of the reasons of why geometry learning is difficult for some students. In order to overcome this difficulty of learning geometry, appropriate teaching of geometry, which incorporates the two aspects, concepts and images, is required.

Furthermore, as the cognitive psychologist Duval (1998) argues, teaching geometry is different, more complex and unfortunately often less successful than teaching other content areas of mathematics such as elementary algebra. Besides, in order to determine the origin of the students' difficulties in comprehending geometry, the cognitive functioning underlying the geometrical processes has to be determined initially. Duval proposes that geometric thinking comprises of three kinds of cognitive processes which are *visualization*, *construction*, and *reasoning*. According to Duval, *visualization* is the "identification of gestalts and configurations in 2D or 3D" (p. 38). Moreover, visualization processes involve the visual representation of a geometrical statement, and the heuristic explanation of a complex geometrical situation. Duval defines *construction* as using tools and likens the construction of configurations to a model in which the actions on the representative and the observed results are connected to the represented mathematical objects. In the construction process, the tools can be ruler, compass or basic construction tools in geometrical softwares. Duval notes that construction processes depend only on connections between mathematical properties and the constraints of the used tools. According to Duval, the *reasoning* processes comprise of discursive processes for the

extension of knowledge, for explanation, and for proof. Duval argues that these three cognitive processes are closely linked and their coordination is essential for proficiency in geometry. The fundamental problem of the teaching of geometry at lower and upper secondary schools is how to guide students in seeing the interrelation between these three kinds of cognitive processes. In relation to the complex cognitive activity required for geometry learning, Duval (2006) points out the large variety of semiotic representations used in geometry:

> Thus, in geometry it is necessary to combine the use of at least two representation systems, one for verbal expression of properties or for numerical expression of magnitude and the other for visualization. What is called "a geometrical figure" always associates both discursive and visual representations, even if only one of them can be explicitly highlighted according to the mathematical activity that is required (p. 108).

This description of Duval resembles Fischbein's (1993) description of a figural concept consisting of figural and conceptual aspects. Duval's visual representations and discursive representations may correspond to Fischbein's figural properties and conceptual properties, respectively. Thus, the theoretical frameworks for the learning of geometrical reasoning may have similarities between Fischbein's and Duval's frameworks.

On the other hand, Smith (2010) considers geometrical reasoning as composed of *proof*, *justification*, and *argumentation*. Smith defines a mathematical *proof* as a "logically correct deductive argument based on given conditions, definitions, theorems, and postulates within an axiom system, which may or may not be apparent to the person building the argument" (p. 2). *Proving* is the process of producing a deductive argument. *Justifying* can be defined as giving reasons for a mathematical action or statement in order to legitimize one's mathematical activity. The outcome of the justifying process is the *justification*. Smith defines a mathematical argument as "a sequence of mathematical statements that aims to convince" (p. 2). On the other hand, *argumentation* is considered as a process in which a logically linked mathematical discourse is generated. For the current study data, claim, and reasons guided the construction of the Geometric Reasoning Test. Proof, justification, and argumentation are not mutually exclusive. According to Smith, argumentation contains justifying, and in turn, justifying contains proving. Based on Smith's work, the relationship between proving, justifying, and argumentation may be showed in a hierarchical triangle in Figure 2.1.

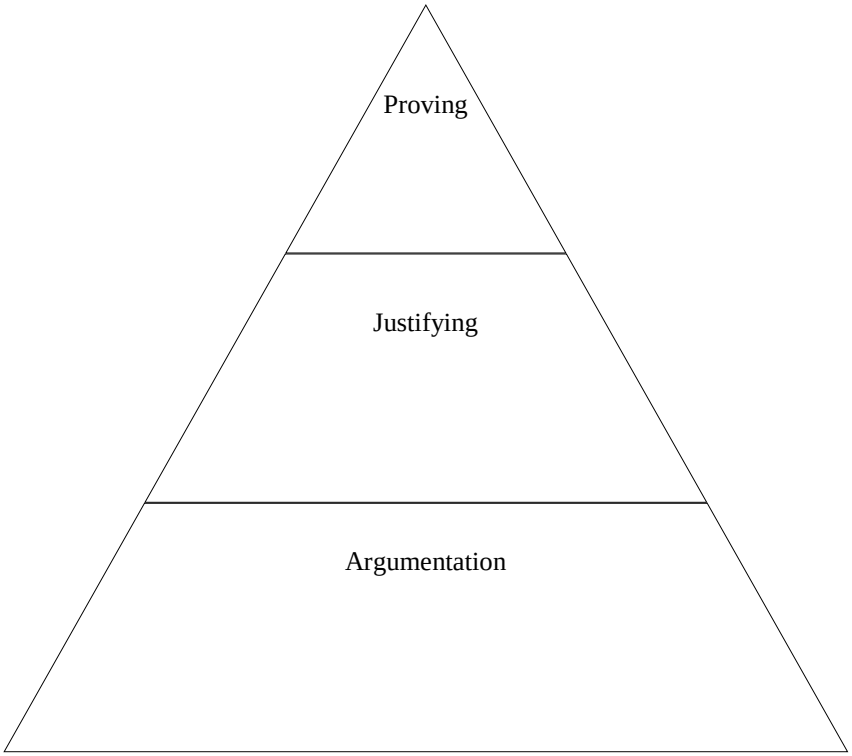


Figure 2.1. The connection between proving, justifying, and argumentation.

Furthermore, according to de Villiers (1999), the roles of proof are explanation, discovery, intellectual challenge, communication, verification, and systematization. Explanation promotes understanding of the claim. As another role of proof, discovery may occur in the process. Proofs may intellectually challenge a person. Proof is also a way of communication among mathematical society. The most familiar function of proof is verification of a claim. Proof may also function as a tool to systematization.

Similar to de Villiers, Jones (2000) mentions that forms of reasoning contain explanation, verification, argument, and proof. Jones (2000) also reports that students usually fail to distinguish between different forms of mathematical reasoning since the focus in the curriculum is just on the result (or on the format of the result). Concerning geometry learning and teaching, Jones notes the necessity of connecting geometrical intuition (or intuitive reasoning) and deductive thinking (formal reasoning).

Manipulatives may serve as a bridge that connects intuitive reasoning and formal reasoning in geometry. Paper, as one of the manipulatives, may help students to proceed from empirical thinking to abstract thinking in geometry. As Goos and her colleagues (2007) have stressed, problems and investigations using paper folding can provide opportunities for students to communicate, explain, convince and develop the skills for proof. Thus, well-designed paper folding activities may be a link between visualization, construction, and reasoning.

All things considered, Duval (1998) and Smith (2010) both emphasized the cognitive side of geometric thinking. However, Duval determined the general frames of geometric reasoning by pointing out the three cognitive processes- visualization, construction and reasoning. Smith, on the other hand, focused on the cognitive progression of geometric thinking. Based on Smith (2010), a student may proceed from argumentation, the base level, to proving, the highest level of geometric thinking.

Another framework for the learning of geometrical reasoning is the van Hiele model of thinking in geometry, which has generated several research studies on geometric thinking. Based on observations of their secondary school students' difficulties in learning geometry, Pierre van Hiele and Dina van Hiele-Geldof have proposed a theoretical model that focuses on levels of geometric thought (Fuys, Geddes and Tischler, 1984). The work of van Hieles is promising because they also emphasize the role of instruction in teaching geometry and in helping students move from one level of geometric level to the next. Thus, the teacher's role and instructional activities are important in the development of students' geometric thinking.

As Knight (2006) and Usiskin (1982) have noted that the van Hiele theory has three aspects, (i) existence of levels of understanding, (ii) properties that are inherent to each level and (iii) movement to a next level after completing the former level. As a defining characteristic of the van Hiele theory, geometric learning is regarded as a discontinuous process. In other words, the theory assumes the existence of discrete, qualitatively differentiated levels of geometric thinking (Clements and Battista, 1992) and the objects of thought, what learners are able to think geometrically, is the considerable difference from one level to the next (Van de Walle, 2007). As a distinction property, each level has its

own vocabulary and set of connections and concepts implicitly understood in the preceding level may be explicitly understood in the current level (Clements and Battista, 1992; Knight, 2006; Usiskin, 1982; Van Hiele, 1959/1985). The sequential levels of geometric thought in the van Hiele theory characterize learners' way of thinking and types of geometric ideas learners think about, rather than the amount of knowledge they have (Van de Walle, 2007). The levels are assumed as being sequential and hierarchical. This means that learners may sufficiently operate at a certain van Hiele level or higher levels if they have mastered most of the requirements for the lower levels (Clements and Battista, 1992).

Before summarizing the specific characteristics of each van Hiele level, it is appropriate to note that the number and names of each level can be seen to vary if the literature concerning the van Hiele levels is reviewed. The van Hiele theory originally described five hierarchical levels of geometric thought but studies of researchers such as Usiskin (1982) and Carroll (1998) imply the necessity of a level below the van Hiele's lowest level in the hierarchy. Clements and Battista (1992) label such a level as the Level 0: Pre-recognition. Van de Walle (2007) named the van Hiele levels as visualization, analysis, informal deduction, deduction, and rigor. Renumbering the labels of van de Walle and adding a level before the visualization level, a new numbering and labeling scheme for the van Hiele levels is constructed for this study. For the sake of consistency, the following scheme will be used in this study: Level 0: Pre-recognition, Level 1: Visualization, Level 2: Analysis, Level 3: Informal Deduction, Level 4: Deduction, Level 5: Rigor.

At Level 0, *pre-recognition*, learners may not be able to identify common shapes since they may not be able to construct the necessary visual images which must be formed as a result of their own actions (Clements and Battista, 1992). Clements and Battista specify the objects of thought and products of thought for the Level 0 (Pre-recognition) as the following, "At this level the 'objects' about which students reason are specific visual or tactile stimuli; the product of this reasoning is a group of figures recognized visually as the same shape" (p. 429).

At Level 1, *visualization*, figures are just judged by their appearance, often comparing them to a known prototype and geometric concepts are seen as total entities

rather than having components or attributes. Geometric figures are identified by their shape as a whole or by their physical appearances (Clements and Battista, 1992; Usiskin, 1982; van de Walle, 2007). A person functioning at this level can learn geometric vocabulary, can identify specified shapes, and given a figure, can reproduce it (Burger and Shaughnessy, 1986; Crowley, 1987; Erdoğan, Akkaya and Akkaya, 2009; Karaman, 2000; Mistretta, 2000; Van Hiele, 1999). The objects of thought for this level are shapes and what they resemble and the products of thought are classes or groupings of shapes that appear to be "alike" (van de Walle, 2007).

At Level 2, *analysis*, a figure is judged according to its properties not just its appearance (Usiskin, 1982; Van Hiele, 1999) and components of a figure such as its sides and angles are focused on. Figures are seen as collection of properties. Students can understand relevant attributes and differentiate them from irrelevant ones. For instance, a student would say that a square has four "equal" sides and four "square" corners as well as turning a square on the page does not change that the shape is a square. But students do not see the relationship between properties. For instance, they cannot say that a figure could belong to many general classes and have several names. A square is also a rectangle since a rectangle has four sides and four square corners but a student at this level thinks that square and rectangle are entirely separate types although they have common attributes (Burger and Shaughnessy, 1986; Crowley, 1987; Erdoğan, Akkaya and Akkaya, 2009; Karaman, 2000; Mistretta, 2000; Van Hiele, 1999). The objects of thought for this level are classes of shapes rather than individual shapes and the products of thought are the properties of shapes (van de Walle, 2007).

At Level 3, *informal deduction*, students can logically order properties of a shape and deduce one property from another (Usiskin, 1982; Van Hiele, 1999). Interrelationships within figures (in quadrilateral, opposite sides being parallel requires opposite angles being congruent) and among figures through implications and class inclusions (a square is a rectangle as it has all the properties of a rectangle) are established. Meaningful definitions and informal arguments for justification can be given but formal deduction is not formed. A proof starting from different or unfamiliar premises could not be constructed (Burger and Shaughnessy, 1986; Crowley, 1987; Erdoğan, Akkaya and Akkaya, 2009; Karaman, 2000; Mistretta, 2000; Van Hiele, 1999). The objects of thought for this level are the

properties of the shapes and the products of thought are relationships among properties of geometric objects (van de Walle, 2007). Van de Walle (2007) states that the key point in this level is the inclusion of logical reasoning (if-then reasoning) and recommend instructors to encourage students to conjecture and to ask the questions of "Why?" or "What if?".

At Level 4, *deduction*, students recognize the need for a system of logic that is based on a minimum set of assumptions and from which other truths can be deduced (van de Walle, 2007). Students can understand the importance of deduction as a way of establishing geometric theory within an axiomatic system as well as the interrelationship and the role of undefined terms, axioms, definitions, theorems and formal proofs (Clements and Battista, 1992; Usiskin, 1982; van de Walle, 2007). Proofs such as those typically found in a high school geometry class should be able to be constructed by students at this level (Burger and Shaughnessy, 1986; Crowley, 1987; Erdoğan, Akkaya and Akkaya, 2009; Karaman, 2000; Mistretta, 2000; Usiskin, 1982; van de Walle, 2007). The objects of thought for this level are relationships among properties of geometric objects and the products of thought are deductive axiomatic systems for geometry (van de Walle, 2007).

At Level 5, *rigor*, postulates or axioms are on the focus of understanding and the study of geometry at this level is highly abstract. The use of indirect proof, proof by contrapositive, non-Euclidean systems can be understood at this level and this level is usually considered to be shown at university or higher levels (Burger and Shaughnessy, 1986; Crowley, 1987; Erdogan, Akkaya and Akkaya, 2009; Karaman, 2000; Mistretta, 2000). The objects of thought for this level are deductive axiomatic systems for geometry and the products of thought are comparisons and contrasts among different axiomatic systems of geometry (van de Walle, 2007).

To sum up the characteristics of van Hiele levels, it can be said that the most prominent feature affecting the progression to higher levels is the geometric experience that a learner is exposed to (van de Walle, 2007). In order to increase the amount of such experience, appropriate instructional activities that allow the learners to explore, talk about, and interact with content at higher levels than the current level can be designed by

teachers. However, there will be a lack of communication when instruction or language is at a level higher than that of the student (Knight, 2006; Usiskin, 1982; van de Walle, 2007). Thus, the teacher plays a crucial role to facilitate this progress.

2.2. Construction

According to Duval (1998), *construction* was a cognitive process of using tools. Constructions can provide students a path for constructing a proof. Battista and Clements (1995) suggested that geometric constructions that were conducted before dealing with formal proofs could help students to make and test conjectures. Battista and Clements also emphasized the importance of geometric constructions by recommending that the secondary school geometry curriculum should have allowed students to use visual justification and empirical thinking since visual and empirical justifications were the fundamentals for advanced geometric thought. While students are folding paper, they can empirically justify certain geometrical relationships. Thus, origami as a geometric construction tool could facilitate students' making generalizations.

Different tools can be used to make constructions in geometry. These tools can be physical or virtual. Straightedge and compass are the basic physical tools for construction in geometry. Dynamic geometry softwares such as Geometer's Sketchpad and GeoGebra are useful virtual tools for geometric constructions. Olson (1975) and Coad (2006) also recommended origami to make geometric constructions. Olson (1975) also argued that many constructions of plane Euclidean geometry could be done through origami and listed some theorems that could be demonstrated by using origami constructions. Pythagorean Theorem was an example that could be done by using origami. Angle bisectors, midpoints, perpendicular bisectors, altitudes were some constructions that could be easily performed by origami (Coad, 2006; Olson, 1975). In addition to geometric aspects of origami, Coad (2006) suggested origami as a useful construction tool in geometry because paper was cheap and easily accessible to students. Coad has stressed that origami could help students to visualize certain geometric concepts such as lines, triangles, parabolas and solve certain construction problems such as trisecting an angle in Euclidean geometry. As Coad also emphasized, origami could be used to improve students' geometric reasoning and visualization skills. By observing the folds of paper in doing origami and questioning the

geometric meaning of the folds, students could be triggered to make and justify geometric conjectures.

In summary, geometric constructions through various manipulatives could facilitate students' visualization of geometric concepts and relationships. Then, the visualization process of geometric connections could help students reason on geometric generalizations, which required a more abstract reasoning. More specifically, constructions through origami may play the role of a bridge that connects empirical and more formal geometric reasoning. Although there were studies that included origami in Turkey (Bayrak, 2008; Çakmak, 2009), there were no studies that specifically investigate constructions through origami and their effect on geometric reasoning and spatial visualization at high school level.

2.3. Spatial Visualization

Although the importance of spatial ability is well-established among researchers, there is not a consensus on the definition of spatial ability. In the literature, different terms such as spatial reasoning, spatial perception, spatial sense and spatial thinking were used to refer to spatial ability (Clements and Battista, 1992; NCTM, 2000; Olkun, 2003). There are different approaches to define spatial ability. For instance, Olkun (2003) defined spatial ability as "the mental manipulation of objects and their parts in 2D and 3D space" (p.8) and pointed out that spatial relations and spatial visualization were the two components of spatial ability. On the other hand, Linn and Petersen (1985) defined spatial ability as the general concept that refers to skills in representing, transforming, generating, and recalling symbolic and nonverbal information.

Spatial visualization is commonly accepted as an essential part of spatial ability so, it is important to define spatial visualization and make studies on spatial visualization. Based on previous factor-analytic studies on spatial ability, McGee (1979) emphasized the existence of at least two factors of spatial ability: spatial visualization and spatial orientation. McGee defined spatial visualization as "the ability to mentally rotate, manipulate, and twist two- and three-dimensional stimulus objects" (p. 909) and spatial orientation as "the comprehension of the arrangement of elements within a visual stimulus

pattern, the aptitude to remain unconfused by the changing orientations in which a spatial configuration may be presented" (p. 909). Connor and Serbin (1980) also noted the existence of two factors of visual-spatial tests: spatial visualization and closure. According to Connor and Serbin, Cube Comparison Test, Paper Folding Test, Card Rotation Test and DAT Space Relations Test were included in spatial visualization category whereas Hidden Patterns, Gestalt Completion and Card Rotations tests were included in the closure category. Moreover, Olkun (2003) defined spatial visualization as "the mental manipulation and integration of stimuli consisting of more than one part or movable parts" (p. 2). Furthermore, Ekstrom, French and Harman (1976) defined spatial visualization as "the ability to manipulate or transform the image of spatial patterns into other arrangements" (p. 173). Ekstrom and his colleagues' definition of spatial visualization may be preferable since this definition necessitates that the figure to be mentally rearranged into different structures or parts.

Spatial visualization is important for many areas such as geometry, science, engineering, architecture (Baki, Kosa and Guven, 2011; Olkun, 2003; Tekin, 2007). The connection between spatial ability/ visualization and geometry achievement at different grade levels was also noted in many research studies (Fennema and Sherman, 1977; Idris, 2005; Karaman, 2000; Ünal, Jakubowski and Corey, 2009). For instance, Ünal and his colleagues (2009) have found that pre-service mathematics teachers who were identified with low spatial abilities had more difficulties in learning geometry compared to those with higher spatial abilities. Fennema and Sherman (1977) also found that spatial visualization and mathematics achievement were strongly correlated for high school students.

Many studies found that spatial ability, in particular spatial visualization, could be developed by training on spatial skills (Baki *et al*., 2011; Bayrak, 2008; Ben-Chaim, Lappan and Houang, 1988; Brinkmann, 1966; Connor and Serbin, 1980; Çakmak, 2009; Idris, 2005; Karaman, 2000; Olkun, 2003). For instance, Brinkmann (1966), Ben-Chaim and his colleagues (1988) found that specifically designed instruction could have a positive effect on improving spatial visualization of middle school students. In addition, there are studies in Turkey that contain both spatial visualization and geometry instruction in which origami is integrated. For instance, Bayrak (2008) investigated the effect of visual treatment which also included origami activities on sixth grade students' spatial ability.

The researcher found a significant effect of visual treatment on students' spatial ability. Furthermore, Çakmak (2009) found the positive effect of origami-based instruction on elementary students' spatial visualization. Thus, well-designed instruction could enhance students' spatial visualization abilities.

Most of the studies concerning spatial ability found the positive effect of training on spatial ability. On the other hand, there were a few of the studies that found no significant of training on spatial ability. For instance, Boyraz (2008) investigated the effect of computer-based instruction on seventh-grade students' spatial ability and found no significant effect on students' spatial ability. Boakes (2006) either found no significant effect of origami lessons on seventh-grade students' spatial visualization.

In summary, although there were some studies that found no significant effect of training on spatial ability (Boakes, 2006; Boyraz, 2008), most studies found a significant effect of training on spatial ability and its factors such as spatial visualization (Baki *et al*., 2011; Bayrak, 2008; Ben-Chaim, Lappan and Houang, 1988; Brinkmann, 1966; Connor and Serbin, 1980; Çakmak, 2009; Idris, 2005; Karaman, 2000; Olkun, 2003). There were studies concerning spatial ability in Turkey but studies including both origami and spatial visualization were limited in Turkey (Bayrak, 2008; Çakmak, 2009).

2.4. Geometry Instruction

The van Hiele levels of geometric thought present teachers a way of understanding of their students' geometric development and teachers play an important role in helping students to raise their levels of geometric understanding. In the process of helping students in the advancement through the van Hiele levels, van Hieles have suggested teachers an instructional sequence which consists of five phases.

The initial phase of learning which the teachers are recommended to start in the van Hiele model is the *inquiry/information phase*. In this phase, students are familiarized with the content to be learned (Clements and Battista, 1992). The teacher learns the students' prior knowledge and current experiences (Mistretta, 2000) so that s/he determines students' geometric thinking levels at hand (Erdoğan *et al*., 2009). The stage for the new topic is set

up by engaging students in investigation (Crowley, 1987; van Hiele, 1999). Thus, the students learn the direction of their further study.

The second phase is the *guided (direct) orientation*. In this phase, the teacher carefully structure and sequence the tasks in order to help students explore the topic of the study (Clements and Battista, 1992; Crowley, 1987). Students are expected to be involved in exploring objects (for instance, folding, measuring) so that they gradually see the main connections of the network of relations that is to be constructed (Clements and Battista, 1992; Crowley, 1987; van Hiele, 1999). The teacher directs students in proper explorations where students manipulate objects and elicit specific responses so that they face with specific concepts and procedures of geometry (Clements and Battista, 1992; Crowley, 1987).

The third phase of learning is the *explicitation*. The teacher introduces the topic and based on their previous experiences, students share their emerging views about the structures that have been observed (Erdoğan *et al*., 2009; Crowley, 1987). Students get explicitly aware of their geometric conceptualizations, express these conceptualizations in their own language, and learn some of the formal mathematical language for the study of the subject matter (Clements and Battista, 1992). The teacher introduces the appropriate mathematical terminology after students have showed the consciousness of the objects of study (geometric objects and ideas, relationships, patterns) and encourages students to use it in their geometrical communication (Clements and Battista, 1992; Crowley, 1987; van Hiele, 1999).

The fourth phase is the *free orientation*. In this phase, students deal with more complex tasks- tasks with many steps, tasks with different solutions, and open-ended tasks (Crowley, 1987; Erdoğan *et al*., 2009; Mistretta, 2000; van Hiele, 1999). Students have to synthesize and use the previously articulated concepts and relations in order to successfully complete these complex tasks. Crowley (1987) gives a paper-folding activity that can be conducted in this phase as the following: Students fold a piece of paper in half, then in half again and predict the type of figure they would get if they cut off the corner constructed by the folds in different angles. Students should also give a justification of their answer before they cut the corner. Therefore, the teacher chooses multiphase tasks, encourages students

to engage in these tasks and introduces terms, concepts, and relevant problem-solving processes in this phase (Clements and Battista, 1992).

The last phase is the *integration*. In this phase, students summarize what they have learned so far about the geometric topic at hand and internalize these in a new framework of relations (Clements and Battista, 1992; Crowley, 1987; Erdoğan *et al*., 2009; Mistretta, 2000; van Hiele, 1999). The role of the teacher in this phase is to help students in articulating and integrating their geometric knowledge with the use of language of mathematics (Clements and Battista, 1992). At the end of this last phase, students should have attained a new level of thought for the topic studied (Clements and Battista, 1992; Crowley, 1987).

The van Hiele levels of geometric thought and the phases of instruction aligned with these levels have implications for teaching geometry. They guide teachers for effective geometry instruction and present an alternative instructional approach to traditional geometry teaching, which is usually based on textbook and direct instruction. For effective geometry instruction, the van Hiele theory suggests using appropriate geometrical terminology as well as exploration and manipulation of geometric objects. Therefore, the use of manipulative models is suggested among the effective teaching strategies for geometry.

2.4.1. The Use of Manipulatives in Geometry Instruction

Both theories of van Hiele and Duval imply the use of manipulatives in teaching geometry. Huetinck and Munshin (2004) define manipulative as "an object that students can handle and move to address mathematical concepts through visual and kinesthetic senses" (p. 77). Similarly, Hynes (1986) uses mathematical manipulatives to refer to "concrete models that incorporate mathematical concepts, appeal to several senses and can be touched and moved around by students" (p. 11). There are a considerable number of studies that support use of manipulatives in the mathematics classroom as concrete models of abstract concepts (Clements and Battista, 1992; Jones, 2010). More than 100 studies have been conducted in the last three decades on the effectiveness of manipulatives on teaching different mathematics topics (Jones, 2010). Based on a review of a number of

studies, Suydam and Higgins (1977) have concluded that elementary level (K-8) students who used manipulatives showed higher achievement scores than students who did not use such materials. Based on meta-analysis of 60 studies, Sowell (1989) concluded that "mathematics achievement is increased through the long-term use of concrete instructional materials and that students' attitudes towards mathematics are improved when they have instruction with concrete materials provided by teachers knowledgeable about their use" (p. 498).

Manipulatives can be useful in different ways in learning geometry. Clements and Battista (1992) have stated that use of manipulatives may help students "try out their ideas, examine and reflect on them, and modify them" (p. 449). They have also noted that this hands-on approach appears to maintain student interest, to aid students in creating definitions and new conjectures, and to help them in gaining insight into new relationships. Based on Sowell's (1989) and Jones' (2010) review of studies concerning use of manipulatives in geometry instruction, it can be concluded that the benefits of manipulatives hold across grade level, ability level, and topic, given that the use of a manipulative is appropriate for that topic.

Although research support that students of all ages and all levels of mathematical achievement can benefit from the use of manipulatives (e.g. Suydam and Higgins, 1977), many studies indicate the lower frequency of use of manipulatives through the elementary school grades and into the higher grades. Jones (2010) has noted the low use of manipulatives in secondary school mathematics lessons in the U.S. by giving the results of the National Survey of Science and Mathematics Education in 1993 as an example. According to these results, it is found that intermediate and secondary school mathematics teachers do not believe in the use of manipulatives for effective mathematics instruction as compared to elementary school teachers.

Thus, more frequent use of manipulatives in high school would more likely to lead to increase in students' achievement and understanding in geometry. Abstract thinking is fundamental in high school geometry but some students may have difficulty in attaining such formal thinking and as Huetinck and Munshin (2004) have argued, manipulatives can be effective tools in advancing students' mathematical understanding by moving from

concrete experiences to abstract thinking. Piaget's research has showed that symbolic representations have meanings for students as old as 12 only if they are closely linked with concrete representations (Huetinck and Munshin, 2004). Concrete experiences through manipulatives are all important in learning stages proposed by educational researchers Piaget, Skemp and the van Hieles. Manipulatives are significant aids to learning in all four of Piaget's developmental stages (sensorimotor, preoperational, concrete operational, and formal operational). Thus, manipulatives can be regarded as fundamental aid to active learning which emphasizes active participation by students through different learning experiences. Manipulatives can enhance the doing, which requires making concrete links to real-world experiences for many students.

Manipulatives extend the teachers' abilities to reach all students by varying the pace, giving necessary reinforcement, and providing a variety of ways for students with different learning abilities and styles to access mathematical knowledge (Huetinck and Munshin, 2004). Thus, using manipulatives in geometry is also consistent with the Multiple Intelligences Theory of Howard Gardner. According to Gardner (1983), each person has varying amounts of different types of intelligences and there is a need to develop rich learning experiences that may help to enhance their levels of intelligences (as cited in Huetinck and Munshin, 2004). Therefore, manipulatives can be effective tools that serve to this purpose since they address mathematical concepts through visual and kinesthetic senses.

Manipulatives such as mirrors, geoboards, tangrams, pattern blocks and paper are especially helpful in visualizing geometric concepts. Among these geometrical manipulatives, paper is easily accessible, relatively cheap, easy to store, simple and familiar to students. Thus, paper folding is suggested to be a useful hands-on tool to teach geometry.

2.5. What is Origami?

Origami literally means the art of *paper folding* coming from the Japanese words of "oru" (to fold) and "kami" (paper) (Wu, 2002). Although the word origami is Japanese, origami is popularly used in the world. The word for paper, kami, is also a homonym for

the word spirit or god (Franco, 1999; Wu, 2006) so origami has religious connotations too. The products of origami are called *models*, the method for folding a model is called a *design*, and drawn instructions for a model are called a set of *diagrams*. An origami artist is usually called a *paper folder*. The only requirement for origami is a piece of paper, which makes it one of the most accessible arts (Wu, 2002). Then, *origami* can be defined as the art of paper folding in which paper is used to create two- or three-dimensional figures, ranging from different models of animals, almost any physical objects including people, faces, flowers, vehicles to abstract geometrical shapes.

2.6. History of Origami

Although the origin of origami was uncertain, origami was believed to be started just after the invention of paper in China and then to be brought to Japan by Buddhist monks (Tuğrul and Kavici, 2002; Wu, 2006). After the introduction of paper into Japan in the late sixth century, origami was adopted by Japanese as a cultural element. At first, paper was an expensive commodity and origami was done by a limited number of people for formal functions. Origami which was used in religion and ceremonies had different symbolic meanings. For example, at the early Japanese weddings, folded butterflies were used to decorate sake bottles and to symbolize happiness (Ritchie, 1976).

In the eighteenth century, textbooks about origami started to be written (Ritchie, 1976). One of these books, *Hiden Senbazuru Orikata* (Folding 1000 Cranes), gave instructions about certain origami models, particularly the popular paper crane (Ritchie, 1976; Tuğrul and Kavici, 2002; Wu, 2006). One portion of the *Kan no Mado*, published around 1845, was about origami and contained a wide collection of traditional Japanese figures (Mitchell, n.d.; Ritchie, 1976; Tuğrul and Kavici, 2002; Wu, 2006).

Origami might have developed independently in different places of the world. Origami might have been introduced to Europe from Asia via the Silk Route (Origami Resource Center, 2010; Tuğrul and Kavici, 2002). The Muslims might have brought paper making to Spain when they conquered Spain (Lister, 2004; Tuğrul and Kavici, 2002). Although it is not clear whether origami was independently invented in Europe or whether it was also introduced from Asia (Origami Resource Center, 2010), origami was done in

Spain and even origami schools were founded. The first origami school in Spain was opened by Miguel de Unamuno (1684-1936) and the school has still been open there (Tuğrul and Kavici, 2002). As an indication of the fact that origami was also known in Europe in the Middle Ages, Lister (2004) and Hatori (n.d.) have noted that John Webster referred to paper fly-traps (which resembles the origami model known as Water Bomb today) in his play "The Duchess of Malfi" in 1614 which was earlier than the appearance of recreational folding in Japan. Furthermore, according to Ritchie (1976), origami existed in America and England long before the twentieth century but under different names such as "paper magic". Ritchie (1976) explains how origami is part of life of people in England and America:

> In Victorian London, street performers in origami drew crowds by exhibiting a most ingenious all-purpose origami piece called "The Magic Fan", or the "Puzzle Wit"…A mere twist of the hand converted it into one of many different shapes. Victorian origami numbered many models just as ingenious as the Magic Fan…They include paper bellows that really blow, a paper Chinese junk, paper hats, paper boxes, paper purses, paper boats, and even a paper kettle in which you could really boil water over the fire (pp. 39-40).

Although origami is known to be a Japanese art, it also exists in different cultures under different names. For example, "papierfalten" in German, "paper folding" in English, "papiroflexia" in Spanish all mean origami. This may indicate that origami is still developing. The existence and interactions of different cultures may also have contributed to the generation of different styles of origami such as modular (unit) origami and kirigami which are considered among modern origami. Modular origami is a paper folding technique where multiple copies of the same unit or module are combined to form geometric models like polyhedra. In kirigami, paper is cut in different forms. Moreover, wet origami, architectural origami are some other examples of different styles of modern origami.

People from different professions and ages are interested in origami. For example, some children can learn to make origami before they learn to read (Franco, 1999). Besides, today, people interested in origami come together in origami associations (like Origami USA or the British Origami Society) which can be encountered in different places around the world. People in different professions ranging from scientists to artists to therapists are interested in origami. Artists and craftspeople may use origami as a way to express themselves creatively. Scientists, architects, and mathematicians may explore the geometry

of origami for its own beauty and practical applications while therapists can use origami as a tool to help their patients (Wu, 2002). Furthermore, teachers can use it to help their students learn the topics better. For example, in the middle of nineteenth century, Friedrich Froebel founded the first kindergarten in the world and origami was an educational tool in Froebel's kindergarten system and was used to teach young children geometry (Hatori, n.d.). Thus, origami may be used for different purposes by different people.

2.7. Applications of Origami

The two fields, origami and mathematical sciences have started to interact within the last few decades (Lang, 2001) and this interaction has generated different applications into different fields. Beyond entertainment, origami is applied into different fields to solve certain complex problems. There are different applications of origami in mathematics, engineering, astronomy, manufacturing, robotics and biology (Cipra, 2001; Demaine, 2004). For example, based on origami, a folding pattern for a space-based telescope is developed (Cipra, 2001). Lang (2001) noted that the use of origami in real-world structural or design problems as the following "Some inhabitants of the boundary between origami art and science have used the techniques of origami to solve real-world structural or design problems" (p. 119). Miura (2001) applied origami to map and atlas design. Moreover, Fuse, Nagashima, Ohara and Okumura (2001) designed a rigid, robust pot which used no adhesive based on an origami twist pattern as a new technique in manufacturing. In automotive industry, the deployment of airbags is based on paper folding (Cipra, 2001).

There is mathematics inherited in origami but this rich intersection area of origami and mathematics has gained acceleration in recent years. Krier (2007) points out the application of origami in different fields of mathematics such as geometry, calculus, and abstract algebra in order to explain mathematical concepts. Similar to the postulates in Euclidean geometry, postulates of origami have been set up and origami has turned out to be an alternative method of solving certain mathematical problems. The first six postulates of origami were developed by Humiaki Huzita and the seventh one by Jacques Justin (Hull, 2001; Krier, 2007). These postulates have pioneered the development of *origami mathematics* (Robert Lang and Thomas Hull use this expression -origami mathematics or origami math- where Haga (2001) uses Origamics) which studies the underlying

mathematics in origami. As Hull (1996; 2001) and many authors such as Geretschläger (1995), Krier (2007), Alperin (1999), Hess (1956), Olson (1975) have pointed out, origami can be a more useful geometric construction tool than the straightedge and compass and can even solve quadratic and cubic equations. Krier (2007) has argued that origami can be helpful in comprehending mathematics. Kawasaki (2001) has also pointed out the connections of origami with geometrical concepts (such as quadrilaterals, circles, bisectors). Therefore, the mathematics of origami guides mathematics educators in designing lessons using origami in mathematics by showing how origami and mathematics are related.

There are books that relate origami with mathematics. Some of them are as the following: *Origami Design Secrets: Mathematical Methods for an Ancient Art* (Lang, 2003), *Genuine Origami: 43 Mathematically-based Models, from Simple to Complex* (Maekawa, 2008), *Origamics: Mathematical Explorations through Paper Folding* (Haga, 2008), *Project Origami: Activities for Exploring Mathematics* (Hull, 2006) and *Unfolding Mathematics with Unit Origami* (Franco, 1999). Lang (2003) explains the mathematical relationships underlying the design of complex origami models where Maekawa (2008) gives the geometry of each origami model in his book. In Maekawa's (2008) book, the rich collection of origami models can be used by mathematics educators to enhance understanding of certain geometry concepts in high school (such as triangles, angle bisectors). Haga (2008) uses the term Origamics to refer to the science of origami where no figures of objects result and shows his mathematical findings of origami in his book. In the introduction to the Haga's (2008) *Origamics: Mathematical Explorations through Paper Folding*, Fonacier (2008) explains the connection of origami with geometry and recommends that origami can be an effective instructional tool for geometry:

> It is easy to see that origami has links with geometry. Creases and edges represent lines, intersecting creases and edges make angles, the intersections themselves represent points. Because of its manipulative and experiential nature, origami could become an effective context for the learning and teaching of geometry…The manipulative nature of origami allows much experimenting, comparing, visualizing, discovering and conjecturing (p. v).

Furthermore, Fonacier (2008) writes that Haga's (2008) *Origamics: Mathematical Explorations through Paper Folding* is a resource book for mathematics educators such

that it guides them in designing novel instructional methods in order to reinforce and apply the theorems of high school geometry.

On the other hand, Hull (2006) presents 22 mathematically-based origami activities which show connections to geometry, algebra, number theory, and combinatorics. Hull (2006) suggests that these origami activities are effective mathematics education tool especially for high school and college teachers. Hull (2006) also says that "…many of these activities can be effectively used in a variety of courses at a variety of levels. The angle trisection activity, for example, has been very popular among high-school geometry teachers, yet it also makes for a very illustrative diversion in an upper-level Galois Theory course." (p. x). Hull (2006) also mentions the active and discovery-based nature of the origami activities. Origami transforms geometry into a concrete entity because origami necessitates hands-on participation by the person who makes origami. The manipulative character of origami may help students concretize the abstract concepts of geometry. Because of this hands-on and discovery-based nature of origami activities, the folding method for an equilateral triangle shown in Hull (2006) is chosen to be used in this thesis. Besides, Franco (1999) also presents sixteen modular origami activities which can be used in high school and middle school geometry and algebra lessons. Franco (1999) mentions the educational benefits of using origami in mathematics lessons as the following:

> Students involved in origami have the opportunity to use the vocabulary of mathematics in context and to discuss geometric concepts as they assemble origami models…There are innumerable mathematical skills and concepts related to origami. Here are some of these skills and concepts: spatial visualization, regular polygons, triangles, congruence and similarity, fractions and ratios, sequences, angles, regular solids, semi regular solids, dual polyhedra, stellated polyhedra, inscribed polyhedra, deltahedra, symmetry, area, volume, and surface area, cross sections, Pythagorean theorem, and Euler formula (p. vii).

As the titles and content of these books suggests, origami can be used as an effective instructional tool in high school geometry.

2.8. Origami in Mathematics Education

Origami, the art of paper folding has been claimed to be as an effective instructional tool in many publications (e.g. Boakes, 2006, 2010; Brumbaugh and Rock, 2001; Cañadas, Molina, Gallardo, Martinez-Santaolalla and Peñas, 2010; Chen, 2005; Cipoletti and Wilson, 2004; Coad, 2006; Cornelius and Tubis, 2006; Çakmak, 2009; Golan

and Jackson, 2009; Higginson and Colgan, 2001; Kavici, 2005; Pope, 2002; Robichaux and Rodrigue, 2003; Russell, 2007; Sağsöz, 2008; Sze, 2005a, Sze, 2005b; Tinsley, 1972; Tuğrul and Kavici, 2002). Origami offers many educational benefits such as behavioral gains, socio-emotional gains, psychomotor developmental gains, language development, multicultural awareness, cooperative learning, cognitive development, and mathematical skills (Tuğrul and Kavici, 2002; Valentini, 2005). Origami incorporates multiple intelligence areas and enriches different areas of learning including social studies, art, language arts, science, special education, therapy, and mathematics (Ho, 2001; Sze, 2005a; Sze, 2005b). The student should listen and follow carefully the instructions in the process of origami and the repeated actions of origami provides schema learning through activating both the right and left hemispheres of the brain by using attention, memory, imagination, memory, nonverbal thinking and 3D-comprehension (Çakmak, 2009; Shumakova and Shumakov, 1998, 2000; Sze, 2005a; Tuğrul and Kavici, 2002). In *Origami 3: Third International Meeting of Origami, Science, Math, and Education*, two authors (Foreman-Takano, 2001; Ho, 2001) have mentioned the use of origami as a tool to facilitate the study of English as a Second Language. Besides, Valentini (2005) has also written the benefits of using origami as an instructional tool in primary English as a Foreign Language (EFL) teaching in Italy. By connecting different conceptual frameworks, Sze (2005a) presents a theoretical model for origami which represents six types of constructive learning: hands-on learning, explicit instruction, higher order thinking, multimodal instruction, social learning, and self-management strategies. Similarly, Tuğrul and Kavici (2002) have reported the relation of origami with learning models such as active learning, cooperative learning, creative learning, project-based learning, and brain-based learning and have recommended the use of origami in education beginning from kindergarten.

Origami is also used in teaching mathematics to disabled children. For example, Chen (2005) has made an overview of the use of origami in teaching mathematics to deaf and hard-of-hearing students. Many mathematical concepts and skills such as analytical and critical thinking skills, spatial visualization, area and volume, mirror images, symmetry, congruence, and angles can be developed through origami (Chen, 2005). According to Chen (2005), origami can empower spatial reasoning for all students and origami mathematics lessons can be designed for all ages such that young students can explore mathematical concepts while developing a mathematical foundation and “older

students can have master mathematical concepts that are more difficult to understand by other ways" (p. 263). Moreover, according to Chen (2005), as a hands-on activity, mathematics origami is appropriate for all ages, inexpensive to do, and may motivate and challenge students of all ability levels. As another application of origami in the mathematics education of disabled children, Tinsley (1972) has used origami to develop geometric concepts for visually impaired students.

As students are doing origami, they build skills such as spatial/geometric reasoning and visualization (Boakes, 2006; Chen, 2005; Cipoletti and Wilson, 2004; Hartzler, 2003; Sze, 2005a; Sze, 2005b) and develop their problem solving skills (Boakes, 2006; Brady, 2008; Chen, 2005; Sze, 2005a). Origami as a hands-on activity increases students' ability to communicate mathematically and promotes their understanding of mathematical concepts (Cipoletti and Wilson, 2004; Sze, 2005a). It also provides a highly motivating environment which encourages higher order thinking (Sze, 2005a; Sze, 2005b), so students can construct their own understanding.

Both mathematicians and mathematics educators use origami. In literature, origami and paper folding are generally used interchangeably, so studies which use "paper folding" in mathematics education are also mentioned here. In the application of origami in mathematics education in literature, the publications can be gathered under two general approaches: activity-based and research-based. The activity-based publications usually explain a specific mathematics lesson which integrates certain origami models and discuss the benefits of these origami integrated mathematics lessons without formally measuring the significant effect of such lessons on students. Thus, these publications can be said to have the mathematics teachers or the practitioners as an audience. On the other hand, the research-based publications mention the studies which search the effect of origami or paper folding lessons on different variables such as spatial ability, visual/spatial perception, geometry knowledge, geometry achievement, engagement, multiplicative thinking, verbal expression, size comparison strategies and disability of the visual sequential memory.

In literature related with the use of origami as an instructional tool in mathematics, there are many instances where origami is used in preschool, elementary or middle school grades (Baicker, 2004; Boakes, 2006, 2008, 2009; Brady, 2008; Costello, 1985; Çakmak,

2009; Erktin, Özkan and Balcı, n.d..; Golan and Jackson, 2009; Heukerott, 1988; Higginson and Colgan, 2001; Kavici, 2005; Pope, 2002; Robichaux and Rodrigue, 2003; Sağsöz, 2008; Senger, Platte and Van Zandt, 1997; Wilson, Flanagan, Gurkewitz and Skrip, 2009; Yuzawa and Bart, 2002).

Golan and Jackson (2009) describe a program called *Origametria* which uses origami to teach geometry from grade one through upwards. In this program, origami is integrated into curriculum for 10000 students who study Origametria every week in Israel. This program was formally approved by the Israeli Ministry of Education in 2008. Furthermore, Golan and Jackson have reported that a pilot course of Origametria would be conducted in a college of teacher training in Israel. According to Golan and Jackson, there are also plans to expand the program into high schools in future.

In Origametria, a geometric topic is initially chosen aligned with the curriculum and grade level. Then, an origami model whose sequence focuses on that topic is determined. The main aim of the program is to improve a student's knowledge of the geometric topic chosen while folding the model. The hands-on way of the program to learning makes it an effective tool with learning problems. Golan and Jackson (2009) argue that the success of Origametria can be described and measured at cognitive, emotional, and motoric levels. The cognitive level contains developing spatial perception, logical thinking, visual perception, improving use of imagination, and building knowledge. The emotional level includes building self-esteem and enhancing feelings of success whereas motoric level includes developing motor ability and improving hand-eye coordination.

The authors noted that Origametria has been piloted and all the classes where Origametria has been taught have had better retention of the geometric topics. Golan and Jackson (2009) commented that although there was informal positive feedback on the success of Origametria, a scientific analysis of the effect of Origametria was needed. Golan and Jackson (2009) hoped that the program would be "adopted as one of the accepted ways to introduce basic geometry topics as an alternative and/or an addition to the traditional ruler-and-straightedge or chalkboard methods" (p. 8).

Concerning the use of origami to teach geometry in middle school, Robichaux and Rodrigue (2003) describe a geometry lesson which integrates origami. Two origami models (an open box and a bench) are used to teach geometric concepts in cooperative groups. The initial sample of students is divided into two. Each group becomes an expert in one of the origami models. Then, they are paired to teach the other model. In 90-minute lesson, the concepts of shape, congruence, similarity, symmetry, fractions, properties of shapes, right triangles, isosceles triangles, acute angles, right angles, polygons such as trapezoids, hexagons, and octagons are emphasized. Robichaux and Rodrigue (2003) point out the role of origami in communicating mathematically, increasing motivation of students toward geometry, and enhancing conceptual understanding in geometry.

On the other hand, there are less publications dealing with the use of origami in high school level. For instance, Cañadas, Molina, Gallardo, Martinez-Santaolalla and Peñas (2010) describe a modular origami activity to teach geometry to guide geometry educators. The geometry lesson which uses origami aims to help high school students learn new geometric concepts and revise concepts already known through the origami activity of building a cube. Concepts of plane geometry and spatial geometry are focused on. In the construction process of polyhedra- a cube- using modular origami; fractions, measurement of surfaces, angles and irrational numbers, parallelism, intersecting lines, Pythagorean Theorem, the sum of the measures of interior angles of a triangle are explored. Cañadas and colleagues emphasized the use of paper for teaching and learning geometry because of the manipulative nature of the paper in helping students to promote giving meaning to geometrical concepts.

In Hull (2001), *Origami 3: Third International Meeting of Origami, Science, Math, and Education*, there are also papers dealing with the use of origami in mathematics lessons in secondary school students. For example, Morrow (2001), Knoll (2001), Cornelius and Tubis (2001) used different origami activities to teach geometry to high school students. The authors taught origami boxes, modular and circular origami activities to help students learn polyhedra, trigonometry and even calculus.

In addition to the activity-based articles about the use of origami as an instructional tool in geometry, there are also research articles at preschool, elementary and middle

school level (Brady, 2008; Boakes, 2006; Erktin *et al*., n.d.; Kavici, 2005; Yuzawa and Bart, 2002). Yuzawa and Bart (2002) have designed an experimental study to see the effect of origami exercises on the learning of size comparison strategies of preschool age children (ages 5 and 6). Origami exercises have helped the children use more effective strategies for size comparison. According to Yuzawa and Bart (2002), two aspects of the instruction with origami may lead the change in size comparison strategies: Children's folding a sheet of paper into two half sheets of the same area, and children's folding a sheet of paper precisely.

In her dissertation study, Boakes (2006) has investigated the effect of origami-mathematics lessons on seventh grade students' spatial visualization skills and achievement levels. A significant interaction effect between group and gender for one of three spatial visualization tests (Card Rotation Test) with males was found by an analysis of covariance (ANCOVA) after initial differences have been controlled. However, no significant differences were found for the Paper Folding Test and geometry achievement. According to the findings of the study, Boakes (2006) argues that origami-mathematics lessons blended within traditional instruction could be as helpful as traditional instruction in terms of mathematics achievement.

A recent study by Boakes (2010) has investigated the effect of origami-mathematics lessons on mathematical and spatial abilities of college students. In all cases there was a significant increase seen in students' mean scores on all three spatial tests. Boakes (2010) concluded that Origami had indeed an effect on students' spatial skills. However, because of the diverse individuals with varied academic backgrounds in the sample, it was difficult to say with certainty that origami was the only cause of such change. But, being conducted over a short period of time, it may be an important contributing factor.

In addition to articles dealing with the use of origami in high school mathematics classrooms, some secondary mathematics teaching textbooks include origami activities to teach mathematics to high school students. For instance, *Teaching Secondary Mathematics* (Brumbaugh and Rock, 2001), *Teaching mathematics for the 21st century: Methods and activities for grades 6*-12 (Huetinck and Munshin, 2004), *Discovering Geometry: An inductive approach* (Serra, 1997), *Teaching secondary school mathematics: Research and*

practice for the 21st century (Goos, Stillman and Vale, 2007) include paper folding activities as a means to teach mathematics. In *Teaching Secondary Mathematics* (Brumbaugh and Rock, 2001), paper folding is connected to perpendicular line segments, parallel line segments, angle bisectors, construction of parabola and regular pentagon. *Geometry* (Jurgensen, Brown and Jurgensen, 1992) contains paper folding activities to construct isosceles triangles or to show the sum of the angle measures of a triangle. *Geometry: Seeing, doing, understanding* (Jacobs, 2003) also includes origami exercises such as an origami duck to reinforce the concepts of angle bisection, type of triangles, congruence, triangle side and angle inequalities as well as an origami frog to reinforce the concepts of parallel lines and types of angles for triangles. In her book, *The Joy of Mathematics* [*Yaşayan Matematik*] Pappas (1993/2007) mentioned the mathematical aspects of paper folding. For example, some geometric concepts such as square, rectangle, right triangle, congruence, similarity, diagonal, midpoint, secant, area, perpendicular mid-segment, trapezoid, Pythagorean Theorem, construction of a parabola, the sum of angle measures in a triangle are shown to be related with paper folding (Pappas, 2007).

The reviewed studies indicate that there are few research studies specifically related with origami and geometry education. Besides, the number of studies conducted in Turkey is very limited. For instance, in her master's thesis study Sağsöz (2008) has searched the effect of the mathematics program supported with origami activities (PSWOA) in teaching fractions at sixth grade in Erzurum, in Turkey. Sağsöz has found out that the experimental group treated with the origami supported program (PSWOA) has been more successful in understanding and application of fractions than the control group who are instructed with the traditional methods only.

In another thesis study, Çakmak (2009) has investigated the effect of origami-based instruction on fourth, fifth, and sixth grade students' spatial ability in mathematics in terms of, as well as spatial orientation and students' self-reported perceptions related to origami-based instruction. The sample of this study consists of 38 fourth, fifth, and sixth grade students chosen from a private school in Ankara. Çakmak has found a significant positive effect of origami-based instruction on elementary students' both spatial visualization and spatial orientation, and students have showed positive attitudes toward origami-based instruction.

Moreover, the study of Bayrak (2008) has investigated the effect of visual treatment on sixth-grade students' spatial ability and attitude toward spatial ability problems. Origami activities are one type of the activities of the visual treatment for that study. Modular origami activities are used in the total of five lectures. Each class hour accounts for 45 minutes and each activity is administered per week. The researcher has found out a significant positive effect on spatial ability.

To sum up, students need relatively strong previous geometrical knowledge in order to be successful in high school geometry because topics in geometry are connected with each other. However, the international comparative studies show that students in Turkey start with a weak mathematics (especially geometry) background to high school. The results of the PISA and TIMSS show that middle grade students in Turkey have performed below the international average for mathematics, in particular geometry. According to the results reported by Mullis, Martin and Foy (2008), Turkey is so below the TIMSS 2007 average scores for all domains in eight-grade mathematics. The lowest content domain score in mathematics is geometry for Turkey. In addition to the inadequate performance in geometry of middle-school Turkish students in TIMSS, ninth-grade Turkish students also performed below the international mean score in mathematical literacy in the PISA. Thus, there is a need to increase the mathematics, in particular geometry, achievement of Turkish high school students. Origami-based geometry lessons may help Turkish high school students to conceptually understand geometric topics and raise their achievement levels in geometry.

The articles and research studies concerning origami and geometry learning have showed that origami could be a useful tool in geometry learning and teaching. The studies up to now indicate that origami may support the three cognitive processes- visualization, construction and reasoning- suggested by Duval (1998). In fact, there should be studies that investigate whether origami supports these cognitive processes or not in Turkish setting.

3. SIGNIFICANCE OF THE STUDY

Studies have indicated that origami has many educational benefits, particularly in learning geometry; and origami may be used as an effective instructional tool in geometry education. Although there are studies to support the benefits of using origami in geometry education (e.g. Bayrak, 2008; Çakmak, 2009), more research studies are needed. In general, there is a limited number of research studies in high school level even if practical references support the use of origami for high school geometry instruction. Moreover, research studies about the use of origami in geometry education in Turkey are mostly about elementary or middle school level. The recent reform movements in high school geometry curriculum in Turkey have also recommended the use of origami in teaching geometry, specifically triangles. In particular, the recent 12^{th} grade geometry curriculum (MEB, 2011) exemplifies the use of origami in geometry lessons and mentions some mathematical aspects of origami. This study was expected to contribute to the literature by investigating the effect of origami-based lessons on tenth grade students' geometry achievement, spatial visualization and geometric reasoning abilities concerning triangles. Triangles were selected as the topic of study based on the literature since triangles were fundamental to learn other topics in Euclidean geometry (such as quadrilaterals).

4. STATEMENT OF THE PROBLEM

The present study questions the effect of using different instructional methods (origami-based instruction and traditional instruction) to teach triangles on tenth grade students' spatial visualization, geometry achievement and geometric reasoning abilities.

4.1. Variables

The independent variables of this study are method of instruction and academic orientation of students. The dependent variables are students' spatial visualization, geometry achievement and geometric reasoning abilities. The following operational definitions should be clarified in order to understand the research questions of the study:

Origami: Origami refers to the art of "paper folding" coming from the Japanese words of "oru" (to fold) and "kami" (paper). In this study, origami is defined as the art of paper folding in which paper is used to create two- or three-dimensional figures, ranging from different models of animals, almost any physical objects including people, faces, flowers, and vehicles to abstract geometrical shapes. Within the literature, paper folding and origami are used interchangeably, and for the sake of consistency, origami is used in this study from now on.

Origami-Based instruction: In the present study, origami-based instruction refers to the set of geometry lessons in which origami activities are used to teach geometry concepts and relationships about triangles. The topics about triangles include elements of a triangle, classification of triangles, angle-side relationships in a triangle, construction of medians, altitudes and angle bisectors and collinearity of medians, altitudes, angle bisectors, and perpendicular bisectors.

Traditional Instruction: The traditional instruction in this study refers to a textbook-based instruction about triangles in the geometry lessons which follow the textbook of Ministry of National Education (MEB, 2010b) without using any origami activities. There are no origami activities in MEB's textbook for the selected topic. The instruction about

triangles include elements of a triangle, classification of triangles, angle-side relationships in a triangle, construction of medians, altitudes and angle bisectors and collinearity of medians, altitudes, angle bisectors, and perpendicular bisectors.

Academic Orientation: Academic Orientation refers to the specialized area of students. There are three academic orientations of the participant students in this study: Science-Mathematics Orientation (SM), Turkish-Mathematics Orientation (TM), and Turkish-Social Orientation (TS).

Spatial Visualization: Ekstrom, French and Harman (1976) defined spatial visualization as "the ability to manipulate or transform the image of spatial patterns into other arrangements" (p. 173). With the purpose of investigating the effect of origami-based instruction on high school students' spatial visualization, the definition of spatial visualization given by Ekstrom and his colleagues (1976) was used in the current study. Spatial visualization ability was measured through the three spatial tests: Card Rotation Test (CRT), Cube Comparison Test (CCT), and Paper Folding Test (PFT).

Geometry Achievement: In this study, geometry achievement score of a student referred to the score obtained from the Geometry Achievement Test on triangles which was prepared by the researcher.

Geometric Reasoning Ability: According to Duval (1998), the reasoning processes comprise of discursive processes for the extension of knowledge, for explanation, and for proof. As a component of geometric thinking, forms of reasoning contain explanation, verification, argument, and proof (Jones, 2000). In the current study, geometric reasoning ability refers to the student's ability to use discursive processes for the extension of knowledge, for explanation, and for proof in geometry about triangles. More specifically, the topics about triangles include elements of a triangle, classification of triangles, angle-side relationships, construction as well as collinearity of medians, altitudes, angle bisectors, and perpendicular bisectors. In Toulmin's (1958) model, the elements of arguments are data, claim, warrants, backing, qualifiers, and rebuttals. In this study, *data*, *claim*, and *reasons* guided the construction of the Geometric Reasoning Test. Items in the test were scored on degree of *argumentation*, *justifying*, and *proving*. The item responses

which contained *proof* were given the highest score range. Then, the item responses which were suitable to *justifying* were given as the middle highest score range. The lowest score range was given to the item responses, which fit into *argumentation*. The reasoning ability score in this study refers to the score obtained in the Reasoning Test on triangles, which was prepared by the researcher.

4.2. Research Questions

The following research questions were investigated in this study:

(i) Is there any significant difference in spatial visualization as measured by Card Rotation Test (CRT), Cube Comparison Test (CCT) and Paper Folding Test (PFT) between tenth-grade students who participate in origami-based instruction and students who participate in traditional instruction?

(ii) Is there any significant difference in geometry achievement concerning triangles, as measured by Geometry Achievement Test (GAT), between tenth-grade students who participate in origami-based instruction and students who participate in traditional instruction?

(iii) Is there any significant difference in geometric reasoning abilities concerning triangles, as measured by Geometric Reasoning Test (GRT), between tenth-grade students who participate in origami-based instruction and students who participate in traditional instruction?

4.3. Statement of the Research Hypotheses

This study hypothesized that there would be significant differences in terms of spatial visualization, geometry achievement, and geometric reasoning abilities concerning triangles between the groups of tenth-grade students who received origami-based instruction and those who did not receive origami-based instruction but only traditional instruction. More specifically, the hypotheses are given for the corresponding research questions as the following:

(i) There will be significant difference in spatial visualization, as measured by the Card Rotation Test (CRT), Cube Comparison Test (CCT) and Paper Folding Test (PFT), between tenth-grade students who receive origami-based instruction and tenth-grade students who receive traditional instruction.

(ii) There will be significant difference in geometry achievement concerning triangles, as measured by Geometry Achievement Test (GAT), between tenth-grade students who receive origami-based instruction and students who receive traditional instruction.

(iii) There will be significant difference in geometric reasoning abilities concerning triangles, as measured by the Geometric Reasoning Test (GRT), between tenth-grade students who receive origami-based instruction and tenth-grade students who receive traditional instruction.

5. METHOD

The present study compares the effectiveness of the two types of instruction (origami-based instruction and traditional instruction) in terms of the spatial visualization, geometry achievement, and geometric reasoning abilities. A quasi-experimental pretest/ posttest design was used in this study. Some important topics of triangles have been chosen as the subject of instruction because triangles are prerequisite for understanding later topics of two-dimensional Euclidean geometry. Also, origami models have been argued to be effective in teaching triangles. The tenth-grade curriculum puts a high emphasis on understanding implications and proving theorems. Geometric reasoning, which is one of the dependent variables in this study, also contains justifying and proving. So, the topics of the study were also aligned with the tenth-grade geometry curriculum in Turkey.

5.1. Design of the Study

A quasi-experimental pretest/ posttest design was used in this study. A control group received traditional instruction during a geometry unit in a tenth grade classroom whereas an experimental group received origami-based instruction. The implementation part of the study lasted four weeks. The experimental group took three-lesson-hour weekly instruction whereas the control group took two-lesson-hour weekly instruction. A lesson hour accounts for 45 minutes. The research design is described in Table 5.1.

Table 5.1. Research design of the study.

Pre-Treatment Measuring Instruments	**Treatment**	**After Treatment Measuring Instruments**
van Hiele Test (vHT)	Traditional instruction with the control group (2 lesson-hours per week) Origami-based instruction with the experimental group (3 lesson-hours per week)	
Geometry Achievement Pretest (PreGAT)		Geometry Achievement Posttest (PostGAT)
Geometric Reasoning Test (GRT)		Geometric Reasoning Test (GRT)
Card Rotation Test (CRT)		Card Rotation Test (CRT)
Cube Comparison Test (CCT)		Cube Comparison Test (CCT)
Paper Folding Test (PFT)		Paper Folding Test (PFT)

5.2. Participants

The student sample for this investigation was drawn from the tenth-grade population of a public high school, Lily (pseudo name) in Linden (pseudo name) town of Tekirdağ city, in Turkey. There were three different academic orientations for classrooms which were the Science-Mathematics Orientation (SM), Turkish-Mathematics Orientation (TM), and Turkish-Social Orientation (TS). There were three tenth-grade classes with SM, four classes with TM, and three classes with TS. In total, there were ten classes at the tenth grade level but eight of them were chosen as the sample. The researcher was the instructor of the geometry lessons in nine of the classes and there was another geometry teacher for one of the TS classes. The TS class with the different geometry teacher and the TM class with the researcher as the teacher of both mathematics and geometry lessons were excluded from the sample in order to control extraneous effects. Therefore, there were three SM classes, three TM classes and two TS classes used for the target population for this study.

Two classes from each orientation group were randomly selected. Then, one of the chosen groups in each orientation was randomly assigned to be included in the

experimental group and the other was included in the control group. Totally, the three classes differing in terms of orientation together formed the control group of the study (94 students). Similarly, three classes formed the experimental group (90 students). The sample had totally 184 students. There were 107 female and 77 male students in the sample. The demographics of the participants were demonstrated in the Table 5.2 (Pseudo names are used for the classes).

The students were generally similar on their socioeconomic backgrounds. Most of the parents had an education level lower than high school. Parents were mostly blue collar workers because Linden was a cosmopolitan, industrial area where many people from different regions in Turkey have been migrating there in order to find employment in the factories.

Table 5.2. Participants' demographics.

Group	Class Name	Orientation	Class Size
Control	CL1	SM	27
	CL2	TM	33
	CL3	TS	34
Experimental	CL4	SM	26
	CL5	TM	31
	CL6	TS	33

In order to assess pre-treatment group differences, the van Hiele Test (vHT) (see Appendix A) was administered to both groups prior to the treatment. The van Hiele Test was translated into Turkish by Duatepe (2004). The vHT was chosen because it has been generally accepted to measure individuals' geometry levels and all dependent variables in this study (geometry achievement, spatial visualization and geometric reasoning abilities) dealt with geometry. In order to compare whether students in control and experimental groups had differences in the van Hiele levels, the vHT, which consisted of 25 multiple choice items, was administered in 35 minutes.

The vHT is organized in blocks of five questions that were created using behaviors identified by the van Hiele theory and the questions are ordered in blocks of 5 questions each, such that questions 1-5 measure student understanding at Level 1, questions 6-10 measure student understanding at Level 2, questions 11-15 measure student understanding at Level 3, questions 16-20 measure student understanding at Level 4, and questions 21-25 measure student understanding at Level 5 (Knight, 2006).

Usiskin's (1982) criterion of assigning van Hiele levels was used. A student was assigned a weighted sum score according to the following scoring guide determined by Usiskin (1982):

(i) If the criterion is met on items 1-5 (Level 1) of the van Hiele test, 1 point is given.
(ii) If the criterion is met on items 6-10 (Level 2) of the van Hiele test, 2 points is given.
(iii) If the criterion is met on items 11-15 (Level 3) of the van Hiele test, 4 points is given.
(iv) If the criterion is met on items 16-20 (Level 4) of the van Hiele test, 8 points is given.
(v) If the criterion is met on items 21-25 (Level 5) of the van Hiele test, 16 points is given.

For each correct answer of an item on the vHT, 1 point is given. For wrong and blank answers of an item on vHT, 0 point is given. A student was considered to be at van Hiele level n if the student correctly responded a fixed percentage of questions at level n and at all lower levels. The criterion percentage 3 of 5 was used. The criterion is met if 3 of 5 items are correctly responded in each five-item block of the vHT. The weighted sum was obtained by adding the points from each five level.

Every student could be assigned to one of the van Hiele levels by using the modified van Hiele levels. Usiskin (1982) defines the modified van Hiele theory as the classical van Hiele theory without Level 5. This level was excluded because high school students were generally not expected to reach it. The modified van Hiele levels with criterion 3 of 5 (M3) was determined for both control and experimental groups. Thus, the following assignment framework for the modified van Hiele level by Usiskin (1982) was used:

Table 5.3. Modified van Hiele levels.

Weighted sum	Level
0 to 16	0
1 or 17	1
3 or 19	2
7 or 23	3
15 or 31	4

Students in both experimental and control groups were similar in terms of their van Hiele levels according to M3. Both groups had similar percentages at van Hiele levels as shown in Table 5.4.

Table 5.4. Van Hiele level M3 and group cross tabulation.

	Group			
	Control		Experimental	
Level M3	Frequency	Per cent	Frequency	Per cent
0	17	18.3	7	7.9
1	66	71.0	59	66.3
2	10	10.8	22	24.7
3	0	0.0	1	1.1
Total	93		89	

The weighted sum scores obtained by M3 assignment for experimental and control groups were compared by using independent samples t-test. According to the t-test results, control group (n = 93, M = 3.52, SD = 5.006) and experimental group (n = 89, M = 3.30, SD = 5.003) do not significantly differ in their van Hiele weighted sum scores, t (180) = .287, p = .775.

5.3. Instruments

5.3.1. Spatial Visualization Instruments

Connor and Serbin (1980) suggested the existence of two factors of visual-spatial tests: spatial visualization and closure. According to Connor and Serbin (1980), the Card Rotation Test (CRT), Cube Comparison Test (CCT) and Paper Folding Test (PFT) were grouped under the spatial visualization factor. Based on Connor and Serbin (1980), the Card Rotation Test (CRT), Cube Comparison Test (CCT) and Paper Folding Test (PFT) were chosen to measure spatial visualization abilities of participants in this study. The tests were originally developed by Ekstrom, French, Harman and Derman (1976) which were translated into Turkish by Delialioğlu (1996). Reliability information regarding spatial visualization tests was found within the study conducted by Delialioğlu. Delialioğlu calculated the reliability coefficients of CRT, CCT, and PFT as .80, .81, and .80, respectively. Ekstrom and his colleagues (1976) defined spatial visualization as "the ability to manipulate or transform the image of spatial patterns into other arrangements" (p.173). In all selected tests, mental manipulation of objects, which spatial visualization required, was involved. Ekstrom and his colleagues have noted that spatial visualization also necessitates mentally restructuring a figure into components for manipulation.

For all three spatial visualization tests, a pilot study was conducted in one of the tenth-grade classes that was not included in experimental or control group. Based on the results of the pilot study, 6 minutes (3 minutes for each section) was determined as appropriate duration for each spatial visualization test. Each spatial visualization test was given to students as pretest before treatment and as posttest after treatment.

5.3.1.1. The Card Rotation Test (CRT). The CRT aims to assess the ability to see similarities and differences between shapes. The CRT requires a respondent to imagine whether a geometric figure has been reflected or rotated. Each item has an original geometric figure on the left and to the right of this shape, there are eight versions of this shape that has been rotated only or reflected. Students are asked to determine whether each of these eight shapes is the same or different with the original shape. The shapes that are only rotated are regarded as the same and others are accepted as different.

The CRT composes of two three-minute sections (see Appendix B for the instrument). Both sections have the same item type and length. In each three-minute sections, there are ten items, each having eight response categories. There are a total of 160 responses participants could respond within the total six minutes for the CRT. After the students completed the CRT, the total number of correct responses was calculated and used in the data analysis to measure one of the components of spatial visualization. Each correct response was regarded as one point and each blank or wrong answer was regarded as zero point. A person could make a maximum score of 160 for the CRT.

5.3.1.2. The Cube Comparison Test (CCT). Each item of the CCT shows two drawings of a cube. Depending on the condition that no cube can have two identical faces, the respondent is expected to determine which items show drawings that can be of the same cube and which show drawings that cannot be of the same cube. There are two three-minute parts of the CCT, each having 21 items (see Appendix C). In total, there are 42 questions in the CCT. Each correct response was graded as "1" point, and "0" point, otherwise. The maximum score of the CCT could be 42. After the students completed the test, the total number of correct responses was calculated and used in the data analysis.

5.3.1.3. The Paper Folding Test (PFT). The PFT necessitates mentally folding and unfolding a square sheet of paper. Each item of the PFT contains a square sheet of paper which is folded and then a hole is punched. The respondent chooses one of the final alternative drawings that may denote how the sheet with holes would seem when unfolded. There are two parts of the PFT, each having ten multiple-choice questions (see Appendix D for the PFT). In total, there are 20 items.

In the assessment of the PFT, one point is given to each correct answer and zero point is given for each wrong or blank answer. Since there are 20 questions in the PFT, the maximum possible score is 20. The PFT has questions with different difficulty levels. Some items necessitate imagining a few folds and others require considering multiple steps of folding and rotations.

5.3.2. Geometry Achievement Test (GAT)

The GAT was used to collect data on geometry achievement about triangles of tenth-grade students (see Appendix E). For content validity, the questions were prepared to cover the related objectives about triangles, which were determined by Ministry of National Education (MEB) for tenth-grade curriculum for the school period 2010-2011. The objectives are about angle-side relationships for a triangle, triangle inequality, triangle types, angle bisector, inscribed circle of a triangle, incenter, centroid, median, circumscribed circle of a triangle, perpendicular bisectors and altitudes in triangles. GAT was developed by the researcher and piloted. The pilot study was conducted in one of the tenth-grade classes that did not involve in the study. Based on the pilot study, wordings of some questions were changed in order to clarify the questions. As a result of the pilot study, the duration for the GAT was determined as a class hour (45 minutes). Parallel forms of GAT were used as pretest and posttest. Both PreGAT (Pretest Geometry Achievement) and PostGAT (Posttest Geometry Achievement) consist of 11 equivalent open-ended items. PreGAT and PostGAT have the same item type but differ in the ordering of questions, orientation of shapes and numerical values.

Based on a rubric prepared by the researcher and an expert in mathematics education, each item was scored within the range from zero point for blank or wrong answers to four points for complete answers. While preparing the rubric, steps to solve each question were initially determined and then points were assigned according to the extent of completeness. Points 1, 2 and 3 were given if responses were incomplete but included some correct parts. The possible maximum score for the test was 44.

For inter-scorer reliability, another high school mathematics teacher and an expert in mathematics education scored GATs using the designed rubric for GATs. As a result, a high consistency was observed with the scoring of the researcher. Furthermore, the Cronbach's alpha for GAT was .86, which is acceptable.

5.3.3. Geometric Reasoning Test (GRT)

The geometric reasoning ability score in this study refers to the score obtained in the Geometric Reasoning Test (GRT) which was prepared and piloted by the researcher. The pilot study for GRT was conducted with one of the tenth-grade classes that did not involve in the study. According to the pilot study, 45 minutes (a class hour) were given for GRT. Based on the results of the study, wordings of some questions and some shapes were modified to form the final version of GRT (see Appendix F). The GRT is supposed to reveal students' geometric reasoning levels about some key topics in triangles. GRT was prepared based on the literature concerning construction and proof questions about triangles (e.g. Choi-Koh, 1999; Jacobs, 2003). The same test was used as both pretest and posttest to measure geometric reasoning levels of participants. There are four main questions and 13 open-ended items for them. Each item was scored from zero point for blank and wrong answers to two points for complete answers. The maximum possible score for GRT is 26.

For content validity, the questions were aligned with the objectives determined by the MEB for the tenth-grade geometry curriculum. The GRT includes items about isosceles triangle, congruence, angle-side relationships, inscribed and circumscribed circles of triangles, incenter and angle bisectors.

Items in GRT were scored on the extent of Smith (2010)'s categories of reasoning (*argumentation*, *justifying* and *proving*). The lowest score range was assigned to the item response which fit to argumentation. The highest score range was assigned to the item response that fit to proving. Proving includes complete geometrical explanation of a geometrical statement. Justifying includes some part of geometrical explanation. Argumentation includes some explanation but this explanation may not be correct or geometrical. In scoring the responses for GRT by using the rubric developed by the researcher and an expert in mathematics education, responses were checked both whether they were correct and whether they had enough explanation.

Two student responses to the third question of GRT could exemplify the way the rubric was used to score the responses. The student response in Figure 5.1 was an example

of complete answer to each of the items 3a and 3b in the GRT. In item 3a of GRT, students were expected to draw the incircle of a given triangle ABC. Zero point would be given if the response was blank or wrong. Two points were given if the incircle was correctly drawn. The circle should have been inscribed in the given triangle and tangent to each side of the given triangle.

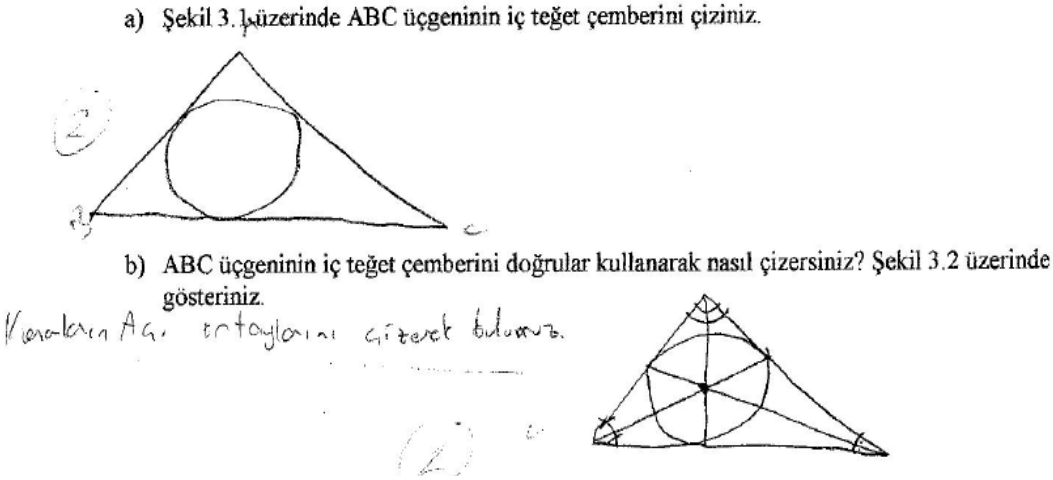


Figure 5.1. Student response from postGRT (complete answer).

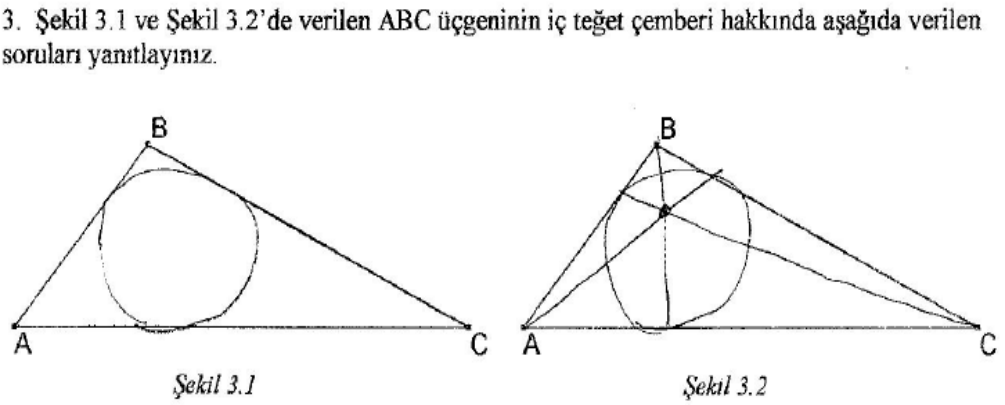


Figure 5.2. Student response from postGRT (incomplete answer).

The student response to the item3a in Figure 5.2 was also complete and was given two points based on the rubric. However, the student responses to the item 3b in Figure 5.1 and Figure 5.2 differ in completeness. In item 3b, students were asked to draw the incircle of the same given triangle ABC but this time with using some lines. Zero point would be given if the response was blank or wrong. One point was given if the student drew the incircle correctly but without using any lines or if the student drew the incircle with some

lines without any justification. Two points were given to the response to the item 3b if students drew the incircle with using angle bisectors. In Figure 5.1, the student response to the item 3b was given two points because the student drew the incircle of the given triangle with using lines and explicitly showed and written that the lines used to construct the incircle were the angle bisectors. Showing or writing the lines as angle bisectors becomes the justification of forming the incircle. Thus, the student response in Figure 5.1 includes *proving* based on Smith's (2010) framework. However, the student response in Figure 5.2 was given one point in scoring since the student just drew the incircle using lines without making any explanations. The student response in Figure 5.2 did not include any argumentation or justifying according to Smith's (2010) framework.

For inter-scorer reliability, one high school mathematics teacher and an expert in mathematics education also scored the responses for GRT. A high consistency with the researcher's scorings was observed. The Cronbach's alpha for GRT was calculated as .701. According to George and Mallery (2003), this value is acceptable for reliability.

5.4. Procedure

The study was conducted in eight weeks including pilot study, pretest and posttest periods. Before the treatment, all participants received pretests. Initially, spatial visualization tests (CRT, CCT and PFT) were administered. Then, GRT and GAT were administered, respectively. Participants were informed about directions and purpose of the tests. The researcher explained that information gathered would remain confidential. The time schedule for the study is presented in Table 5.5.

Table 5.5. The time schedule for the study.

Time	Administration
14 - 18 February 2011	Pilot studies for the tests
21 - 25 February 2011	Van Hiele Test (vHT)
28 February - 4 March 2011	Administration of Pretests
7 - 11 March 2011	Instruction on elements of a triangle, classification of triangles
14 - 18 March 2011	Instruction on angle-side relationships in a triangle
21 - 25 March 2011	Instruction on angle bisectors in a triangle
28 March - 1 April 2011	Instruction on median, altitude and angle bisector relationships; construction of medians and altitudes as well as intersections of these elements; perpendicular bisectors
4 - 8 April 2011	Administration of Posttests

After the pretests were completed, the treatment began. The researcher instructed both the experimental and control groups. Both groups used the tenth-grade geometry textbook of MEB (2010b). The experimental group additionally used origami during the treatment. In order to be consistent with the Ministry of National Education (MEB) geometry curriculum, the objectives of the tenth-grade geometry curriculum of MEB were taken into consideration in designing the instruction in both groups. The treatment lasted four weeks. The control group received traditional instruction for two class hours in each week on their regular scheduled meeting. On the other hand, the experimental group received origami-based instruction for three class hours in each week. In total, the experimental group had instruction of twelve class hours while the control group had instruction of eight class hours during four weeks. After the completion of the lessons, posttest data were collected by administering GAT and re-administering spatial visualization tests (CRT, CCT and PFT) and GRT to all participants.

5.5. Treatment

Lesson plans were prepared for each lesson in experimental and control groups (see Appendix G for the sample lesson plan for the origami-based instruction and Appendix H for the sample lesson plan for the traditional instruction). As part of the lesson plan for origami-based instruction, the researcher prepared the following sections for each lesson: the unit (triangles), the grade level (tenth-grade), topic, related MEB objectives, the materials needed, geometrical objectives, instructional sequence and key questions to be asked at each step of the folding process with potential responses expected by students. The related MEB (2010a) curriculum objectives are listed as the following:

- The student should be able to explain basic and auxiliary elements of a triangle,
- The student should be able to prove the angle-side relationships in a triangle and make related applications,
- The student should be able to determine that angle bisectors are concurrent and make related applications,
- The student should be able to determine that medians in a triangle are concurrent and make related applications,
- The student should be able to calculate lengths of altitudes in a triangle.

The experimental group had origami-based instruction in which origami was integrated into the geometry lessons. For the experimental group, a brief introduction session about the description and the historical development of origami was conducted in the first lesson of the instruction. The origami models (see Appendix I for the sample origami diagrams) in the study were chosen based on alignment with the key topics about triangles that were covered during the treatment, alignment with the objectives of the tenth-grade geometry curriculum, difficulty level of the origami model and time restriction due to the lesson hour. The origami models should have been easy enough to fold in at most 30 minutes because the class hour was 45 minutes. Furthermore, the main aim of integrating folding origami models was to make students explore the underlying geometric relationships about the topics covered. A set of written and illustrated instructions was

developed after determining which origami models to be used. Table 5.6 presents origami activities used in each week of the treatment.

Table 5.6. Origami activities used in origami-based instruction.

Week	**Topic**	**Origami Activity**
1	Elements of a triangle, classification of triangles	Dart; equilateral triangle
2	Angle-side relationships in a triangle	Swan
3	Angle bisectors in a triangle	Whale
4	Median, altitude and angle bisector relationships; construction of medians and altitudes as well as intersections of these elements; perpendicular bisectors	Folding a square paper

Students made constructions of geometric concepts through origami in the experimental group and through pencil, ruler and/or protractor in the control group. After students inductively had explored geometric relationships about the topic of the week, they were guided to deductively prove related theorems of the triangles in that week. In origami-based instruction, theorems were proved through origami. In the control group, on the other hand, proofs were shown without using origami.

For origami-based instruction, the first lesson of each week was devoted to folding an origami model in at most 30 minutes. In the first lesson of each weekly origami-based instruction, the researcher showed the origami model step by step. The researcher folded the model simultaneously with the students by using two-colored papers (with A4 size). The researcher encouraged students to work in pairs and helped them if they had difficulties in folding by showing the folding steps again. Some students more quickly finished folding the origami model so that the researcher directed them to cooperate with

their friends who had difficulties in folding. After the model was completed, the researcher wanted students to unfold the model and draw certain key steps of folding and transform these steps into geometrical figures. The first step of the first origami model (dart) was drawn by the researcher to exemplify the drawing process. In the second lesson of each weekly origami-based instruction, the researcher distributed the diagrams of the origami model that was folded in the previous lesson and wanted students to compare their drawings and the diagrams in at most five minutes. The remaining two lessons focused on the geometrical exploration and discussion of each origami model. Students were encouraged to reveal geometric relations underlying the origami model at hand by asking questions concerning the related folding steps. In origami-based instruction, students were supposed to explain their reasoning processes behind their answers. After the geometric discussion and proofs had been completed in the last lesson of each week, students solved exercises about related topics in the experimental group as similar to the control group.

6. RESULTS

Results are organized to address each of the primary research questions in order to compare groups regarding their spatial visualization skills, geometry achievement levels, and geometric reasoning levels. Table 6.1 outlines the data collection and data analysis process for each research question.

Table 6.1. Data collection and analysis methods by research questions.

Research Questions	**Data Collection**	**Analysis Techniques**
1. Is there any significant difference in spatial visualization as measured by CRT, CCT and PFT between tenth-grade students who participate in origami-based instruction and students who participate in traditional instruction?	Pre and post measures of spatial visualization from the Card Rotation (CR), Cube Comparison (CC) and Paper Folding (PF) tests	Repeated measures ANOVA for each spatial visualization test scores
2. Is there any significant difference in geometry achievement concerning triangles, as measured by GAT, between tenth-grade students who participate in origami-based instruction and students who participate in traditional instruction?	Pre and post measures of geometry achievement from the Geometry Achievement Test (GAT)	Repeated measures ANOVA for GAT scores
3. Is there any significant difference in geometric reasoning abilities concerning triangles, as measured by GRT, between tenth-grade students who participate in origami-based instruction and students who participate in traditional instruction?	Pre and post measures of geometric reasoning from the Geometric Reasoning Test (GRT)	Repeated measures ANOVA for GRT scores

All collected data were analyzed by using the Statistical Package for the Social Sciences (SPSS). The significance level was kept on .05. Repeated Measures Analysis of

Variance (ANOVA) was used to determine whether there was a significant difference between mean pretest scores, between mean posttest scores, between mean pretest and mean posttest scores of experimental group and control group. Stevens (2002) has called the design as *two-way ANOVA with repeated measures on one factor* when there is one between-subjects factor and one within-subjects factor in the design. Since there was a grouping factor and participants were measured at different times in this study, *two-way ANOVA with repeated measures on one factor* design was used to analyze the collected data. In each repeated measures ANOVA performed, the between-subjects factor was specified as Group and the within-subjects factor was specified as Time.

6.1. Results for Spatial Visualization Tests Analyses

Three tests were used to determine students' spatial visualization abilities: the Card Rotation Test (CRT), the Cube Comparison Test (CCT) and the Paper Folding Test (PFT). The following analyses present the results of each test.

6.1.1. Results for the Card Rotation Test (CRT) Analyses

The Card Rotation Test (CRT) assessed students' ability to notice whether a figure had been rotated only or not. Scores were calculated by determining the number of correct answers. Descriptive statistics for scores of the Pretest Card Rotation (PreCRT) and Posttest Card Rotation (PostCRT) were calculated for experimental and control groups. The results are shown in Table 6.2.

Table 6.2. Descriptive statistics for the preCRT and postCRT scores.

Group	**N**	**PreCRT Mean**	**SD**	**PostCRT Mean**	**SD**
Experimental	85	68.61	30.14	84.94	34.48
Control	89	79.90	34.24	76.97	23.26

A two-way ANOVA with repeated measures on one factor (time) was conducted to assess the effectiveness of origami-based instruction on CRT scores. The Table 6.3 shows

the results of Repeated Measures ANOVA for CRT. ANOVA results from the PreCRT and PostCRT revealed a significant interaction effect between group and time, $F(1,172) = 19.35$, $p < .001$ with an effect size (partial eta squared = .10). The main effect of Time was also statistically significant $F(1,172) = 9.36$, $p < .005$ with an effect size (partial eta squared = .05). However, the main effect of Group on CRT was not statistically significant, $F(1,172) = .16$, $p = .689$.

Table 6.3. Results of repeated measures ANOVA for Card Rotation Test (CRT) scores.

Source	**Sum of Squares**	**df**	**F**	**p**	**Partial Eta Squared**	**Observed Power**
Time	3901.52	1	9.36	.003**	.052	.86
Group	238.49	1	.16	.689	.001	.06
Time×Group	8065.52	1	19.35	.000***	.101	.99
Error (Time)	71680.19	172				

*p <.05, **p < .005, ***p < .001

For further investigation, the significant interaction effect on CRT, pairwise comparisons were conducted by group and time and these pairwise comparisons were shown in Table 6.4 (ctr and exp refer to the control group and experimental group, respectively). Pairwise comparisons indicated that PostCRT mean scores were significantly higher than PreCRT mean scores for experimental group, Wilks' lambda = .863, $F(1,172) = 27.19$, $p < .001$ although there was no significant mean difference between PostCRT and PreCRT scores for control group, Wilks' lambda= .995, $F(1,172) = .92$, $p = .339$. Pretest scores for Card Rotation (CR) were also compared between groups. There was a significant mean difference between PreCRT scores in groups in favor of control group $F(1,172) = 5.31$, $p < .05$. Besides, posttest scores for Card Rotation (CR) were also

compared between groups. However, there was no significant difference between mean posttest scores for CR, $F(1,172) = 3.22$, $p = .074$ between the two compared groups.

Table 6.4. Pairwise comparisons of Card Rotation Test (CRT) scores.

				95% Confidence Interval for Difference	
Comparisons	**Mean Difference**	**Std. Error**	***p***	**Lower Bound**	**Upper Bound**
PostCRTctr-PreCRTctr	-2.93	3.06	.339	-8.97	3.11
PostCRTexp-PreCRTexp	16.33	3.13	.000***	10.15	22.51
PreCRTexp-PreCRTctr	-11.29	4.90	.022*	-20.96	-1.62
PostCRTexp-PostCRTctr	7.97	4.44	.074	-.79	16.74

*p <.05, **p < .005, ***p < .001

6.1.2. Results for the Cube Comparison Test (CCT) Analyses

The Cube Comparison Test (CCT) assessed students' ability to identify whether two given cubes could be the same. The test had a total of 42 items. The scores were calculated by determining the number answered correctly out of the 42 items. Descriptive statistics for scores of the Pretest Cube Comparison (PreCCT) and Posttest Cube Comparison (PostCCT) were calculated for experimental and control groups. The results were found in Table 6.5.

Table 6.5. Descriptive statistics for the preCCT and postCCT scores.

Group	N	PreCCT Mean	SD	PostCCT Mean	SD
Experimental	85	18.52	5.87	20.72	5.80
Control	89	21.11	5.06	21.91	4.61

A two-way ANOVA with repeated measures on one factor (time) was conducted to assess the effectiveness of origami-based instruction on CCT scores. The results of ANOVA for Cube Comparison Test (CCT) were summarized in Table 6.6. ANOVA results from the PreCCT and PostCCT revealed no significant interaction effect between group and time, $F(1, 172) = 2.83$, $p = .094$. However, the main effect of time on CCT scores was statistically significant, $F(1,172) = 12.94$, $p < .001$ with an effect size (partial eta squared = .070). Moreover, the main effect of group on CCT scores was statistically significant, $F(1, 172) = 7.40$, $p < .05$ with an effect size (partial eta squared = .041).

Table 6.6. Results of repeated measures ANOVA for Cube Comparison Test (CCT) scores.

Source	Sum of Squares	*df*	*F*	*p*	Partial Eta Squared	Observed Power
Time	195.35	1	12.94	.000***	.070	.95
Group	311.79	1	7.40	.007*	.041	.77
Time×Group	42.74	1	2.83	.094	.016	.39
Error (Time)	2596.98	172				

*p <.05, **p < .005, ***p < .001

Furthermore, each of PreCCT mean scores and PostCCT mean scores were compared between experimental and control groups. Pairwise comparisons for CCT were given in the Table 6.7 (ctr and exp refer to the control group and experimental group, respectively). Based on one-way ANOVA results, there was a significant mean difference

between PreCCT scores in favor of the control group, $F(1,172) = 9.78$, $p < .05$. However, there was no significant mean difference between PostCCT scores for experimental and control groups, $F(1,172) = 2.26$, $p = .134$. Pretest and posttest scores for Cube Comparison Test were also compared within each group by using multivariate tests. For experimental group, PostCCT mean scores were significantly higher than PreCCT mean scores, Wilks' lambda = .927, $F(1,172) = 13.62$, $p < .001$ with an effect size (partial eta squared = .073). However, for control group, there was no significant mean difference between PostCCT and PreCCT scores, Wilks' lambda = .989, $F(1,172) = 1.87$, $p = .173$.

Table 6.7. Pairwise comparisons of Cube Comparison Test (CCT) scores.

				95% Confidence Interval for Difference	
Comparisons	**Mean Difference**	**Std. Error**	***p***	**Lower Bound**	**Upper Bound**
PostCCTctr-PreCCTctr	.80	.58	.173	-.35	1.95
PostCCTexp-PreCCTexp	2.20	.60	.000***	1.02	3.38
PreCCTexp-PreCCTctr	-2.60	.83	.002**	- 4.23	-.96
PostCCTexp-PostCCTctr	-1.19	.79	.134	- 2.76	.37

*p <.05, **p < .005, ***p < .001

6.1.3. Results for the Paper Folding Test (PFT) Analyses

The Paper Folding Test (PFT) assessed students' ability to imagine folding and unfolding. The test had a total of 20 items. The scores were calculated by determining the number answered correctly out of the 20 items. Descriptive statistics for scores of the

Pretest Paper Folding (PrePFT) and Posttest Paper Folding (PostPFT) were calculated for experimental and control groups. The results were found in Table 6.8.

Table 6.8. Descriptive statistics for the prePFT and postPFT scores.

Group	**N**	**PrePFT Mean**	**SD**	**PostPFT Mean**	**SD**
Experimental	85	7.73	3.17	9.89	3.43
Control	89	8.09	3.62	9.43	3.44

A two-way ANOVA with repeated measures on one factor (time) was conducted to assess the effectiveness of origami-based instruction on PFT scores. The results of ANOVA for PFT were given in the Table 6.9. ANOVA results from the PrePFT and PostPFT scores revealed no significant interaction effect between group and time, $F(1,172) = 2.48$, $p = .117$. The main effect of group on PFT scores was either not significant, $F(1,172) = .01$, $p = .905$. However, the main effect of time on PFT scores was significant, $F(1,172) = 44.35$, $p < .001$ with an effect size (partial eta squared = .205).

Table 6.9. Results of repeated measures ANOVA for Paper Folding Test (PFT) scores.

Source	**Sum of Squares**	***df***	***F***	***p***	**Partial Eta Squared**	**Observed Power**
Time	266.57	1	44.35	.000***	.205	1
Group	.247	1	.01	.905	.000	.05
Time×Group	14.89	1	2.48	.117	.014	.35
Error (Time)	1033.79	172				

$^{*}p < .05$, $^{**}p < .005$, $^{***}p < .001$

One-way ANOVAs were also performed on group and time for PFT scores. Pretest PFT scores as well as posttest PFT scores were compared within each group. One-way

ANOVA results showed that there was no significant mean difference between PrePFT scores in experimental and control groups, $F(1,172) = .49$, $p = .487$. The mean difference between PostPFT scores in experimental and control groups was either not significant, $F(1,172) = .80$, $p = .371$.

Table 6.10. Pairwise comparisons of Paper Folding Test (PFT) scores.

				95% Confidence Interval for Difference	
Comparisons	**Mean Difference**	**Std. Error**	***p***	**Lower Bound**	**Upper Bound**
PostPFTctr-PrePFTctr	1.34	.37	.000***	.61	2.06
PostPFTexp-PrePFTexp	2.16	.38	.000***	1.42	2.91
PrePFTexp-PrePFTctr	-.36	.52	.49	-1.38	.66
PostPFTexp-PostPFTctr	.47	.52	.37	-.56	1.50

*p <.05, **p < .005, ***p < .001

Pretest and posttest scores for Paper Folding Test were also compared within each group by using multivariate tests from the repeated measures ANOVA. The pairwise comparisons for PFT were given in the Table 6.10 (ctr and exp refer to the control group and experimental group, respectively). ANOVA results indicated that for experimental group, PostPFT mean scores were significantly higher than PrePFT mean scores, Wilks'lambda = .838, $F(1,172) = 33.13$, $p < .001$ with an effect size (partial eta squared = .162). For control group, PostPFT mean scores were also significantly higher than PrePFT mean scores, Wilks'lambda = .929, F (1,172) = 13.24, $p < .001$ with an effect size (partial eta squared = .071). However, for experimental group, the mean difference between PostPFT and PrePFT scores was higher than the mean difference for control group.

As a summary, the results concerning spatial visualization tests indicated that there was a statistically significant change in students' spatial visualization scores across two time periods (from pretest to posttest) for students who received origami-based instruction. Furthermore, students who received origami-based instruction showed more change in spatial visualization average scores than those who received traditional instruction. Thus, the hypothesis concerning spatial visualization was supported.

6.2. Results for Geometry Achievement Test (GAT) Analyses

The Geometry Achievement Test (GAT) aimed to evaluate students' achievement level about certain key topics in triangles. The test had a total of 11 items. Each item was scored between zero and four points according to the completeness of the responses by using a rubric. Descriptive statistics for the Pretest Geometry Achievement (PreGAT) and Posttest Geometry Achievement (PostGAT) scores were calculated for experimental and control groups. The results were found in Table 6.11.

Table 6.11. Descriptive statistics for the preGAT and postGAT scores.

Group	**N**	**PreGAT Mean**	**SD**	**PostGAT Mean**	**SD**
Experimental	79	2.13	3.56	12.80	11.05
Control	88	1.52	2.54	5.95	6.79

A two-way ANOVA with repeated measures on one factor (Time) was conducted to assess the effectiveness of origami-based instruction on GAT scores. ANOVA results from the PreGAT and PostGAT scores revealed a significant interaction effect between Group and Time, $F(1,165) = 25.02$, $p < .001$ with an effect size (partial eta squared = .132). The main effect of Group on GAT scores was also statistically significant, $F(1,165) = 19.53$, $p < .001$ with an effect size (partial eta squared = .106). Moreover, the main effect of Time on GAT scores was significant, $F(1,165) = 146.60$, $p < .001$ with an effect size (partial eta squared = .470). The results of repeated measures ANOVA for Geometry Achievement Test (GAT) scores were presented in the Table 6.12.

Table 6.12. Results of repeated measures ANOVA for Geometry Achievement Test (GAT) scores.

Source	**Sum of Squares**	***df***	***F***	***p***	**Partial Eta Squared**	**Observed Power**
Time	4747.58	1	146.60	.000***	.47	1
Group	1154.25	1	19.53	.000***	.11	.99
Time×Group	810.22	1	25.02	.000***	.13	1
Error (Time)	5343.52	165				

*p <.05, **p < .005, ***p < .001

Pretest GAT scores as well as posttest GAT scores were also compared within each group. Pairwise comparisons for GAT were given in the Table 6.13 (ctr and exp refer to the control group and experimental group, respectively). One-way ANOVA results showed that there was no significant mean difference between PreGAT scores in experimental and control groups, $F(1,165) = 1.61$, $p = .206$. However, one-way ANOVA results indicated that there was a significant mean difference between PostGAT scores in groups in favor of the experimental group, $F(1,165) = 23.75$, $p < .001$.

Pretest and posttest scores for Geometry Achievement Test were also compared within each group by using multivariate tests from the repeated measures ANOVA. The results indicated that for experimental group, PostGAT mean scores were significantly higher than PreGAT mean scores, Wilks'lambda = .543, $F(1,165) = .01$, $p < .001$. For the control group, PostGAT mean scores were also significantly higher than PreGAT mean scores, Wilks'lambda = .861, $F(1,165) = 26.68$, $p < .001$. However, the mean difference between PostGAT and PreGAT scores was higher in experimental group than the mean difference in control group.

Table 6.13. Pairwise comparisons of Geometry Achievement Test (GAT) scores.

				95% Confidence Interval for Difference	
Comparisons	**Mean Difference**	**Std. Error**	***p***	**Lower Bound**	**Upper Bound**
PostGATctr-PreGATctr	4.43	.86	.000***	2.74	6.13
PostGATexp-PreGATexp	10.67	.90	.000***	8.88	12.46
PreGATexp-PreGATctr	.60	.47	.206	-.33	1.54
PostGATexp-PostGATctr	6.84	1.40	.000***	4.07	9.61

*p <.05, **p < .005, ***p < .001

As a summary, the results concerning geometry achievement indicated that there was a statistically significant change in students' GAT scores across two time periods (from pretest to posttest) for students in both groups. Furthermore, when groups were compared based on mean difference scores between posttest and pretest, students who received origami-based instruction showed more change in geometry achievement average scores than those who received traditional instruction. Thus, the hypothesis concerning geometry achievement was supported.

6.3. Results for Geometric Reasoning Test (GRT) Analyses

The Geometric Reasoning Test (GRT) aimed to evaluate students' geometric reasoning ability concerning certain key topics in triangles. The test had a total of 13 items. Each item was scored between zero point and two points according to the degree of proving. Descriptive statistics for the Pretest Geometric Reasoning (PreGRT) and Posttest

Geometric Reasoning (PostGRT) scores were calculated for experimental and control groups. The results were found in Table 6.14.

Table 6.14. Descriptive statistics for the preGRT and postGRT scores.

Group	**N**	**PreGRT Mean**	**SD**	**PostGRT Mean**	**SD**
Experimental	77	3.14	2.38	6.42	3.78
Control	90	3.79	1.97	4.79	2.16

A two-way ANOVA with repeated measures on one factor (time) was conducted to assess the effectiveness of origami-based instruction on GRT scores. ANOVA results for GRT were shown in Table 6.15. ANOVA results from the PreGRT and PostGRT scores revealed a significant interaction effect between group and time, $F(1,165) = 29.10$, $p < .001$ with an effect size (partial eta squared = .150). The main effect of time on GRT scores was also statistically significant, $F(1,165) = 102.86$, $p < .001$ with an effect size (partial eta squared = .384). However, the main effect of group on GRT scores was not significant, $F(1,165) = 1.97$, $p = .162$.

Table 6.15. Results of repeated measures ANOVA for Geometric Reasoning Test (GRT) scores.

Source	**Sum of Squares**	***df***	***F***	***p***	**Partial Eta Squared**	**Observed Power**
Time	378.79	1	102.86	.000***	.384	1
Group	19.95	1	1.97	.162	.012	.29
Time×Group	107.17	1	29.10	.000***	.150	1
Error (Time)	607.64	165				

*p <.05, **p < .005, ***p < .001

For further analysis on GRT scores, one-way ANOVAs performed on group and time. Pretest GRT scores as well as posttest GRT scores were compared within each group. One-way ANOVA results showed that there was no significant mean difference between PreGRT scores in experimental and control groups, $F(1,165) = 3.68$, $p = .057$. However, one-way ANOVA results indicated that there was a significant mean difference between PostGRT scores in groups in favor of the experimental group, $F(1,165) = 12.06$, $p < .005$.

Pretest and posttest scores for Geometric Reasoning Test were also compared within each group by using multivariate tests in the repeated measures ANOVA. Pairwise comparisons for GRT were presented in Table 6.16 (ctr and exp refer to the control group and experimental group, respectively). The results indicated that for experimental group, PostGRT mean scores were significantly higher than PreGRT mean scores, Wilks's lambda = .596, $F(1,165) = .01$, $p < .001$. For the control group, PostGRT mean scores were also significantly higher than PreGRT mean scores, Wilks'lambda= .931, $F(1,165) = 12.22$, $p < .005$. However, the mean difference between PostGRT and PreGRT scores was higher in experimental group than the mean difference in control group.

Table 6.16. Pairwise comparisons of Geometric Reasoning Test (GRT) scores.

				95% Confidence Interval for Difference	
Comparisons	**Mean Difference**	**Std. Error**	***p***	**Lower Bound**	**Upper Bound**
PostGRTctr-PreGRTctr	1	.29	.001**	.43	1.56
PostGRTexp-PreGRTexp	3.27	.31	.000***	2.66	3.88
PreGRTexp-PreGRTctr	-.65	.34	.057	-1.31	.02
PostGRTexp-PostGRTctr	1.63	.47	.001**	.70	2.55

*p <.05, **p < .005, ***p < .001

To sum up, the results concerning geometric reasoning showed that there was a statistically significant change in students' GRT scores across two time periods (from pretest to posttest) for students in both groups. Furthermore, when groups were compared based on mean difference scores between posttest and pretest, students who received origami-based instruction showed more change in geometric reasoning average scores than those who received traditional instruction. This indicated that the hypothesis concerning geometric reasoning was supported.

7. DISCUSSION AND CONCLUSION

The current study aimed to investigate the effect of the origami-based instruction concerning triangles on tenth-grade students' spatial visualization, geometry achievement and geometric reasoning. The repeated measures ANOVAs were used to test the hypotheses concerning students' spatial visualization, geometry achievement and geometric reasoning. The results of each repeated measures ANOVA were discussed separately.

7.1. Spatial Visualization

The Card Rotation Test (CRT), the Cube Comparison Test (CCT) and the Paper Folding Test (PFT) were used in the analysis concerning spatial visualization. The results of each repeated measures ANOVA on each spatial visualization test were discussed to further examine the effect of instruction on students' spatial visualization.

The results concerning the Card Rotation Test (CRT) indicated that there was no statistically significant change in students' CRT scores across two time periods (from pretest to posttest) for the group who received traditional instruction. On the other hand, there was a statistically significant change in students' CRT scores across two time periods (from pretest to posttest) for the group who received origami-based instruction. There was also a similar trend concerning the results of the repeated measures ANOVA on the Cube Comparison Test (CCT) scores. There was no statistically significant change in students' CCT scores across two time periods (from pretest to posttest) for the group who received traditional instruction. On the other side, there was a statistically significant change in students' CCT scores across two time periods (from pretest to posttest) for the group who received origami-based instruction. Furthermore, the results concerning the Paper Folding Test (PFT) revealed that there was a significant change in students' PFT scores across two time periods (from pretest to posttest) for the group who received origami-based instruction and the group who received traditional instruction. Besides, the results concerning PFT scores demonstrated that the average PFT scores of students who received origami-based instruction changed more when compared to the average test scores of students who received traditional instruction.

Results related to the spatial tests indicated that origami-based instruction might have had a positive effect on participants' spatial visualization abilities. There was a statistically significant change in students' spatial visualization scores across two time periods (from pretest to posttest) for experimental group. In all three spatial visualization tests, on the average, students who received origami-based instruction scored higher in posttest than they did in pretest. For CRT and CCT, there was no significant mean difference between posttest and pretest scores for students who received traditional instruction. For PFT, students who received traditional instruction also scored significantly higher in posttest than they did in pretest. However, students who received origami-based instruction showed more change in PFT average scores than those who received traditional instruction. Thus, origami-based instruction was better in improving students' spatial visualization ability than traditional instruction.

The results indicated that origami-based instruction had a positive effect on tenth-grade participants' spatial visualization abilities. Many studies found that spatial ability, in particular spatial visualization, could be improved by instruction on spatial skills (Baki *et al*., 2011; Bayrak, 2008; Ben-Chaim, Lappan and Houang, 1988; Brinkmann, 1966; Connor and Serbin, 1980; Çakmak, 2009; Idris, 2005; Karaman, 2000; Olkun, 2003). Besides, origami-based instruction also addresses spatial visualization because origami forces a person to transform the images of paper into various arrangements through folding a model. Therefore, it is aligned with the literature to investigate the effect of origami-based instruction on students' spatial visualization abilities. One of the possible explanations for the improvement in spatial visualization abilities of students in this study could be related to the nature of the origami-based instruction. Origami is spatial in nature. While students were making origami, they modified paper into different arrangements by creating two- and three-dimensional objects. The experiential and spatial nature of origami was also emphasized in research studies (e.g. Bayrak, 2008; Boakes, 2006; Çakmak, 2009). In many articles and research studies related with origami, origami was recommended as a tool in teaching geometry, especially related with spatial visualization (Bayrak, 2008; Coad, 2006; Çakmak, 2009; Kavici, 2005). Thus, manipulative and visual nature of origami could have contributed to enhance spatial visualization abilities of students when origami was used in conjunction with geometry instruction.

7.2. Geometry Achievement

The second aspect of this study was to determine whether origami-based instruction would affect geometry achievement of the participants. Geometry achievement was measured by researcher-designed test called Geometry Achievement Test (GAT). Parallel forms of GAT were used as pretest and posttest. Results indicated that there was a statistically significant change in students' geometry achievement across two time periods (from pretest to posttest) for both groups. Furthermore, the mean difference between posttest and pretest for GAT in experimental group was significantly higher than that in control group. On the average, tenth-grade students who received origami-based instruction performed better on GAT than those who received traditional instruction. Thus, origami-based instruction had a significant positive effect on geometry achievement of tenth-grade participants in the study.

In the literature concerning geometry achievement and instruction, there were different results about the effect of the type of instruction on students' geometry achievement levels. For instance, there were few studies that found no significant effect of origami on geometry achievement (Boakes, 2006). The difference of the results concerning geometry achievement might stem from the difference of the grade level of participants. Furthermore, Boakes (2006) used a multiple-choice geometry test adapted from a standardized test. On the other hand, this study had open-ended questions developed by the researcher aligned with the curriculum. There were also studies that found significant positive effects of certain instructional activities on increasing geometry achievement levels of students. For example, Idris (2005) compared two groups of students on the ages of 13 and 14 years old in terms of their geometry achievement and spatial visualization abilities. The researcher developed a paper-and-pencil Geometry Test to measure geometry achievement of participants. Idris found that the specifically designed instructional activities were effective in improving spatial visualization abilities and geometry achievement of participants. About geometry achievement, Karaman (2000) found a significant relationship between spatial visualization abilities and performances in plane geometry of the six graders. Besides, Sağsöz (2008) found that origami could be a useful tool in teaching fractions to six-grade students. In an experimental study, Yuzawa and Bart (2002) have also found the positive effect of origami exercises on the learning of size

comparison strategies of preschool age children (ages 5 and 6). Even though, those studies were not all focused on geometry achievement after an origami instruction, they provided information on practices to increase geometry achievement levels of students. The results of the current research are parallel with the literature.

Based on reform movements, the geometry curricula for high school students in Turkey have been changed since 2010. MEB (2011) has recommended using origami to enhance geometric understanding of high school students. For instance, there is a section on the use of origami in teaching geometry in the reformed geometry curriculum for the 12^{th} grade. Benefits of origami (behavioral, psychomotor, social and emotional as well as linguistic effects of origami) and the use of origami in mathematics teaching as a manipulative are mentioned in the new 12^{th} grade geometry curriculum (MEB, 2011). Origami has also been included in the 11^{th} grade geometry textbook of MEB (Aydın, Biberoğlu and Camus, 2011) for the 2011-2012 school period. Some constructions such as a kite have been done by origami and some questions about quadrilaterals included paper folding in the 11^{th} grade geometry textbook. The results of this study about the effect of origami on enhancing geometry achievement provide scientific evidence to the suggestions made by MEB (2011).

7.3. Geometric Reasoning

The third aspect of this study was to determine whether origami-based instruction would have an effect on geometric reasoning of the participants. Geometric reasoning was measured by the researcher-designed Geometric Reasoning Test (GRT). GRT was administered as pretest and posttest to see whether there would be any change from pretest scores to posttest scores after treatment. The repeated measures ANOVA results revealed that there was a statistically significant change in students' geometric reasoning across two time periods (from pretest to posttest) for the group that received origami-based instruction and the group that received traditional instruction. Furthermore, the average difference between posttest and pretest scores for GRT in the group that received origami-based instruction was significantly higher than that in the group that received traditional instruction. On the average, tenth-grade students who received origami-based instruction performed better on GRT than those who received traditional instruction. Thus, origami-

based instruction had a significant positive effect on geometric reasoning of tenth-grade participants in the study.

The results of this study concerning geometric reasoning may provide evidence to the claims that origami enhances geometric reasoning (Cañadas *et al*., 2010; Cipoletti and Wilson, 2004; Coad, 2006; Golan and Jackson, 2009; Goos *et al*., 2007; Robichaux and Rodrigue, 2003; Sze, 2005a). Furthermore, intuition and formal reasoning in geometry should be connected in instruction (Jones, 2000). Origami may provide an appropriate classroom environment that students can connect visual reasoning based on folding paper and formal geometrical reasoning which requires abstract thinking. Questioning the geometrical relationships emerged from folding helps students to reason more abstractly in geometry. Thus, this study implied that origami-based instruction could help students to construct the connection between intuitive reasoning and formal reasoning in geometry.

The positive effect of origami-based instruction on geometric reasoning might have stemmed from the definition of geometric reasoning, which consists of constructions, generating and justifying certain conjectures. Duval (1998) has defined *construction* as a cognitive process of using tools. In this study, *construction* refers to a series of geometrically connected steps that result in a geometrical configuration by using different tools such as origami, ruler and protractor. The origami models were selected to make students to form triangles. Coad (2006) has emphasized the role of paper folding in making constructions and developing conceptual understanding of certain geometrical relations. Coad (2006) has also noted that paper folding could promote students to make conjectures and advance the development of explanatory proof. The current research could provide evidence to that constructions made through origami might have strengthened geometrical reasoning concerning triangles.

The new geometry curriculum for 12^{th} graders in Turkey has also recommended using origami in fostering geometric reasoning. An example of origami use in teaching geometry (trisecting a rectangular region through origami) was presented in the 12^{th} curriculum (MEB, 2011, p.25). This origami activity is followed up with a proof of why the rectangular region is trisected. Thus, origami was used to proceed to geometric proofs. The activity of trisecting a rectangular region through origami in the new curriculum may

exemplify how origami can be used to enhance geometric reasoning. The results of the current study about geometric reasoning also support the suggestions of the new geometry curricula of MEB by providing statistical evidence to their suggestions.

All results concerning spatial visualization, geometric reasoning abilities and geometry achievement indicated that there were statistically significant differences on average test scores from the beginning of the treatment to the period after the treatment in the group that received origami-based instruction. The results showed that spatial visualization and geometric reasoning abilities could be improved through the implementation of an origami-based instruction on triangles. Moreover, the results also implied that geometry achievement levels of students concerning triangles could be increased through the origami-based instruction for tenth-grade students.

7.4. Limitations

There are a couple of limitations of this study. For instance, purposive sampling was conducted in this study. The school for this study was not selected randomly but selected for convenience because the researcher was the teacher in this school. Besides, the classes were randomly assigned to be included in control or experimental group because it is not possible in real-life settings to mix students in classes.

As another constraint, students were unfamiliar to using origami to learn geometry. To mitigate students' unfamiliarity to origami, an introductory session which described origami and mentioned the history of origami was conducted before starting to do origami activities. Some students in the experimental group did not perceive origami as part of geometry lessons; instead they wanted to solve more exam-like exercises. At the end of each origami lesson, geometrical exercises that were more familiar to students were also solved.

The type of tests and the time pressure, especially on spatial visualization tests could be considered as a final limitation. Students were not much familiar to questions that require justification. They were mostly result-oriented and accustomed to multiple-choice

questions. Thus, this may have affected their performances in the Geometric Reasoning Test.

In summary, non-random selection of the school, unfamiliarity to origami, the type of tests and the time pressure of tests are the main limitations of the study.

7.5. Implications and Recommendations for Further Research

The results of the current research have also implications for mathematics educators and curriculum planners. The origami-based instruction in this study may be implemented with some modifications on lower grades by excluding the formal presentation of proofs. The origami-based instruction may be also a guide to mathematics teachers and pre-service mathematics teachers to design similar instructional sequences that integrate origami to teach geometry. Mathematics educators can be informed about the effectiveness of origami-based instruction in geometry through workshops on origami, integrating origami into courses of majors in mathematics education. Curriculum designers should take into account the effect of origami in designing high school geometry curriculum and textbooks. Although origami recently has started to be mentioned in high school geometry curriculum programs in Turkey, origami could be included more in curriculum and textbooks.

This study revealed that origami-based instruction in tenth-grade geometry classes had both statistically and practically significant effect on geometry achievement level, spatial visualization and geometric reasoning abilities of the participants. To further investigate the effect of origami-based instruction, the study may be replicated with larger sample size, different grade levels and with other content areas and topics other than triangles. The study may be replicated by using random selection of students to make generalizations. The high school students in this study have low van Hiele level (mostly second van Hiele level). For further research, the effect of origami-based instruction on participants with higher van Hiele levels could be studied. In another study, the van Hiele levels of students may be compared before and after treatment to see the effect of origami-based instruction on geometric thinking.

The duration and design of the origami-based instruction may be changed to investigate further the effect in different contexts. The study used three class hours for each weekly origami-based instruction for four weeks. The effect of origami-based instruction that lasts longer than four weeks should be investigated. Different origami models may also be selected to teach triangles or other geometry topics in future research. Computers and geometry softwares may also be integrated to origami-based instruction and their combined effect may be searched. Moreover, the connection between origami and proof or geometric reasoning may be further investigated in different settings. Besides, other spatial visualization tests may be used to investigate the effect of instruction. The gender effect of instruction may also be investigated in future research.

APPENDIX A: THE VAN HIELE TEST

Adı Soyadı:
Sınıfı/No:

VAN HIELE GEOMETRİ TESTİ

YÖNERGE

Bu test 25 sorudan oluşmaktadır. Sizden testteki her soruyu bilmeniz beklenmemektedir.

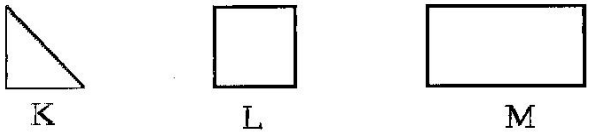
Kitapçığı açtığınızda;

1. Bütün soruları dikkatlice okuyunuz.
2. Doğru olduğunu düşündüğünüz seçenek üzerinde düşününüz. Her soru için tek bir doğru cevap vardır. Doğru olduğunu düşündüğünüz seçeneği işaretleyiniz.
3. İşaretlemiş olduğunuz cevabı değiştirmek isterseniz, ilk işareti tamamen siliniz.
4. Bu test için size verilecek süre 35 dakikadır.

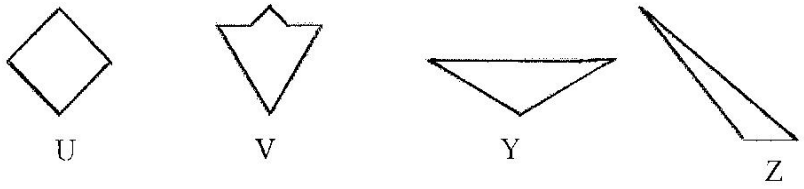
VAN HIELE GEOMETRİ TESTİ

1. Aşağıdakilerden hangisi ya da hangileri karedir?

a) Yalnız K
b) Yalnız L
c) Yalnız M
d) L ve M
e) Hepsi karedir.

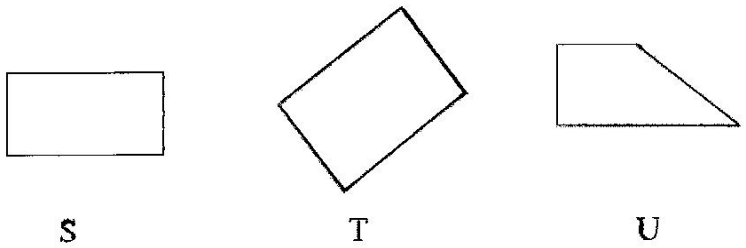


2. Aşağıdakilerden hangisi ya da hangileri üçgendir?



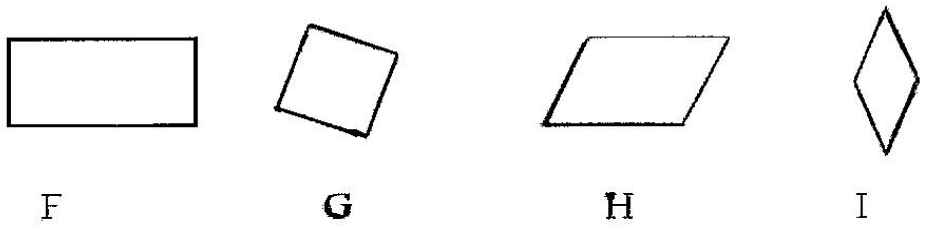
a) Hiçbiri üçgen değildir.
b) Yalnız V
c) Yalnız Y
d) Y ve Z
e) V ve Y

3. Aşağıdakilerden hangisi ya da hangileri dikdörtgendir?



a) Yalnız S
b) Yalnız T
c) S ve T
d) S ve U
e) Hepsi dikdörtgendir.

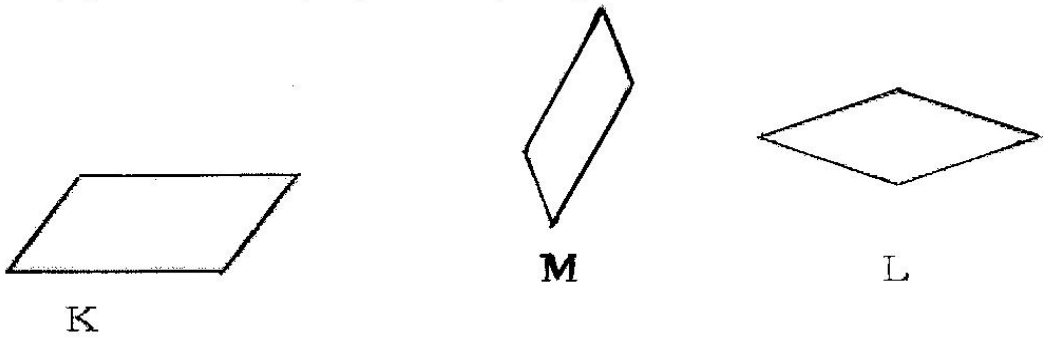
4. Aşağıdakilerden hangisi ya da hangileri karedir?



F G H I

a) Hiçbiri kare değildir.
b) Yalnız G
c) F ve G
d) G ve I
e) Hepsi karedir.

5. Aşağıdakilerin hangisi ya da hangileri paralelkenardır?

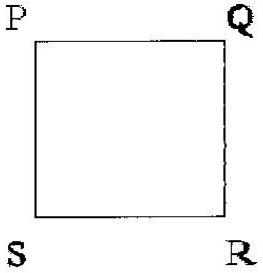


K M L

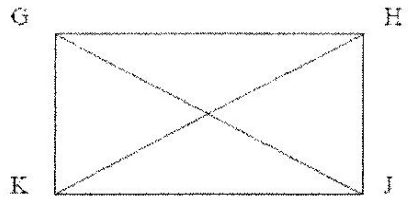
a) Yalnız K
b) Yalnız L
c) K ve M
d) Hiçbiri paralelkenar değildir.
e) Hepsi paralelkenardır.

6. PORS bir karedir. Aşağıdakilerden hangi özellik her kare için doğrudur?

a) [PR] ve [RS] eşit uzunluktadır.
b) [QS] ve [PR] diktir.
c) [PS] ve [QR] diktir.
d) [PS] ve [QS] eşit uzunluktadır.
e) Q açısı R açısından daha büyüktür.

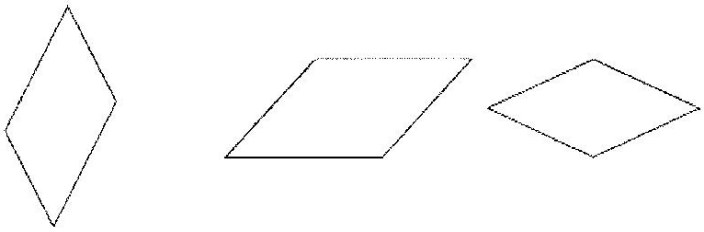


7. Bir GHJK dikdörtgeninde, [GJ] ve [HK] köşegendir. Buna göre, aşağıdakilerden hangis her dikdörtgen için doğru değildir?



a) 4 dik açısı vardır.
b) 4 kenarı vardır.
c) Köşegenlerin uzunlukları eşittir.
d) Karşılıklı kenarların uzunlukları eşittir.
e) $|GJ|$, $|GH|$ den kısadır.

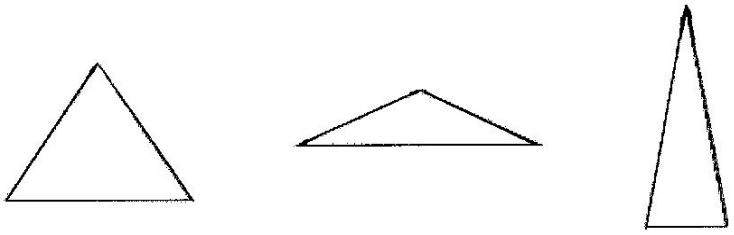
8. Eşkenar dörtgen, tüm kenar uzunlukları eşit olan 4 kenarlı bir şekildir. Aşağıda 3 tane eşkenar dörtgen verilmiştir.



Aşağıdakilerden hangisi her eşkenar dörtgen için her zaman doğru değildir?

a) İki köşegenin uzunlukları eşittir.
b) Her köşegen, aynı zamanda açıortaydır.
c) Köşegenleri birbirine diktir.
d) Karşılıklı açılarının ölçüleri eşittir.
e) Yukarıdaki seçeneklerin hepsi ((a)dan (d)ye kadar), her eşkenar dörtgen için doğru olabilir.

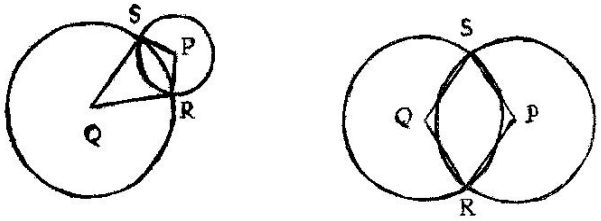
9. İkizkenar üçgen, iki kenarı eşit olan üçgendir. Aşağıda üç ikizkenar üçgen verilmiştir.



Aşağıdaki seçeneklerden hangisi her ikizkenar üçgen için doğrudur?

a) Üç kenarı eşit uzunlukta olmalıdır.
b) Bir kenarının uzunluğu, diğerinin iki katı olmalıdır.
c) Ölçüsü eşit olan en az iki açısı olmalıdır.
d) Üç açısının da ölçüsü eşit olmalıdır.
e) Yukarıdaki seçeneklerin hiçbiri, her ikizkenar üçgen için doğru değildir.

10. Merkezleri P ve Q olan iki çember, 4 kenarlı PRQS şeklini oluşturmak üzere R ve S noktalarında kesişirler. Aşağıda iki örnek verilmiştir.



Aşağıdaki seçeneklerden hangisi her zaman doğru değildir?

a) PRQS şeklinin iki kenarı eşit uzunlukta olacaktır.
b) PRQS şeklinin en az iki açısının ölçüsü eşit olacaktır.
c) [PQ] ve [RS] dik olacaktır.
d) P ve Q açılarının ölçüleri eşit olacaktır.
e) $|PQ|$, $|QR|$ den daha uzundur.

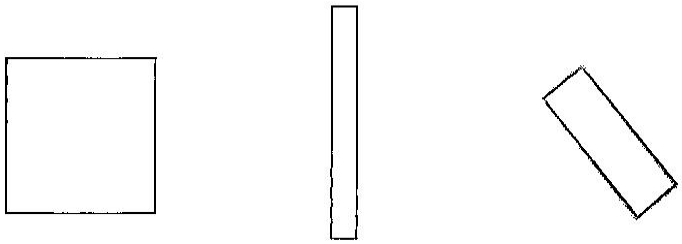
11. Önerme 1: F şekli bir dikdörtgendir.
Önerme 2: F şekli bir üçgendir.
Verilen bu iki önermeye göre aşağıdakilerden hangisi doğrudur?

a) Eğer 1 doğruysa, 2 de doğrudur.
b) Eğer 1 yanlışsa, 2 doğrudur.
c) 1 ve 2 aynı anda doğru olamaz.
d) 1 ve 2 aynı anda yanlış olamaz.
e) Yukarıdaki seçeneklerin hiçbiri doğru değildir.

12. Önerme S: ABC üçgeninin üç kenarı eşit uzunluktadır.
Önerme T: ABC üçgeninde, B ve C açılarının ölçüleri eşittir.
Buna göre, aşağıdakilerden hangisi doğrudur?

a) S ve T önermeleri ikisi de aynı anda doğru olamaz.
b) Eğer S doğruysa, T de doğrudur.
c) Eğer T doğruysa, S de doğrudur.
d) Eğer S yanlışsa, T de yanlıştır.
e) Yukarıdaki seçeneklerin hiçbiri doğru değildir.

13. Aşağıdaki şekillerden hangisi ya da hangileri dikdörtgen olarak adlandırılabilir?



P Q R

a) Hepsi
b) Yalnız Q
c) Yalnız R
d) P ve Q
e) Q ve R

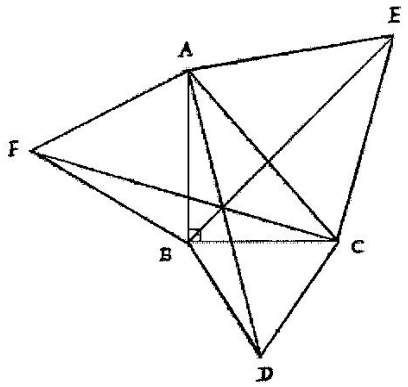
14. Aşağıdakilerden hangisi doğrudur?

a) Dikdörtgenlerin tüm özellikleri, tüm kareler için geçerlidir.
b) Karelerin tüm özellikleri, tüm dikdörtgenler için de geçerlidir.
c) Dikdörtgenin tüm özellikleri, tüm paralelkenarlar için geçerlidir.
d) Karelerin tüm özellikleri, tüm paralelkenarlar için geçerlidir.
e) Yukarıdaki seçeneklerin hiçbiri doğru değildir.

15. Tüm dikdörtgenlerde olup, bazı paralelkenarlarda olmayan özellik nedir?

a) Karşılıklı kenarları eşittir.
b) Köşegenler eşittir.
c) Karşılıklı kenarlar paraleldir.
d) Karşılıklı açıları eşittir.
e) Yukarıdaki seçeneklerin hiçbiri doğru değildir.

16. Aşağıda bir ABC dik üçgeni verilmiştir. ABC üçgeninin kenarları üzerinde, ACE, ABF ve BCD eşkenar üçgenleri çizilmiştir.



Bu bilgilerden, [AD], [BE] ve [CF] nin ortak bir noktadan geçtikleri kanıtlanabilir. Bu kanıt size neyi ifade eder?

a) Yalnızca bu üçgen için; [AD], [BE] ve [CF] nin ortak bir noktası olduğundan emin olabiliriz.
b) Sadece bazı dik üçgenlerde; [AD], [BE] ve [CF] nin ortak bir noktası vardır.
c) Herhangi bir dik üçgende, [AD], [BE] ve [CF] nin ortak bir noktası vardır.
d) Herhangi bir üçgende, [AD], [BE] ve [CF] nin ortak bir noktası vardır.
e) Herhangi bir eşkenar üçgende, [AD], [BE] ve [CF] nin ortak bir noktası vardır.

17. Aşağıda bir şeklin üç özelliği verilmiştir.
Özellik D: Köşegenleri eşit uzunluktadır.
Özellik S: Bir karedir.
Özellik R: Bir dikdörtgendir.
Bu özellikler dikkate alındığında aşağıdakilerden hangisi doğrudur?

a) D gerektirir S, o da gerektirir R.
b) D gerektirir R, o da gerektirir S.
c) S gerektirir R, o da gerektirir D.
d) R gerektirir D, o da gerektirir S.
e) R gerektirir S, o da gerektirir D.

18. Aşağıda iki önerme verilmiştir.
I: Eğer bir şekil dikdörtgense, köşegenleri birbirini ortalayarak keser.
II: Eğer bir şeklin köşegenleri birbirini ortalayarak kesiyorsa şekil, dikdörtgendir.
Buna göre aşağıdakilerden hangisi doğrudur?

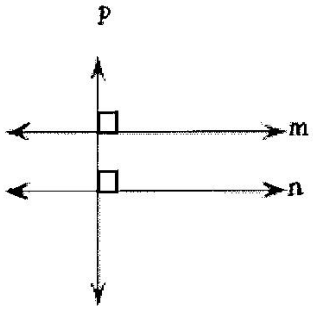
a) I'in doğru olduğunu kanıtlamak için, II'nin doğru olduğunu kanıtlamak yeterlidir.
b) II'nin doğru olduğunu kanıtlamak için, I'in doğru olduğunu kanıtlamak yeterlidir.
c) II'nin doğru olduğunu kanıtlamak için, köşegenleri birbirini ortalayan bir dikdörtgen bulmak yeterlidir.
d) II'nin yanlış olduğunu kanıtlamak için, köşegenleri birbirini ortalayan dikdörtgen olmayan bir şekil bulmak yeterlidir.
e) Yukarıdaki seçeneklerin hiçbiri doğru değildir.

19. Aşağıdaki ifadelerden hangisi doğrudur? Geometride;

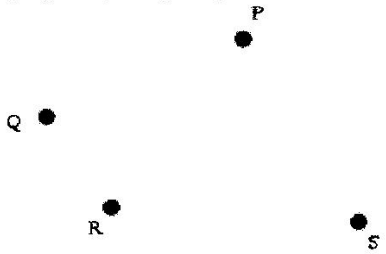
a) Her terim tanımlanabilir ve her doğru önermenin doğru olduğu kanıtlanabilir.
b) Her terim tanımlanabilir; ama bazı önermelerin doğru olduğunu varsaymak gerekir.
c) Bazı terimler tanımsız kalmalıdır; ama bütün doğru önermelerin doğruluğu kanıtlanabilir.
d) Bazı terimler tanımsız kalmalıdır ve doğru olduğu varsayılmış bazı önermelere gerek vardır.
e) Yukarıdaki seçeneklerin hiçbiri doğru değildir.

20. Aşağıdaki üç ifadeyi inceleyiniz.
{1} Aynı doğruya dik olan iki doğru paraleldir.
{2} İki paralel doğrudan birine dik olan doğru, diğerine de diktir.
{3} Eğer iki doğru eş uzaklıktaysa paraleldir.
Aşağıdaki şekilde, m ve p ile n ve p doğrularının birbirine dik olduğu verilmiştir. Buna göre, yukarıdaki cümlelerden hangisi ya da hangileri m doğrusunun n doğrusuna paralel olmasının nedeni olabilir?

a) Yalnız {1}
b) Yalnız {2}
c) Yalnız {3}
d) {1} veya {2}
e) {2} veya {3}



21. F-geometrisinde her şey alışık olduğumuzdan farklıdır. Burada sadece 4 nokta ve 6 doğru vardır. Her doğru, iki nokta içerir. Eğer P, Q, R ve S nokta ise, {P,Q}, {P,R}, {P, S}, {Q, R}, {Q, S} ve {R, S} doğrulardır.



Kesişme ve paralel terimlerinin F-geometrisindeki kullanımı şöyledir: {P,Q}ve {P,R} doğruları P'de kesişirler; çünkü P, {P,Q}ve{P,R}doğrularının ortak noktasıdır. {P,Q} ve {R, S} doğruları paraleldir; çünkü ortak hiçbir noktaları yoktur.
Buna göre, aşağıdakilerden hangisi doğrudur?

a) {P,R} ve {Q, S}kesişirler.
b) {P,R} ve {Q, S}paraleldir.
c) {Q, R} ve {R, S} paraleldir.
d) {P, S} ve {Q, R} kesişirler.
e) Yukarıdaki seçeneklerin hiçbiri doğru değildir.

22. Bir açıyı üçlemek demek onu üç eşit parçaya bölmek demektir. 1847 yılında, P. L. Wantzel bir açının yalnızca pergel ve işaretlenmemiş cetvel kullanılarak üçlenemeyeceğini kanıtlamıştır. Bu kanıttan nasıl bir sonuca varabilirsiniz?

a) Açılar yalnızca pergel ve işaretlenmemiş cetvel kullanılarak iki eş parçaya ayrılamaz.
b) Açılar yalnızca pergel ve işaretlenmiş cetvel kullanılarak üçlenemez.
c) Açılar herhangi bir çizim aracı kullanılarak üçlenemez.
d) Gelecekte, birinin yalnız pergel ve işaretlenmemiş cetvel kullanarak açıları üçlemesi mümkün olabilir.
e) Hiç kimse, açıları yalnızca pergel ve işaretlenmemiş cetvel kullanarak üçleyecek genel bir yöntem bulamayacaktır.

23. Ali adlı bir matematikçinin kendi tanımladığı geometriye göre, aşağıdaki önerme doğrudur:

Bir üçgenin iç açılarının ölçüleri toplamı 180 dereceden azdır.

Buna göre, aşağıdakilerden hangisi doğrudur?

a) Ali üçgenin açılarını ölçerken hata yapmıştır.
b) Ali mantıksal bir hata yapmıştır.
c) Ali doğru sözcüğünün anlamını bilmiyordur.
d) Ali bilinen geometrilerden farklı varsayımlarla başlamıştır.
e) Yukarıdaki seçeneklerin hiçbiri doğru değildir.

24. İki ayrı geometri kitabı, 'dikdörtgen' sözcüğünü iki farklı şekilde tanımlamıştır. Buna göre, aşağıdakilerden hangisi doğrudur?

a) Kitaplardan birinde hata vardır.
b) Tanımlardan biri yanlıştır. Dikdörtgen için iki farklı tanım olamaz.
c) Bir kitapta tanımlanan dikdörtgenin özellikleri diğer kitaptakinden farklı olmalıdır.
d) Bir kitapta tanımlanan dikdörtgenin özellikleri diğer kitaptakiyle aynı olmalıdır.
e) Kitaplarda tanımlanan dikdörtgenlerin farklı özellikleri olabilir.

25. Aşağıdaki önerme I ve II'yi kanıtladığımızı varsayalım.

I: Eğer p ise q.
II: Eğer s ise q değildir.

Buna göre, önerme I ve II'den aşağıdakilerden hangisi çıkartılabilir?

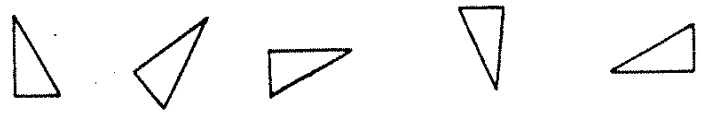
a) Eğer p ise s dir.
b) Eğer p değil ise q değildir.
c) Eğer p veya q ise s dir.
d) Eğer s ise p değildir.
e) Eğer s değil ise p dir.

APPENDIX B: THE CARD ROTATION TEST

Adı Soyadı:_____

KART ÇEVİRME TESTİ

Bu test şekiller arasındaki farkı görebilme yeteneğini ölçmek için geliştirilmiştir. Aşağıdaki üçgen şeklindeki 5 kartı inceleyiniz.



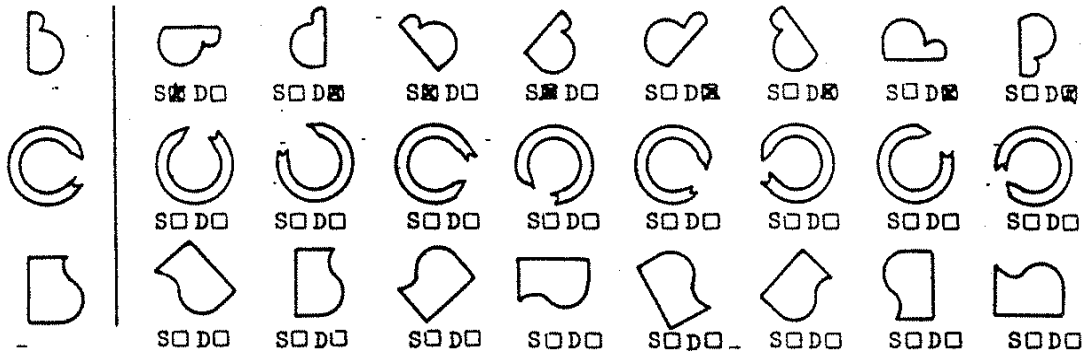
Fark edeceğiniz gibi tüm şekiller baştaki kartın döndürülmüş (yuvarlanmış) halleridir.Şimdi aşağıdaki iki kartı inceleyiniz.



Gördüğünüz gibi bu kartlar aynı değildir.İlk kart döndürme (yuvarlama) yoluyla ikincisine dönüştürülemez. Ancak yüzü tam çevrilirse ilkine dönüşebilir. Dolayısıyla bu kartlar farklıdır diyebiliriz.

Bu testte yapmanız gereken dikey çizginin solundaki şekille sağındaki sekiz şekili karşılaştırıp aynı olup olmadıklarını tespit etmektir. Sağdaki şekillerden herhangi birisi soldakiyle aynı ise şeklin altındaki S (Sabit); farlı ise D (Değişik) şıklarını işaretleyiniz.

Aşağıdaki örnekleri inceleyip çözünüz. İlk sıra sizin için doğru olarak çözülmüştür.

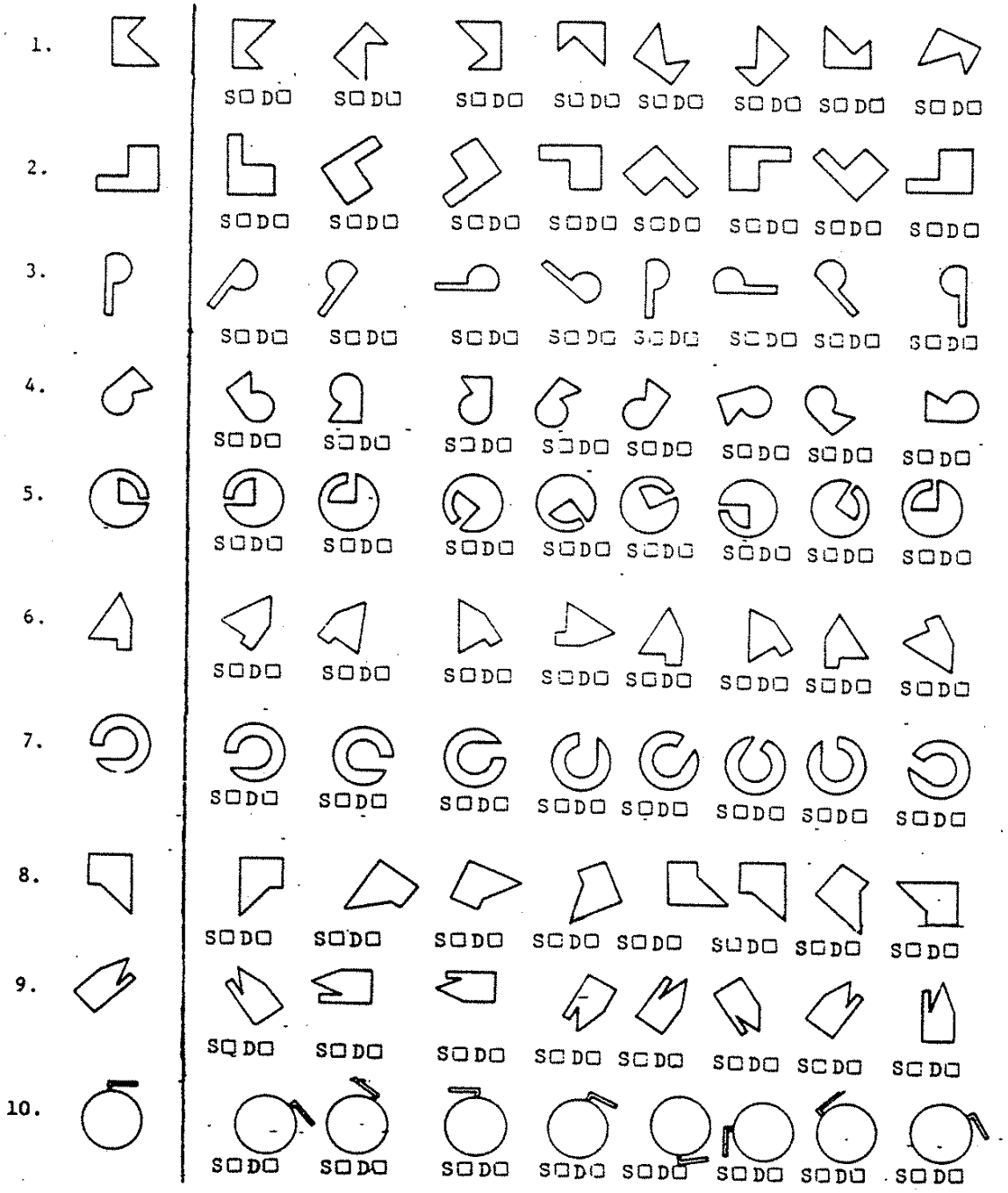


Test iki bölümden oluşmaktadır ve her bölüm için 3 dakikanız vardır. Süre dolduğunda lütfen 1. Bölümü cevaplandırmayı bırakıp 2. Bölümün dağıtılmasını bekleyiniz. Başarılar;

LÜTFEN SÖYLENMEDEN SAYFAYI ÇEVİRMEYİNİZ.

Sayfa 2

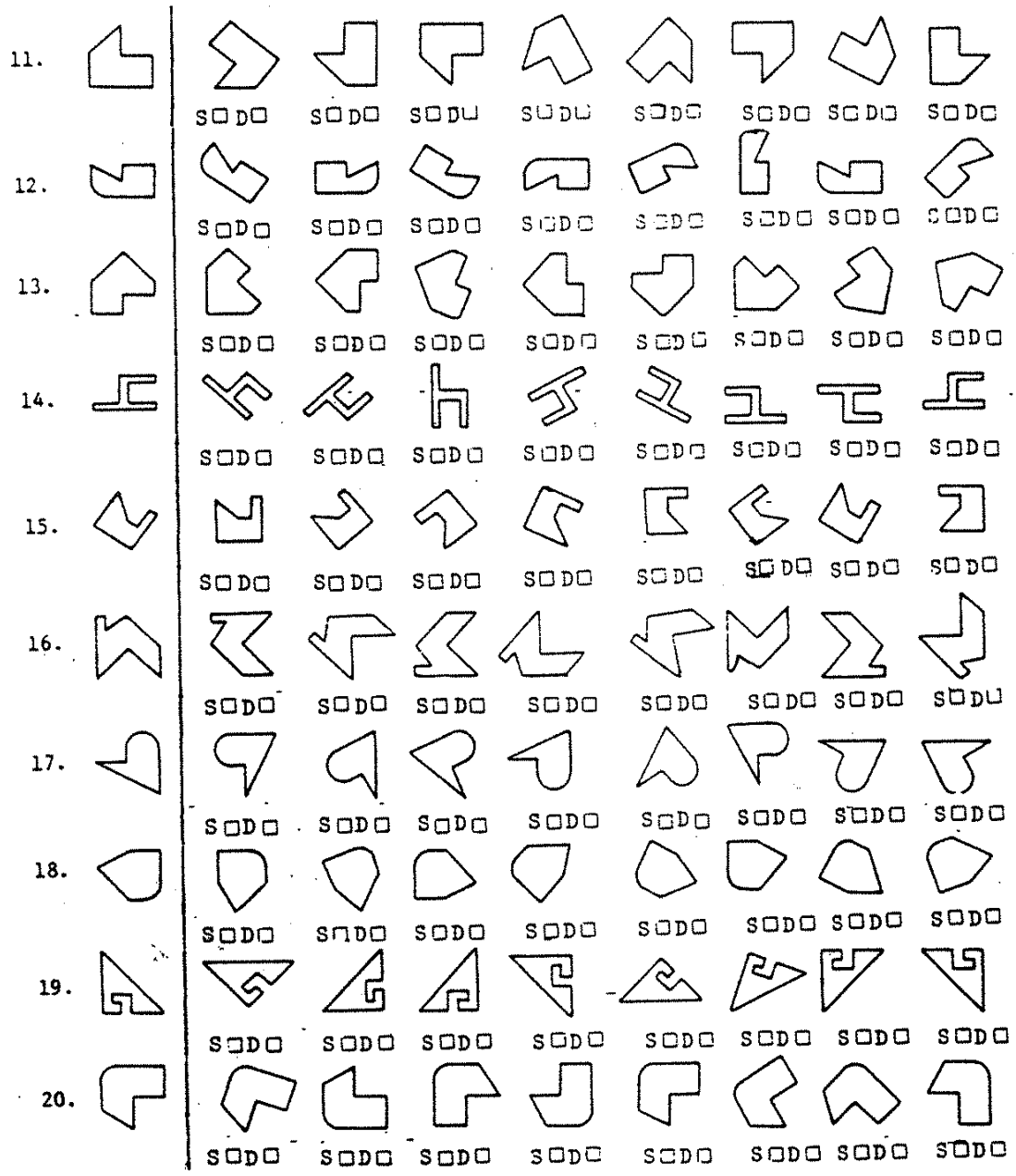
1. BÖLÜM (3 Dakika)



LÜTFEN 2.BÖLÜMÜN DAĞITILMASINI BEKLEYİNİZ

Sayfa 3

2. BÖLÜM (3 Dakika)



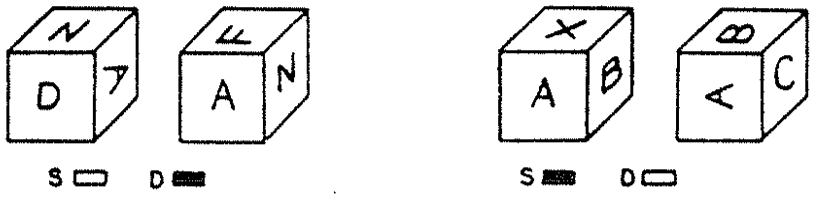
LÜTFEN SÜRENİZ BİTENE KADAR BEKLEYİNİZ

APPENDIX C: THE CUBE COMPARISON TEST

Adı Soyadı:_____

KÜP KARŞILAŞTIRMA TESTİ

Bu testteki tüm problemlerde üzerlerinde harf, rakam veya şekil bulunan 6 yüzü (alt yüz, üst yüz ve dört yan yüz) olan küpler verilmiştir ve küplerin birbirlerinin aynı olup olmadığını bulmanız istenmektedir. Aşağıdaki iki küp çiftini inceleyiniz.

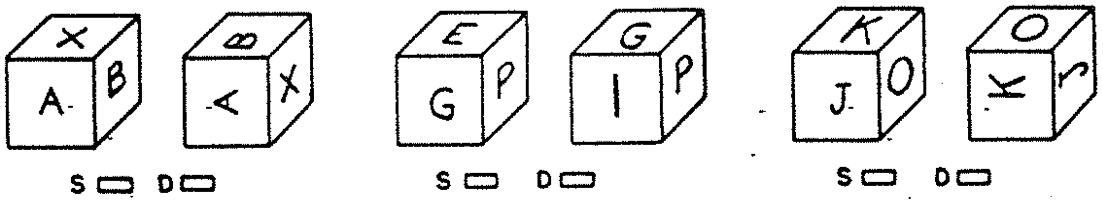


İlk çift için D şıkkı seçilmiştir çünkü küpler birbirinden **farklıdırlar** (Değişik). Soldaki küpün A harfi bulunan yüzü size bakacak şekilde çevrildiğinde, N harfi bulunan yüzü A harfi bulunan yüzün soluna ve görünmeyecek konuma gelir. Oysa sağdaki küpün N harfli yüzü A harfli yüzün sağında ve görünür haldedir, dolayısıyla bu küpler farklıdırlar.

İkinci çiftte ise S şıkkı seçilmiştir çünkü küpler **aynı olabilir.** A harfli yüzey yana çevrildiğinde X harfli yüzey görünmez konuma, B harfli yüzey üste gelir ve görünmez konumdaki C harfli yüzey görünür konuma gelir. Buda küplerin aynı olabileceğini gösterir.

Not: Bütün harf, rakam ve şekiller bir küpte birden fazla bulunamaz, fakat görünmeyecek konumda olabilir.

Aşağıdaki üç örneği inceleyiniz.



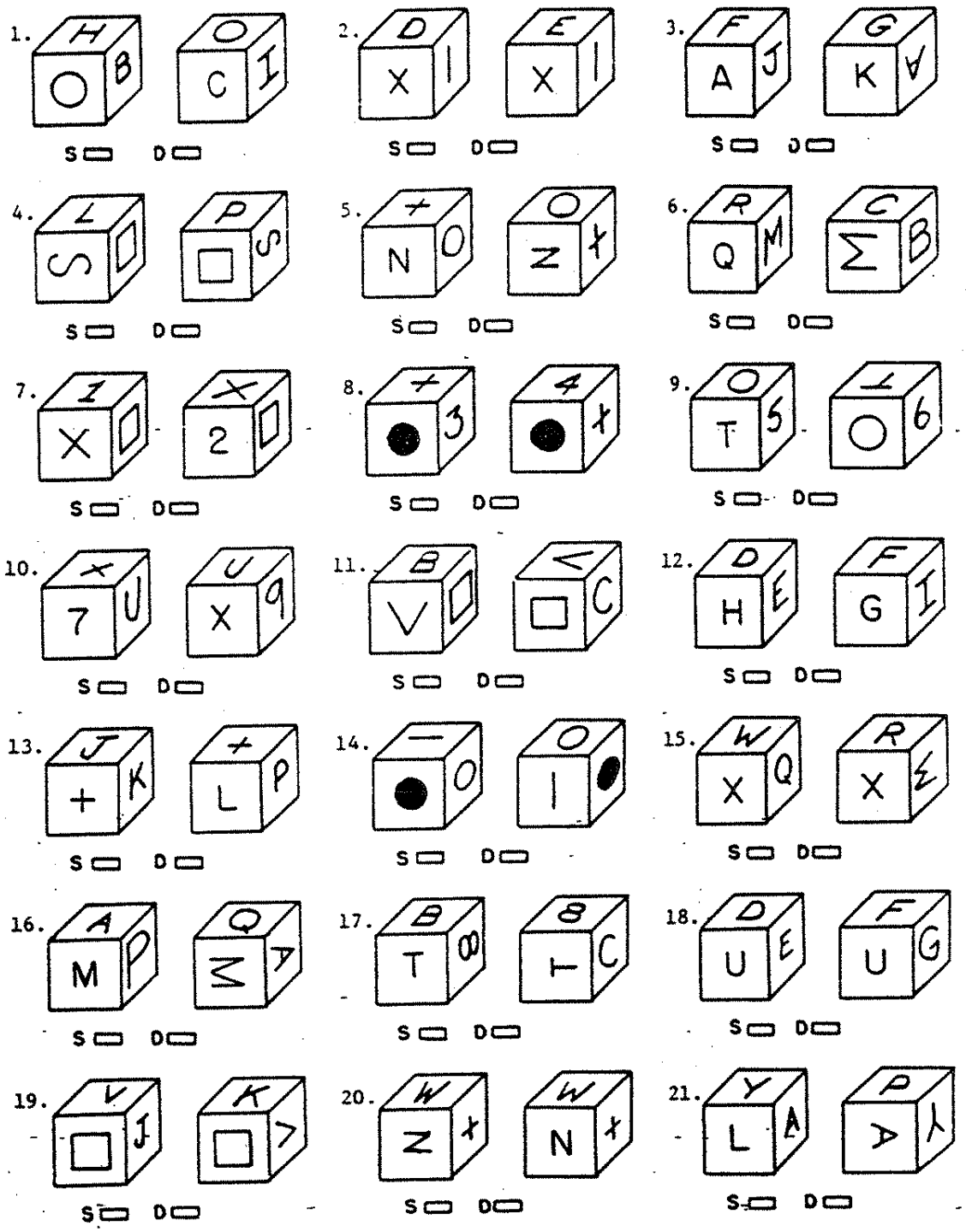
İlk çift hemen D işaretlenmelidir çünkü X harfi bir küpte iki defa bulunamaz.İkinci ve üçüncü çiftleri inceleyip cevaplandırınız.

Test iki bölümden oluşmaktadır ve her bölüm için 3 dakikanız vardır. Süre dolduğunda lütfen 1. Bölümü cevaplandırmayı bırakıp 2. Bölümün dağıtılmasını bekleyiniz. Başarılar;

LÜTFEN SÖYLENMEDEN SAYFAYI ÇEVİRMEYİNİZ.

Sayfa 2

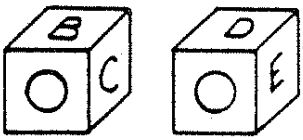
1. BÖLÜM (3 Dakika)

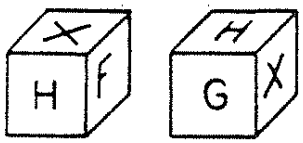


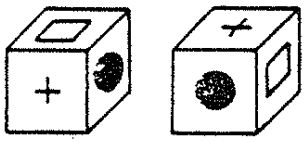
LÜTFEN 2.BÖLÜMÜN DAĞITILMASINI BEKLEYİNİZ

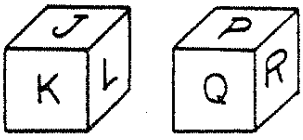
Sayfa 3

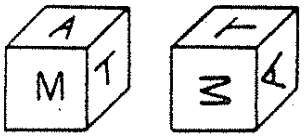
2. BÖLÜM (3 Dakika)

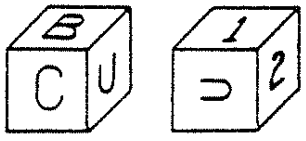
22.  S ▭ D ▭

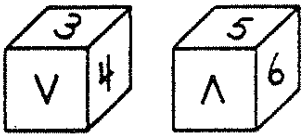
23.  S ▭ D ▭

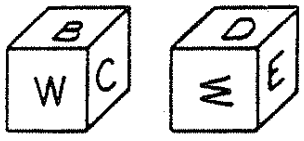
24.  S ▭ D ▭

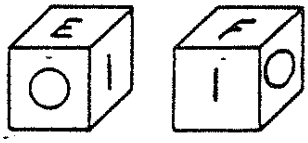
25.  S ▭ D ▭

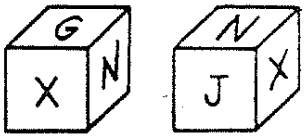
26.  S ▭ D ▭

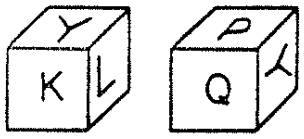
27.  S ▭ D ▭

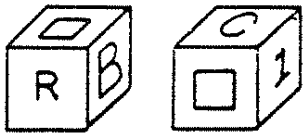
28.  S ▭ D ▭

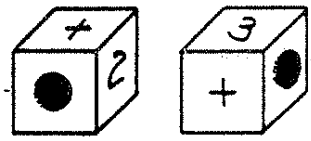
29.  S ▭ D ▭

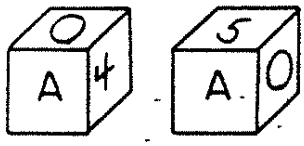
30.  S ▭ D ▭

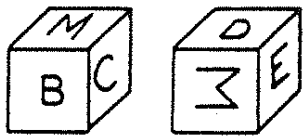
31.  S ▭ D ▭

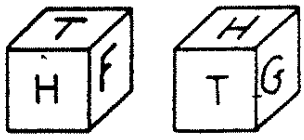
32.  S ▭ D ▭

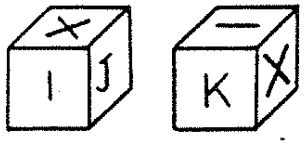
33.  S ▭ D ▭

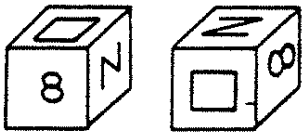
34.  S ▭ D ▭

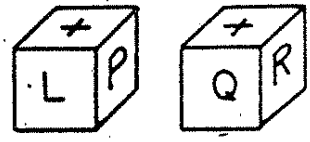
35.  S ▭ D ▭

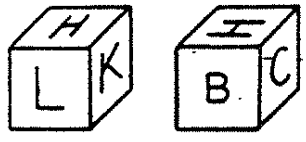
36.  S ▭ D ▭

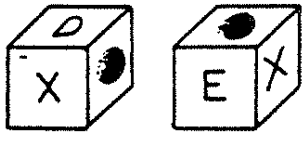
37.  S ▭ D ▭

38.  S ▭ D ▭

39.  S ▭ D ▭

40.  S ▭ D ▭

41.  S ▭ D ▭

42.  S ▭ D ▭

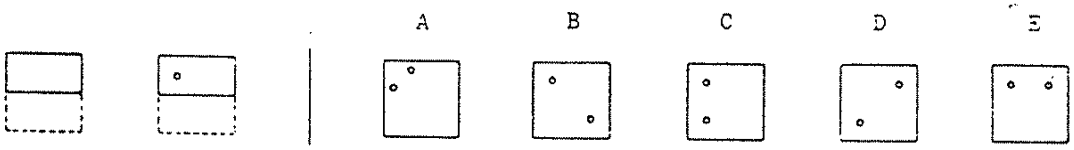
LÜTFEN SÜRENİZ BİTENE KADAR BEKLEYİNİZ

APPENDIX D: THE PAPER FOLDING TEST

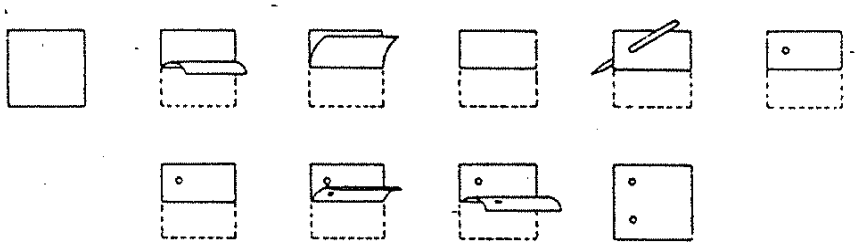
Adı Soyadı:_____

KAĞIT KATLAMA TESTİ

Bu testte bir parça kağıdın katlanıp açılmasını hayal etmeniz gerekmektedir. Aşağıdaki örnekte dikey çizginin solundaki kare şeklindeki kağıt, katlanıp bir noktadan deliniyor. Kağıt açıldıktan sonra sağdaki şekillerden hangisinin oluşacağını bulunuz.



Yukardaki örnekte doğru cevap C şıkkıdır. Kağıdın nasıl katlandığını ve doğru cevabın neden C şıkkı olduğunu gösteren şekilleri inceleyiniz.



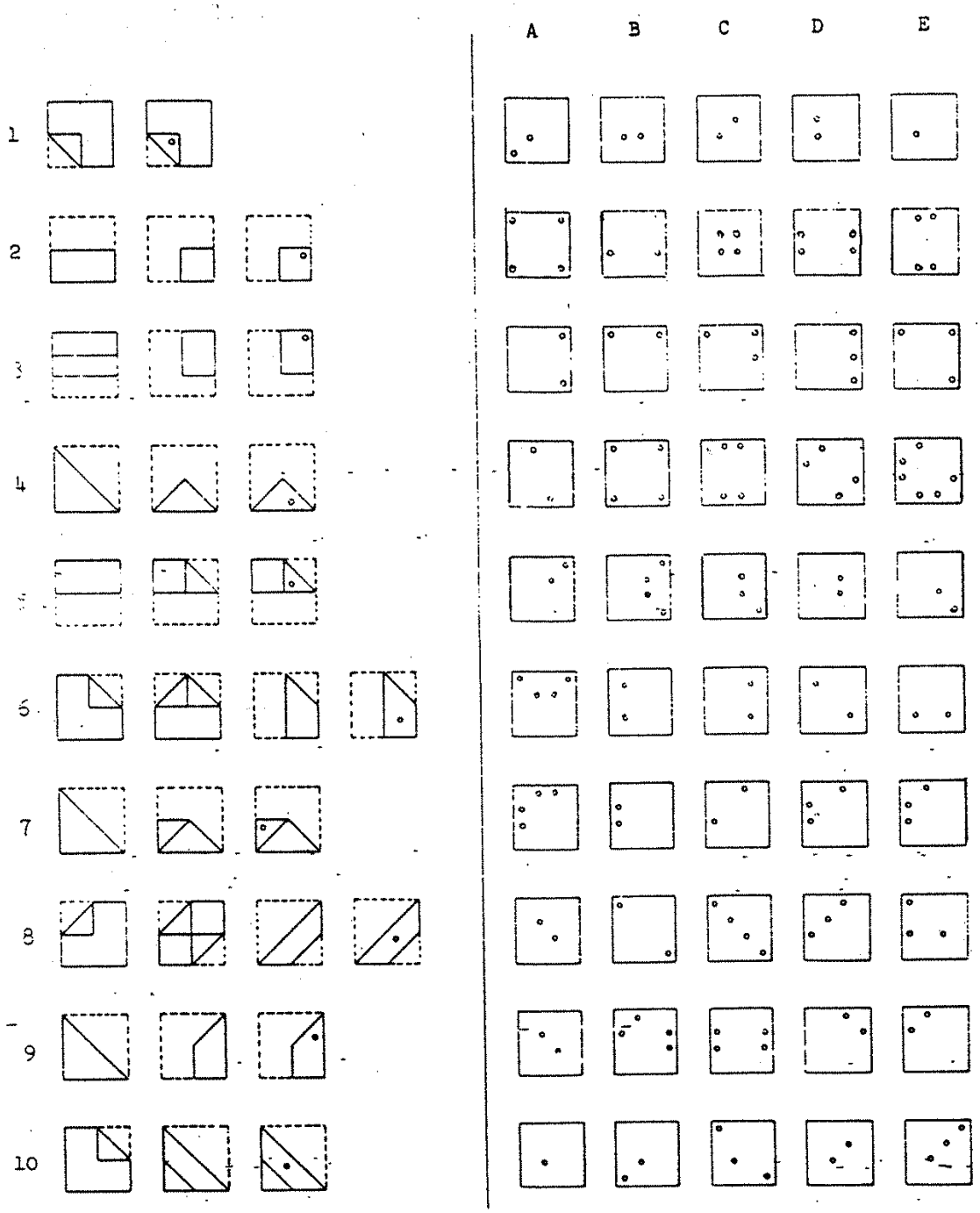
Tüm problemlerde katlamalar dikey çizginin solunda yapılmaktadır. Ayrıca kağıt hiç bir yöne çevrilmemekte sadece katlanmaktadır. Doğru cevabın kağıdın tamamen açıldıktan sonraki deliklerin yerini gösteren seçenek olduğunu unutmayınız.

Test iki bölümden oluşmaktadır ve her bölüm için 3 dakikanız vardır. Süre dolduğunda lütfen 1. Bölümü cevaplandırmayı bırakıp 2. Bölümün dağıtılmasını bekleyiniz. Başarılar;

LÜTFEN SÖYLENMEDEN SAYFAYI ÇEVİRMEYİNİZ.

Sayfa 2

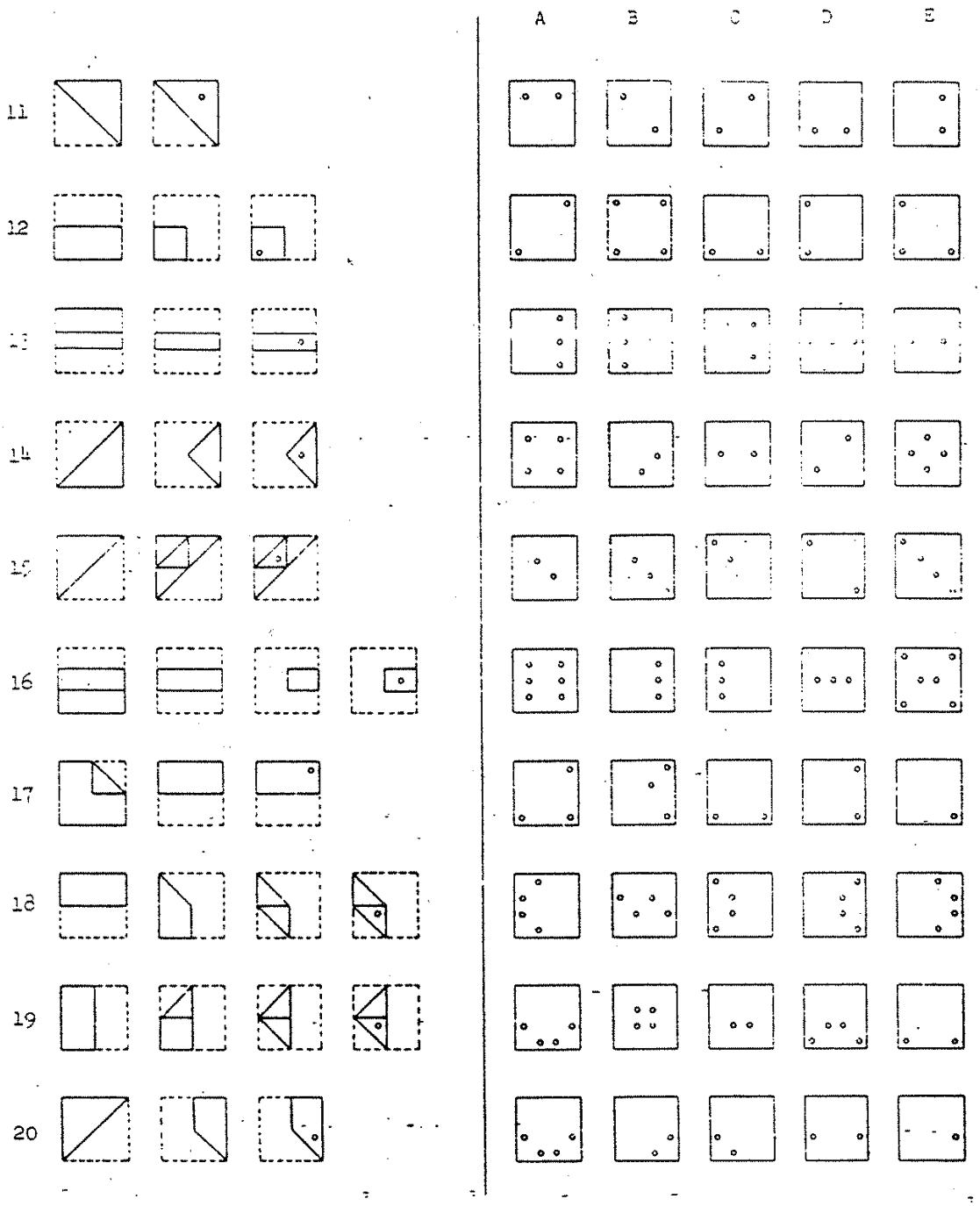
1. BÖLÜM (3 Dakika)



LÜTFEN 2.BÖLÜMÜN DAĞITILMASINI BEKLEYİNİZ

Sayfa 3

2. BÖLÜM (3 Dakika)



LÜTFEN SÜRENİZ BİTENE KADAR BEKLEYİNİZ

APPENDIX E: GEOMETRY ACHIEVEMENT POSTTEST

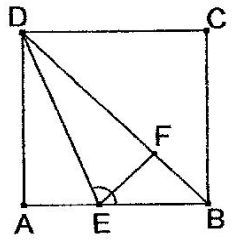
Adı Soyadı:
Sınıfı/ No:

A

GEOMETRİ BAŞARI SON TESTİ (GBST)

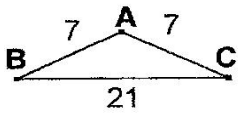
Açıklama: *Yanıtlarınızı matematiksel olarak ifade ediniz.*

1.



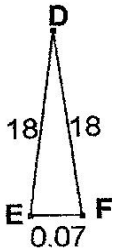
<u>*Kare*</u> bir kâğıt önce C noktasından A noktası üzerine katlanıyor. Sonra [DA] kenarı, [DB] üzerine katlanıyor. Buna göre, $m(\hat{DEB})$ açısının ölçüsü kaç derecedir?

2.



Şekil 2.1

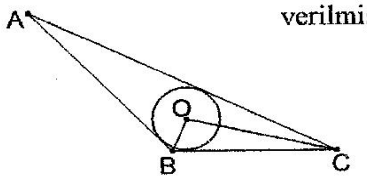
a) Şekil 2.1'de kenar uzunlukları 7,7,21 birim olarak verilen bir ABC üçgeninin var olması mümkün müdür? Açıklayınız.



Şekil 2.2

b) Şekil 2.2'de kenar uzunlukları 18,18,0.07 birim olarak verilen bir DEF üçgeninin var olması mümkün müdür? Açıklayınız.

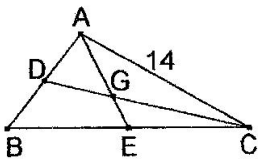
3.



Şekil 3.1

Şekil 3.1'de, bir ABC üçgeni ve O merkezli <u>iç teğet çemberi</u> verilmiştir. $m(\hat{OBC}) = 65^\circ$, $m(\hat{OCA}) = 10^\circ$ ise $m(\hat{A}) = ?$

4.

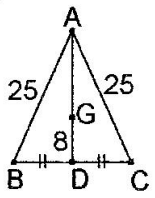


Şekil 4.1

Şekil 4.1'deki ABC üçgeninde G ağırlık merkezi, $|AC|$=14 cm, $|AE|$=9 cm, $|CD|$=15 cm ise ACG üçgeninin çevresi kaç cm dir?

5. Çevresi 9 birim ve kenar uzunlukları tam sayı olan en fazla kaç tane ikizkenar üçgen oluşturulabilir?

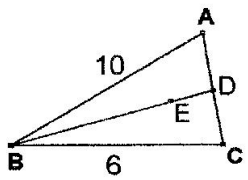
6.



Şekil 6.1

Şekil 6.1'deki ABC üçgeninde G ağırlık merkezi, $|AB| = |AC|$=25 cm, $|GD|$= 8 cm ise $|BC|$ kaç cm dir?

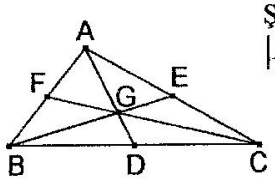
7.



Şekil 7.1

Şekil 7.1'de E noktası, ABC üçgeninin iç teğet çemberinin merkezidir. $|AB|$=10 birim, $|BC|$=6 birim, ABC üçgeninin çevresi 24 birim ise $|AD|$ kaç birimdir?

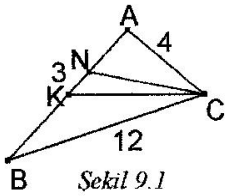
8.



Şekil 8.1

Şekil 8.1'deki ABC üçgeninde G ağırlık merkezi,
$|AG|+|BG|+|CG|=8$ cm ise $|GD|+|GF|+|GE|$ kaç cm dir?

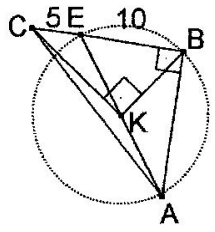
9.



Şekil 9.1

Şekil 9.1'deki ABC üçgeninde [CN] açıortay ve [CK] kenarortaydır.
$|BC|=12$ cm, $|AC|=4$ cm, $|NK|=3$ cm, ise $|AB|$ kaç cm dir?

10.



Şekil 10.1

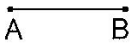
Şekil 10.1'de $m(B\hat{K}C)=m(A\hat{B}C)=90^{\circ}$ ve K noktası, ABE üçgeninin çevrel çemberinin merkezidir. $|BE|=2.|EC|=10$ birim ise $|AB|$ kaç birimdir?

APPENDIX F: GEOMETRIC REASONING TEST

GEOMETRİK AKIL YÜRÜTME TESTİ

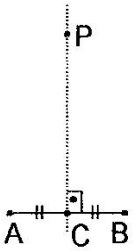
Yönerge: Yanıtlarınızı matematiksel olarak gerekçelendiriniz.

1. Aşağıda bir üçgeni oluşturmak için adımlar verilmektedir.



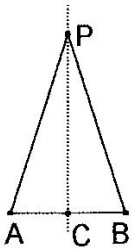
Şekil 1.1

1. Bir doğru parçası çizelim.



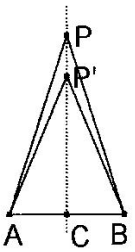
Şekil 1.2

2. Bu doğru parçasına orta dikme doğrusunu çizelim ve bunun üzerinde P ve C noktalarını alalım.



Şekil 1.3

3. P ve A ile P ve B noktalarını birleştiren doğru parçalarını çizelim.



Şekil 1.4

4. P ve P' noktaları, orta dikme doğrusu üzerindedir.

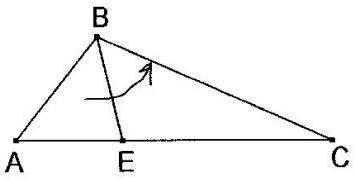
a) APB üçgeninin temel ve yardımcı elemanları hakkında neler söyleyebilirsiniz?

b) Şekil 1.3'teki APB üçgeninin çeşidi hakkında neler söyleyebilirsiniz?

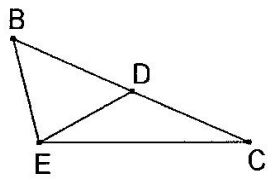
c) Şekilde seçilen P noktasının sadece kesik çizgi ile gösterilen orta dikme doğrusu üzerinde *hareketli* bir nokta olduğunu varsayarsak, oluşabilecek üçgenlerin çeşitleri hakkında neler söyleyebilirsiniz? (Şekil 1.4'e bakınız.)

GEOMETRİK AKIL YÜRÜTME TESTİ

2. Şekil 2.1'deki ABC üçgeninin kısa kenarı olan [AB], uzun kenarı olan [BC] üzerine katlanıyor. A ve C açılarını karşılaştırarak bu açılar ve bu açıları gören kenarlar arasındaki ilişkiyi bulabiliriz. Bu soruda, üçgende bir açı-kenar ilişkisini göstermek için aşağıda verilen soruları yanıtlayarak bu ilişkiye ulaşmanız beklenmektedir.



Şekil 2.1



Şekil 2.2

a) ABC üçgeni Şekil 2.1'deki gibi katlanıp Şekil 2.2 oluşturulduğunda BAC açısı Şekil 2.2'deki hangi açıya eş olur?

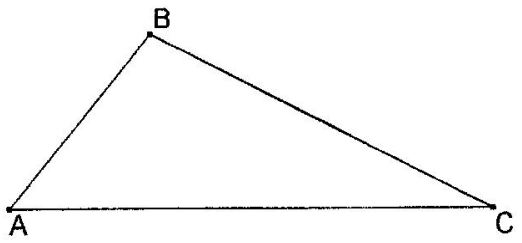
b) Şekil 2.2'deki BDE açısı, DEC üçgeninin açıları ile nasıl ilişkilendirilebilir?

c) BDE açısı ile C açısının ölçülerini karşılaştırdığınızda bunlar hakkında neler söyleyebilirsiniz?

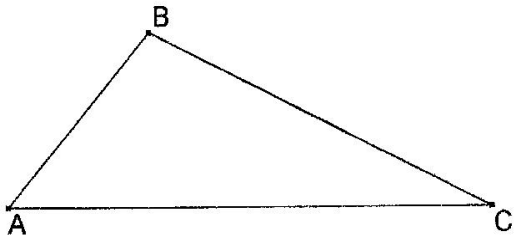
d) Yukarıda izlediğiniz adımlarla ABC üçgenindeki A ve C açıları ve onları gören [BC] ve [AB] kenarları arasında nasıl bir ilişkiye ulaştınız?

GEOMETRİK AKIL YÜRÜTME TESTİ

3. Şekil 3.1 ve Şekil 3.2'de verilen ABC üçgeninin iç teğet çemberi hakkında aşağıda verilen soruları yanıtlayınız.



Şekil 3.1



Şekil 3.2

a) Şekil 3.1 üzerinde ABC üçgeninin iç teğet çemberini çiziniz.

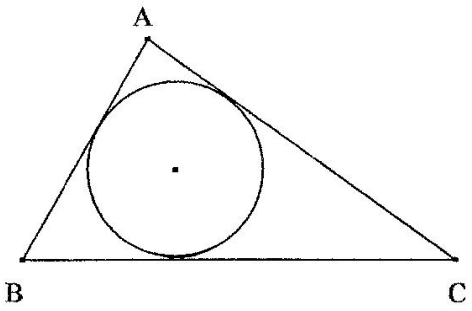
b) ABC üçgeninin iç teğet çemberini doğrular kullanarak nasıl çizersiniz? Şekil 3.2 üzerinde gösteriniz.

c) Bu doğrular üçgenin hangi yardımcı elemanlarına karşılık gelmektedir?

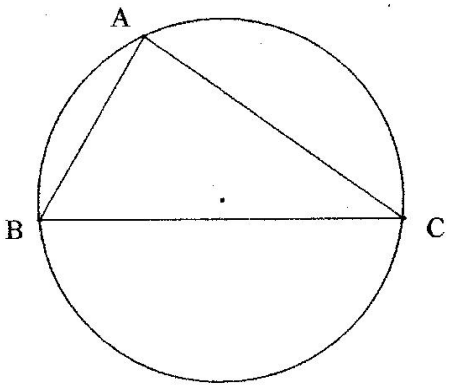
d) İç teğet çemberin oluşturulmasında *en az* kaç doğruya ihtiyaç vardır?

GEOMETRİK AKIL YÜRÜTME TESTİ

4. Aşağıdaki şekillerde ABC üçgeninin iç teğet çemberi (Şekil 4.1) ve çevrel çemberi (Şekil 4.2) çizilmiştir.



Şekil 4.1



Şekil 4.2

a) Bu çemberlerin merkezlerini nasıl bulabilirsiniz? Şekil 4.1 ve Şekil 4.2 üzerinde gösteriniz.

b) Bu çemberlerde merkez, üçgenin hangi temel elemanlarına eşit uzaklıktadır?
İpucu: Çemberlerin yarıçaplarını düşününüz.

APPENDIX G: SAMPLE ORIGAMI-BASED INSTRUCTION LESSON PLAN

Origami 2. Hafta Ders Planı

Ünite : Üçgenler

Sınıf : 10

Konu : Üçgende kenar-açı ilişkileri

Kazanım: Üçgenin kenarları ve açıları arasındaki ilişkileri ispatlar, uygulamalar yapar.

Malzemeler: Her öğrenciye birer kare kâğıt

Ders Akışı:

1. **Ders (45 dakika)**:
   - Kuğu yapımı **(20-25 dakika)**
   - Kuğu modelindeki ilk 4 adımın çizimi **(10-15 dakika)**
   - İlk iki adım için üçgenlerdeki açı ölçülerinin açıölçer kullanmadan bulunması
   - Kuğu modelindeki 2. adımdaki üçgenler için kenar-açı ilişkilerinin bulunması

2. **Ders (45 dakika)**: Kuğu yapımındaki üçgenlere yoğunlaşarak üçgenlerle ilgili geometrik tartışmanın yapılması (özellikle üçgende kenar-açı ilişkileri)
   - Kuğu modelindeki 3. adımdaki üçgenler için kenar-açı ilişkilerinin bulunması (bir üçgenin kenar uzunluklarının sıralaması ile iç açılarının ölçüleri arasındaki sıralamanın ilişkilendirilmesi)
   - Bir üçgende uzun kenar karşısında daha büyük açının olduğunun ispatının yapılması
   - Bir üçgende üçgen eşitsizliğinin ispatının yapılması

3. **Ders (45 dakika)**:
   - Üçgende kenar-açı ilişkileriyle ilgili uygulamaların yapılması
   - Bir üçgende bir iç açının dik açı, dar açı veya geniş açı olmasına göre kenarlar arasındaki ilişkinin bulunması

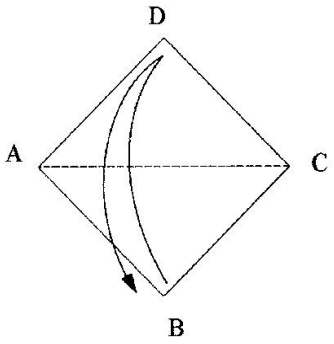
2. Hafta 1. Ders: Kuğu Yapımı

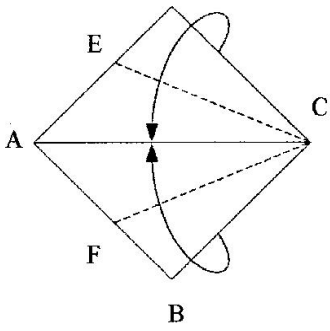
Matematiksel hedefler: Öğrenciler kuğu yapımında

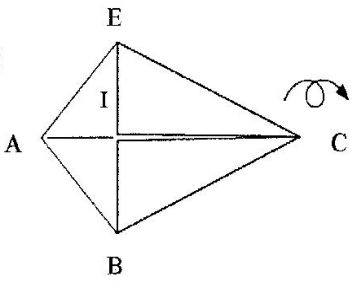
- Oluşan geometrik şekilleri çizebileceklerdir.
- Oluşan üçgenlerin açılarının ölçülerini açıölçer kullanmadan yazabileceklerdir.
- Kuğu modelindeki üçgenler için kenar-açı ilişkileri bulabileceklerdir.

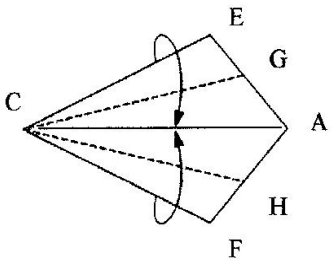
Ders Akışı

Öğretmen kuğu yapımı için adımları teker teker göstererek öğrencilerin kuğu yapmalarını ister (20-25 dakika). Sonra yaptıkları kuğu modelini tamamen açmaları istenir. Kuğunun ilk adımı örnek amaçlı öğrencilerle beraber çizilebilir. 2., 3. ve 4. adımları öğrencilerin kendilerinin çizmeleri istenir (10-15 dakika).



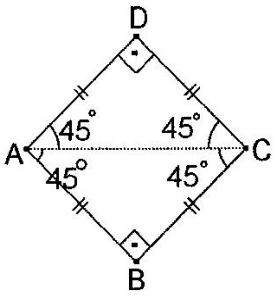






Yanlış çizimler olabileceği için öğrenciler çizimlerini bitirdikten sonra öğretmen doğru çizimleri tahtaya çizer. Asıl diyagramlar dağıtılır. Öğrencilerin kendi çizimleriyle dağıtılan şekilleri karşılaştırmaları ve geometrik açıdan bu şekilleri incelemeleri için onlara birkaç dakika süre tanınır.

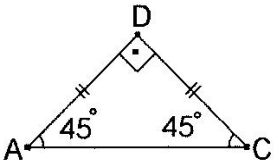
1. adım için:



Öğretmen, öğrencilere "ADC (veya ABC) üçgeninin kenarları ve açıları arasında nasıl bir ilişki vardır?" diye sorar. Bu ilişkileri keşfetmeleri için en az 5 dakika süre tanır ve buna yardımcı olmak için şu soruları sorar:

- "ADC üçgeninin iç açılarının ölçülerini karşılaştırınız."
- "ADC üçgeninin kenar uzunluklarını karşılaştırınız."

ADC üçgeninin iç açılarının ölçülerinin karşılaştırılması:



En başta kâğıdımız kareydi (ABCD) ve karenin her bir iç açısının ölçüsü 90°dir. Bu yüzden

$m(\hat{A}) = m(\hat{B}) = m(\hat{C}) = m(\hat{D}) = 90^\circ$

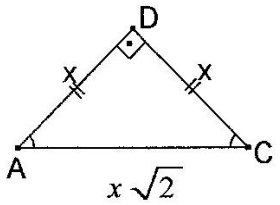
Ayrıca ABCD karesi [AC] köşegeninden yarıya katlandığı ve ADC üçgeni, ABC üçgeninin tam üstüne katlandığı için ADC ve ABC eş üçgenler olur. Buradan;

$m(D\hat{A}C) = m(C\hat{A}B) = \frac{90^\circ}{2} = 45^\circ$

$m(D\hat{C}A) = m(A\hat{C}B) = \frac{90^\circ}{2} = 45^\circ$ elde edilir. (Aynı zamanda [AC], $\hat{A}$ ve $\hat{C}$ açıları için açıortaydır.) Buna göre;

$m(D\hat{A}C) = m(D\hat{C}A) = 45^\circ < m(\hat{D}) = 90^\circ$

ADC üçgeninin kenar uzunluklarının karşılaştırılması:



ADC dik üçgeninde Pisagor Bağıntısı'ndan

$|AC|^2 = |AD|^2 + |DC|^2$

$|AD| = |DC| = x$ ise $|AC|^2 = x^2 + x^2 = 2x^2 \Rightarrow |AC| = x\sqrt{2}$

$\Rightarrow |AD| = |DC| < |AC|$

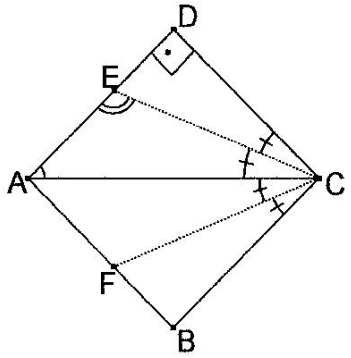
Öğretmen, "ADC üçgeninin kenar uzunlukları arasındaki sıralama ile iç açıların ölçüleri arasındaki sıralamayı nasıl ilişkilendirirsiniz?" sorusunu öğrencilere sorar.

- $|AD| = |DC| \Rightarrow m(D\hat{C}A) = m(D\hat{A}C)$ (Bir üçgende eş kenarlar karşısındaki açılar da eştir.)
- $|DC| < |AC| \Rightarrow m(D\hat{A}C) = m(\hat{D})$ (Bir üçgende uzun kenar karşında büyük açı vardır.)

$\Rightarrow |AD| = |DC| < |AC| \Rightarrow m(D\hat{A}C) = m(D\hat{C}A) < m(\hat{D})$

"O zaman bir dik üçgende en uzun kenar hangisidir?" sorusu öğrencilere yöneltilir ve öğrencilerden "Dik açıyı gören kenar yani hipotenüstür" yanıtı beklenmektedir.

2. adım için:



"2. adımda oluşan kat izi yani [EC], ADC üçgeninde hangi yardımcı elemana karşılık gelmektedir?" sorusu öğrencilere sorularak "açıortay" yanıtı öğrencilerden beklenmektedir. Bundan sonra öğrencilerden AEC üçgeninin iç açılarının ölçülerini karşılaştırmaları istenir.

[EC] açıortay olduğundan $m(A\hat{C}E) = m(E\hat{C}D) = \frac{m(A\hat{C}D)}{2} = \frac{45^\circ}{2} = 22,5^\circ$

Ayrıca 1. adımda $m(D\hat{A}C) = 45^\circ$ bulunmuştu.

AEC üçgeninde iç açıların ölçüleri toplamından $m(A\hat{E}C) + 45^\circ + 22,5^\circ = 180^\circ$ ve

$m(A\hat{E}C) = 112,5^\circ$

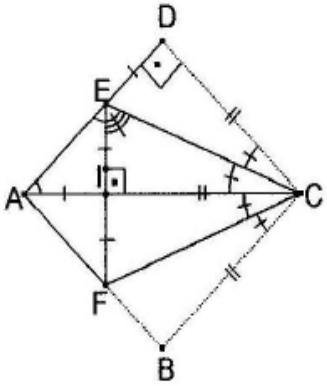
$\therefore m(A\hat{E}C) = 112,5^\circ > m(E\hat{A}C) = 45^\circ > m(E\hat{C}A) = 22,5^\circ$

2. Hafta 2. Ders: Kuğu Yapımının Geometrik Açıdan İncelenmesi

Matematiksel hedefler: Öğrenciler

- Oluşan üçgenlerin açılarının ölçülerini açıölçer kullanmadan yazabileceklerdir.
- Kuğu modelinde 3. adımdaki üçgenlerin iç açılarının ölçülerini açıölçer kullanmadan karşılaştırabileceklerdir.
- Kuğu modelinde 3. adımdaki üçgenlerin kenar uzunluklarını karşılaştırabileceklerdir.
- Kuğu modelinde 3. adımdaki üçgenler için kenar-açı ilişkilerini bulabileceklerdir.
- Bir üçgende uzun kenar karşısında daha büyük açının olduğunu gösterebileceklerdir.
- Bir üçgende üçgen eşitsizliğini gösterebileceklerdir.

3. adım için:



Öğretmen, öğrencilerden önce AIE ve EIC üçgeninin açı ölçülerini bulmalarını ister.
Geçen ders, 2. adım için AEC üçgeninin açı ölçüleri bulunmuştu.
$m(E\hat{A}C) = 45^\circ, m(E\hat{C}A) = 22,5^\circ, m(A\hat{E}C) = 112,5^\circ$
EDC üçgeni, EIC üçgeni üzerine katlandığı ve bu üçgenler tam üst üste geldikleri için EDC ve EIC eş üçgenlerdir. Buna göre, $m(E\hat{D}C) = m(E\hat{I}C) = 90^\circ$
EIC üçgeninde iç açıların ölçüleri toplamından;
$m(I\hat{E}C) + 90^\circ + 22,5^\circ = 180^\circ \Rightarrow m(I\hat{E}C) = 67,5^\circ$

$m(A\hat{E}C) = m(A\hat{E}I) + m(I\hat{E}C) \Rightarrow 112,5^\circ = m(A\hat{E}I) + 67,5^\circ \Rightarrow m(A\hat{E}I) = 45^\circ$

Bundan sonra, öğretmen öğrencilerin AEC üçgeninin kenar uzunluklarını karşılaştırmalarını ister.

EDC üçgeni, EIC üçgeni üzerine katlandığı ve bu üçgenler tam üst üste geldikleri için EDC ve EIC eş üçgenlerdir ($E\overset{\Delta}{D}C \cong E\overset{\Delta}{I}C$). Buna göre $|IE| = |ED|$ (veya [EC] açıortayının kollarına inen dikmelerin eşitliğinden). Eş üçgenlerden, $|DC| = |IC|$ de elde edilir.
AIE ikizkenar dik üçgen olduğundan $|IA| = |IE|$ olur.

$|IA| = |IE| = |ED| = a$ ise AIE ikizkenar dik üçgeninde Pisagor Bağıntısı'ndan

$|AE|^2 = |AI|^2 + |IE|^2 = a^2 + a^2 = 2a^2 \Rightarrow |AE| = a\sqrt{2}$

$|AD| = |AE| + |ED| = a\sqrt{2} + a$ ve ADC ikizkenar üçgen olduğundan

$|AD| = |DC| = a + a\sqrt{2}$

$E\hat{D}C \cong E\hat{I}C$ olduğundan $|DC| = |IC| = a + a\sqrt{2}$ (veya açıortayın özelliğinden) bulunur.

Buna göre, $|AC| = |AI| + |IC| = a + (a + a\sqrt{2}) = 2a + a\sqrt{2} > |AE| = a\sqrt{2}$

$\Rightarrow |AC| > |AE|$

EIC dik üçgen olduğundan en uzun kenar hipotenüstür yani [EC]dir. Pisagor Bağıntısı'ndan;

$|EC|^2 = |IE|^2 + |IC|^2 = a^2 + (a + a\sqrt{2})^2 = a^2 + a^2 + 2\sqrt{2}.a^2 + 2a^2 = a^2(4 + 2\sqrt{2})$

$\Rightarrow |EC| = a\sqrt{4 + 2\sqrt{2}}$

$|EC| > |IC| = a + a\sqrt{2} > |EA| = a\sqrt{2} \Rightarrow |EC| > |EA|$

$|AC| = 2a + a\sqrt{2} = a.(2 + \sqrt{2})$

$|EC| = a.\sqrt{4 + 2\sqrt{2}}$

$|EA| = a\sqrt{2}$

$\therefore |AC| > |EC| > |EA|$ (EAC üçgeninin kenar uzunluklarının sıralaması)

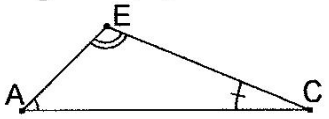
Bunun ardından, öğretmen öğrencilere "AEC üçgeni için kenarlar ve açılar arasında nasıl bir ilişkiye ulaştınız?" diye sorar.

$|AC| > |EC| > |EA| \Leftrightarrow m(A\hat{E}C) > m(E\hat{A}C) > m(E\hat{C}A)$

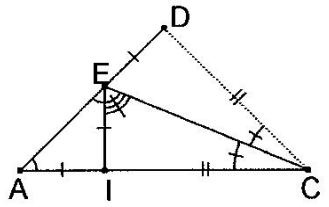
Bir üçgenin iç açıları arasındaki sıralama, bu açıları gören kenarlar arasındaki gibidir.

Bundan sonra öğretmen, öğrencilere bir üçgende uzun kenarın karşısındaki açının daha büyük olduğunun ispatını kuğu yapımındaki kâğıt katlamayla ilişkilendirerek verir.

Öğretmen, "Şimdi elde ettiğimiz bu sonucu genelleyelim" diyerek ispata başlar.



(Kuğu yapımındaki gibi) Çeşitkenar bir AEC üçgeni alalım ve bu üçgenin bir kenarı diğerinden daha uzun olsun.



Verilen: AEC üçgeni; $|AC| > |EC|$

İstenen: $m(A\hat{E}C) > m(E\hat{A}C)$

Kuğuyu katlarken yaptığımız adımlardan (özellikle 3. adımdan) yararlanalım.

$|EI| = |AI|$ olacak şekilde [AC] üzerinde bir I noktası alalım.

$m(E\hat{A}I) = m(A\hat{E}I) = \alpha, m(I\hat{E}C) = \beta$ olsun.

Buna göre, $m(A\hat{E}C) = m(A\hat{E}I) + m(I\hat{E}C) = \alpha + \beta$ ve $m(E\hat{A}C) = \alpha$

$\alpha + \beta > \alpha$ ($\alpha > 0, \beta > 0$ olduğundan ve eşitsizliğin özelliğinden)

$\Rightarrow m(A\hat{E}C) > m(E\hat{A}C)$

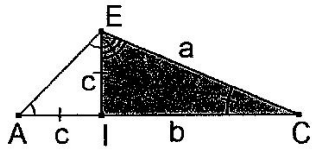
Bu ifade, bir üçgende uzun kenarın karşısındaki açının ölçüsünün üçgenin diğer iç açılarından daha büyük olduğunu söylemektedir (uzun kenar $\Rightarrow$ büyük açı).

Not: Bu ifadenin karşıtı da doğrudur (Büyük açı $\Rightarrow$ uzun kenar).

Sonuç olarak, bir üçgende kenar uzunlukları arasındaki sıralama, bu kenarları gören iç açı ölçülerinin arasındaki sıralama gibidir.

Üçgen Eşitsizliğinin İspatı

Öğretmen, öğrencilere üçgen eşitsizliğinin ispatını kuğu yapımındaki kâğıt katlamayla ilişkilendirerek gösterir. Bunu yaparken kuğu yapımındaki özellikle 3. adımdaki (veya 4. adımdaki) üçgenlerden yararlanıldığı öğrencilere söylenir.



Verilen: EIC üçgeni; $|EI| = c, |IC| = b, |EC| = a$

İstenen: $|b - c| < a < b+c$

[IC] kenarının uzantısı üzerinde $|AI| = |EI|$ olacak şekilde bir A noktası alalım.

$m(E\hat{A}C) = \alpha, m(I\hat{E}C) = \beta$ olsun.

AEC üçgeninde $m(A\hat{E}C) = \alpha + \beta > m(E\hat{A}C) = \alpha \Rightarrow |AC| > |EC|$

$\Rightarrow$ b+c > a .....(I)

Benzer yöntemle, b< a+c ve c< a+b bulunur. Buradan; b-c< a ve c-b< a elde edilir. Böylece, $|b - c| < a$ .....(II) bulunur.

(I) ve (II) eşitsizliklerinden, $|b - c| < a < b + c$ elde edilir.

Benzer şekilde $|a - c| < b < a + c$ ve $|a - b| < c < a + b$ bulunur.

2. Hafta 3. Ders: Üçgende Açı-Kenar Bağıntıları ile İlgili Uygulamalar Yapılması

Matematiksel hedefler: Öğrenciler

- Bir üçgende bir iç açının dik açı, dar açı veya geniş açı olmasına göre kenarlar arasındaki ilişkiyi gösterebileceklerdir.
- Üçgende kenar-açı ilişkileriyle ilgili uygulamalar yapabileceklerdir.

Öğretmen, önce öğrencilere bir üçgende bir iç açının dik açı, dar açı veya geniş açı olmasına göre kenarlar arasındaki ilişkinin nasıl değiştiğini sorar. Öğrencilerin yanıtlarından sonra bu ilişki öğretmen tarafından verilir:

Bir ABC üçgeninde $|BC| = a, |AC| = b, |AB| = c$ olmak üzere;

- $m(\hat{A}) = 90^\circ \Rightarrow a^2 = b^2 + c^2$ (Pisagor Teoremi)
- $m(\hat{A}) < 90^\circ \Rightarrow a^2 < b^2 + c^2$
- $m(\hat{A}) > 90^\circ \Rightarrow a^2 > b^2 + c^2$

Bu ifadenin ispatı, öğrencilere derste verilmeyip onlara ödev olarak verilebilir. İspat için ipucu olarak Kosinüs Teoremi'ni kullanmaları söylenebilir.

İspat: Herhangi bir ABC üçgeni için $|BC| = a, |AC| = b, |AB| = c$ ve $m(\hat{A}) = \alpha$ olmak üzere; Kosinüs Teoremi'nden $a^2 = b^2 + c^2 - 2bc.\cos\alpha$ elde edilir.

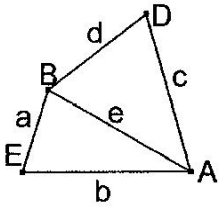
Durum 1: $\alpha = 90^\circ \Rightarrow \cos 90^\circ = 0 \Rightarrow a^2 = b^2 + c^2$ (Pisagor Teoremi)

Durum 2: $\alpha < 90^\circ \Rightarrow \cos\alpha > 0 \Rightarrow a^2 = b^2 + c^2 - 2\underbrace{bc}_{>0}.\underbrace{\cos\alpha}_{>0} \Rightarrow a^2 < b^2 + c^2$

Durum 3: $\alpha > 90^\circ \Rightarrow \cos\alpha < 0 \Rightarrow a^2 = b^2 + c^2 \underbrace{-2\underbrace{bc}_{>0}.\underbrace{\cos\alpha}_{<0}}_{>0} \Rightarrow a^2 > b^2 + c^2$

Bundan sonra üçgende kenar-açı bağıntıları ile ilgili uygulamalara yer verilir:

Soru 1:



Yandaki şekilde EBA ve BAD üçgenleri için $m(\hat{E}) = 62^\circ, m(E\hat{A}B) = 58^\circ, m(B\hat{A}D) = 60^\circ, m(\hat{D}) = 59^\circ$ ve $|EA| = b, |EB| = a, |AD| = c, |BD| = d, |AB| = e$ verildiğine göre bu kenar uzunlukları arasındaki sıralamayı yazınız.

Çözüm 1: EBA üçgeninde iç açıların ölçüleri toplamından $m(E\hat{B}A) = 60^\circ$ ve ABD üçgeninde iç açıların ölçüleri toplamından $m(A\hat{B}D) = 61^\circ$ bulunur.

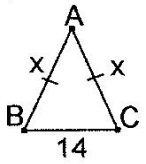
EBA üçgeninde; $62^\circ > 60^\circ > 58^\circ \Rightarrow e > b > a$

BAD üçgeninde; $61^\circ > 60^\circ > 59^\circ \Rightarrow c > d > e$

$\therefore c > d > e > b > a$

Soru 2: Bir ikizkenar üçgenin taban uzunluğu 14 cm ise eş kenarlardan birinin uzunluğu hakkında neler söyleyebilirsiniz?

Çözüm 2:



Şekildeki ABC üçgeninde üçgen eşitsizliğinden

$|x - x| < 14 < x + x \Rightarrow 0 < 14 < 2x \Rightarrow 7 < x$

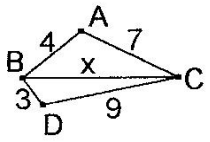
Soru 3: Bir üçgenin aşağıda verilen kenar uzunluklarına sahip olması mümkün müdür? Neden?

a) 15,10,5
b) 17,17,0.08
c) 8,8,21
d) 9,9,18.1
e) 0.7,0.4,1

Çözüm 3:

a) Hayır; çünkü 15=15+5 üçgen eşitsizliği ile çelişmektedir.
b) Evet; çünkü kenar uzunlukları arasındaki ilişkiler, üçgen eşitsizliğini sağlar. $|17 - 17| < 0.08 < 17 + 17$ ve $|17 - 0.08| < 17 < 17 + 0.08$
c) Hayır; çünkü 8+8<21 eşitsizliği, üçgen eşitsizliği ile çelişir.
d) Hayır; çünkü 9+9<18.1 eşitsizliği, üçgen eşitsizliği ile çelişir.
e) Evet; çünkü kenar uzunlukları arasındaki ilişkiler, üçgen eşitsizliğini sağlar. $|0.7 - 0.4| < 1 < 0.7 + 0.4$ ve $|1 - 0.7| < 0.4 < 1 + 0.7$ ve $|1 - 0.4| < 0.7 < 1 + 0.4$

Soru 4:

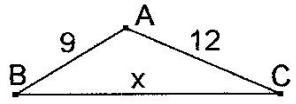


Yandaki şekilde verilenlere göre, $|BC| = x$ uzunluğunun alabileceği değer aralığını bulunuz.

Çözüm 4: ABC üçgeninde üçgen eşitsizliğinden; $|7 - 4| < x < 7 + 4 \Rightarrow 3<x<11$ ve BDC üçgeninde üçgen eşitsizliğinden; $|9 - 3| < x < 9 + 3 \Rightarrow 6<x<12$

$\therefore 6<x<11$

Soru 5:



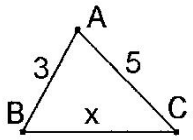
Yandaki şekilde $m(\hat{A}) > 90^\circ$, $|AB| = 9$ birim, $|AC| = 12$ birim ise $|BC| = x$ uzunluğunun alabileceği değer aralığını bulunuz.

Çözüm 5: $m(\hat{A}) > 90^\circ \Rightarrow x^2 > 9^2 + 12^2 \Rightarrow x^2 > 225 \Rightarrow x > 15$ .....(I)

ABC üçgeninde üçgen eşitsizliğine göre $|12-9| < x < 9+12 \Rightarrow 3 < x < 21$ ...(II)

(I) ve (II) den 15<x<21 Çözüm Kümesi: (15,21)

Soru 6:



Yandaki şekilde $m(\hat{A}) < 90^\circ$, $|AB| = 3$ birim, $|AC| = 5$ birim ise $|BC| = x$ uzunluğunun alabileceği en küçük ve en büyük tam sayı değerlerini bulunuz.

Çözüm 6: $m(\hat{A}) < 90^\circ \Rightarrow x^2 < 3^2 + 5^2 \Rightarrow x^2 < 34 \Rightarrow x < \sqrt{34}$ ....(I)

ABC üçgeninde üçgen eşitsizliğine göre $|5-3| < x < 5+3 \Rightarrow 2 < x < 8$ ...(II)

(I) ve (II) den 2<x< $\sqrt{34}$ Buna göre, en küçük x tam sayı değeri 3 ve en büyük x tam sayı değeri 5 olur.

APPENDIX H: SAMPLE TRADIONAL INSTRUCTIONAL LESSON PLAN

Kontrol Grubu 2. Hafta Ders Planı

Ünite : Üçgenler

Sınıf : 10

Konu : Üçgende kenar-açı ilişkileri

Kazanım: Üçgenin kenarları ve açıları arasındaki ilişkileri ispatlar, uygulamalar yapar.

Malzemeler: Ders kitabı, kareli defter, kalem

Ders Akışı:

1. Ders (45 dakika):

- Bir üçgende uzun kenar karşısında daha büyük açının olduğunun ispatının yapılması
- Bir üçgende üçgen eşitsizliğinin ispatının yapılması

2. Ders (45 dakika):

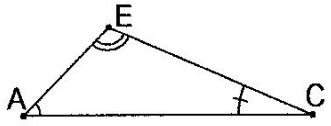
- Üçgende kenar-açı ilişkileriyle ilgili uygulamaların yapılması
- Bir üçgende bir iç açının dik açı, dar açı veya geniş açı olmasına göre kenarlar arasındaki ilişkinin bulunması

Ders 1:

Matematiksel hedefler: Öğrenciler

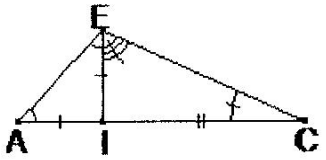
- Bir üçgende uzun kenar karşısında daha büyük açının olduğunu gösterebileceklerdir.
- Bir üçgende üçgen eşitsizliğini gösterebileceklerdir.

Öğretmen, öğrencilere bu derste üçgenin kenarları ve açıları arasındaki ilişkileri inceleyeceklerini söyler. Öğretmen, çeşitkenar bir üçgen için üçgenin kenar uzunlukları ve açı ölçüleri arasında nasıl bir ilişki olabileceğini sınıfa sorar. Öğrencilerin yanıtları dinlendikten sonra, üçgenin kenar uzunlukları arasındaki ilişki verildiğinde açıları arasındaki ilişkiyi elde ettiğimiz teoremin ispatı öğrencilerle beraber yapılır. İspattaki adımların nerden elde edildiği sorularak ispat tamamlanır.



Verilen: AEC üçgeni; $|AC| > |EC|$

İstenen: $m(A\hat{E}C) > m(E\hat{A}C)$



$|EI| = |AI|$ olacak şekilde [AC] üzerinde bir I noktası alalım.

$m(E\hat{A}I) = m(A\hat{E}I) = \alpha, m(I\hat{E}C) = \beta$ olsun.

Buna göre, $m(A\hat{E}C) = m(A\hat{E}I) + m(I\hat{E}C) = \alpha + \beta$ ve $m(E\hat{A}C) = \alpha$

$\alpha + \beta > \alpha$ ($\alpha > 0, \beta > 0$ olduğundan ve eşitsizliğin özelliğinden)

$\Rightarrow m(A\hat{E}C) > m(E\hat{A}C)$

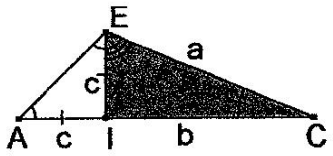
Bu ifade, bir üçgende uzun kenarın karşısındaki açının ölçüsünün üçgenin diğer iç açılarından daha büyük olduğunu söylemektedir (uzun kenar $\Rightarrow$ büyük açı).

Not: Bu ifadenin karşıtı da doğrudur (Büyük açı $\Rightarrow$ uzun kenar).

Sonuç olarak, bir üçgende kenar uzunlukları arasındaki sıralama, bu kenarları gören iç açı ölçülerinin arasındaki sıralama gibidir.

Üçgen Eşitsizliğinin İspatı

Üçgen eşitsizliğinin ispatı, bir önceki teoremle bağlantı kurularak ve ispatın adımlarında öğrencilere bu adımların gerekçeleri sorularak verilir.



Verilen: EIC üçgeni; $|EI| = c, |IC| = b, |EC| = a$

İstenen: $|b - c| < a < b+c$

[IC] kenarının uzantısı üzerinde $|AI| = |EI|$ olacak şekilde bir A noktası alalım.

$m(E\hat{A}C) = \alpha, m(I\hat{E}C) = \beta$ olsun.

AEC üçgeninde $m(A\hat{E}C) = \alpha + \beta > m(E\hat{A}C) = \alpha \Rightarrow |AC| > |EC|$

$\Rightarrow$ b+c > a .....(I)

Benzer yöntemle, b< a+c ve c< a+b bulunur. (Bu kısımları öğrencilerin çıkarması beklenir). Buradan; b-c< a ve c-b< a elde edilir. Böylece, $|b - c| < a$ .....(II) bulunur.

(I) ve (II) eşitsizliklerinden, $|b - c| < a < b + c$ elde edilir.

Benzer şekilde $|a - c| < b < a + c$ ve $|a - b| < c < a + b$ bulunur.

Ders 2:

Matematiksel hedefler: Öğrenciler

- Bir üçgende bir iç açının dik açı, dar açı veya geniş açı olmasına göre kenarlar arasındaki ilişkiyi gösterebileceklerdir.
- Üçgende kenar-açı ilişkileriyle ilgili uygulamalar yapabileceklerdir.

Öğretmen, önce öğrencilere bir üçgende bir iç açının dik açı, dar açı veya geniş açı olmasına göre kenarlar arasındaki ilişkinin nasıl değiştiğini sorar. Öğrencilerin yanıtlarından sonra bu ilişki öğretmen tarafından verilir:

Bir ABC üçgeninde $|BC| = a, |AC| = b, |AB| = c$ olmak üzere;

- $m(\hat{A}) = 90^\circ \Rightarrow a^2 = b^2 + c^2$ (Pisagor Teoremi)
- $m(\hat{A}) < 90^\circ \Rightarrow a^2 < b^2 + c^2$
- $m(\hat{A}) > 90^\circ \Rightarrow a^2 > b^2 + c^2$

Bu ifadenin ispatı, öğrencilere derste verilmeyip onlara ödev olarak verilebilir. Ispat için ipucu olarak Kosinüs Teoremi'ni kullanmaları söylenebilir.

İspat: Herhangi bir ABC üçgeni için $|BC| = a, |AC| = b, |AB| = c$ ve $m(\hat{A}) = \alpha$ olmak üzere; Kosinüs Teoremi'nden $a^2 = b^2 + c^2 - 2bc.\cos\alpha$ elde edilir.

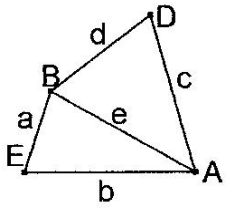
Durum 1: $\alpha = 90^\circ \Rightarrow \cos 90^\circ = 0 \Rightarrow a^2 = b^2 + c^2$ (Pisagor Teoremi)

Durum 2: $\alpha < 90^\circ \Rightarrow \cos\alpha > 0 \Rightarrow a^2 = b^2 + c^2 - 2\underbrace{bc}_{>0}.\underbrace{\cos\alpha}_{>0} \Rightarrow a^2 < b^2 + c^2$

Durum 3: $\alpha > 90^\circ \Rightarrow \cos\alpha < 0 \Rightarrow a^2 = b^2 + c^2 \underbrace{-2\underbrace{bc}_{>0}.\underbrace{\cos\alpha}_{<0}}_{>0} \Rightarrow a^2 > b^2 + c^2$

Bundan sonra üçgende kenar-açı bağıntıları ile ilgili uygulamalara yer verilir:

Soru 1:



Yandaki şekilde EBA ve BAD üçgenleri için

$m(\hat{E}) = 62^\circ, m(E\hat{A}B) = 58^\circ, m(B\hat{A}D) = 60^\circ, m(\hat{D}) = 59^\circ$ ve

$|EA| = b, |EB| = a, |AD| = c, |BD| = d, |AB| = e$

verildiğine göre bu kenar uzunlukları arasındaki sıralamayı yazınız.

Çözüm 1: EBA üçgeninde iç açıların ölçüleri toplamından $m(E\hat{B}A) = 60^\circ$ ve ABD üçgeninde iç açıların ölçüleri toplamından $m(A\hat{B}D) = 61^\circ$ bulunur.

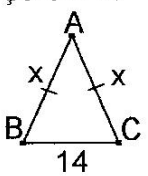
EBA üçgeninde; $62^\circ > 60^\circ > 58^\circ \Rightarrow e > b > a$

BAD üçgeninde; $61^\circ > 60^\circ > 59^\circ \Rightarrow c > d > e$

$\therefore c > d > e > b > a$

Soru 2: Bir ikizkenar üçgenin taban uzunluğu 14 cm ise eş kenarlardan birinin uzunluğu hakkında neler söyleyebilirsiniz?

Çözüm 2:



Şekildeki ABC üçgeninde üçgen eşitsizliğinden

$|x - x| < 14 < x + x \Rightarrow 0 < 14 < 2x \Rightarrow 7 < x$

Soru 3: Bir üçgenin aşağıda verilen kenar uzunluklarına sahip olması mümkün müdür? Neden?

a) 15,10,5
b) 17,17,0.08
c) 8,8,21
d) 9,9,18.1
e) 0.7,0.4,1

Çözüm 3:

a) Hayır; çünkü 15=15+5 üçgen eşitsizliği ile çelişmektedir.

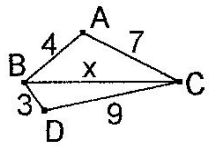
b) Evet; çünkü kenar uzunlukları arasındaki ilişkiler, üçgen eşitsizliğini sağlar. $|17-17|<0.08<17+17$ ve $|17-0.08|<17<17+0.08$

c) Hayır; çünkü 8+8<21 eşitsizliği, üçgen eşitsizliği ile çelişir.

d) Hayır; çünkü 9+9<18.1 eşitsizliği, üçgen eşitsizliği ile çelişir.

e) Evet; çünkü kenar uzunlukları arasındaki ilişkiler, üçgen eşitsizliğini sağlar. $|0.7-0.4|<1<0.7+0.4$ ve $|1-0.7|<0.4<1+0.7$ ve $|1-0.4|<0.7<1+0.4$

Soru 4:

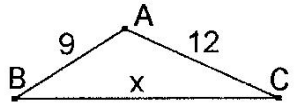


Yandaki şekilde verilenlere göre, $|BC|=x$ uzunluğunun alabileceği değer aralığını bulunuz.

Çözüm 4: ABC üçgeninde üçgen eşitsizliğinden; $|7-4|<x<7+4 \Rightarrow 3<x<11$ ve BDC üçgeninde üçgen eşitsizliğinden; $|9-3|<x<9+3 \Rightarrow 6<x<12$

$\therefore 6<x<11$

Soru 5:



Yandaki şekilde $m(\hat{A})>90^{\circ}$, $|AB|=9$ birim, $|AC|=12$ birim ise $|BC|=x$ uzunluğunun alabileceği değer aralığını bulunuz.

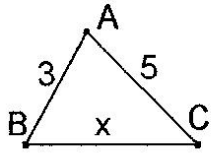
Çözüm 5: $m(\hat{A}) > 90^\circ \Rightarrow x^2 > 9^2 + 12^2 \Rightarrow x^2 > 225 \Rightarrow x > 15 \ldots$(I)

ABC üçgeninde üçgen eşitsizliğine göre

$|12-9| < x < 9+12 \Rightarrow 3 < x < 21 \ldots$(II)

(I) ve (II) den 15<x<21 Çözüm Kümesi: (15,21)

Soru 6:



Yandaki şekilde $m(\hat{A}) < 90^\circ$, $|AB| = 3$ birim, $|AC| = 5$ birim ise $|BC| = x$ uzunluğunun alabileceği en küçük ve en büyük tam sayı değerlerini bulunuz.

Çözüm 6: $m(\hat{A}) < 90^\circ \Rightarrow x^2 < 3^2 + 5^2 \Rightarrow x^2 < 34 \Rightarrow x < \sqrt{34} \ldots$(I)

ABC üçgeninde üçgen eşitsizliğine göre

$|5-3| < x < 5+3 \Rightarrow 2 < x < 8 \ldots$(II)

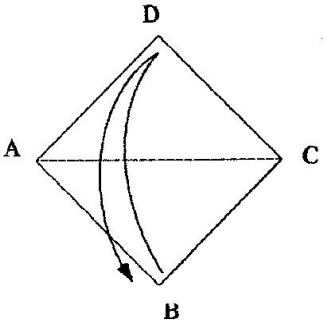
(I) ve (II) den 2<x<$\sqrt{34}$

Buna göre, en küçük x tam sayı değeri 3 ve en büyük x tam sayı değeri 5 olur.

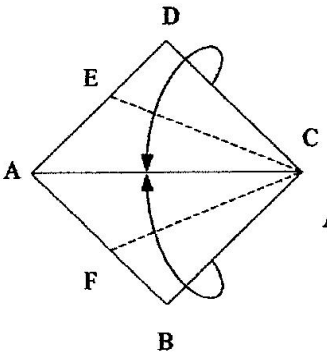
Sevil ARICI
Matematik Öğretmeni

APPENDIX I: SAMPLE ORIGAMI DIAGRAM (SWAN)

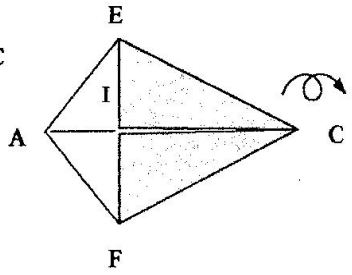
KUĞU



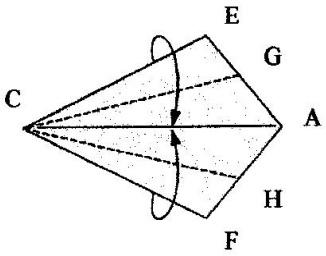
1. Kare bir kâğıdı, beyaz tarafı yukarı olacak şekilde köşegeninden yarıya katlayınız.



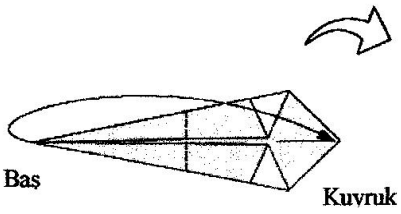
2. Ardışık iki kenarı, şekildeki gibi ortaya doğru katlayınız.



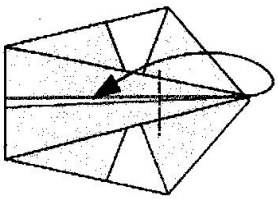
3. Arkasını çeviriniz.



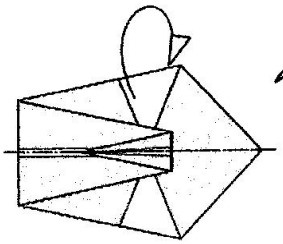
4. Şekildeki gibi ortaya doğru katlayınız.



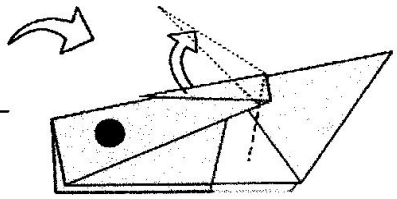
5. Baş kısmı kuyruğa getiriniz.



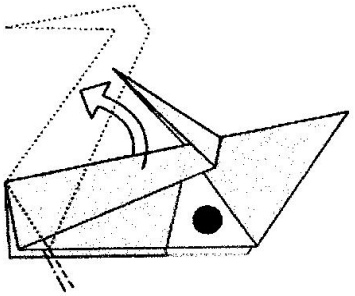
6. Başı, şekildeki gibi kat yerinden katlayınız.



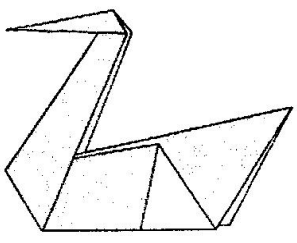
7. Ortadan arkaya doğru katlayınız.



8. Siyah noktadan tutarak başı çekiniz. Başın arka kısmını yeni kat izi oluşturacak şekilde bastırınız.



9. Siyah noktadan tutarak boynu yukarıya çekiniz. Şekildeki gibi yeni kat izi oluşturacak biçimde gövdeyi katlayınız.



10. Kuğu artık hazır!

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