SPACE-TIME-POLARIZATION DIVERSITY IN MULTIPLE INPUT MULTIPLE OUTPUT COMMUNICATION SYSTEMS

by

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ABSTRACT

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In wireless communications systems, diversity techniques are widely used to combat the effects of wireless fading channels. Multiple-input multiple-output (MIMO) systems have better performance under fading channels than single-input single-output (SISO) systems. In most scattering environments, antenna diversity is a practical, effective and a widely used technique for reducing the effect of multipath fading.

Previous analyses of MIMO systems concentrate on uncorrelated antennas. However, antenna spacing and properties of scattering environment can create correlation between channel coefficients that affects the system performance remarkably. The use of dual-polarized antennas (*polarization diversity*) is a cost and space effective alternative where two spatially separated uni-polarized antennas are replaced by a single antenna structure employing orthogonal polarizations. This work presents a comprehensive analysis of MIMO systems under polarization diversity. Pairwise error probability (PEP) expressions for quasi-static fading under Rayleigh channel conditions are calculated. Then these PEP expressions are used to calculate union bounds on the performance of orthogonal space-time block code for 4Tx-2Rx MIMO system. Using polarization diversity leads to performance loss, but 2Tx-1Rx (dual polarized) MIMO system reduces to 4Tx-1Rx MIMO system which gives us advantage of having one receive uni-polarized antenna at the receiver instead of having one dual polarized antenna. Iterative detection and decoding technique for MIMO systems are focused and it is applied to 2Tx-1Rx (dual polarized) MIMO system.

ÖZET

ÇOK GİRDİLİ ÇOK ÇIKTILI HABERLEŞME SİSTEMLERİNDE UZAY-ZAMAN-KUTUPLANMA ÇEŞİTLİLİĞİ

Sönümlemeli kanallarda çok girdili çok çıkışlı sistemlerin başarımları tek girdili tek çıktılı sistemlere göre daha iyidir. Anten çeşitliliği saçılımlı telsiz iletişim ortamında sönümlemenin etkisini azaltmak için kullanılan etkili bir yöntemdir.

Çok girdili çok çıktılı sistemler üzerindeki analizlerin çoğu ilintisiz antenler düşünülerek yapılmıştır. Bununla birlikte, antenler arası uzaklık ve ortamın saçılım özellikleri, analizi yapılan iletişim ortamını karakterize eden katsayılar arasında ilinti oluşturabilir. Verici ve alıcı antenler arasında ilintisiz kanallar elde edilmek istenirse, antenler arasında yeterince açıklık bırakılmalıdır.

Bu çalışma ilk olarak çok girdili çok çıktılı sistemlerin ilintisiz sönümlemeli ortamdaki analizini sunar. Tezde, Rayleigh kanal koşulu altında hata olasığını çıkartıldı ve bu hata olasığını dört verici ve iki alıcı antenden oluşan çok girdili çok çıktılı sistemde kullanılan dikgen uzay-zaman blok kodlarının başarımının üst sınırını bulmak için kullanıldı. Ayrıca, kutuplanma-çeşitliliği durumunda iki verici - bir alıcı antenden ve iki verici - iki alıcı antenden oluşan çok girdili çok çıktılı sistemlerde kullanılan Alamouti'nin yönteminin başarımı ile iki verici - bir alıcı antenden oluşan sistemde kullanılan dikgen uzay-zaman blok kodlarının başarımını karşılaştırıldı. Genel olarak, kutuplanma çeşitliliği başarım kaybına neden olmaktadır. Bununla birlikte, alıcı ve vericide kutuplanma yönlerinin tam olarak ayrıldığı durumda iki verici - bir alıcı antenden oluşan çok girdili çok çıkışlı sistem dört verici - bir alıcı antenden oluşan çok girdili çok çıktılı sisteme dönüşmektedir.

TABLE OF CONTENTS

ACKNOWLEDGEMENTS iii
ABSTRACT iv
ÖZET v
LIST OF FIGURES viii
LIST OF TABLES xi
LIST OF SYMBOLS/ABBREVIATIONS xii
1. INTRODUCTION 1
1.1. Diversity Techniques 1
1.2. MIMO Communication System 2
1.3. Space-time Signaling 4
1.4. Literature Review 5
1.5. Scope of the Thesis 6
1.6. Outline of the Thesis 8
2. SPACE-TIME CODES 9
2.1. Multiple-Input Multiple-Output (MIMO) channels 9
2.2. Capacity of Multiple-Input Multiple-Output (MIMO) Communication . 10
2.3. Space-Time Coding 17
2.3.1. Space-Time Block Codes 21
2.3.1.1. Alamouti Space-Time Code 21
2.3.1.2. General OSTBC design 24
2.4. Decoding of STBC 28
2.5. Pairwise Error Probability Analysis 30
3. SPACE-TIME-POLARIZATION CODES 38
3.1. Capacity of MIMO Systems In The Presence Of Polarization Diversity 38
3.2. Performance of Alamouti Scheme In Case of Polarization Diversity . . 42
3.3. 4 Tx-2 Rx MIMO System Channel Model For Polarization Diversity . . 46
3.4. Pairwise Error Probability Analysis 50
4. ITERATIVE DETECTION AND DECODING OF SPACE-TIME-POLARIZATION CODES 59

4.1. MAP Receiver 59
5. CONCLUSIONS 69
REFERENCES 70

LIST OF FIGURES

Figure 2.1. Channel capacity curves for receive diversity on block Rayleigh fading channel with maximum ratio diversity combining 16

Figure 2.2. Channel capacity curves for receive diversity on block Rayleigh fading channel with selection diversity combining 17

Figure 2.3. Channel capacity curves for transmit diversity on block Rayleigh fading channel 18

Figure 2.4. Space - time coding diagram 19

Figure 2.5. The BER performance of Alamouti scheme for 2Tx-1Rx MIMO system in Rayleigh fading 36

Figure 2.6. Symbol error rate comparison of Alamouti scheme with 4Tx - 2Rx and 4Tx - 1Rx channel scenarios 37

Figure 3.1. Channel capacity curve as a function of receive correlation on block Rayleigh fading channel, for SNR=0 dB, $n_r = 2$,$n_t = 2$,$\alpha = 0.1$,$\alpha_f = 0$ and $K = 0$ 42

Figure 3.2. Symbol error rate for the Alamouti scheme as a function of SNR for varying K-factor. The estimate closely matches the actual symbol error rate (even for small SNR values) 44

Figure 3.3. Symbol error rate for the Alamouti scheme as a function of α for varying receive correlation. The use of dual-polarized antennas leads to performance loss 45

Figure 3.4. Symbol error rate for the Alamouti scheme as a function of α for varying transmit correlation. The use of dual-polarized antennas leads to performance loss 46

Figure 3.5. Symbol error rate for rate 1/2 OSTBC used in 4Tx-2Rx MIMO channel as a function of SNR (dB) 54

Figure 3.6. 4Tx-2Rx MIMO system (above, in case of full polarization diversity), 2Tx-1Rx MIMO system with dual polarized antennas (below) 55

Figure 3.7. Channel model in case of full polarization diversity (received signals) 56

Figure 3.8. Received signals from vertical polarization in each symbol period . 56

Figure 3.9. Received signals from horizontal polarization in each symbol period 57

Figure 3.10. Sent symbols in case of full polarization diversity 57

Figure 3.11. Symbol error rate for rate 1/2 OSTBC as a function of α for varying receive correlation. The use of dual-polarized antennas leads to performance loss. 58

Figure 3.12. Symbol error rate for rate 1/2 OSTBC as a function of α for varying transmit correlation. The use of dual-polarized antennas leads to performance loss 58

Figure 4.1. Transmitter, MIMO channel, and receiver with iterative detection and decoding. The subscript "1" denotes variables associated with inner code, and the subscript "2" denotes variables associated with the outer channel code. 61

Figure 4.2. The SER performance of comparison of ML and MAP receivers . . 66

Figure 4.3. The SER performance of comparison of ML and MAP receivers in case of correlation 66

Figure 4.4. The SER performance of comparison of ML and MAP receivers in case of high correlation 67

Figure 4.5. The SER performance of comparison of ML and MAP receivers in case of very high correlation 68

LIST OF TABLES

LIST OF SYMBOLS/ABBREVIATIONS

B	Bandwidth
d_h^2	Euclidean distance between two matrices
E	Expected value
h	Fading coefficient
$\mathbf{H}$	Channel matrix
$\|\cdot\|_F$	Frobenius norm
$\mathbf{n}$	Noise vector
n_r	Number of receive antennas
n_t	Number of transmit antennas
P	Total transmit power
r	Receive correlation coefficient
R	Code rate
$\mathbf{R}_{Tx}$	Transmit correlation matrix
$\mathbf{R}_{Rx}$	Receive correlation matrix
$\mathbf{t}$	Transmit correlation coefficient
$\mathbf{X}$	Space-time code transmission matrix
α	Polarization discrimination coefficient
η	Spectral efficiency
APP	A Posteriori Probability
AWGN	Additive White Gaussian Noise
BLAST	Bell Labs Layered Space Time
BER	Bit Error Rate
BPSK	Binary Phase Shift Keying
CSI	Channel State Information
exp	Exponential function
i.i.d.	Independent Identically Distributed
LLR	Log Likelihood ratio

LOS	Line Of Sight
MAP	Maximum A Posteriori
ML	Maximum Likelihood
MRC	Maximal Ratio Combining
MIMO	Multiple Input Multiple Output
MMSE	Minimum Mean Square Error
MSE	Mean Square Error
OSTBC	Orthogonal Space-Time Block Code
PEP	Pairwise Error Probability
XPD	Polarization Discrimination Factor
PD	Polarization Diversity
pdf	probability density function
QAM	Quadrature Amplitude Modulation
Rx	Receive
RD	Receive Diversity
SER	Symbol Error Rate
sgn	Sign function
SISO	Single Input Single Output
SNR	Signal to Noise Ratio
SVD	Singular Value Decomposition
STBC	Space-time Block Code
Tx	Transmit
TD	Transmit Diversity

1. INTRODUCTION

1.1. Diversity Techniques

Due to the new emerging technologies, demand for high data rates in wireless communications has increased. The available bandwidth or spectrum is often limited and the process of new spectrum allotment by the federal government is often slow. Also, devices should use as little power as possible to conserve battery life and keep the products small. Therefore, designers for wireless systems face a big challenge, to improve the performance and increase the data rates while keeping the same bandwidth and the power. The wireless channel is random and unpredictable, and in general channel error rates are poorer over a wireless channel than over a wired channel. In wireless channel, out-of-phase reception of multipath signals causes deep attenuation in the received signal, which is known as *fading*. The distortion induced by the time-varying fading is caused by the superposition of delayed, reflected, scattered and diffracted signal components. Instantaneous decrease in signal-to-noise ratio (SNR) results in error bursts which degrades the performance significantly.

Diversity techniques are widely used to combat the effects of wireless fading channels. The replicas of the same information bits are received in the receiver. It is less likely that all replicas of the same information will be in deep fading at the same time. Therefore, receiver can determine the transmitted bits based on less attenuated received signal which improves the performance of the system. The most commonly used diversity techniques [25] are

1. *Frequency Diversity* where signals are transmitted on different frequencies,
2. *Time Diversity* where signals are transmitted at different times,
3. *Space Diversity* where signals are transmitted and/or received by multiple antennas.
4. *Polarization Diversity* where signals are transmitted/received with different polarization directions of antennas,

5. *Angle Diversity* where signals are received by directional antennas.

All of the diversity techniques can be used in wireless communication systems to achieve the best performance, but not all diversity forms are applicable in every channel environment. For instance, frequency diversity can not be applied in frequency selective channels. The most obvious approach to diversity is the use of spatially separated antennas (*space diversity*). In a fading environment, the receiver cannot recover an information symbol affected by a deep fade, one can improve the performance (data rate) by providing the receiver with different replicas of the transmitted symbols which can be done with multi-receive or multi-transmit antennas with sufficient spacing between antennas. Therefore, *space diversity (antenna diversity)* is one of the key techniques to reduce the effect of multipath fading in wireless systems. One can use multiple antennas at the transmitter/receiver which is called *transmit diversity* and *receive diversity* respectively. When two antennas are separated far enough, the channel fading for each antenna becomes independent and different fading conditions are observed for the same symbol which provides additional information about the transmitted signal in detection. Hence, the incorporation of the diversity dimension helps to cope with some fundamental issues of wireless communications: *signal fading*, *throughput* and *co-channel interference (CCI)*. The availability of multiple transmit antennas allows for two-dimensional channel codes that exploit the spatial transmit diversity.

1.2. MIMO Communication System

The use of wireless communications showed its greatest increase in the last ten years, during which new methods were introduced and new devices were invented. Today, we are surrounded by wireless devices and networks in our everyday lives: cellular phone, handheld PDA, wireless INTERNET, walkie-talkie, etc. The ultimate goal of wireless communications is to communicate with anybody from anywhere at anytime for anything. The capacity of wireless communication systems is a big problem that prevents us to reach this ultimate goal. Increasing use of diverse wireless facilities makes the demand for bandwidth or capacity more and more urgent, especially for power and complexity limited systems. This means that we can not increase capacity

by simply increasing the transmit power. Communication systems are predominantly single-antenna systems. Because of the multiple-path propagation in wireless channels, the capacity of a single wireless channel can be very low. Research efforts have focused on ways to make more efficient use of this limited capacity and have accomplished remarkable progresses. However, single-antenna systems suffer another great disadvantage: its high error rate. In an additive white Gaussian noise (AWGN) channel, which models a typical wired channel, the pairwise error probability (PEP), the probability of mistaking the transmitted signal with another one, decreases exponentially with the signal-to-noise ratio (SNR), while due to the fading effect, the average PEP for wireless single-antenna systems only decreases linearly with SNR. This means that achieving the same performance requires much higher transmit power single-antenna wireless communication systems [17].

Given the above disadvantages, single-antenna systems are not good candidates to meet the needs of future wireless communications. Therefore, new communication systems superior in capacity and error rate must be introduced and consequently, new communication theories for these systems are of great importance at the present time.

Recently, digital communication systems using multiple-input-multiple- output (MIMO) wireless links, that is, using multiple antennas at the transmitter and the receiver, has emerged. This is one of the most significant technical breakthroughs in modern communications. The key feature of a multiple-antenna system is its ability to turn multiple-path propagation, which is a disadvantage to wireless communications, into a benefit to the users.

MIMO systems have two major attractive advantages that conventional single-input single-output (SISO) systems do not have. These are:

1. Multiplexing gain (or spectral efficiency gain): The channel capacity of a multiple-antenna system is considerably higher than that of a single-antenna system. In particular, it is widely understood that channel capacity increases asymptotically linearly with the minimum number of transmit and receive antennas when chan-

nel knowledge is available at the receiver. Therefore, the degree of freedom for communications is increased. As a result, the transmission rate increases linearly without an increase in the total transmission power or channel bandwidth.

2. Diversity gain: If the antennas at both ends have no, or very low, correlation, the signaling branches between different transmit-receive antenna pairs in a MIMO system can be assumed to be statistically independent. These independent branches create diversity gain. By transmitting the same data (in the same, or different, representations) over multiple independent branches, fading can be effectively mitigated and, hence, link reliability significantly improved.

MIMO systems also provide other types of gains such as array gain and interference suppression gain. Consequently, multiple antennas are expected to play an important role in advanced wireless systems, for example, 3G and beyond.

1.3. Space-time Signaling

The large capacity of multiple-antenna systems and the desire to transmit at higher data rates with better performance over wireless channels have motivated research on signal processing over multiple transmit-receive antennas. Signal processing methods that use diversity are commonly referred to as *space processing*. Coding schemes which jointly utilizes the spatial and temporal dimension are known as *space-time coding schemes* [31]. The term Space-Time Code (STC) was originally appeared in 1998 by Tarokh to describe a new two-dimensional way of encoding and decoding signals transmitted over wireless fading channels using multiple transmit antennas [3]. The other scheme employing multiple antenna arrays was also developed at about the same time, e.g., the simple and popular Alamouti STBC [2]. Since then, the term STC has been used more generally to refer to transmit diversity techniques in which the transmitted signals and corresponding receiver are designed to exploit spatial diversity.

Space-time coding has been gaining more and mote attention recently due to its attractive characteristics to provide diversity at the receiver and coding gain over an uncoded system without sacrificing the bandwidth, and increase the effective trans-

mission rate as well as the potential system capacity. *Space-time coding* introduces temporal and spatial correlation into signals transmitted from different antennas. The diversity is obtained at the transmitter by spacing the transmit antennas sufficiently apart and introducing a code between the transmitted symbols over M transmit antennas (space) and T symbol periods (time). Theoretical investigations of M transmit and N receive antennas system showed that if the receiver knows the channel, then the capacity of the system increases linearly with the minimum number of transmit and receive antennas [31].

1.4. Literature Review

Receive diversity takes its base from 1960s [20]. Although receive diversity is suitable for uplink in mobile communications, it is not appropriate for downlink. Therefore, transmit diversity has attracted attention. Alamouti found a two transmit diversity scheme [1] and Tarokh [26] generalized the Alamouti scheme, which is called *orthogonal space-time block codes (OSTBC)*. OSTBC over fading channel can be viewed as an equivalent number of SISO channels which is proven in [2]. Union bounds for Alamouti scheme for temporarily correlated and i.i.d. channel is given in [19].

Trellis based space-time codes is another class of space-time codes which can be divided into two subclasses which are space-time trellis codes [24] and super-orthogonal space-time trellis codes [12]. These codes include cooperative design of channel coding, modulation, transmit diversity and receive diversity. There are some space-time signaling schemes aiming at increasing the rate of the system which is known as Bell Labs Layered Space-time Architecture (BLAST) [7] that provides a high data rate but no diversity gain. In [10], a new scheme that provides both high data rate and diversity gain are found which is called Linear dispersion (LD) codes. Much of the analysis of these systems concentrates on uncorrelated antennas [28]. Exploring the performance of MIMO systems under correlated antennas is important for practical system design. Hence, the effect of parameters such as angle spread and Ricean factor on the performance of different space-time signaling schemes is analyzed in [5]. In [3],[23] and [9], Chernnoff bounds are found under the quasi-static channel assumption. Pairwise error

probability expressions for transmit antenna correlation are derived in [29].

In [15], the effects of polarization diversity on 2Tx - 2Rx MIMO channel are analyzed. An upper bound for the probability of error is computed and the performance degradation of system is investigated in case of polarization diversity. The upper bound in [15] is completely determined by properties of channel matrix and there is no information about space-time signaling method. In [24] new upper bounds for pairwise error probability which takes into account the properties of space-time signaling for both correlated and uncorrelated channels are proposed.

Securing high reliability of the data transmission requires special attention to receiver design. Nonetheless, good decoding methods may result in high complexity of the receiver. In [30], sphere decoding technique was proposed for lattice decoding and adapted for space-time codes. The sphere decoder provides the maximum likelihood estimated of the transmitted signal sequence. This implies that in many cases of interest ML performance can be obtained with complexity similar to nulling and cancelling technique [7]. Soft iterative decoding techniques are very famous research area and these techniques have been reported to achieve important results for codes with long codeword length. The key idea in iterative decoding techniques is the use of probabilistic (soft) information about each bit in the transmitted sequence. In case of multiantenna systems, however, the best way of calculating of soft information with low complexity is not very accurate. If the soft information is obtained by means of an exhaustive search, the computational complexity grows exponentially as indicated in [21], where turbo-coded modulation for multiantenna systems has been studied. In [21] and [11], two different ways of the sphere decoding algorithm have been proposed for estimating the soft information. Iterative methods that combine channel codes and space-time processing are studied in [27]-[18].

1.5. Scope of the Thesis

In this work, our purpose is to make comprehensive analysis of MIMO systems in case of polarization diversity. First, capacity of general MIMO system was calculated,

MIMO system model for any number of transmit-receive antennas was given and properties of space-time codes were presented especially for orthogonal space-time block codes. We assume that the channel coefficients are known to the receiver which means that we have perfect channel state-information (CSI).

Second, the same capacity calculation in case of polarization diversity was made and we concentrated on uncorrelated and correlated channel conditions for each of which error probability calculation was presented. Careful consideration was given to 4Tx-2Rx MIMO system, performance results in terms of symbol error rate were given, and this system was analyzed when we use dual-polarized (polarization diversity) antennas at transmitter and receiver side. Using polarization diversity needs careful attention in practical MIMO system design because this method introduces correlation between channel coefficients that degrades the system performance, which is proven by computer simulations. Based on computer simulations and mathematical analysis, in case of full polarization diversity, 2Tx-1Rx (dual-polarized antennas) MIMO system will reduce to 4Tx-1Rx MIMO system which has an advantage of having one uni-polarized antenna instead of having one dual-polarized antenna. In the receiver side, this result is important because of *space* and *power* limitations.

Third, we changed receiver mechanism which is linear in case of orthogonal space-time block codes to iterative decoder and detection method for MIMO system that improves the system performance undoubtedly. Because of having correlation between channel coefficients, we changed the signal model that we used before in order to include the effects of correlation parameters. In iterative detection, receiver side consists of two parts. One part is MIMO detector that uses soft information that comes from the other part of the receiver and the second part is decoder for channel code used in the transmitter side. In MIMO detection part MAP and MMSE algorithms were implemented that uses probability information about each bit and this parts outputs soft information that is used in the decoder. In decoder part, channel decoding is made by using the soft information taken from the MIMO detector. Two parts of the receiver transfer soft information between each-other to make better bit decisions.

1.6. Outline of the Thesis

The outline of this thesis is as follows: We review in Chapter 1 some background knowledge, including diversity techniques, MIMO communication, and space-time signaling. Chapter 2 presents the theory of MIMO communication and space-time codes with emphasis on space-time block codes over Rayleigh fading channel followed by simulation results. Chapter 3, we present the first part of our research. It starts with the derivation of channel capacity of MIMO communication system in the presence of polarization diversity followed by the channel model for 4Tx-2Rx MIMO system, PEP analysis, performance of Alamouti scheme in case of polarization diversity, and simulation results. The focus of chapter 4 is on iterative detection and decoding of space-time-polarization codes. We first present the basic framework and then develop iterative detection mechanism for space-time-polarization codes. Finally, we provide some concluding remarks and suggestions for future research in Chapter 5.

2. SPACE-TIME CODES

2.1. Multiple-Input Multiple-Output (MIMO) channels

In order to evaluate the performance of a MIMO transmission scheme effectively, one should have a model that accounts for all the major effects of wireless channel on various signals. The most commonly used channel model for MIMO systems in *quasi-static flat Rayleigh fading* at all antenna elements. The advantage of this channel model is to provide less complicated performance analysis. There are basic assumptions behind the quasi-static flat Rayleigh fading channel which are given as [31],

1. A large number of scatterers are present in wireless channel so that the signal at any receive antenna of the MIMO system is the sum of several multipath components. In this case the distribution of the received signal at each antenna will be complex Gaussian. The amplitude of such complex Gaussian distributed signals is *Rayleigh* distributed.
2. The channel delay spread (time difference between multipath signals) is less than the symbol rate, so that one have *flat-fading*.
3. The channel remains constant for one frame length of signal which gives *quasi-static fading*.

In MIMO system, the receiver can use one of three combining techniques to improve the quality of received signal [32],

1. *Selection Combining*: Receiver selects the signal that has the highest SNR.
2. *Switching Combining*: Receiver selects different signals according to some SNR threshold.
3. *Maximal Ratio Combining (MRC)*: Receiver combines the signals linearly to increase SNR.

In the MRC method, it is assumed that the receiver has the perfect channel knowledge. If transmitted signal at time t is $x(t)$, the received signal at *ith* receiver is given by,

$$r_i(t) = x(t)h_i(t) + n_i(t) \tag{2.1}$$

where $n_i(t)$ is the complex gaussian noise variable. The receiver combining scheme is

$$\widetilde{r_i}(t) = \sum_{i=1}^{N} h_i^* r_i(t) = x(t) \sum_{i=1}^{N} |h_i|^2 + n'(t). \tag{2.2}$$

This detected symbol is sent to the Maximum Likelihood Detector to produce estimate of the signal $s(t)$. MRC provides full diversity, but complexity is high because of channel estimation.

2.2. Capacity of Multiple-Input Multiple-Output (MIMO) Communication

Channel capacity is defined as the highest rate at which information can be sent with arbitrarily low probability of error [4]. It is assumed that the random channel matrix is constant for such a large number of symbol transmission times and it is known only to the receiver but not the transmitter. If the path gains between transmit and receive antennas are assumed to be i.i.d., capacity grows linearly with the number of antennas n ($n_T = n_R = n$).

Consider a MIMO system with an array of n_T transmit antennas and n_R receive antennas. The transmitted signals in each symbol period are represented by an $n_R \times 1$ column vector $\mathbf{x}$, where x_i is the transmitted signal from the ith antenna. The covariance matrix of the transmitted signal $\boldsymbol{x}$ is given by $\mathbf{R}_{xx} = E\{\mathbf{x}\mathbf{x}^H\}$ and the total transmitted power is given by $P = tr(\mathbf{R}_{xx})$. Channel is described by the $n_R \times n_T$ complex matrix $\mathbf{H}$ [31]. The ij-th component ($h_{i,j}$) of the channel matrix $\mathbf{H}$ represents the *channel fading coefficients*. The channel matrix can be estimated at the receiver by transmitting a training sequence. By using the linear model, the received vector

can be represented as [15]-[3],

$$\mathbf{r} = \mathbf{Hx} + \mathbf{n}. \tag{2.3}$$

The covariance matrix of received signal $\boldsymbol{r}$ is given by,

$$\begin{aligned} E\{\mathbf{r}\mathbf{r}^{\mathbf{H}}\} = \mathbf{R}_{rr} &= E\{(\mathbf{Hx} + \mathbf{n})(\mathbf{x}^H\mathbf{H}^H + \mathbf{n}^H)\} \\ &= \mathbf{H}\mathbf{R}_{xx}\mathbf{H}^H + \mathbf{R}_{nn} \end{aligned} \tag{2.4}$$

where $\mathbf{R}_{nn}$ is the noise covariance matrix. The total received signal power is represented as $tr(\mathbf{R}_{rr})$. The channel capacity is defined as the maximum possible transmission rate such that the probability of error is being arbitrarily small with coding at the transmitting side. If $\mathbf{H}_{n_R \times n_T}$ matrix is decomposed by using SVD,

$$\begin{aligned} \mathbf{H} &= \mathbf{UDV}^H \\ \mathbf{UU}^H &= \mathbf{I}_{n_R} \\ \mathbf{VV}^H &= \mathbf{I}_{n_T} \\ \mathbf{D} &= n_R \times n_T \end{aligned} \tag{2.5}$$

where $\mathbf{I}_{n_R}$ and $\mathbf{I}_{n_T}$ are $n_R \times n_R$ and $n_T \times n_T$ unitary matrices respectively, and $\mathbf{D}$ is a non-negative diagonal matrix. Diagonal entries of D are non-negative square roots of the eigenvalues of the matrix $\mathbf{HH}^H$.

The eigenvalues of $\mathbf{HH}^H$, denoted by λ, are defined as

$$\mathbf{HH}^H\mathbf{y} = \lambda\mathbf{y}, \qquad \mathbf{y} \neq 0 \tag{2.6}$$

where $\mathbf{y}$ is an $n_R \times 1$ vector associated with λ, called an eigenvector. By using our

received signal model and replacing $\mathbf{H}$ with its SVD,

$$\mathbf{r} = \mathbf{U}\mathbf{D}\mathbf{V}^H\mathbf{x} + \mathbf{n} \tag{2.7}$$

multiplying both sides with $\mathbf{U}^H$ reduces,

$$\begin{aligned} \mathbf{U}^H\mathbf{r} &= \mathbf{D}\mathbf{V}^H\mathbf{x} + U^H n \\ \mathbf{U}^H\mathbf{r} &= \mathbf{r}^{'}, \mathbf{V}^H\mathbf{x} = \mathbf{x}^{'}, \mathbf{U}^H\mathbf{n} = \mathbf{n}^{'} \\ \mathbf{r}^{'} &= \mathbf{D}\mathbf{x}^{'} + \mathbf{n}^{'}. \end{aligned} \tag{2.8}$$

The number of non-zero eigenvalues of matrix $\mathbf{H}\mathbf{H}^H$ is equal to the rank of matrix $\mathbf{H}$ (number of nonzero diagonal elements of $\mathbf{D}$). For the $n_R \times n_T$ matrix $\mathbf{H}$, the rank is at most m=min(n_R, n_T), which means that at most m of its singular values are nonzero. If the singular values of $\mathbf{H}$ are denoted by $\sqrt{\lambda_i}$, the received signal components are [31]

$$r_i^{'} = \sqrt{\lambda_i} x_{i'} + n_{i'}, \quad i = 1, 2, ..., r \tag{2.9}$$

$$r_i^{'} = n_{i'}, \quad i = r+1, r+2, ..., n_R \tag{2.10}$$

As indicated above, received components $r_i^{'}$ $(i = r+1, r+2, ..., n_R)$ do not depend on the transmitted signal, i.e. the channel gain is zero. On the other hand, received components $r_i^{'}$, for i=1,2,...,r depend only on the transmitted component $x_i^{'}$.

Assuming that the transmitted power from each antenna in the equivalent MIMO channel model is P/n_t, the overall channel capacity can be estimated by using the Shannon capacity formula [4],[31]

$$C = W \sum_{i=1}^{r} \log_2 \left(1 + \frac{P_{ri}}{\sigma^2}\right) \tag{2.11}$$

where W is the bandwidth of each sub-channel and P_{ri} is the received signal power in

the ith sub-channel. It is given by

$$P_{ri} = \frac{\lambda_i P}{n_T} \tag{2.12}$$

where $\sqrt{\lambda_i}$ is the singular value of channel matrix $\mathbf{H}$. Thus the channel capacity can be written as

$$C = W \sum_{i=1}^{r} \log_2 \left(1 + \frac{\lambda_i P}{n_T \sigma^2}\right) = W \log_2 \prod_{i=1}^{r} \left(1 + \frac{\lambda_i P}{n_T \sigma^2}\right). \tag{2.13}$$

If there is only one transmit and n_R receive antennas, the channel matrix can be represented by the vector

$$\mathbf{H} = \begin{bmatrix} h_1 & h_2 & \dots & h_{n_R} \end{bmatrix}^T \tag{2.14}$$

the capacity becomes

$$C = W \log_2 \left(1 + \sum_{i=1}^{n_R} |h_i|^2 \frac{P}{\sigma^2}\right). \tag{2.15}$$

In the case when the channel matrix elements are equal and normalized as follows,

$$|h_1|^2 = |h_2|^2 = ... = |h_{n_R}|^2 = 1 \tag{2.16}$$

the capacity becomes

$$C = W \log_2 \left(1 + n_R \frac{P}{\sigma^2}\right) \tag{2.17}$$

and the system achieves the diversity gain of n_R relative to a single antenna channel.

Now, MIMO system consists of n_T transmit antennas and only one receive an-

tenna. The channel is represented by the vector

$$\mathbf{H} = \begin{bmatrix} h_1 & h_2 & \dots & h_{n_R} \end{bmatrix}^T \tag{2.18}$$

If the channel coefficients are equal and normalized as it was done before, the transmit diversity capacity becomes

$$C = W \log_2 \left(1 + \frac{P}{\sigma_2}\right) \tag{2.19}$$

where the capacity does not increase with the number of transmit antennas.

Now, more realistic case could be where the channel matrix entries are *random variables*. The elements of matrix are assumed to be zero mean Gaussian complex random variables, each with variance of 1/2. Each entry of the channel matrix thus has a Rayleigh distributed magnitude, uniform phase and expected magnitude square equal to unity, $E[|h_{ij}|^2] = 1$.

According to frequency of channel coefficients changes, there are three different scenarios [31].

1. Its entries change randomly at the beginning of each symbol interval T and are constant during one symbol interval. This channel model is referred to as *fast fading* channel.
2. Its entries are random and are constant during a fixed number of symbol intervals, which is much shorter than the total transmission duration. This channel model is called as *block fading*.
3. The entries of matrix **H** are selected at the start of transmission and kept constant all the time. This channel model is referred to as *slow* or *quasi-static fading* model.

For a receive diversity system with one transmit and n_R receive antennas on *fast*

Rayleigh fading channel, specified by the channel matrix

$$\mathbf{H} = \begin{bmatrix} h_1 & h_2 & \dots & h_{n_R} \end{bmatrix}^T. \tag{2.20}$$

This formula gives the capacity expression for maximum ratio combining at the receiver under block Rayleigh fading channel [31]

$$C = E\left[W \log_2\left(1 + \frac{P}{\sigma^2}\chi^2_{2n_R}\right)\right] \tag{2.21}$$

where

$$\chi^2_{2n_R} = \sum_{i=1}^{n_R} |h_i|^2 \tag{2.22}$$

is a chi-squared random variable with $2n_R$ degress of freedom. It can be represented as [31]

$$y = \chi^2_{2n_R} = \sum_{i=1}^{2n_R} z_i^2. \tag{2.23}$$

Channel capacity curves for different values of SNR are shown in Figure 2.1. If a selection diversity is used, the capacity is given by,

$$C = E\left[W \log_2\left(1 + \frac{P}{\sigma^2}\max_i(|h_i|^2)\right)\right] \tag{2.24}$$

For a transmit diversity system with n_T transmit and one receive antenna on a fast Rayleigh fading channel, specified by the channel matrix,

$$\mathbf{H} = \begin{bmatrix} h_1 & h_2 & \dots & h_{n_R} \end{bmatrix}^T \tag{2.25}$$

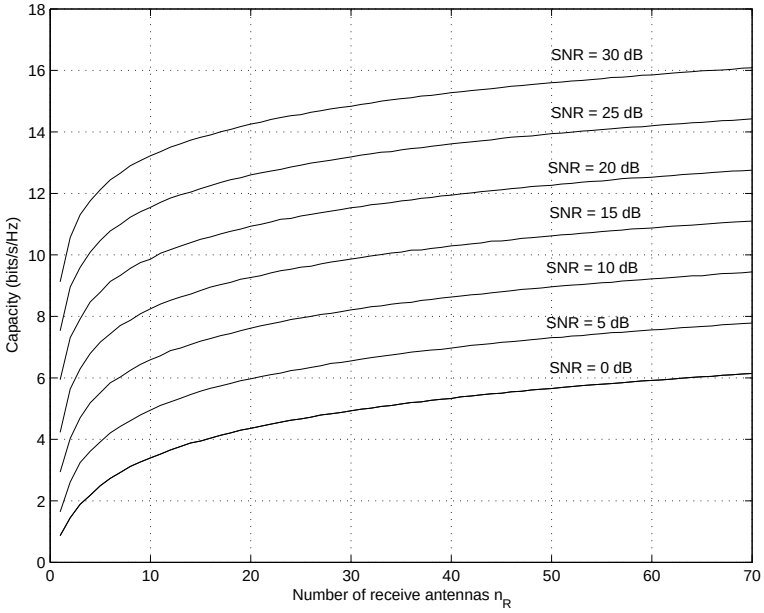


Figure 2.1. Channel capacity curves for receive diversity on block Rayleigh fading channel with maximum ratio diversity combining

The capacity expression is given by [31],

$$C = E\left[W \log_2\left(1 + \frac{P}{n_T \sigma^2}\chi^2_{2n_R}\right)\right] \tag{2.26}$$

where

$$\chi^2_{2n_T} = \sum_{j=1}^{n_T} |h_j|^2 \tag{2.27}$$

is a chi-squared random variable with $2n_T$ degress of freedom. As the number of transmit antennas increases, the capacity approaches the asymptotic value

$$\lim_{n_T \to \infty} C = W \log_2\left(1 + \frac{P}{\sigma^2}\right). \tag{2.28}$$

The system behaves as if the total power is transmitted over a single unfaded channel which means that the transmit diversity has a capable of removing the effect of fading for a large number of antennas which is shown in Figure 2.3.

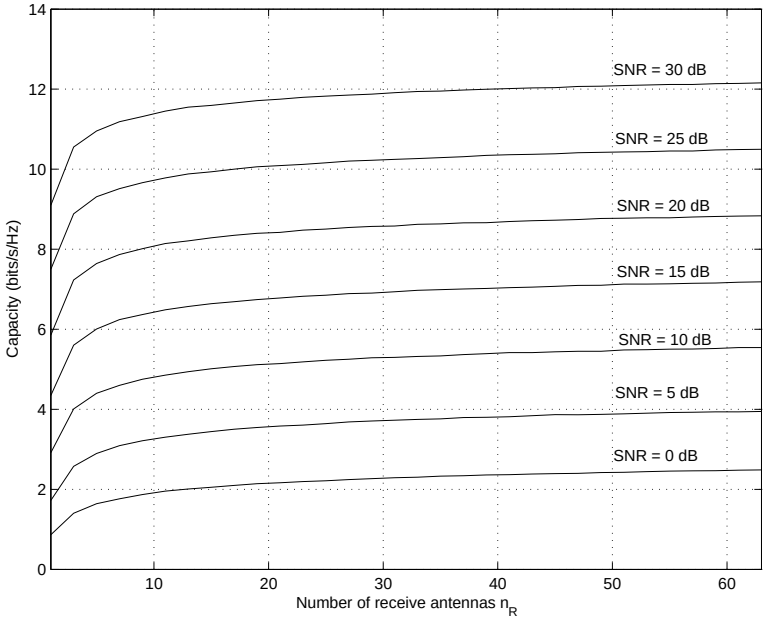


Figure 2.2. Channel capacity curves for receive diversity on block Rayleigh fading channel with selection diversity combining

2.3. Space-Time Coding

An effective and practical way to approach the capacity of MIMO wireless channels is to employ *space-time (ST) coding*. Space-time coding is a coding technique designed for use with multiple transmit antennas. *Coding* is performed in both *spatial* and *temporal* domains to introduce correlation between signals transmitted from various antennas at various time periods[24]-[3]. There are various approaches in coding structures, including *space-time block codes(STBC)*, *space-time trellis codes (STTC)*, and *layered space-time (LST) codes*. The main idea of all these methods is the exploitation of multipath effects in order to achieve high spectral efficiencies and performance gains.

Because of being constructed of multipath waves, the amplitude fluctuation of the received signal is called *signal fading*. It is caused by the time-variant multipath characteristics of the channel. In a narrowband system, the transmitted signal usually occupy a bandwidth smaller than the channel's *coherence bandwidth*, which is defined as the frequency range over which the channel fading process is correlated. That is, all spectral components of the transmitted signal are subject to the same fading

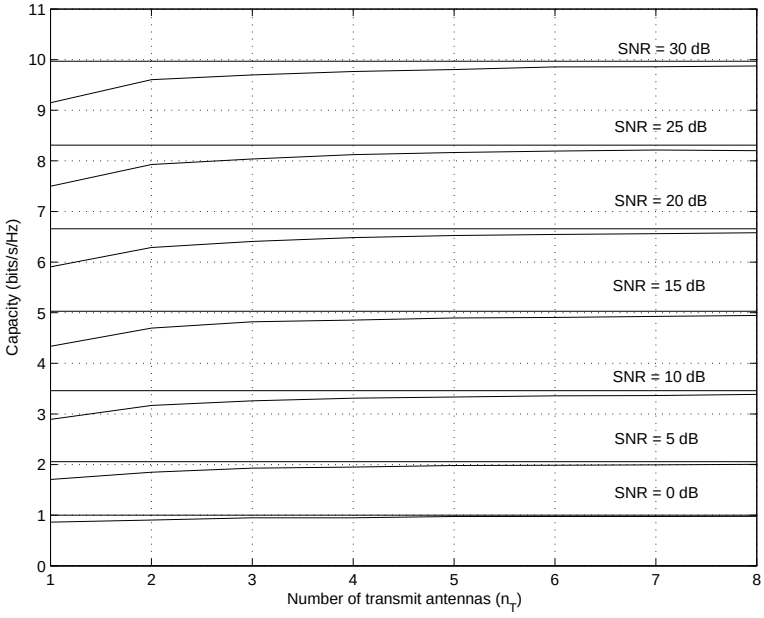


Figure 2.3. Channel capacity curves for transmit diversity on block Rayleigh fading channel

attenuation. This type of fading is called *frequency flat*.

On the other hand, if the transmitted signal bandwidth is greater than the channel coherence bandwidth, the spectral components of the transmitted signal with a frequency separation larger than the coherence bandwidth are faded independently. As a result, the received signal spectrum becomes distorted. This phenomenon is known as *frequency selective* fading. When the signal bandwidth is much smaller than the coherence bandwidth *Rayleigh* and *Rician* distributions are used to describe the received signal amplitude.

In general, the performance of communication systems with diversity techniques depends on how multiple signals are combined at the receiver to increase the overall received SNR [16]. There are four main types of combining techniques, including *selection combining*, *switched combining*, *equal-gain combining* (EGC), and *maximal ratio cobining*(MRC) [32][31].

In contrast to receive diversity which is widely used in cellular mobile systems, transmit diversity has received little attention as the behavior of transmit antenna

is different from that of receive antenna diversity and it is more difficult to exploit diversity. The difficulties mainly include that:

1. Since the transmitted signals from multiple antennas are mixed spatially before they arrive at the receiver, some additional signal processing is required at both the transmitter and receiver in order to separate the received signals and exploit diversity.
2. Unlike the receiver that can usually estimate fading channels, the transmitter does not have instantaneous information about the channel unless the information is fed back from the receiver to the transmitter.

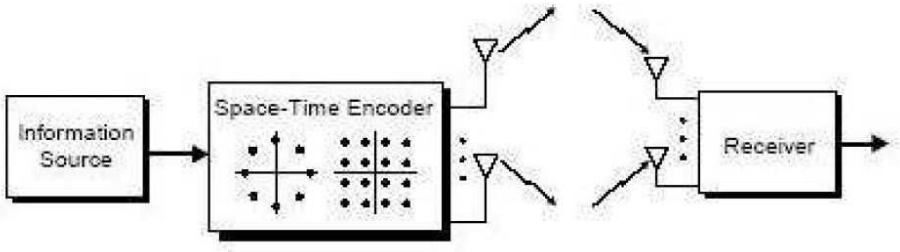


Figure 2.4. Space - time coding diagram

A baseband space-time coded communication system which is shown in Figure 2.4 with $\mathbf{n}_T$ transmit antennas and $\mathbf{n}_R$ receive antennas are focused. The transmitted data are encoded by a space-time encoder. At each time instant t, a block of m binary symbols, denoted by [31]

$$\mathbf{H} = \begin{bmatrix} c_t^1 & c_t^2 & \dots & c_t^m \end{bmatrix}^T \tag{2.29}$$

is fed into the space-time encoder. The space-time encoder maps the block of m binary input data into n_T modulation symbols from a signal set of $M = 2^m$ points. A sequence of n_T data symbols are produced from the coded data, which is represented as an $n_T \times 1$ column vector

$$\mathbf{H} = \begin{bmatrix} x_t^1 & x_t^2 & \dots & x_t^m \end{bmatrix}^T . \tag{2.30}$$

The n_T parallel outputs are transmitted by the n_T transmit antennas at the same time. The vector of coded modulation symbols from different antennas is called a *space-time symbol*. The spectral efficiency of the system is

$$\eta = \frac{r_b}{B} = m \qquad bits/sec/Hz \tag{2.31}$$

where r_b is the data rate and B is the channel bandwidth.

The channel matrix is given by,

$$\mathbf{H}_t = \begin{bmatrix} h^t_{1,1} & h^t_{1,2} & \dots & h^t_{1,n_T} \\ h^t_{2,1} & h^t_{2,2} & \dots & h^t_{2,n_T} \\ \vdots & \vdots & \ddots & \vdots \\ h^t_{n_R,1} & h^t_{n_R,2} & \dots & h^t_{n_R,n_T} \end{bmatrix} \tag{2.32}$$

where the *ji-th* element, denoted by $h^t_{j,i}$, is the fading attenuation coefficient for the path from transmit antenna i to receive antenna j [24]-[3]. Assume that the fading coefficients $h^t_{j,i}$ are independent complex Gaussian variables with mean $\mu_h^{j,i}$ and variance 1/2 per dimension, implying that the amplitude of the path coefficients are modeled as Rayleigh fading.

At the receiver, the signal at each of the n_R receive antennas is a noisy superposition of the n_T transmitted signals degraded by the channel fading. At time t, the received signal at antenna j, j=1,2,...,n_R, denoted by r^j_t, is given by

$$r^j_t = \sum_{i=1}^{n_T} h^t_{j,i} x^i_t + n^i_t \tag{2.33}$$

where n^t_j is the noise component of receive antenna j at time t, which is independent sample of the zero-mean complex Gaussian random variable with the one sided power spectral density of N_0.

In the performance analysis of space-time codes, $n_T \times L$ *space-time codeword*

matrix should be defined, assuming that the transmitted data frame length is L symbols for each antenna,

$$\mathbf{X} = \begin{bmatrix} x_1^1 & x_2^1 & \dots & x_L^1 \\ x_1^2 & x_2^2 & \dots & x_L^2 \\ \vdots & \vdots & & \vdots \\ x_1^{n_T} & x_2^{n_T} & \dots & x_L^{n_T} \end{bmatrix} \tag{2.34}$$

where the *i-th* row

$$\mathbf{H}_i = \begin{bmatrix} x_1^i & x_2^i & \dots & x_L^i \end{bmatrix}^T \tag{2.35}$$

is the data sequence transmitted from the *i-th* antenna, and the *t-th* column

$$\mathbf{H}^t = \begin{bmatrix} x_t^1 & x_t^2 & \dots & x_t^{n_t} \end{bmatrix}^T \tag{2.36}$$

is the space-time symbol at time *t*.

The variable $d_h^2(\mathbf{X}, \hat{\mathbf{X}})$ is a Euclidean distance between the two space-time codeword matrices $\mathbf{X}$ and $\hat{\mathbf{X}}$, which is formulated as [24]

$$d_h^2(\mathbf{X}, \hat{\mathbf{X}}) = ||\mathbf{H}(\hat{\mathbf{X}} - \mathbf{X})||^2 \tag{2.37}$$

The pairwise error probability conditioned on $\mathbf{H}$ is given by [31]

$$P(\mathbf{X}, \hat{\mathbf{X}}|\mathbf{H}) = Q\left(\sqrt{\frac{E_s}{2N_0} d_h^2(\mathbf{X}, \hat{\mathbf{X}})}\right). \tag{2.38}$$

2.3.1. Space-Time Block Codes

2.3.1.1. Alamouti Space-Time Code. Alamouti found the first space-time block code to provide full transmit diversity for systems with two transmit antennas. The key

feature of the scheme is that it achieves a full diversity gain with a simple maximum-likelihood decoding algorithm [1].

Assume that an M-ary modulation scheme is used. In the Alamouti space-time encoder, each group of m information bits is first modulated, where $m = \log_2 M$. Then, the encoder takes a block of two modulated symbols x_1 and x_2 in each encoding operation and maps them to the transmit antennas according to the code matrix given by [1]

$$\mathbf{X} = \begin{bmatrix} x_1 & -x_2^* \\ x_2 & x_1^* \end{bmatrix}. \tag{2.39}$$

The encoder outputs are transmitted in two consecutive transmission periods from two transmit antennas. During the first transmission period, two signals x_1 and x_2 are transmitted from antenna one and antenna two, at the same time. In the second transmission period, signal $-x_2^*$ is transmitted from transmit antenna one and signal x_1^* from transmit antenna two. The key feature of the Alamouti scheme is that the transmit sequences from the two transmit antennas are orthogonal. Thus, the code matrix has the following structure [31],

$$\mathbf{X}\mathbf{X}^{\mathbf{H}} = \begin{bmatrix} |x_1|^2 + |x_2|^2 & 0 \\ 0 & |x_1|^2 + |x_2|^2 \end{bmatrix}. \tag{2.40}$$

The fading channel coefficients from the first and second transmit antennas to the receive antenna at time t are denoted by $h_1(t)$ and $h_2(t)$ and assume that the fading coefficients are constant across two consecutive symbol transmission periods, they can be represented as,

$$h_1(t) = h_1(t+T) = h_1 = |h_1|e^{j\theta_1} \tag{2.41}$$

$$h_2(t) = h_2(t+T) = h_2 = |h_2|e^{j\theta_2} \tag{2.42}$$

where $|h_i|$ and θ_i, i=0,1, are the amplitude gain and phase shift for the path from transmit antenna i to the receive antenna, and T is the symbol duration [1].

At the receive antenna, the received signals over two consecutive symbol periods, denoted by r_1 and r_2 for the time t and $t+T$ can be expressed as

$$r_1 = h_1 x_1 + h_2 x_2 + n_1 \tag{2.43}$$

$$r_2 = -h_1 x_2^* + h_2 x_1^* + n_2 \tag{2.44}$$

where n_1 and n_2 are independent complex variables with zero mean and power spectral density $N_0/2$ per dimension, representing additive white Gaussian noise samples at time t and $t+T$ respectively [1].

For a coherent detection scheme where receiver knows the channel path gains h_1 and h_2, the maximum-likelihood detection requires minimizing the decision metric,

$$|r_1 - h_1 x_1 - h_2 x_2|^2 + |r_2 + h_1 x_2^* - h_2 x_1^*|^2 \tag{2.45}$$

for all possible values of x_1 and x_2. As it is seen, such a process requires a full search over all possible (x_1,x_2) values, and complexity grows exponentially with the number of antennas. Above expression can be separated as follows,

$$|x_1|^2 \sum_{n=1}^{2} |h_n|^2 - [r_1 h_1^* x_1^* + r_1^* h_1 x_1 + r_2 h_2^* x_1 + r_2 h_2^* x_1^*] \tag{2.46}$$

which is only function of x_1, and the other one

$$|x_2|^2 \sum_{n=1}^{2} |h_n|^2 - [r_2 h_2^* x_2^* + r_2^* h_2 x_1 + r_1 h_1^* x_2 + r_1 h_1^* x_2^*] \tag{2.47}$$

that is only function of x_2. In the receiver, these two functions can be minimized separately which results in separate decoding of the two symbols. Therefore, *decoding complexity* increases linearly, instead of exponentially, by the number of transmit

antennas [31].

2.3.1.2. General OSTBC design. A question that may come to mind is: Is it possible to design similar codes for more number of transmit antennas? In [24], this question was answered. Basically they have shown that the good properties mentioned about the Alamouti scheme come from the fact that,

$$\mathbf{X}^H\mathbf{X} = \left(|x_1^2| + |x_2^2|\right) I_2. \tag{2.48}$$

Every matrix with matrix with this property can be generator of a code that provides full diversity as well as simple decoding. Therefore, Alamouti scheme was generalized to any number of transmit antennas by applying the theory of *orthogonal designs* [24]. The next step is to investigate the possibility of similar generator matrices for more than two transmit antennas.

Achieving the full transmit diversity, STBC's allow a very simple decosing algorithm based only on linear processing of the received signals [24]. Assume that the signal constellation consists of 2^m points. Space-time encoder receives $k.m$ information bits in each encoding operation in order to select $x_1, x_2, ..., x_k$ k modulated signals each of which is the symbol from signal constellation. These k modulated signals are encoded to generate n_t parallel signal sequences of length T which are transmitted through antennas simultaneously in T time periods.

In general case, the entries of the transmission matrix $\mathbf{X}$ are linear combinations of the k modulated symbols and their conjugates. As it is outlined above, in order to achieve full diversity, the transmission matrix $\mathbf{X}$ is constructed based on orthogonal designs such that [24]

$$\mathbf{X}\mathbf{X}^H = c\left(|x_1|^2 + |x_2|^2 + ... + |x_k|^2\right)\mathbf{I}_{n_T} \tag{2.49}$$

The ith row of $\mathbf{X}$ represents the symbols transmitted from the ith transmit antenna consecutively in T transmission periods and the jth column of $\mathbf{X}$ represents the symbols transmitted from n_t transmit antennas at time j. The jth column of $\mathbf{X}$ is regarded as a space-time symbol transmitted at time j. The rows of the transmission matrix $\mathbf{X}$ are orthogonal to each other which means that signals sequences from each antenna are orthogonal [31].

For example, if one transmits $\mathbf{x}_i = (x_{i,1}, x_{i,2}, ..., x_{i,p})$ from the i-th antenna, $i = 1, 2, ..., n_T$, one has,

$$< \mathbf{x}_i \mathbf{x}_j >= \sum_{t=1}^{T} x_{i,t}.x_{j,t}^{*}, \quad i \neq j, \quad i, j \in \{1, 2, ..., n_T\} \tag{2.50}$$

where $< \mathbf{x}_i \mathbf{x}_j >$ denotes the inner product of the sequences $\mathbf{x}_i$ and $\mathbf{x}_j$. The orthogonality enables to achieve the full transmit diversity for a given number of transmit antennas.

Space-time block codes can be classified into space-time block codes with *real* signals and space-time block codes with *complex* signals based on the type of the signal constellations. For real numbers, matrices satisfying full rank property are called *orthogonal designs* only exist for $N = 2, 4, 8$ where N is the dimension of the square matrix. This means that the original designs offered only provide an orthogonal code for the case of N=2,4,8 transmit antennas [24]. These codes are of full rate R=1 and offer the full transmit diversity of n_T [31].

The transmission matrices are given by [31],

$$\mathbf{X}_2 = \begin{bmatrix} x_1 & -x_2 \\ x_2 & x_1 \end{bmatrix} \tag{2.51}$$

$$\mathbf{X}_4 = \begin{bmatrix} x_1 & -x_2 & -x_3 & -x_4 \\ x_2 & x_1 & x_4 & -x_3 \\ x_3 & -x_4 & x_1 & x_2 \\ x_4 & x_3 & -x_2 & x_1 \end{bmatrix} \tag{2.52}$$

$$\mathbf{X}_8 = \begin{bmatrix} x_1 & -x_2 & -x_3 & -x_4 & -x_5 & -x_6 & -x_7 & -x_8 \\ x_2 & x_1 & -x_4 & x_3 & -x_6 & x_5 & x_8 & -x_7 \\ x_3 & x_4 & x_1 & -x_2 & -x_7 & -x_8 & x_5 & x_6 \\ x_4 & -x_3 & x_2 & x_1 & -x_8 & x_7 & -x_6 & x_5 \\ x_5 & x_6 & x_7 & x_8 & x_1 & -x_2 & -x_3 & -x_4 \\ x_6 & -x_5 & x_8 & -x_7 & x_2 & x_1 & x_4 & -x_3 \\ x_7 & -x_8 & -x_5 & x_6 & x_3 & -x_4 & x_1 & x_2 \\ x_8 & x_7 & -x_6 & -x_5 & x_4 & x_3 & -x_2 & x_1 \end{bmatrix} \tag{2.53}$$

for $n_T = 2$, 4, and 8 transmit antennas, respectively.

The Alamouti scheme can be regarded as a STBC for complex signals for two transmit antennas and provides the full diversity of 2 and the full rate of 1 [1],[31]. For an arbitrary complex signal constellation, there are STBCs that can achieve a rate of 1/2 for any given number of transmit antennas. For example, complex transmission matrices $\mathbf{X}_3^c$ and $\mathbf{X}_4^c$ are designed for three and four transmit antennas which are given below [31]

$$\mathbf{X}_3^c = \begin{bmatrix} x_1 & -x_2 & -x_3 & -x_4 & -x_1^* & -x_2^* & -x_3^* & -x_4^* \\ x_2 & x_1 & x_4 & -x_3 & x_2^* & x_1^* & x_4^* & -x_3^* \\ x_3 & -x_4 & x_1 & x_2 & x_3^* & -x_4^* & x_1^* & x_2^* \end{bmatrix} \tag{2.54}$$

$$\mathbf{X}_4^c = \begin{bmatrix} x_1 & -x_2 & -x_3 & -x_4 & -x_1^* & -x_2^* & -x_3^* & -x_4^* \\ x_2 & x_1 & x_4 & -x_3 & x_2^* & x_1^* & x_4^* & -x_3^* \\ x_3 & -x_4 & x_1 & x_2 & x_3^* & -x_4^* & x_1^* & x_2^* \\ x_4 & x_3 & -x_2 & x_1 & x_4^* & x_3^* & -x_2^* & x_1^* \end{bmatrix}. \tag{2.55}$$

It can be shown that the inner product of any two rows of these matrices is zero that proves the orthogonality of these structures. If matrix $\mathbf{X}_3^c$ is used, space-time encoder takes four symbols at each encoding operation and will transmit these symbols in eight symbol periods via three antennas which makes the transmission rate 1/2. Similarly, for the matrix $\mathbf{X}_4^c$ encoder again takes four symbols and transmit these

symbols in eight symbol periods via four antennas which results rate 1/2 code.

A more involved linear processing results in a higher rate for space-time block codes with a complex constellation and more than two antennas. The following matrix $\mathbf{X}_3^h$ is complex orthogonal design for space-time block codes with rate 3/4 [24]

$$\mathbf{X}_3^h = \begin{bmatrix} x_1 & -x_2^* & x_3^*/\sqrt{2} & x_3^*/\sqrt{2} \\ x_2 & x_1^* & x_3^*/\sqrt{2} & -x_3^*/\sqrt{2} \\ x_3/\sqrt{2} & x_3/\sqrt{2} & (-x_1 - x_1^* + x_2 - x_2^*)/2 & (x_2 + x_2^* + x_1 - x_1^*)/2 \end{bmatrix} \quad (2.56)$$

Another rate 3/4 space-time block code with three antennas over complex signal constellations is given by [31]

$$\mathbf{X}_3^{h'} = \begin{bmatrix} x_1 & x_2^* & x_3^* & 0 \\ -x_2 & x_1^* & 0 & -x_3^* \\ -x_3 & 0 & x_1^* & x_2^* \end{bmatrix}. \quad (2.57)$$

Researchers generalized the meaning of orthogonal design to non-square matrices. They showed for every N, there exits T where $T \times N$ matrix X with real entries $s_1, -s_1, s_2, -s_2, ..., s_k, -s_k$

$$\mathbf{X}^T\mathbf{X} = k.\left(x_1^2 + x_2^2 + ...x_k^2\right) I_N \quad (2.58)$$

where $\mathbf{I}_N$ is the $N \times N$ identity matrix and k is a constant. In a similar way, a complex orthogonal design is defined by a $T \times N$ matrix $\mathbf{G}$ with complex entries $\pm x_1, \pm x_1^*, \pm x_2, \pm x_2^*, ..., \pm x_k, \pm x_k^*$ such that,

$$\mathbf{X}^T\mathbf{X} = k.\left(|x_1^2| + |x_2^2| + ...|x_k^2|\right) I_N. \quad (2.59)$$

A matrix that satisfies the above requirement can be the generator matrix for a

complex code that provides full diversity as well as simple maximum-likelihood decoding. Unfortunately, codes for complex signals are limited to rate 3/4 for more than two antennas. There are two design criteria for orthogonal space time block codes [24],

1. *Rank criterion*: In order to achieve maximum possible diversity, code difference matrix should be full rank.
2. *Determinant criterion*: To maximize coding gain minimum determinant of code difference matrix $\mathbf{I}_N$ should be maximized over all codewords.

2.4. Decoding of STBC

We will analyze STBC described by a square transmission matrix over real signal constellation, such as $\mathbf{X}_2$,$\mathbf{X}_4$ and $\mathbf{X}_8$. In this case, the first column of the transmission matrix is a vector $[x_1, x_2, ..., x_{n_T}]^T$. The other columns of $\mathbf{X}_{n_T}$ are all permutations of the first column with possible different signs. Let $\in_t$ denote the permutations of the symbols from the first column to the t-th column. The row position of x_i in the t-th column is represented by $\in_t (i)$ and the sign of x_i in the t-th column is denoted by $sgn_t(i)$[31]. Assume that the channel coefficients $h_{j,i}(t)$ are constant over p symbol periods

$$h_{j,i}(t) = h_{j,i}, \quad t = 1, 2, ..., p. \tag{2.60}$$

In deriving the maximum likelihood decoding, similar to the one for the Alamouti scheme, the decision statistics for the transmitted signal x_i can be constructed as in [31],

$$\widetilde{x}_i = \sum_{t=1}^{n_T} \sum_{j=1}^{n_R} sgn_t(i).r_t^j.h^*_{j,\in_t(i)} \tag{2.61}$$

where $i = 1, 2, ..., n_T$. Because of the orthogonality of pairwise rows of the transmission

matrix, minimizing the maximum likelihood metric

$$\sum_{t=1}^{n_T}\sum_{j=1}^{n_R}|r_t^j-\sum_{t=1}^{n_T}h_{j,i}x_t^i|^2 \tag{2.62}$$

is equivalent to minimizing the joint decision metric

$$\sum_{t=1}^{n_T}\left[|\widetilde{x}_i-x_i|^2+(\sum_{t=1}^{n_T}\sum_{j=1}^{n_R}|h_{j,t}|^2-1)|x_i|^2\right] \tag{2.63}$$

the value of $\widetilde{x}_i$ only depends on the code symbol x_i, given the received signals, the path coefficients and the structure of the orthogonal transmission matrix. Hence, minimizing the joint decision metric is equivalent to minimizing each individual decision metric [31]

$$|\widetilde{x}_i-x_i|^2+\left(\sum_{t=1}^{n_T}\sum_{j=1}^{n_R}|h_{j,t}|^2-1\right)|x_i|^2 \tag{2.64}$$

this algorithm simplifies the joint decoding significantly by performing separate decoding for each transmitted signal. Due to the orthogonality, the decision statistics for the desired transmitted signal x_i is independent of the other transmitted signals x_j, $j=1,2,...,n_T$, $j\neq i$. The decoding metric for each signal x_i is based on linear processing of its decision statistics $\widetilde{x}_i$.

For the STBC with non-square transmission matrix over real signal constellations, the decision statistics at the receiver can be given as [31]

$$\widetilde{x}_i=\sum_{t\in\eta(i)}\sum_{j=1}^{n_R}sgn_t(i).r_t^j.h_{j,\in_t(i)}^* \tag{2.65}$$

where $i=1,2,...,p$, and $\eta(i)$ is the set of columns of the transmission matrix, in which x_i appears. For instance, consider the transmission matrix $\mathbf{X}_3$ with three transmit antennas

$$\eta(1) = 1,2,3; \quad \eta(2)=1,2,4 \tag{2.66}$$

$$\eta(3) = 1,3,4; \quad \eta(4)=2,3,4. \tag{2.67}$$

The decision metric for each individual signal x_i is given by [31] as

$$|\widetilde{x}_i - x_i|^2 + \left(\sum_{t=1}^{n_T} \sum_{j=1}^{n_R} |h_{j,t}|^2 - 1 \right) |x_i|^2 \tag{2.68}$$

in [31], similar decoding algorithms can be derived for STBC with complex signal constellations. For the rate 1/2 STBC $\mathbf{X}_3^c$ and $\mathbf{X}_4^c$, the decision statistics $\widetilde{x}_i$ can be represented by,

$$\widetilde{x}_i = \sum_{t \in \eta(i)} \sum_{j=1}^{n_R} sgn_t(i).\widetilde{r}_t{}^j(i).\widetilde{h}_{j,\in_t(i)} \tag{2.69}$$

where

$$\widetilde{r}_t{}^j(i) = \begin{cases} r_t^j & if \quad x_i \in (\mathbf{X}_{n_T}^c)^{cl} \\ (r_t^j)^* & if \quad x_i^* \in (\mathbf{X}_{n_T}^c)^{cl} \end{cases}$$

$$\widetilde{h}_{j,\in_t(i)} = \begin{cases} h_{j,\in_t(i)}^* & if \quad x_i \in (\mathbf{X}_{n_T}^c)^{cl} \\ h_{j,\in_t(i)} & if \quad x_i^* \in (\mathbf{X}_{n_T}^c)^{cl} \end{cases}$$

where $(\mathbf{X}_{n_T}^c)^{cl}$ represents t-th column of $\mathbf{X}_{n_T}^c$. The decision metric is given by,

$$|\widetilde{x}_i - x_i|^2 + \left(2 \sum_{t=1}^{n_T} \sum_{j=1}^{n_R} |h_{j,t}|^2 - 1 \right) |x_i|^2. \tag{2.70}$$

2.5. Pairwise Error Probability Analysis

In this section, the orthogonal STBC will be reviewed and then the corresponding uncoded symbol error rate estimate as a function of the channel statistics will be presented. Consider a baseband space-time coded communication system with $\mathbf{n_T}$ transmit antennas and $\mathbf{n_R}$ receive antennas. The transmitted data is encoded by a

space-time encoder. At each time instant t, a block of m binary symbols, denoted by

$$\mathbf{c}_T = \begin{bmatrix} c_t^1 & c_t^2 & \dots & c_t^m \end{bmatrix}^T \tag{2.71}$$

is fed into the space-time encoder. The space-time encoder maps the block of m binary input data into n_T modulation symbols from a signal set of $M = 2^m$ points. A sequence of n_T data symbols are produced from the coded data, which is represented as an $n_T \times 1$ column vector

$$\mathbf{c}_T = \begin{bmatrix} x_t^1 & x_t^2 & \dots & x_t^m \end{bmatrix}^T. \tag{2.72}$$

The n_T parallel outputs are transmitted by the n_T transmit antennas at the same time. The vector of coded modulation symbols from different antennas is called a *space-time symbol*. The spectral efficiency of the system is

$$\eta = \frac{r_b}{B} = m \quad bits/sec/Hz \tag{2.73}$$

where r_b is the data rate and B is the channel bandwidth [31].

The channel matrix is defined as

$$\mathbf{H_t} = \begin{bmatrix} h_{1,1}^t & h_{1,2}^t & \dots & h_{1,n_T}^t \\ h_{2,1}^t & h_{2,2}^t & \dots & h_{2,n_T}^t \\ \vdots & \vdots & \ddots & \vdots \\ h_{n_R,1}^t & h_{n_R,2}^t & \dots & h_{n_R,n_T}^t \end{bmatrix} \tag{2.74}$$

where the *ji-th* element, denoted by $h_{j,i}^t$, is the fading attenuation coefficient for the path from transmit antenna i to receive antenna j. Assume that the fading coefficients $h_{j,i}^t$ are independent complex Gaussian variables with mean $\mu_h^{j,i}$ and variance 1/2 per dimension, implying that the amplitude of the path coefficients are modeled as Rayleigh fading [15],[1].

At the receiver, the signal at each of the n_R receive antennas is a noisy superposition of the n_T transmitted signals degraded by the channel fading. At time t, the received signal at antenna j, j=1,2,...,n_R, denoted by r_t^j, is given by

$$r_t^j = \sum_{i=1}^{n_T} h_{j,i}^t x_t^i + n_t^i \tag{2.75}$$

where n_j^t is the noise component of receive antenna j at time t, which is independent sample of the zero-mean complex Gaussian random variable with the one sided power spectral density of N_0.

In the performance analysis of space-time codes, one should define an $n_T \times L$ *space-time codeword matrix*, assuming that the transmitted data frame length is L symbols for each antenna [24],

$$\mathbf{X} = \begin{bmatrix} x_1^1 & x_2^1 & \dots & x_L^1 \\ x_1^2 & x_2^2 & \dots & x_L^2 \\ \vdots & \vdots & & \vdots \\ x_1^{n_T} & x_2^{n_T} & \dots & x_L^{n_T} \end{bmatrix} \tag{2.76}$$

where the *i-th* row

$$\mathbf{x}^i = \begin{bmatrix} x_1^i & x_2^i & \dots & x_L^i \end{bmatrix} \tag{2.77}$$

is the data sequence transmitted from the *i-th* antenna, and the *t-th* column

$$\mathbf{x}^i = \begin{bmatrix} x_t^1 & x_t^2 & \dots & x_t^{n_t} \end{bmatrix}^T \tag{2.78}$$

is the space-time symbol at time t.

In matrix form, received signal is expressed as

$$\mathbf{r} = \mathbf{HX} + \mathbf{N} \tag{2.79}$$

where **N** is the independent zero-mean Gaussian noise vector, with variance normalized to give out the desired signal to noise ratio. In order to find probability of error, the effects of mistaking two codewords with eachother should be first found. Assume that a codeword **X** is transmitted and decoder decides that another codeword **Y** has been transmitted. Rate 1/2 orthogonal space-time code is used for complex signal constellation which is given by the code matrix [31],

$$\mathbf{X} = \begin{bmatrix} x_1 & x_2 & x_3 & x_4 \\ -x_2 & x_1 & -x_4 & x_3 \\ -x_3 & x_4 & x_1 & x_2 \\ -x_4 & -x_3 & x_2 & x_1 \\ x_1^* & x_2^* & x_3^* & x_4^* \\ -x_2^* & x_1^* & -x_4^* & x_3^* \\ -x_3^* & x_4^* & x_1^* & -x_2^* \\ -x_4^* & -x_3^* & x_2^* & x_1^* \end{bmatrix} \quad \text{and} \quad \mathbf{Y} = \begin{bmatrix} y_{1,1} & y_{1,2} & y_{1,3} & y_{1,4} \\ y_{2,1} & y_{2,2} & y_{2,3} & y_{2,4} \\ y_{3,1} & y_{3,2} & y_{3,3} & y_{3,4} \\ y_{4,1} & y_{4,2} & y_{4,3} & y_{4,4} \\ y_{5,1} & y_{5,2} & y_{5,3} & y_{5,4} \\ y_{6,1} & y_{6,2} & y_{6,3} & y_{6,4} \\ y_{7,1} & y_{7,2} & y_{7,3} & y_{7,4} \\ y_{8,1} & y_{8,2} & y_{8,3} & y_{8,4} \end{bmatrix}. \quad (2.80)$$

The pairwise error probability of transmitting **X** and detecting it is denoted by **Y** by P($\mathbf{X} \rightarrow \mathbf{Y}$) [24],[13]. The probability of error is upper bounded by P(error| **X** is sent)=$\bigcup_{(Y \neq X)} P(\mathbf{X} \rightarrow \mathbf{Y})$. In order to calculate the pairwise error probability P($\mathbf{X} \rightarrow \mathbf{Y}$), assume a known channel matrix **H** and then calculate the average error by computing the expected value over the distribution of the **H**. Since **N** is gaussian noise with independent components, for a fixed **X** and **H**, the received vector **r** is also a Gaussian random variable. The probability distribution function of **X** , **H** is given by f(**X**,**H**) which is the following

$$\begin{aligned} f(\mathbf{X}, \mathbf{H}) &= \frac{1}{(\pi N_0)^{N_r x N_r}} \exp\left\{ - \frac{Tr\left[(\mathbf{r} - \mathbf{XH})^H (\mathbf{r} - \mathbf{XH})\right]}{N_0} \right\} \\ &= \frac{1}{(\pi N_0)^{N_r x N_r}} \exp\left\{ ||\mathbf{r} - \mathbf{XH}||_F^2 \right\}. \end{aligned} \quad (2.81)$$

The pairwise error probability is calculated by,

$$\begin{aligned} P(\mathbf{X} \to \mathbf{Y}|\mathbf{H}) &= P\left(\|\mathbf{r} - \mathbf{X}\mathbf{H}\|_F^2 - \|\mathbf{r} - \mathbf{Y}\mathbf{H}\|_F^2 > 0\right) \\ &= P\left(Tr\left[(\mathbf{r} - \mathbf{X}\mathbf{H})^H(\mathbf{r} - \mathbf{X}\mathbf{H})\right] - Tr\left[(\mathbf{r} - \mathbf{Y}\mathbf{H})^H(\mathbf{r} - \mathbf{Y}\mathbf{H})\right]\right) \\ &= P\left(Tr\left[(\mathbf{r} - \mathbf{X}\mathbf{H})^H(\mathbf{r} - \mathbf{X}\mathbf{H}) - (\mathbf{r} - \mathbf{Y}\mathbf{H})^H(\mathbf{r} - \mathbf{Y}\mathbf{H})\right] > 0\right) \\ &= P\left(Tr\left[\mathbf{N}^{\mathbf{H}}\mathbf{N} - (\mathbf{X}\mathbf{H} + \mathbf{N} - \mathbf{Y}\mathbf{H})^{\mathbf{H}}(\mathbf{X}\mathbf{H} + \mathbf{N} - \mathbf{Y}\mathbf{H})\right] > 0\right) \\ &= P\left(Tr\left[((\mathbf{X} - \mathbf{Y})\mathbf{H} + \mathbf{N})^{\mathbf{H}}((\mathbf{X} - \mathbf{Y})\mathbf{H} + \mathbf{N}) - \mathbf{N}^{\mathbf{H}}\mathbf{N} < 0\right]\right) \\ &= P\left(Tr\left[\mathbf{H}^{\mathbf{H}}(\mathbf{X} - \mathbf{Y})^{\mathbf{H}}(\mathbf{X} - \mathbf{Y})\mathbf{H}\right] - \mathbf{M} < 0\right) \end{aligned} \tag{2.82}$$

where $\mathbf{M} = Tr\left[\mathbf{H}^{\mathbf{H}}(\mathbf{X} - \mathbf{Y})^{\mathbf{H}}\mathbf{N} + \mathbf{N}^{\mathbf{H}}(\mathbf{X} - \mathbf{Y})\mathbf{H}\right]$ is a zero-mean Gaussian random variable with variance $2N_0\|(\mathbf{X} - \mathbf{Y})\mathbf{H}\|_F^2$. Then, pairwise error probability (2.83) can be calculated by using the Q function

$$P(\mathbf{X} \to \mathbf{Y}|\mathbf{H}) = Q\left(\sqrt{\frac{1}{2N_0}}\|(\mathbf{X} - \mathbf{Y})\mathbf{H}\|_F\right). \tag{2.83}$$

The code difference matrix should be calculated in order to calculate probability of error. Let's say a matrix $\mathbf{D}$ is defined as $\mathbf{D}(\mathbf{X}, \mathbf{Y}) = (\mathbf{X} - \mathbf{Y})^{\mathbf{H}}(\mathbf{X} - \mathbf{Y})$ [24],[13]. Using singular value decomposition, one can decompose $\mathbf{D}(\mathbf{X}, \mathbf{Y})$ as $\mathbf{D}(\mathbf{X}, \mathbf{Y}) = \mathbf{V}^{\mathbf{H}}\mathbf{\Lambda}\mathbf{V}$, where $\mathbf{\Lambda} = diag(\lambda_1, \lambda_1, ..., \lambda_N)$, $\lambda_i's$ are eigenvalues of the code difference matrix $\mathbf{D}(\mathbf{X}, \mathbf{Y})$.

$$\begin{aligned} \|(\mathbf{X} - \mathbf{Y})\mathbf{H}\|_{\mathbf{F}}^{\mathbf{2}} &= Tr\left[\mathbf{H}^{\mathbf{H}}(\mathbf{X} - \mathbf{Y})^{\mathbf{H}}(\mathbf{X} - \mathbf{Y})\mathbf{H}\right] \\ &= Tr\left[\mathbf{H}^{\mathbf{H}}\mathbf{D}(\mathbf{X}, \mathbf{Y})\mathbf{H}\right] \\ &= Tr\left[\mathbf{H}^{\mathbf{H}}\mathbf{V}^{\mathbf{H}}\mathbf{\Lambda}\mathbf{V}\mathbf{H}\right]. \end{aligned} \tag{2.84}$$

Since the elements of $\mathbf{H}$ are independent Gaussian random variables, the elements

of $\mathbf{V} \cdot \mathbf{H}$ are also Gaussian. (2.85) can be written as,

$$\| (\mathbf{X}-\mathbf{Y})\mathbf{H} \|_F^2 = Tr \begin{pmatrix} h_{1,1}^* & h_{1,2}^* & \dots & h_{1,N}^* \\ h_{2,1}^* & h_{2,2}^* & \dots & h_{2,N}^* \\ \vdots & \vdots & \ddots & \vdots \\ h_{M,1}^* & h_{M,2}^* & \dots & h_{M,N}^* \end{pmatrix} \cdot \begin{pmatrix} \lambda_1 & 0 & \dots & 0 & 0 \\ 0 & \lambda_2 & \dots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \dots & 0 & \lambda_N \end{pmatrix} \cdot \begin{pmatrix} h_{1,1} & h_{2,1} & \dots & h_{M,1} \\ h_{1,2} & h_{2,2} & \dots & h_{M,2} \\ \vdots & \vdots & \ddots & \vdots \\ h_{1,N} & h_{2,N} & \dots & h_{M,N} \end{pmatrix} \quad (2.85)$$

If we calculate the trace above, we will have the following expression

$$\| (\mathbf{X}-\mathbf{Y})\mathbf{H} \|_F^2 = \sum_{m=1}^{M} \sum_{n=1}^{N} \lambda_n \mid h_{n,m} \mid^2 \quad (2.86)$$

Using this result,

$$P(\mathbf{X} \to \mathbf{Y} \mid \mathbf{H}) = Q\left(\sqrt{\frac{1}{2N_0} \sum_{m=1}^{M} \sum_{n=1}^{N} \lambda_n \mid h_{n,m} \mid^2} \right) \quad (2.87)$$

and the upper bound for Q function saying $Q(x) < e^{-x^2/2}$, an upper bound on the conditional pairwise error probability can be calculated as follows

$$P(\mathbf{X} \to \mathbf{Y} \mid \mathbf{H}) \leq \frac{1}{2} \exp\left(-\frac{1}{4N_0} \sum_{m=1}^{M} \sum_{n=1}^{N} \lambda_n \mid h_{n,m} \mid^2 \right) \quad (2.88)$$

It is known that $\mid \alpha_{n,m} \mid$ is Rayleigh distributed, the expected value of the pairwise error probability can be calculated

$$P(\mathbf{X} \to \mathbf{Y}) = E[P(\mathbf{X} \to \mathbf{Y}|\mathbf{H})] \leq \frac{1}{\prod_{n=1}^{N} [1 + (\lambda_n / 4N_0)]^{N_r}} \quad (2.89)$$

In computer simulations, we used BPSK modulation method unless otherwise stated. We send 1000 packets each of which consists of 1000 bits. In the receiver ML decoding

is used.

Simulation 1: Figure 2.5 simulation shows the BER performance comparison of Alamouti's method for 2Tx-1Rx and 2Tx-2Rx scenarios with Maximal Ratio Receive combining scheme. For theorical results of Alamouti scheme, pairwise error probability formula is given in [31].

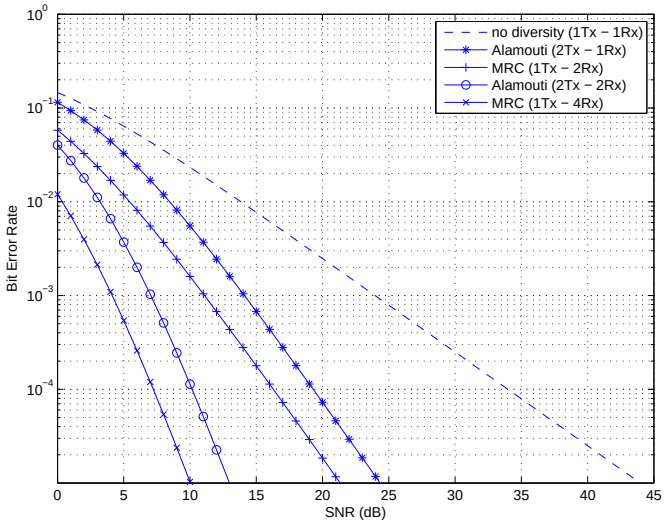


Figure 2.5. The BER performance of Alamouti scheme for 2Tx-1Rx MIMO system in Rayleigh fading

Simulation 2: Figure 2.6 shows the SER performance comparison of Alamouti's method for 2Tx-1Rx and 2Tx-2Rx scenarios with 4Tx-2Rx and 4Tx-1Rx scenarios for which rate 1/2 orthogonal space-time block code is used.

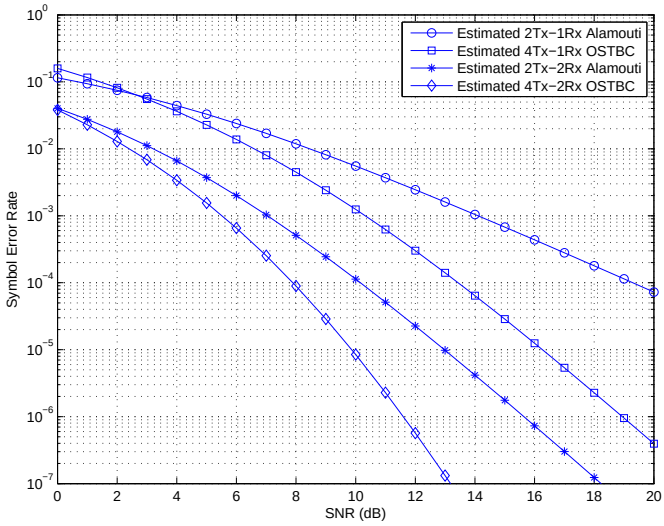


Figure 2.6. Symbol error rate comparison of Alamouti scheme with 4Tx - 2Rx and 4Tx - 1Rx channel scenarios

3. SPACE-TIME-POLARIZATION CODES

3.1. Capacity of MIMO Systems In The Presence Of Polarization Diversity

In practice there is a correlation between transmitted or received signals at different antenna elements. If the antenna elements are not located far apart from each other, spatial correlation occurs. As it is pointed out in [14], obtaining correlation coefficient less than *0.7* at adjacent elements requires *15-20* wavelengths separation in the broadside case and *70* wavelengths in the inline case. Therefore, in practical MIMO systems, there is a performance degradation and capacity decrement due to the correlation between the elements of the system.

In this section, we will prove that *why* the capacity of the MIMO communication system decreases with the receive correlation between receiving antennas. We will analyze MIMO system that consists of one dual-polarized transmit antenna and one dual-polarized receive antenna.

In the case of *polarization diversity*, the channel matrix $\mathbf{H}$ can be decomposed into fixed and variable component [15],

$$\mathbf{H} = \sqrt{\frac{K}{1+K}}\bar{\mathbf{H}} + \sqrt{\frac{1}{1+K}}\tilde{\mathbf{H}} \tag{3.1}$$

where K specifies fading characteristic of the channel that takes value between 0 and 10. K=0 corresponds to the pure *Rayleigh* fading ang K $\neq$ 0 corresponds to the *Ricean* Fading.

If Hermitian of the channel matrix is taken (for now, we omit $\sqrt{\frac{K}{1+K}}$ and $\sqrt{\frac{1}{1+K}}$ terms)

$$\mathbf{H^H} = \bar{\mathbf{H}}^H + \tilde{\mathbf{H}}^H \tag{3.2}$$

In this derivation, the matrix $\mathbf{H}\mathbf{H}^H$ will first be constructed in order to see the effects of correlation and antenna polarization.

If the Channel matrix $\mathbf{H}$ is decomposed by using SVD, it will result in three matrices and matrix in the middle will represent the singular values of $\mathbf{H}$ or eigenvalues of $\mathbf{H}\mathbf{H}^H$. Therefore instead of applying SVD to the matrix H, Eigen Value Decomposition can be applied to the $\mathbf{H}\mathbf{H}^H$

$$\begin{aligned} E(\mathbf{H}\mathbf{H}^H) &= E\left((\bar{\mathbf{H}}+\tilde{\mathbf{H}})(\bar{\mathbf{H}}^H+\tilde{\mathbf{H}}^H)\right) \\ &= E(\bar{\mathbf{H}}\bar{\mathbf{H}}^H)+E(\bar{\mathbf{H}}\tilde{\mathbf{H}}^H)+E(\tilde{\mathbf{H}}\bar{\mathbf{H}}^H)+E(\tilde{\mathbf{H}}\tilde{\mathbf{H}}^H) \end{aligned} \tag{3.3}$$

The elements of $\tilde{\mathbf{H}}$ matrix are circularly symmetric zero-mean gaussian random variables which make the terms $E(\bar{\mathbf{H}}\tilde{\mathbf{H}}^H)$ and $E(\tilde{\mathbf{H}}\bar{\mathbf{H}}^H)$ zero. By this gaussian assumption, we have

$$\begin{aligned} E(\mathbf{H}\mathbf{H}^H) &= E\left((\bar{\mathbf{H}}+\tilde{\mathbf{H}})(\bar{\mathbf{H}}^{\mathbf{H}}+\tilde{\mathbf{H}}^H)\right) \\ &= E(\bar{\mathbf{H}}\bar{\mathbf{H}}^H)+E(\tilde{\mathbf{H}}\tilde{\mathbf{H}}^H) \end{aligned} \tag{3.4}$$

The elements of $\bar{\mathbf{H}}$ matrix are expressed as

$$\bar{\mathbf{H}} = \begin{bmatrix} \bar{h}_{00} & \bar{h}_{01} \\ \bar{h}_{10} & \bar{h}_{11} \end{bmatrix} = \begin{bmatrix} 1 & \sqrt{\alpha_f} \\ \sqrt{\alpha_f} & 1 \end{bmatrix} \tag{3.5}$$

where $\sqrt{\alpha_f}$ is directly related to the XPD for the fixed component of the channel. The elements of $\tilde{\mathbf{H}}$ can be expressed as

$$\tilde{\mathbf{H}} = \begin{bmatrix} \tilde{h}_{00} & \tilde{h}_{01} \\ \tilde{h}_{10} & \tilde{h}_{11} \end{bmatrix} \tag{3.6}$$

where

$$E(|\tilde{h}_{00}|^2) = E(|\tilde{h}_{11}|^2) = 1 \tag{3.7}$$

$$E(|\tilde{h}_{01}|^2) \;=\; E(|\tilde{h}_{10}|^2) = \alpha \tag{3.8}$$

$$\tilde{\mathbf{H}}\tilde{\mathbf{H}}^H = \begin{bmatrix} \tilde{h}_{00} & \tilde{h}_{01} \\ \tilde{h}_{10} & \tilde{h}_{11} \end{bmatrix} \begin{bmatrix} \tilde{h}^*_{00} & \tilde{h}^*_{10} \\ \tilde{h}^*_{01} & \tilde{h}^*_{11} \end{bmatrix} = \begin{bmatrix} \tilde{h}_{00}\tilde{h}^*_{00} + \tilde{h}_{01}\tilde{h}^*_{01} & \tilde{h}_{00}\tilde{h}^*_{10} + \tilde{h}_{01}\tilde{h}^*_{11} \\ \tilde{h}_{10}\tilde{h}^*_{00} + \tilde{h}_{11}\tilde{h}^*_{01} & \tilde{h}_{10}\tilde{h}^*_{10} + \tilde{h}_{11}\tilde{h}^*_{11} \end{bmatrix} \tag{3.9}$$

$$E(\tilde{\mathbf{H}}\tilde{\mathbf{H}}^H) = \begin{bmatrix} E(|\tilde{h}_{00}|^2) + E(|\tilde{h}_{00}|^2) & r\sqrt{\alpha} + r\sqrt{\alpha} \\ r^*\sqrt{\alpha} + r^*\sqrt{\alpha} & E(|\tilde{h}_{10}|^2) + E(|\tilde{h}_{11}|^2) \end{bmatrix} \tag{3.10}$$

where r is the receive correlation coefficient and is defined

$$r = \frac{E(\tilde{h}_{00}\tilde{h}^*_{10})}{\sqrt{\alpha}} = \frac{E(\tilde{h}_{01}\tilde{h}^*_{11})}{\sqrt{\alpha}}. \tag{3.11}$$

If the scaling factors $\sqrt{\frac{K}{1+K}}$ and $\sqrt{\frac{1}{1+K}}$ terms are added, we have

$$E(\tilde{\mathbf{H}}\tilde{\mathbf{H}}^H) \;=\; \left(\frac{1}{1+K}\right) \begin{bmatrix} 1+\alpha & 2r\sqrt{\alpha} \\ 2r^*\sqrt{\alpha} & 1+\alpha \end{bmatrix} \tag{3.12}$$

$$E(\bar{\mathbf{H}}\bar{\mathbf{H}}^H) \;=\; \left(\frac{K}{1+K}\right) \begin{bmatrix} 1+\alpha_f & 2r\sqrt{\alpha_f} \\ 2r\sqrt{\alpha_f} & 1+\alpha_f \end{bmatrix} \tag{3.13}$$

from the channel definition, the channel can be defined in terms of receive correlation coefficient r, α (XPD factor for the variable component of the channel) and α_f (XPD factor for the fixed component of the channel)

$$E(\mathbf{H}\mathbf{H}^H) = \frac{1}{(1+K)} \begin{bmatrix} (1+\alpha) + K(1+\alpha_f) & 2(r\sqrt{\alpha} + K\sqrt{\alpha_f}) \\ 2(r^*\sqrt{\alpha} + K\sqrt{\alpha_f}) & (1+\alpha) + K(1+\alpha_f) \end{bmatrix}. \tag{3.14}$$

If Eigen Value Decomposition is used , the eigen values of this matrix can be

found and these eigen values can be used in the capacity formula which is defined as

$$\begin{aligned} C &= W\sum_{i=1}^{r}\log_2\left(1+\frac{\lambda_i P}{n_T\sigma^2}\right) \\ &= W\log_2\prod_{i=1}^{r}\left(1+\frac{\lambda_i P}{n_T\sigma^2}\right). \end{aligned} \tag{3.15}$$

For the analysis of one dual-polarized transmit and one dual-polarized receive MIMO communication system, take r=2. If the capacity formula above is written in open form as

$$\begin{aligned} C &= W\sum_{i=1}^{2}\log_2\left(1+\frac{\lambda_i P}{n_T\sigma^2}\right) \\ &= W\log_2\left((1+\frac{\lambda_1 P}{n_T\sigma^2})(1+\frac{\lambda_2 P}{n_T\sigma^2})\right) \\ &= W\log_2\left((1+\lambda_1 G)(1+\lambda_2 G)\right) \\ &= W\log_2\left(1+\lambda_1 G+\lambda_2 G+\lambda_1\lambda_2 G^2\right). \end{aligned} \tag{3.16}$$

Now, we want to see the effect of receive correlation coefficient on the capacity of MIMO system. If **r** value in the diagonal of the $\mathbf{HH}^H$ matrix is increased, off-diagonal terms increase and diagonal terms remain unchanged. This means that the sum of the eigenvalues in the open capacity formula does not change, and value of multiplication term decreases. Therefore, argument of the *log* function decreases as r increases that brings us the capacity decrement of MIMO system.

In practice, if fixed component (K) of the transmitted signal increases or polarization diversity is used, the capacity of system increases. However, if there is a big correlation in the receiver side (r), decisions are made by using insufficient data (correlation means that there is no new information to support our decisions) which means decrement in the capacity of the system.

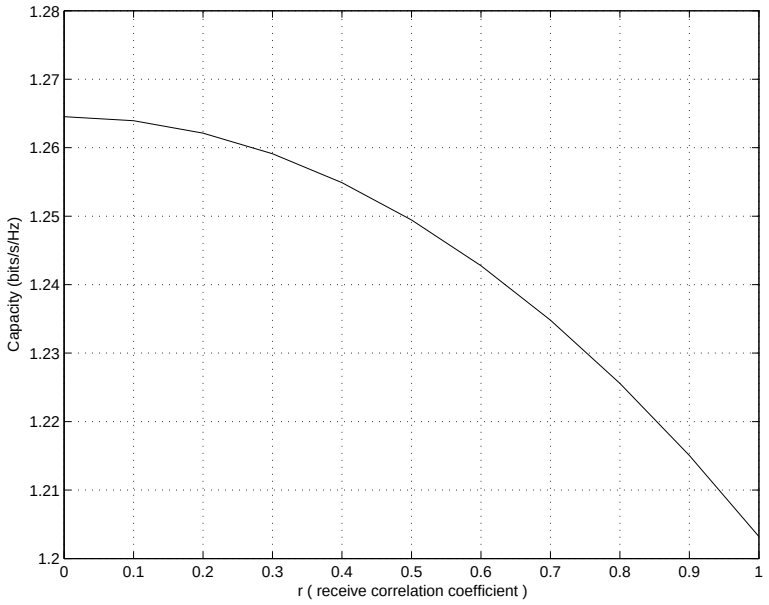


Figure 3.1. Channel capacity curve as a function of receive correlation on block Rayleigh fading channel, for SNR=0 dB, $n_r = 2$,$n_t = 2$,$\alpha = 0.1$,$\alpha_f = 0$ and $K = 0$

3.2. Performance of Alamouti Scheme In Case of Polarization Diversity

In this section, the performance of Alamouti scheme for the polarization diversity channel are focused. If the symbols s_0 and s_1 are transmitted at $+45^\circ$ and -45°, respectively, during one symbol period, then during the next symbol period $-s_1^*$ and s_0^* are transmitted at the same polarizations directions. Because of being orthogonal space-time block code, Alamouti code simplifies the detection problem at the receiver and vector detection problem turns to the scalar detection problems.

The input-output relation is given as [15],

$$\widetilde{r}_i = \sqrt{E_s}||\mathbf{H}||_F^2 s_i + \widetilde{n}_i, \qquad i = 0, 1 \tag{3.17}$$

where $\widetilde{r}_i$ is the scalar processed received signal corresponding to transmitted symbol s_i (i=0,1), and $\widetilde{n}_i$ is a scalar zero-mean complex gaussian noise with variance $\varepsilon|\widetilde{n}_i|^2 = ||\mathbf{H}||_F^2\sigma_n^2$, and $||\mathbf{H}||_F^2$ is the squared Frobenius norm of the channel matrix [15].

Standard scalar ML detection is given by [15],

$$\widehat{s}_i = arg \min_{s_i} |\widetilde{r}_i - \sqrt{E_s}||\mathbf{H}||_F^2 s_i| \tag{3.18}$$

where $\widehat{s}_i$ is the estimated data symbol. In order to include the effects of correlation we decompose the channel matrix as in equation (3.1). For 2 Tx-2Rx MIMO system, the elements of $\mathbf{H}$ matrix are given as,

$$\mathbf{H} = \begin{bmatrix} h_{00} & h_{01} \\ h_{10} & h_{11} \end{bmatrix} = \sqrt{\frac{K}{1+K}} \begin{bmatrix} \bar{h}_{00} & \bar{h}_{01} \\ \bar{h}_{10} & \bar{h}_{11} \end{bmatrix} + \sqrt{\frac{1}{1+K}} \begin{bmatrix} \tilde{h}_{00} & \tilde{h}_{01} \\ \tilde{h}_{10} & \tilde{h}_{11} \end{bmatrix} \tag{3.19}$$

In [15], the error probability for Alamouti scheme is given by,

$$P(\widehat{s}_i \neq s_i|\mathbf{H}) \approx \overline{N}_e e^{-E_s d_{min}^2/(4\sigma_n^2||\mathbf{y}||^2)} \tag{3.20}$$

where $\mathbf{y} = vec(\mathbf{H})$ and

$$\overline{\mathbf{y}} = vec\left(\sqrt{\frac{K}{1+K}}\overline{\mathbf{H}}\right) \tag{3.21}$$

$$\widetilde{\mathbf{y}} = vec\left(\sqrt{\frac{1}{1+K}}\widetilde{\mathbf{H}}\right). \tag{3.22}$$

In [15], since $\mathbf{H}$ is complex gaussian, the vector $\mathbf{y}$ is also complex gaussian and the right-hand side of error probability equation averaged over all channel realizations is completely characterized by the eigenvalues of the 4 x 4 covariance matrix $\mathbf{C}_{\widetilde{\mathbf{y}}} = \epsilon\{\widetilde{\mathbf{y}}\widetilde{\mathbf{y}}^{\mathbf{H}}\} = \mathbf{U}\mathbf{\Lambda}\mathbf{U}^{\mathbf{H}}$, where $\Lambda = diag\{\lambda_i\}_{i=1}^4$ and by the fixed component $\overline{\mathbf{y}}$. If we define,

$$\mathbf{d} = \begin{bmatrix} d_1 & d_2 & d_3 & d_4 \end{bmatrix} \tag{3.23}$$

we get,

$$P(\widehat{s}_i \neq s_i) \leq \overline{N}_e e^{-E_s d_{min}^2/(4\sigma_n^2)||\overline{\mathbf{y}}||^2} \prod_{i=1}^{r(\mathbf{C}_{\overline{\mathbf{y}}})} \frac{e^{((E_s d_{min}^2/(4\sigma_n^2))|d_i|\lambda_i)^2/(1+(E_s d_{min}^2/(4\sigma_n^2))\lambda_i)}}{1+\frac{E_s d_{min2}}{4\sigma_n^2}\lambda_i} \quad (3.24)$$

where $P(\widehat{s}_i \neq s_i) = \epsilon_H\{P(\widehat{s}_i \neq s_i)|\mathbf{H}\}$ is the probability of symbol error averaged over all channel realizations. Simulations results for Alamouti scheme in case of polarization diversity is shown below.

Simulation 1: Figure 3.2 shows the performance of Alamouti scheme in the presence of polarization diversity, for t (transmit correlation) = 0.7, r (receive correlation) = 0.1, $\alpha = 0.2$, $\alpha_f = 0.6$, and scaling constant k = 0.25.

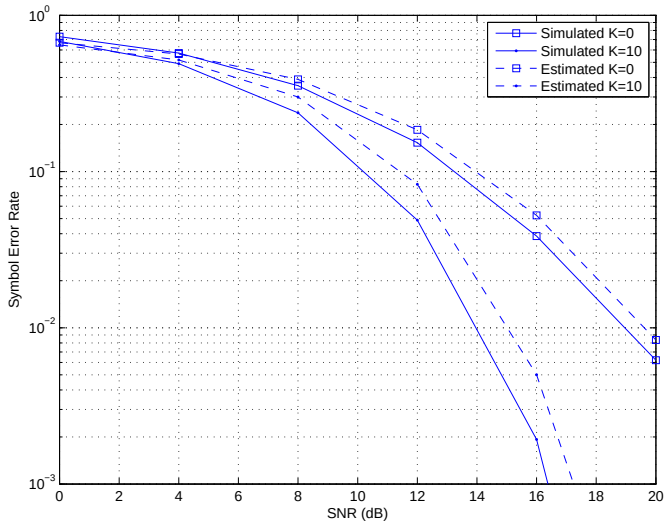


Figure 3.2. Symbol error rate for the Alamouti scheme as a function of SNR for varying K-factor. The estimate closely matches the actual symbol error rate (even for small SNR values)

Simulation 2: Figure 3.3 shows the performance degradation of Alamouti scheme in the context of TD. This figure shows the symbol error rate as a function of α for K=0 and SNR=17 dB. It is clearly seen that $\alpha = 1$ results in the lowest symbol error rate for all scenarios.

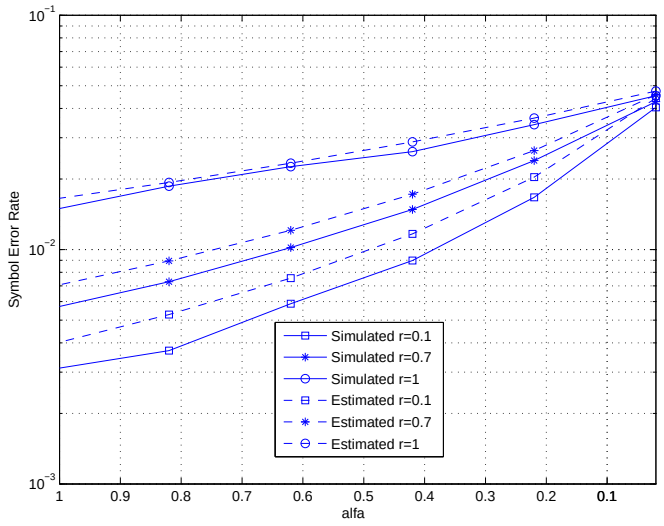


Figure 3.3. Symbol error rate for the Alamouti scheme as a function of α for varying receive correlation. The use of dual-polarized antennas leads to performance loss

Simulation 3: Figure 3.4 shows the performance degradation of Alamouti scheme in the context of TD. This figure shows the symbol error rate as a function of α for K=0 and SNR=17 dB. It is clearly seen that $\alpha = 1$ results in the lowest symbol error rate for all scenarios.

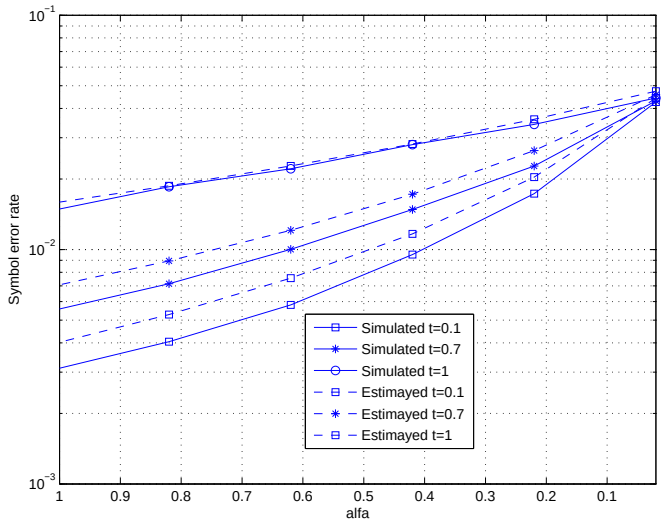


Figure 3.4. Symbol error rate for the Alamouti scheme as a function of α for varying transmit correlation. The use of dual-polarized antennas leads to performance loss

3.3. 4 Tx-2 Rx MIMO System Channel Model For Polarization Diversity

Consider a system with two dual-polarized transmit and one dual-polarized receive antenna. The channel is assumed frequency-flat over the band of interest. The system has the following input-output relation [1],[15]

$$\mathbf{r}=\mathbf{H}\mathbf{x}+\mathbf{n} \tag{3.25}$$

where $\mathbf{x} = [x_0 \quad x_1 \quad x_2 \quad x_3]^T$ is the 4 x 1 transmit signal vector whose elements are taken from a finite (complex) constellation chosen such that the average energy of the constellation elements is 1. $\mathbf{r} = [r_0 \quad r_1 \quad r_2 \quad r_3]^T$ is the 4 x 1 received signal vector. $\mathbf{n}$ is the 4 x 1 temporarly i.i.d. zero-mean complex Gaussian noise vector. $\mathbf{n}$ is the 4 x 1 temporarly i.i.d. zero-mean complex Gaussian noise vector.

$$\mathbf{H} = \begin{bmatrix} h_{0,0} & h_{0,1} \\ h_{1,0} & h_{1,1} \\ h_{2,0} & h_{2,1} \\ h_{3,0} & h_{3,1} \end{bmatrix} \quad or \quad \mathbf{H} = \begin{bmatrix} h_{0,0} & h_{1,0} & h_{2,0} & h_{3,0} \\ h_{0,1} & h_{1,1} & h_{2,1} & h_{3,1} \end{bmatrix} \tag{3.26}$$

is the 4 x 2 or 2 x 4 channel transfer matrix (*polarization matrix* [15]). Although MIMO system has a two dual-polarized transmit antennas and one dual-polarized receive antenna, it can be modeled as four-input two-output channel, since each polarization direction can be thought of as a separate physical channel.

The elements of $\mathbf{H}$ are complex gaussian random variables. The channel matrix can be decomposed into the sum of an average component and variable component as,

$$\mathbf{H} = \sqrt{\frac{K}{K+1}}\overline{\mathbf{H}} + \sqrt{\frac{1}{1+K}}\widetilde{\mathbf{H}} \tag{3.27}$$

where $E\{\mathbf{H}\} = \sqrt{K/(K+1)}\overline{\mathbf{H}}$ and $\sqrt{1/(1+K)}\widetilde{\mathbf{H}}$ are the average and variable components of the channel matrix respectively.

The elements of the matrix $\widetilde{\mathbf{H}}$ are denoted as $\widetilde{h}_{i,j}$ (i,j=0,1,2,3), and they are zero-mean circularly symmetric complex gaussian random variables whose variances depend on both propagation conditions and the antenna characteristics [15]. In this thesis, we set

$$E\{|\widetilde{h}_{0,0}|^2\} = E\{|\widetilde{h}_{1,1}|^2\} = E\{|\widetilde{h}_{2,0}|^2\} = E\{|\widetilde{h}_{3,1}|^2\} = 1 \tag{3.28}$$

$$E\{|\widetilde{h}_{0,1}|^2\} = E\{|\widetilde{h}_{1,0}|^2\} = E\{|\widetilde{h}_{2,1}|^2\} = E\{|\widetilde{h}_{3,0}|^2\} = \alpha \tag{3.29}$$

where $0 < \alpha < 1$ is directly related to the XPD for the variable component of the channel.

The elements of the matrix $\overline{\mathbf{H}}$ are denoted as $\overline{h}_{i,j}$ (i,j=0,1,2,3) and they are fixed components (complex numbers) of the channel matrix

$$E\{|\overline{h}_{0,0}|^2\} = E\{|\overline{h}_{1,1}|^2\} = E\{|\overline{h}_{2,0}|^2\} = E\{|\overline{h}_{3,1}|^2\} = 1 \tag{3.30}$$

$$E\{|\overline{h}_{0,1}|^2\} = E\{|\overline{h}_{1,0}|^2\} = E\{|\overline{h}_{2,1}|^2\} = E\{|\overline{h}_{3,0}|^2\} = \alpha_f \tag{3.31}$$

where $0 < \alpha_f < 1$ is directly related to the XPD for the fixed component of the channel

[15].

The Ricean K-factor for a fading channel is defined as the ration of the power in the fixed component to the power in the variable component. The K matrix of the channel can be defined as

$$\mathbf{K} = \begin{bmatrix} K_{1,1} & K_{1,2} & K_{1,3} & K_{1,4} \\ K_{2,1} & K_{2,2} & K_{2,3} & K_{2,4} \end{bmatrix} \tag{3.32}$$

where the components are

$$K_{1,1} = K_{1,3} = K_{2,2} = K_{2,4} = \frac{\alpha_f}{\alpha} K \tag{3.33}$$

$$K_{1,2} = K_{1,4} = K_{2,1} = K_{2,3} = K \tag{3.34}$$

throughout this thesis, K means the K-factor of the channel. K=0 condition means that we have pure Rayleigh fading.

The transmit and receive correlation coefficients, t and r, respectively are defined as,

$$r = \frac{E\{\widetilde{h}_{1,1}\widetilde{h}^*_{0,1}\}}{\sqrt{\alpha}} = \frac{E\{\widetilde{h}_{1,0}\widetilde{h}^*_{0,0}\}}{\sqrt{\alpha}} = \frac{E\{\widetilde{h}_{3,1}\widetilde{h}^*_{2,1}\}}{\sqrt{\alpha}} = \frac{E\{\widetilde{h}_{3,0}\widetilde{h}^*_{2,0}\}}{\sqrt{\alpha}} \tag{3.35}$$

$$t = \frac{E\{\widetilde{h}_{1,1}\widetilde{h}^*_{1,0}\}}{\sqrt{\alpha}} = \frac{E\{\widetilde{h}_{0,1}\widetilde{h}^*_{0,0}\}}{\sqrt{\alpha}} = \frac{E\{\widetilde{h}_{3,1}\widetilde{h}^*_{3,0}\}}{\sqrt{\alpha}} = \frac{E\{\widetilde{h}_{2,1}\widetilde{h}^*_{2,0}\}}{\sqrt{\alpha}} \tag{3.36}$$

For the sake of simplicity, we assume that,

$$E\{\widetilde{h}_{1,1}\widetilde{h}^*_{0,0}\} = E\{\widetilde{h}_{0,1}\widetilde{h}^*_{1,0}\} = E\{\widetilde{h}_{3,1}\widetilde{h}^*_{2,0}\} = E\{\widetilde{h}_{2,1}\widetilde{h}^*_{3,0}\} = 0. \tag{3.37}$$

In order to analyze the effects of receive correlation coefficient (r), transmit correlation coefficient (t), and diversity parameter α, a matrix that contains multiplication of all channel coefficients with each other should be available so that these multiplica-

tion terms can be replaced with MIMO system correlation parameters (r, t, α). For this purpose, we will first have $\mathbf{y} = vec(\mathbf{H})$. The vector $\mathbf{y}$ can be decomposed into a fixed component $\overline{\mathbf{y}}$ and a variable component $\widetilde{\mathbf{y}}$ as in [15]

$$\overline{\mathbf{y}} = vec\left(\sqrt{\frac{K}{1+K}}\overline{\mathbf{H}}\right) \quad , \quad \widetilde{\mathbf{y}} = vec\left(\sqrt{\frac{1}{1+K}}\widetilde{\mathbf{H}}\right) \tag{3.38}$$

$$\overline{\mathbf{H}} = \begin{bmatrix} \overline{h}_{0,0} & \overline{h}_{0,1} \\ \overline{h}_{1,0} & \overline{h}_{1,1} \\ \overline{h}_{2,0} & \overline{h}_{2,1} \\ \overline{h}_{3,0} & \overline{h}_{3,1} \end{bmatrix} \quad , \quad \overline{\mathbf{H}} = \begin{bmatrix} \widetilde{h}_{0,0} & \widetilde{h}_{0,1} \\ \widetilde{h}_{1,0} & \widetilde{h}_{1,1} \\ \widetilde{h}_{2,0} & \widetilde{h}_{2,1} \\ \widetilde{h}_{3,0} & \widetilde{h}_{3,1} \end{bmatrix}. \tag{3.39}$$

If the matrix $\mathbf{C}_{\widetilde{\mathbf{y}}} = E\{\widetilde{\mathbf{y}}\widetilde{\mathbf{y}}^{\mathbf{H}}\}$ are constructed all correlation terms can be represented as an element of this matrix. For this channel model, correlation matrix is defined as

$$\mathbf{C}_{\widetilde{\mathbf{y}}} = \begin{bmatrix} 1 & r^*\sqrt{\alpha} & s & s & t^*\sqrt{\alpha} & 0 & s & s \\ r\sqrt{\alpha} & \alpha & s & s & 0 & t^*\sqrt{\alpha} & s & s \\ s & s & 1 & r^*\sqrt{\alpha} & s & s & t^*\sqrt{\alpha} & 0 \\ s & s & r\sqrt{\alpha} & \alpha & s & s & 0 & t^*\sqrt{\alpha} \\ t\sqrt{\alpha} & 0 & s & s & \alpha & r^*\sqrt{\alpha} & s & s \\ 0 & t\sqrt{\alpha} & s & s & r\sqrt{\alpha} & 1 & s & s \\ s & s & t\sqrt{\alpha} & 0 & s & s & \alpha & r^*\sqrt{\alpha} \\ s & s & 0 & t\sqrt{\alpha} & s & s & r\sqrt{\alpha} & 1 \end{bmatrix} \tag{3.40}$$

By using SVD, $\mathbf{C}_{\widetilde{\mathbf{y}}}$ is decomposed as,

$$\mathbf{C}_{\widetilde{\mathbf{y}}} \;=\; \mathbf{A}\mathbf{A}^H = \mathbf{U}\boldsymbol{\Delta}\mathbf{U}^H = \mathbf{U}(\boldsymbol{\Delta}^{1/2})(\boldsymbol{\Delta}^{1/2})^H\mathbf{U}^H = (\mathbf{U}\boldsymbol{\Delta}^{\mathbf{1/2}})(\mathbf{U}\boldsymbol{\Delta}^{\mathbf{1/2}})^H \tag{3.41}$$

where

$$\mathbf{A} = \mathbf{U}\boldsymbol{\Delta}^{\mathbf{1/2}} \tag{3.42}$$

Therefore, it reduces to $\tilde{\mathbf{y}} = \mathbf{U}\boldsymbol{\Delta}^{\mathbf{1/2}}\mathbf{x}$ that contains information from the $\mathbf{C}_{\tilde{\mathbf{y}}}$ matrix. Now, we have correlated channel coefficients where correlation degree depends on eigenvectors and eigenvalues of the $\mathbf{C}_{\tilde{\mathbf{y}}}$matrix, elements of which depends on receive correlation coefficient (r),transmit correlation coefficient (t), antenna's XPD factor α (polarization discrimination coefficient) and spatial correlation coefficient (s).

3.4. Pairwise Error Probability Analysis

In case of dependent channel coefficients, the method that is explained in [24] is followed, the formula below is used for the Symbol error rate performance of 4Tx-1Rx MIMO communication system that uses the code given by [31],

$$P(\mathbf{x} \to \mathbf{y}|h_i^j, i = 0, 1, ..., n_t, j = 0, 1, ..., n_r) \leq exp(-\boldsymbol{\Omega}\mathbf{Y}(\mathbf{x}, \mathbf{y})\boldsymbol{\Omega}^* E_s/4N_0) \tag{3.43}$$

where

$$Y(\mathbf{x}, \mathbf{y}) = \begin{bmatrix} A(\mathbf{x}, \mathbf{y}) & \mathbf{0} & \dots & \mathbf{0} \\ \mathbf{0} & A(\mathbf{x}, \mathbf{y}) & \dots & \mathbf{0} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{0} & \mathbf{0} & \dots & A(\mathbf{x}, \mathbf{y}) \end{bmatrix} \tag{3.44}$$

is $n_R n_T \times n_R n_T$ matrix and

$$B(\mathbf{x}, \mathbf{y}) = \begin{bmatrix} y_1^1 - x_1^1 & y_2^1 - x_2^1 & \dots & \dots & y_l^1 - x_l^1 \\ y_1^2 - x_1^2 & y_2^2 - x_2^2 & \dots & \dots & y_l^2 - x_l^2 \\ y_1^3 - x_1^3 & y_2^3 - x_2^3 & \dots & \dots & y_l^3 - x_l^3 \\ \vdots & \vdots & \ddots & \ddots & \vdots \\ y_1^n - x_1^n & y_2^n - x_2^n & \dots & \dots & y_l^n - x_l^n \end{bmatrix} \tag{3.45}$$

$$A(\mathbf{x},\mathbf{y}) = B(\mathbf{x},\mathbf{y})B(\mathbf{x},\mathbf{y})^* \tag{3.46}$$

$$\mathbf{\Omega} = (\mathbf{\Omega}_1,...,\mathbf{\Omega}_m)_{n_T\times n_T} \tag{3.47}$$

$$\mathbf{\Omega}_j = (h_{1,j},...,h_{n,j}), \qquad j=0,1,...,n_R. \tag{3.48}$$

The formulas for both uncorrelated and correlated channels are for any kind of space-time codes. In this thesis, orthogonal space-time block codes is used which requires some little modifications in order to have correct pairwise error probability. The probability that a certain "true" transmitted code matrix X_0 is mistaken for a different matrix $X \neq X_0$ at the receiver is $P(\mathbf{X_0} \rightarrow \mathbf{X})$ [24]. Clearly this error measure depends on which particular matrices X and X_0 one chooses to study. In general, the system performance is dominated by the error events where X and X_0 are close and hence the pairwise error probability is usually a meaningful quantity.

Assuming that all code matrices X are equally likely to be transmitted, the average error rate is upper bounded by ,

$$P_{tot} \leq \frac{1}{|\mathbf{X}|}\sum_{\mathbf{X}_n\in\mathbf{X}}\sum_{\mathbf{X}_k\in\mathbf{X}} P(\mathbf{X_n}\rightarrow\mathbf{X_k}), \qquad k\neq n \tag{3.49}$$

where $P(\mathbf{X_n}\rightarrow\mathbf{X_k})$ denotes the probability that a transmitted code matrix is mistaken for a different code matrix X_k, and $\mid \mathbf{X} \mid$ is the number of elements in the matrix constellation $\mathbf{X}$ [13]. For the set of transmitted codewords a distance profile can be defined. The distance profile D for a given code matrix X_0 is defined as the set of the squared distances between that code matrix X_0 and all other code matrices $X_0 \neq X$. In mathematical terms [24],[13],

$$D(\mathbf{X_0}) = \{\parallel \mathbf{X}_k - \mathbf{X}_0 \parallel^2 \colon \mathbf{X}_k \neq \mathbf{X}_0\}. \tag{3.50}$$

A set of code matrices is said to constitute a uniform constellation if its members

have the same distance profile. This means that the minimum Euclidean distance,

$$d^2_{min} = \min_{\mathbf{X}_k \neq \mathbf{X}_0} \| \mathbf{X}_k - \mathbf{X}_0 \|^2 \tag{3.51}$$

as well as the number of code matrices achieving this minimum distance is the same for all codewords; hence the average error probability is approximately the same for all codewords [24].

An example of a uniform constellation in the context of OSTBC is the set of OSTBC matrices where the scalar symbols $\{s_k\}$ are taken from an MPSK constellation. This is so because in the case of MPSK all the signal points are uniformly distributed over the unit circle and therefore the constellation looks same from each point, but this property does not hold for QAM. Therefore OSTBC using QAM symbols does not form a uniform constellation. Because of having different average error probability for different codewords, $P(\mathbf{X_0} \rightarrow \mathbf{X})$ have to be calculated for every different code word and taken the average of error probabilities in order to include effects of all code word pairs.

The simulations results below show BER performance comparison for various space-time block codes for different MIMO systems under correlated channel conditions. We made monte carlo simulations in order to see the effects of polarization diversity. We used BPSK modulation method. We send 1000 packets each of which consists of 1000 bits and symbol errors were counted. In the receiver ML decoding is used and we have perfect channel knowledge. MRC (maximal ratio combining) is used in order to produce the estimates of transmitted symbols. The SNR was defined as $10\log(4E_s/\sigma_n^2)$ for 4 Tx MIMO system and $10\log(2E_s/\sigma_n^2)$ for 2 Tx MIMO system. The channel is defined by 3.26. For estimating probability of error, union bound was used which was defined by 3.49.

Simulation 1: Figure 3.5 shows the performance of orthogonal space-time block code in case of polarization diversity for 4Tx-2Rx channel model. This figure shows the symbol error rate as a function of SNR for $K = 0$ (Ricean factor of the channel),

$\alpha_f = 0$ (XPD factor for the fixed component of the channel), $r = 0$ (receive antenna correlation coefficient) and $t = 0$ (transmit antenna correlation coefficient). In case of polarization diversity, MIMO system has two dual-polarized antennas in the transmitter and one dual-polarized antenna at the receiver. Polarization discrimination ability of antennas is given by the XPD parameter (separation of orthogonal polarizations). In simulations, in order to include effects of XPD factor α and α_f parameters are used for variable and fixed components of the channel coefficients respectively as explained in [15]. Good discrimination of orthogonal polarizations amounts to small values of α or vice versa. But, there is a misunderstanding when $\alpha = 1$. This situation does not mean that discrimination is bad in transmit and receive antennas. $\alpha = 1$ corresponds to a different state of the system which indicates that MIMO system consists of four transmit antennas at the transmitter and two receive antennas at the receiver. This is also proven by the figure below where the performance for $\alpha = 1$ in 2Tx-1Rx (dual polarized) MIMO channel model is the same as 4Tx-2Rx MIMO channel model.

$$E\{|\widetilde{h}_{0,0}|^2\} = E\{|\widetilde{h}_{1,1}|^2\} = E\{|\widetilde{h}_{2,0}|^2\} = E\{|\widetilde{h}_{3,1}|^2\} = 1 \tag{3.52}$$

$$E\{|\widetilde{h}_{0,1}|^2\} = E\{|\widetilde{h}_{1,0}|^2\} = E\{|\widetilde{h}_{2,1}|^2\} = E\{|\widetilde{h}_{3,0}|^2\} = \alpha = 1 \tag{3.53}$$

For $\alpha < 1$ values, symbol error rate performance increases as α decreases towards zero. In this case, a different state of the system occurs which is 2Tx-1Rx dual polarized system model .

Now, let's analyze why 2Tx-1Rx dual-polarized model reduces 4Tx-1Rx MIMO system when MIMO system uses maximum polarization diversity. Four different symbols are sent in every symbol period from each of four transmit antennas. When $\alpha \neq 1$ and $\alpha \ll 1$, the channel coefficients are given

$$E\{|\widetilde{h}_{0,1}|^2\} = E\{|\widetilde{h}_{1,0}|^2\} = E\{|\widetilde{h}_{2,1}|^2\} = E\{|\widetilde{h}_{3,0}|^2\} = \alpha \approx 0 \tag{3.54}$$

and we only have $h_{00}, h_{11}, h_{20}, h_{31}$ coefficients as it is seen from the Figure 3.6.

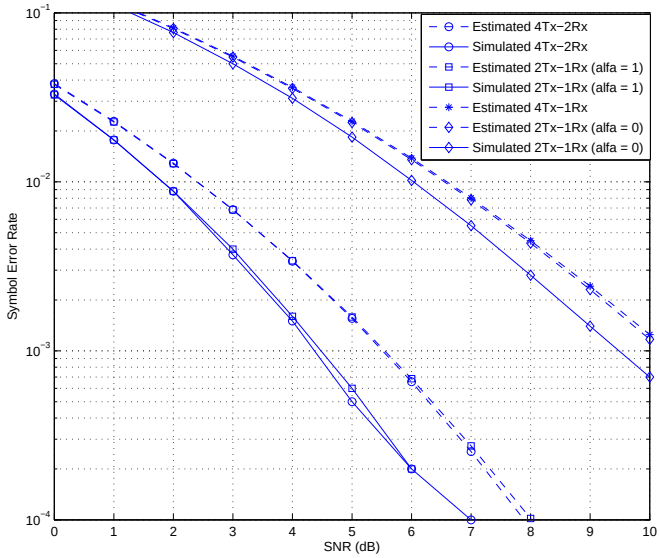


Figure 3.5. Symbol error rate for rate 1/2 OSTBC used in 4Tx-2Rx MIMO channel as a function of SNR (dB)

Again, four symbols are sent in four time intervals. In order to obtain orthogonal space-time block code, two more rows should be added that make products of each row with others is zero. If it is done, the orthogonal space-time block code used for 4Tx-2Rx MIMO system emerges. Using complex constellation requires adding conjugates of signals which again should make the code orthogonal. As a result, we will have the rate 1/2 orthogonal space-time block code (for complex signal constellations) as it is mentioned before. If special attention is given to orthogonal space-time block code below,

$$\mathbf{X} = \begin{bmatrix} x_1 & -x_2 & -x_3 & -x_4 \\ x_2 & x_1 & x_4 & -x_3 \\ x_3 & -x_4 & x_1 & x_2 \\ x_4 & x_3 & -x_2 & x_1 \end{bmatrix}. \tag{3.55}$$

It is seen that the first and the third row of the code matrix are the same symbols that are received from vertical polarization of receive antenna. Similarly, the second and the third rows of the code matrix are the same symbols that are received from

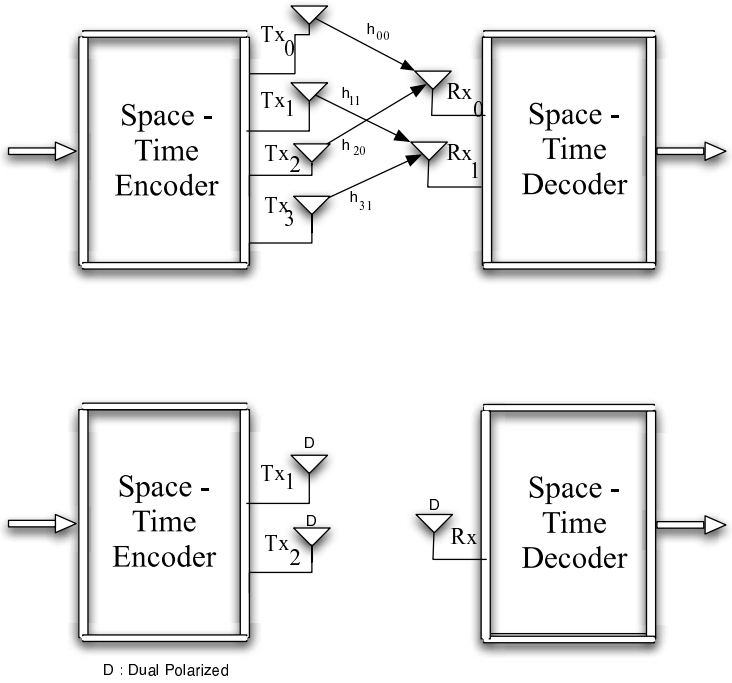


Figure 3.6. 4Tx-2Rx MIMO system (above, in case of full polarization diversity), 2Tx-1Rx MIMO system with dual polarized antennas (below)

horizontal polarization of the receive antenna. Received code symbols should be made orthogonal to benefit from the full diversity. As it is seen from the code, it does not matter which of the received symbols are chosen. If the symbols from vertical polarization of the receive antenna are selected, other two columns (second and fourth) have to be added to make the code matrix orthogonal. Likewise, if the symbols from horizontal polarization of the receive antenna are chosen, the first and the third rows have to be added to the code matrix. Hence, it does not matter to which polarization direction is focused, the same orthogonal code matrix results in any one of two cases. In case of complex signal constellations, the rate 1/2 orthogonal space-time block code should be used as it is given below that means complex symbols have to be added to make code matrix orthogonal,

$$\mathbf{X}_4^c = \begin{bmatrix} x_1 & -x_2 & -x_3 & -x_4 & -x_1^* & -x_2^* & -x_3^* & -x_4^* \\ x_2 & x_1 & x_4 & -x_3 & x_2^* & x_1^* & x_4^* & -x_3^* \\ x_3 & -x_4 & x_1 & x_2 & x_3^* & -x_4^* & x_1^* & x_2^* \\ x_4 & x_3 & -x_2 & x_1 & x_4^* & x_3^* & -x_2^* & x_1^* \end{bmatrix}. \tag{3.56}$$

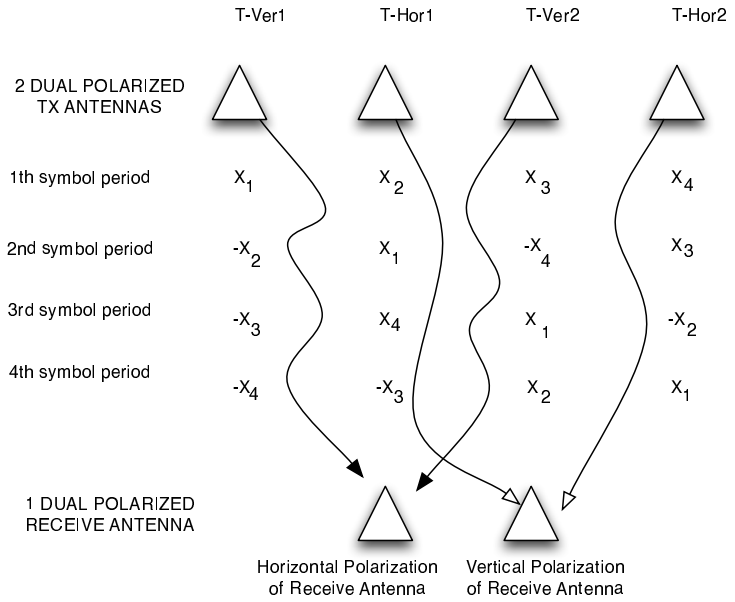


Figure 3.7. Channel model in case of full polarization diversity (received signals)

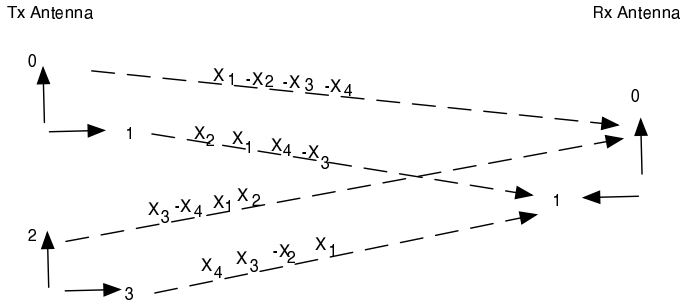
	Received symbols from vertical polarization	
1th Symbol Period	X_1	X_3
2nd Symbol Period	$-X_2$	$-X_4$
3rd Symbol Period	$-X_3$	X_1
4th Symbol Period	$-X_4$	X_2

Figure 3.8. Received signals from vertical polarization in each symbol period

Because of the reasons given above, 2Tx - 1Rx (dual polarized) MIMO system reduces to 4Tx - 1Rx MIMO system in case of full polarization diversity.

	Received symbols from horizontal polarization	
1th Symbol Period	X_2	X_4
2nd Symbol Period	X_1	X_3
3rd Symbol Period	X_4	$-X_2$
4th Symbol Period	$-X_3$	X_1

Figure 3.9. Received signals from horizontal polarization in each symbol period



	Received Symbols
Vertical Polarization of Receive Antenna	X_1 $-X_2$ $-X_3$ $-X_4$ X_3 $-X_4$ X_1 X_2
Horizontal Polarization of Receive Antenna	X_2 X_1 X_4 $-X_3$ X_4 X_3 $-X_2$ X_1

Figure 3.10. Sent symbols in case of full polarization diversity

Simulation 2: Figure 3.11 shows the performance degradation of rate 1/2 Orthogonal Space-Time Block Code for four inputs two outputs MIMO channel that consists of 2Tx-1Rx dual polarized antennas. This figure shows the symbol error rate as a function of α for K=0 and SNR=4 dB. It is clearly seen that $\alpha = 1$ results in the lowest symbol error rate for all scenarios.

Simulation 3: Figure 3.12 shows the performance degradation of rate 1/2 Orthogonal Space-Time Block Code for 4Tx - 2Rx MIMO channel that consists of 2Tx-1Rx dual polarized antennas. This figure shows the symbol error rate as a function of α for K=0 and SNR=4 dB. It is clearly seen that $\alpha = 1$ results in the lowest symbol error rate for all scenarios.

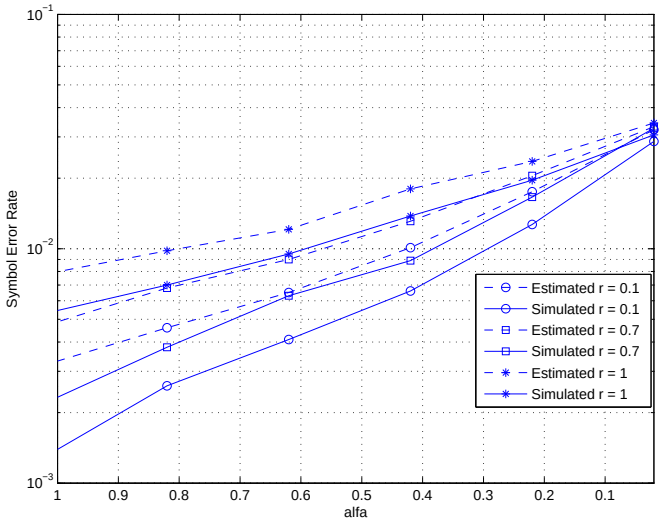


Figure 3.11. Symbol error rate for rate 1/2 OSTBC as a function of α for varying receive correlation. The use of dual-polarized antennas leads to performance loss.

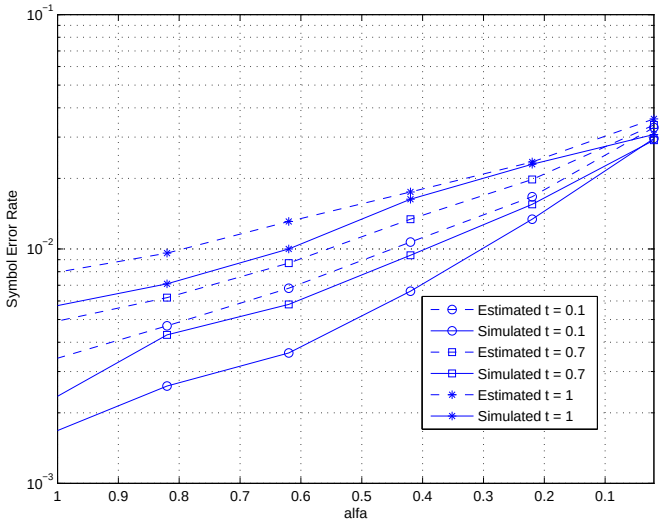


Figure 3.12. Symbol error rate for rate 1/2 OSTBC as a function of α for varying transmit correlation. The use of dual-polarized antennas leads to performance loss

4. ITERATIVE DETECTION AND DECODING OF SPACE-TIME-POLARIZATION CODES

Securing high reliability of the data transmission requires special attention to receiver design. Nonetheless, good decoding methods may result in high complexity of the receiver. In [30], sphere decoding technique was proposed for lattice decoding and adapted for space-time codes. The sphere decoder provides the maximum likelihood estimated of the transmitted signal sequence. This implies that in many cases of interest ML performance can be obtained with complexity similar to nulling and cancelling technique[7]. Soft iterative decoding techniques are very famous research area and these techniques have been reported to achieve important results for codes with long codeword length. The key idea in iterative decoding techniques is the use of probabilistic (soft) information about each bit in the transmitted sequence. In case of multiantenna systems, however, the best way of calculating of soft information with low complexity is not very accurate. If the soft information is obtained by means of an exhaustive search, the computational complexity grows exponentially as indicated in [21], where turbo-coded modulation for multiantenna systems has been studied. In [21] and [11], two different way of the sphere decoding algorithm have been proposed for estimating the soft information. Iterative methods that combine channel codes and space-time processing are studied in [27]-[18].

4.1. MAP Receiver

Combining iterative processing with multiple-antenna channels is particularly challenging because the channel capacities can be a factor of ten or more higher than their single-antenna counterparts. However, any effort to achieve capacity on a channel usually requires some form of outer channel code that provides redundancy. In this case, the space-time encoder or mapper acts like an inner code that transmits symbols that have redundancy introduced by the outer code. A method to iteratively detect and decode any space-time mapper that is combined with an outer channel code is analyzed.

Space-time mapper means that the symbols to be transmitted on each antenna should be functions of the encoded data stream. The channel code can be any code that may be decoded using soft inputs and outputs.

In this section, the following model for MIMO channel is used [2],[19]. Let s be an M×1 vector of symbols whose entries are chosen from some complex constellation and let y be an N×1 vector of received signals given by,

$$\mathbf{y} = \mathbf{Hs} + \mathbf{n} \tag{4.1}$$

where $\mathbf{H}$ is a complex matrix, known perfectly to the receiver and $\mathbf{n}$ is a vector of independent zero-mean complex Gaussian noise entries with variance σ^2 per real component. It is assumed that the vector $\mathbf{s} = [s_1, ..., s_M]^T$ obeys the component-wise energy constraint $E||s_m||^2 = E_s/M$. The vector model (4.1) is used repeatedly to transmit a continuous stream of data bits, separated into blocks representing uses of the channel. For any given block, the components of the symbol vector $\mathbf{s}$ be obtained using the mapping function $s_m = map(\mathbf{x}^{<m>})$, m=1,...,M, where $\mathbf{x}^{<m>}$ is an $M_c \times 1$ vector of data bits, and M_c is the number of bits per constellation symbol. The vector of bits transmitted during one application of the model (4.1) is denoted by $\mathbf{x}^{<1>}, ..., \mathbf{x}^{<m>}$ such that the transmitted vector constellation symbol is $\mathbf{s} = map(\mathbf{s})$.

If the information bits in the blocks $\mathbf{x}^1, \mathbf{x}^2, ...$ are uncoded, the decisions on the bits can be made ML on a block-by-block basis. It is, however, considered $\mathbf{x}^1, \mathbf{x}^2, ...$ to themselves be the output of a channel code of rate $R \leq 1$ that introduces redundancy and correlation between its entries[11].

Figure 4.1 below gives a flowchart of the iterative algorithm. The detector takes channel observations $\mathbf{y}$ and a priori knowledge L_{A1} on the inner coded bits and computes new (extrinsic) information L_{E1} for each of the $M.M_c$ coded bits per vector channel symbol $\mathbf{y}$. Then L_{E1} is deinterleaved to become a priori input L_{A2} to the outer soft-in/soft-out decoder (maximum a posteriori (MAP), a posteriori probability (APP),Bahl-Cocke-Jelinek-Raviv (BCJR) algorithm) which calculates extrinsic infor-

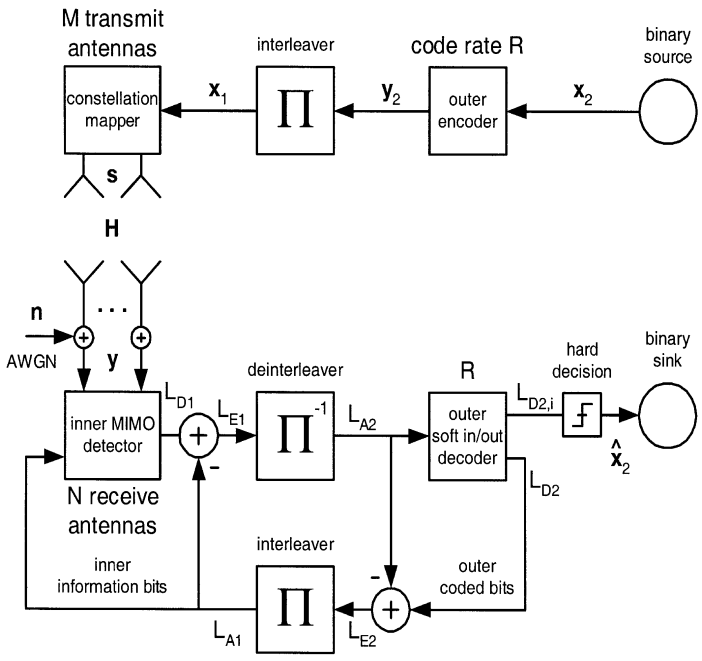


Figure 4.1. Transmitter, MIMO channel, and receiver with iterative detection and decoding. The subscript "1" denotes variables associated with inner code, and the subscript "2" denotes variables associated with the outer channel code.

mation L_{E2} on the outer coded bits. Then L_{E2} is reinterleaved and fed back as a priori knowledge L_{A1} to the inner detector, thus completing a cycle or iteration. Each iteration reduces the bit-error rate (BER) by this exchange of information.

The APP is usually expressed as a log-likelihood ratio value(L-value). The magnitude of the L-value indicates the reliability of the decision. L-values near zero correspond to unreliable bits.

The a posteriori L-value of the bit $x_k, k = 0, ..., M.M_c - 1$, conditioned on the received vector channel symbol $\mathbf{y}$, is

$$L_D(x_k|\mathbf{y}) = \ln \frac{P[x_k = +1|\mathbf{y}]}{P[x_k = -1|\mathbf{y}]} \tag{4.2}$$

It is assumed that the bits in $\mathbf{x}$ have been encoded with a channel code, but that an interleaver at the encoder is used to scramble the bits from other blocks into our block,

so that the bits within $\mathbf{x}$ are approximately statistically independent of one another. Therefore, by using Bayes' theorem, (4.2) can be written as [11]

$$L_D(x_k|\mathbf{y}) = L_A(x_k) + \ln \frac{\sum_{\mathbf{x}\in\mathbf{X_{k,+1}}} p(\mathbf{y}|\mathbf{x}) \exp \sum_{j\in J_{k,x}} L_A(x_j)}{\sum_{\mathbf{x}\in\mathbf{X_{k,-1}}} p(\mathbf{y}|\mathbf{x}) \exp \sum_{j\in J_{k,x}} L_A(x_j)} \tag{4.3}$$

where $X_{k,+1}$ is the set of $2^{M.M_c-1}$ bit vectors having $x_k = +1$; that is

$$X_{k,+1} = \{\mathbf{x}|x_k = +1\} \ , \ X_{k,-1} = \{\mathbf{x}|x_k = -1\}$$

$J_{k,\mathbf{x}}$ is the set of indices j with

$$J_{k,\mathbf{x}} = \{j|j = 0, ..., M.M_c - 1, j \neq k, x_j = 1\}$$

and

$$L_A(x_j) = \ln \frac{P[x_j = +1]}{P[x_j = -1]}. \tag{4.4}$$

By multiplying the numerator and denominator with $\exp[-1/2 \sum_{k=0}^{M.M_c-1} L_A(x_k)]$, we may write (4.3) as

$$L_D(x_k|\mathbf{y}) = L_A(x_k) + \ln \frac{\sum_{\mathbf{x}\in\mathbf{X_{k,+1}}} p(\mathbf{y}|\mathbf{x}) \exp\{\frac{1}{2}\mathbf{x}_{[k]}^T \mathbf{L}_{A,[k]}\}}{\sum_{\mathbf{x}\in\mathbf{X_{k,-1}}} p(\mathbf{y}|\mathbf{x}) \exp\{\frac{1}{2}\mathbf{x}_{[k]}^T \mathbf{L}_{A,[k]}\}} \tag{4.5}$$

where $\mathbf{x}_{[k]}$ denotes the subvector of $\mathbf{x}$ obtained by omitting its kth element x_k and $\mathbf{L_{A,[k]}}$ denotes the vector of all L_A values, also omitting x_k. Thus L_D can be written as a sum of a priori L-value L_A and extrinsic L-value L_E. Eqn. 4.5 can be rewritten using the subscripts used in the figure above,

$$L_D(x_1|\mathbf{y}) = L_A(x_{1,[k]}) + \ln \frac{\sum_{\mathbf{x}_1 \in \mathbf{X_{k,+1}}} p(\mathbf{y}|\mathbf{x}_1) \exp\{\frac{1}{2}\mathbf{x}_{1,[k]}^T \mathbf{L}_{A_1,[k]}\}}{\sum_{\mathbf{x}_1 \in \mathbf{X_{k,-1}}} p(\mathbf{y}|\mathbf{x}_1) \exp\{\frac{1}{2}\mathbf{x}_{1,[k]}^T \mathbf{L}_{A_1,[k]}\}} \tag{4.6}$$

where $\mathbf{x}_1$ represents the coded bits to be transmitted and $\mathbf{y}$ is the vector measurement obtained at the receiver [11].

The channel decoder processing can also be decomposed into a priori and extrinsic components, which is given by,

$$L_D(x_{2,k}|\mathbf{L}_{A_2}) = L_{A_2}(x_{2,[k]}) + \ln \frac{\sum_{\mathbf{x}_2 \in \mathbf{X_{k,+1}}} \exp\{\frac{1}{2}\mathbf{x}_{2,[k]}^T \mathbf{L}_{A_2,[k]}\}}{\sum_{\mathbf{x}_2 \in \mathbf{X_{k,-1}}} \exp\{\frac{1}{2}\mathbf{x}_{2,[k]}^T \mathbf{L}_{A_2,[k]}\}} \tag{4.7}$$

An important part of computing the L-value (4.5) for the detector is computing the likelihood function $p(\mathbf{y}|\mathbf{x})$, which can be calculated by the formula,

$$p(\mathbf{y}|\mathbf{s} = map(\mathbf{x})) = \frac{\exp[\frac{-1}{2\sigma^2}.||\mathbf{y} - \mathbf{H}.\mathbf{s}||^2]}{(2\pi\sigma^2)^N}. \tag{4.8}$$

For MIMO systems employing polarization diversity method in both transmitter and receiver, likelihood function has to be modified in order to increase the performance of overall system. In case of using polarization diversity, received data symbols will be correlated according to receive and transmit correlation parameters of the system. In practice, there are no such parameters in MIMO system. These parameters are the result of antenna spacings and degree of discrimination of polarization directions of transmit and receive antennas.

In order to include the effects of correlation parameters, our channel have to be modeled in a different way[14]. As it is known from previous sections, the following input-output relationship specifies MIMO system,

$$\mathbf{y} = \mathbf{H}\mathbf{s} + \mathbf{n} \tag{4.9}$$

where $\mathbf{H}$ is an $N_r \times N_t$ complex propagation matrix.Element (q,p) of $\mathbf{H}$ contains the flat-fading channel coefficient from TX antenna p to RX antenna q. The vector $\mathbf{n}$ represents zero-mean, complex Additive White Gaussian Noise with covariance matrix $\mathbf{R}_{nn} = E[\mathbf{n}\mathbf{n}^H] = {\sigma_n}^2\mathbf{I}$. Model that includes correlation for a narrowband flat-fading MIMO channel $\mathbf{H}$ is defined as

$$\mathbf{R}_H = E\left[vec(\mathbf{H})vec(\mathbf{H})^H\right] \tag{4.10}$$

where $vec(\mathbf{H})$ denotes the $N_rN_t \times 1$ vector composed by stacking the columns of $\mathbf{H}$. In a correlated channel $\mathbf{R}_H$ can be decomposed into two terms $\mathbf{R}_{TX}$ and $\mathbf{R}_{RX}$ which are called transmit and receive correlation matrices respectively[14]. The channel matrix $\mathbf{H}$ can be rewritten as

$$\mathbf{R}_H = {\mathbf{R}_{TX}}^T \otimes \mathbf{R}_{RX} \tag{4.11}$$

with $\otimes$ denoting the Kronecker product. $\mathbf{R}_{TX}$ and $\mathbf{R}_{RX}$ are defined as

$$\mathbf{R}_{TX} = E\left[(\mathbf{h}^q)^H\mathbf{h}^q\right], for\ all\ q = 1, ..., N_r \tag{4.12}$$

$$\mathbf{R}_{RX} = E\left[\mathbf{h}_p({\mathbf{h}_p}^H)\right], for\ all\ p = 1, ..., N_t \tag{4.13}$$

where $\mathbf{h}_q$ is the q-th row of $\mathbf{H}$, and $\mathbf{h}_p$ is the p-th column of $\mathbf{H}$ Receive correlation matrix $\mathbf{C}_y$ can be found or the received signal in correlated channel. This model will be different in order to include effects of correlation parameters at transmitter and receiver. We should start by rewriting received signal $\mathbf{y}$

$$\mathbf{y} = \mathbf{H}\mathbf{s} + \mathbf{n} = (\mathbf{s}^T \otimes \mathbf{I}_{Nr})vec(\mathbf{H}) + \mathbf{n}. \tag{4.14}$$

The correlation matrix of received signal can be computed as,

$$\mathbf{C}_y = E\left[\mathbf{y}\mathbf{y}^H\right]$$

$$
\begin{aligned}
&= (\mathbf{s}^T \otimes \mathbf{I}_{Nr}) vec(\mathbf{H}) vec(\mathbf{H})^H (\mathbf{s}^* \otimes \mathbf{I}_{Nr}) + \mathbf{R}_{nn} \\
&= (\mathbf{s}^T \otimes \mathbf{I}_{Nr}) \mathbf{R}_H (\mathbf{s}^* \otimes \mathbf{I}_{Nr}) + \mathbf{R}_{nn} \\
&= (\mathbf{s}^T \otimes \mathbf{I}_{Nr}) \mathbf{R}_{TX}^T \otimes \mathbf{R}_{RX} (\mathbf{s}^* \otimes \mathbf{I}_{Nr}) + \mathbf{R}_{nn} \\
&= \left[(\mathbf{s}^T {\mathbf{R}_{TX}}^T) \otimes \mathbf{R}_{RX} \right] [\mathbf{s}^* \otimes \mathbf{I}_{Nr}] + \mathbf{R}_{nn} \\
&= \left[\mathbf{s}^T {\mathbf{R}_{TX}}^T \mathbf{s}^* \right] \otimes \mathbf{R}_{RX} + \mathbf{R}_{nn} \\
&= \rho \mathbf{R}_{RX} + \mathbf{R}_{nn}
\end{aligned}
$$

where $\rho = \mathbf{s}^T {\mathbf{R}_{TX}}^T \mathbf{s}^*$. We modified (4.8) by multiplying $||\mathbf{y} - \mathbf{H}\mathbf{s}||^2$ with ${\mathbf{C}_y}^{-1}$ matrix such that Therefore, our new formula becomes,

$$
p(\mathbf{y}|\mathbf{s} = map(\mathbf{x})) = \frac{\exp[\frac{-1}{2\sigma^2}.Tr\{{\mathbf{C}_y}^{-1}(\mathbf{y} - \mathbf{H}\mathbf{s})(\mathbf{y} - \mathbf{H}\mathbf{s})^H\}]}{(2\pi\sigma^2)^N}. \tag{4.15}
$$

We made monte carlo simulations in order to see the effects of MAP receiver. We used BPSK modulation method unless otherwise stated. We send 1000 packets each of which consists of 1000 bits and symbol errors were counted. In the receiver we have perfect channel knowledge. MRC(maximal ratio combining) is used in order to produce the estimates of transmitted symbols. The SNR was defined as $10\log(4E_s/\sigma_n^2)$, and the channel is defined by 3.26. For estimating probability of error, union bound was used which was defined by 3.49.

Simulation 1: Figure 4.2 shows the SER performance comparison of ML receiver and MAP receiver. Values of simulation parameters are $\alpha = 0.02$ (polarization coefficient),t (transmit antenna correlation coefficient)=0, r (receive antenna correlation coefficient)=0 and s (spatial correlation coefficient)=0. As it is seen from the figure, we have about 3dB SNR gain if we use MAP receiver. When we have full polarization discrimination, we have a SER performance between 4Tx-1Rx and 4Tx-2Rx MIMO system. At the final iteration of MAP receiver, 2Tx-1Rx (dual-polarized antennas) MIMO system performance is near to the performance of 4Tx-2Rx MIMO system.

Simulation 2: Figure 4.3 shows the SER performance comparison of ML receiver

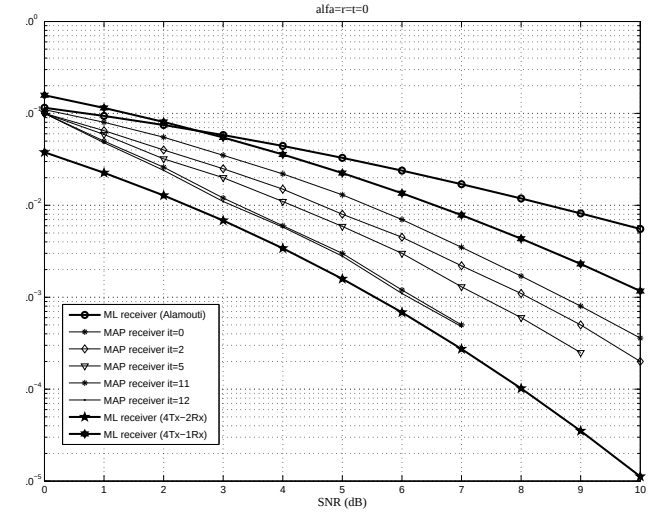


Figure 4.2. The SER performance of comparison of ML and MAP receivers

and MAP receiver in presence of correlation in the wireless channel. Values of simulation parameters are $\alpha = 0.1$ (polarization coefficient),t (transmit antenna correlation coefficient)=0.1, r (receive antenna correlation coefficient)=0.1 and s (spatial correlation coefficient)=0.1. We can have better performance than 4Tx-1Rx MIMO system if we use MAP receiver as it is shown in the figure (*11*th iteration). When there is

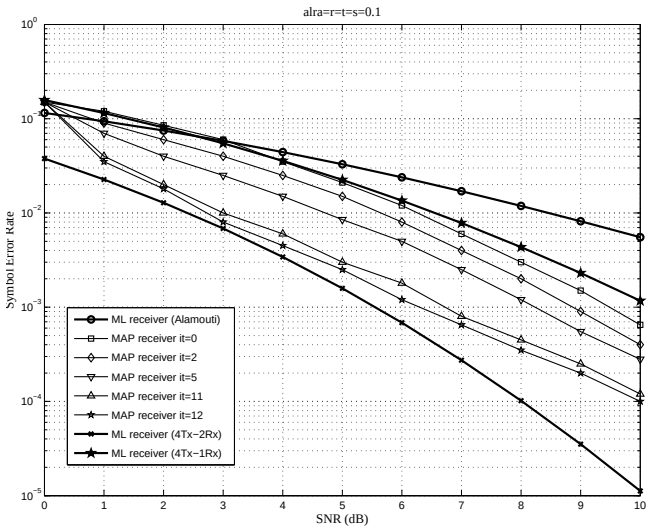


Figure 4.3. The SER performance of comparison of ML and MAP receivers in case of correlation

a correlation between transmit and between receive antennas, we have a performance degradation. If we have a correlation between channel coefficients, we can compensate

it by using MAP receiver. As it is seen from the figure, at the final iteration of MAP receiver our 2Tx-1Rx (dual-polarized) MIMO system has a performance near to the 4Tx-2Rx MIMO system.

Simulation 3: Figure 4.4 shows the SER performance comparison of ML receiver and MAP receiver in presence of high correlation in the wireless channel. Values of simulation parameters are $\alpha = 0.1$ (polarization coefficient),t (transmit antenna correlation coefficient)=0.5, r (receive antenna correlation coefficient)=0.5 and s (spatial correlation coefficient)=0.1. We can have better performance than 4Tx-1Rx MIMO system if we use MAP receiver as it is shown in the figure (*11*th iteration). In this

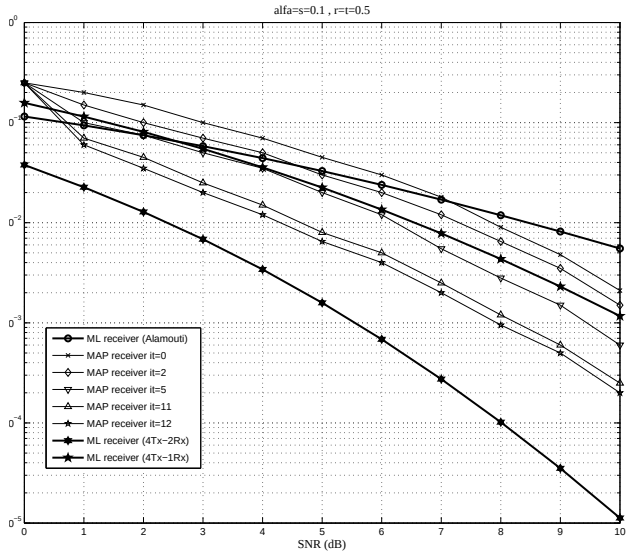


Figure 4.4. The SER performance of comparison of ML and MAP receivers in case of high correlation

simulation, we have a high correlation between channel coefficients which is dictated by transmit antenna and receive antenna coefficient. At the $5th$ iteration, 2Tx-1Rx (dual-polarized) system performance is near to 4Tx-1Rx system performance and it exceeds 4Tx-1Rx system performance at the final iteration of MAP algorithm which means that if we have high correlation in wireless channel, using MAP receiver increases the system performance.

Simulation 4: Figure 4.5 shows the SER performance comparison of ML receiver

and MAP receiver in presence of very high correlation in the wireless channel. Values of simulation parameters are $\alpha = 0.1$ (polarization coefficient),t (transmit antenna correlation coefficient)=0.7, r (receive antenna correlation coefficient)=0.7 and s (spatial correlation coefficient)=0.1. We can not reach the performance of 4Tx-1Rx MIMO system if we use MAP receiver as it is shown in the figure (*11*th iteration) because there is a very high correlation between transmit and receive antennas. In this simulation,

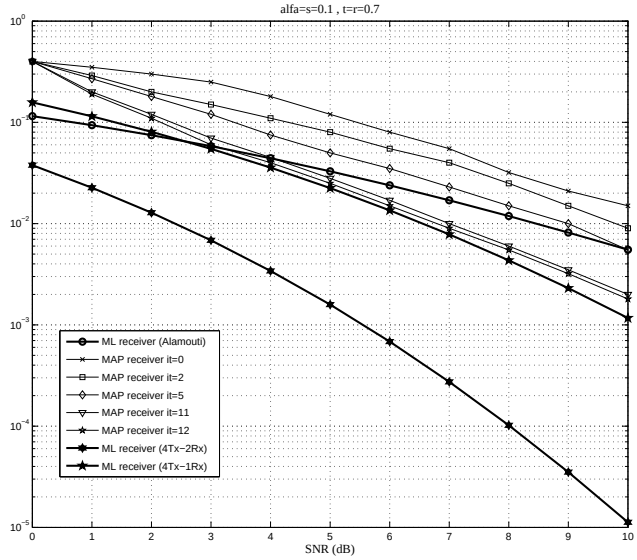


Figure 4.5. The SER performance of comparison of ML and MAP receivers in case of very high correlation

we have a very high correlation between channel coefficients. As it is seen from the figure 2Tx-1Rx (dual polarized) system can reach 4Tx-1Rx system performance at the final iteration of MAP algorithm. We can not compensate very high correlation even if we use MAP receiver.

5. CONCLUSIONS

In this thesis we discussed performance of orthogonal space-time block codes for multiple antenna wireless channels without information of the channel at the transmitter. We focused on space-time block codes and explained the benefits and limitations of primary designs. We first discussed uncorrelated multi-antenna systems. Because of independent channels between transmitter and receiver, performance increase is expectable in uncorrelated systems and it is proven by computer simulations that contains both estimated and simulated results.

Second, we analyze correlated multi-antenna systems which is the result of using polarization diversity at both transmitter and receiver. The transmit diversity of any scheme in these channels was shown to be limited to the rank of the correlation matrix. When the rank of the correlation matrix is not full, we cannot get diversity equal to the number of transmit antennas. We compare the performance of Alamouti code with orthogonal space-time block code in the presence of polarization diversity. Although, polarization diversity leads to performance loss in both MIMO system, we have an advantage of having one uni-polarized antenna instead of having one dual-polarized antenna at the receiver if we use orthogonal space-time block code which is an important because of high correlation between polarization directions of antenna of receiver.

Finally, different receiver structures for MIMO system are emphasized. ML receivers are mostly used because of its simlicity, but their performance depends on the orthogonality of transmitted symbols which are not the case when we have correlation in wireless channel. Therefore, MAP and MMSE receivers are better because they use soft information of each bit to detect it. In other words, we use maximum information available about each transmitted bit in order to make better bit decisions. We have about 3dB and 1-1.5dB SNR gain when we use MAP receiver and MMSE receiver structure respectively.

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