PRESERVICE MATHEMATICS TEACHERS' PROBABILISTIC KNOWLEDGE AND THEIR ATTITUDES TOWARD PROBABILITY AND TEACHING OF PROBABILITY

by

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ABSTRACT

PRESERVICE MATHEMATICS TEACHERS' PROBABILISTIC KNOWLEDGE AND THEIR ATTITUDES TOWARD PROBABILITY AND TEACHING OF PROBABILITY

This study investigates preservice mathematics teachers' (PSMT) probabilistic knowledge and their attitudes toward probability and teaching probability. The target population was PSMTs in the third and fourth grades in Turkey. The participants were seniors and juniors who volunteered to participate. Firstly, preservice teachers completed attitudes toward probability and its teaching scale (APT), which consists of 27 items. The number of participants was 195 for APT. Then, they completed the probability concept inventory (PCI), which includes 12 probability questions regarding different meanings and different knowledge types of probability. In total, 95 PSMTs took PCI. After the analysis of the results of PCI and APT, 13 participants were interviewed via Zoom. The results showed that PSMTs have a low level of Advanced Content Knowledge (ACK) while they have adequate Common Content Knowledge (CCK) among three types of probabilistic knowledge. The examination of knowledge of three meanings of probability indicated that PSMTs' knowledge of frequentist meaning (FM) is weak. The results of APT showed that PSMTs have positive attitudes toward probability and its teaching. Uncertainty and lack of understanding of probabilistic concepts were observed as common reasons for negative attitudes toward probability and teaching of probability. Also, it was examined that there was no statistically significant relationship between probabilistic knowledge and attitudes.

ÖZET

MATEMATİK ÖĞRETMEN ADAYLARININ OLASILIK BİLGİSİ VE OLASILIK VE OLASILIK ÖĞRETİMİNE KARŞI TUTUMLARI

Bu çalışma, matematik öğretmen adaylarının olasılıksal bilgilerini ve onların olasılık ve olasılık öğretimine yönelik tutumlarını araştırmaktadır. Bu araştırmanın hedef kitlesi Türkiye'deki matematik öğretmen adaylardır. Katılımcılar, katılmaya gönüllü olan son sınıf ve üçüncü sınıf üniversite öğrencileridir. İlk olarak öğretmen adayları 27 maddeden oluşan olasılık ve öğretimine karşı tutum ölçeğini cevaplandırdılar. Bu ölçek için katılımcı sayısı 195 kişiydi. Daha sonrasında ise olasılığın farklı anlamlarına ve farklı bilgi türlerine ilişkin 12 olasılık sorusu içeren olasılık kavramı envanterini tamamladılar. Toplamda 95 matematik öğretmeni adayı bu kavram envanterini cevaplandırdı. Bu iki ölçeğin sonuçlarının analizinin ardından çevrimiçi toplantı platformu üzerinden 13 katılımcıyla görüşme yapıldı. Sonuçlar, matematik öğretmeni adaylarının düşük düzeyde Gelişmiş İçerik Bilgisine sahip olduklarını; fakat üç tür olasılık bilgisi arasında yeterli Ortak İçerik Bilgisine sahip olduklarını gösterdi. Olasılığın üç anlamı üzerine adayların bilgilerinin düzeyi incelendiğinde, katılımcıların frekans anlamı bilgisinin zayıf olduğunu gözlenmiştir. Tutum ölçeği sonuçları ise öğretmen adaylarının olasılık ve öğretimine karşı olumlu tutumlara sahip olduğunu göstermiştir. Olumlu ve olumsuz tutumların nedenleri görüşme bulguları üzerinden incelenmiştir. Olasılık kavramlarına ilişkin belirsizlik ve bilgi eksikliği, olasılığa ve olasılığın öğretilmesine yönelik olumsuz tutumların ortak nedeni olarak gözlenmiştir. Ayrıca, incelemeler sonucunda, olasılık bilgisi ile tutumlar arasında istatistiksel olarak anlamlı bir ilişki bulunamamıştır.bulunamamıştır.

TABLE OF CONTENTS

ACKNOWLEDGEMENTS iii
ABSTRACT v
ÖZET vi
LIST OF FIGURES xii
LIST OF TABLES xvi
LIST OF ACRONYMS/ABBREVIATIONS xvii
1. INTRODUCTION 1
1.1. Significance of the Study 2
2. LITERATURE REVIEW 5
2.1. The Nature of Probability 5
2.1.1. Different Meanings of Probability 6
2.1.1.1. Classical Meaning 6
2.1.1.2. Frequentist Meaning 6
2.1.1.3. Subjective Meaning 7
2.1.2. Misconceptions and Biases of Probability 7
2.1.2.1. Representativeness 8
2.1.2.2. Availability of Instances 9
2.1.2.3. Adjustment from an Anchor 9
2.1.2.4. Gambler's Fallacy 10
2.1.2.5. Law of Small Numbers 10
2.1.2.6. Equiprobability Bias 10
2.1.2.7. Falk Paradox/ Time Axis Fallacy 11
2.1.2.8. Outcome Approach 11
2.2. Teachers' Knowledge 11
2.2.1. Types of Content Knowledge 12
2.2.1.1. Common Content Knowledge 12
2.2.1.2. Specialized Content Knowledge (SCK) 12
2.2.1.3. Advanced Content Knowledge (Horizon Knowledge) . 13

2.2.2. Probabilistic Knowledge 13
2.2.2.1. The Role of Technology for Improving Probabilistic Knowledge 15
2.3. Teachers' Attitudes towards Teaching Probability 16
2.3.1. Review of Teachers' Attitudes in General 16
2.3.2. Teachers' Attitudes toward Teaching Probability 19
2.4. Theoretical Framework 20
2.4.1. The Relationship between Knowledge of Probability and Attitudes Towards Probability and Teaching of Probability 24
2.5. Statement of the Problem 28
3. METHODOLOGY 29
3.1. The Framework 29
3.2. Sample 30
3.3. Instruments 32
3.3.1. Probability Concept Inventory (PCI) 32
3.3.2. Attitude Scale Toward Probability and Probability Teaching . . 33
3.4. Semi-Structured Interview 36
3.5. Data Collection 37
3.5.1. Data Source 37
4. DATA ANALYSIS 40
5. RESULTS 42
5.1. Probability Concept Inventory (PCI) 42
5.1.1. General Scores of PCI 42
5.1.2. Components of Probabilistic Knowledge 43
5.1.2.1. Common Content Knowledge (CCK) 43
5.1.2.2. Advanced Content Knowledge (ACK) 44
5.1.3. Specialized Content Knowledge (SCK) 45
5.1.3.1. Classical Meaning (CM) 46
5.1.3.2. Frequentist Meaning (FM) 46
5.1.3.3. Subjective Meaning (SM) 47
5.1.4. The differences in the PCI Scores 48

5.1.4.1. Junior vs. Senior 48
5.1.4.2. A Probability Course Taken or Not 50
5.1.4.3. A Statistics Course Taken or Not 52
5.1.4.4. Grade Points Average (GPA) 54
5.2. Attitudes toward Probability and Its Teaching Scale (APT) 55
5.2.1. General Results 55
5.2.2. Components of APT 56
5.2.3. Results Based on Descriptive Properties 57
5.2.3.1. Junior vs Senior 58
5.2.3.2. A Probability Course Taken or Not 60
5.2.3.3. A Statistics Course Taken or Not 61
5.2.3.4. Grade Points Average (GPA) 63
5.3. Probabilistic Knowledge and Attitudes toward Probability and Teaching Probability 63
5.4. Findings from Interviews 65
5.4.1. Probability Concept Inventory (PCI) 65
5.4.1.1. Equiprobability Bias 66
5.4.1.2. Outcome Approach 68
5.4.1.3. Gambler's Fallacy 70
5.4.1.4. Representative Heuristics: Law of Small Numbers . . . 72
5.4.1.5. Other Problems in the Answers 74
5.4.1.6. The Reasons for Difficulties While Solving PCI 75
5.4.2. Attitude toward Probability and Teaching Probability (APT) . 77
5.4.2.1. Positive Attitudes 78
5.4.2.2. Positive Attitudes Toward Probability 78
5.4.2.3. Probability is Life Itself 78
5.4.2.4. Probability is A Part of Mathematics 79
5.4.2.5. Probability Includes Complexity 80
5.4.2.6. PSMTs have Positive Past Experiences 81
5.4.2.7. Positive Attitudes Toward Teaching Probability 82
5.4.2.8. Conceptual Understanding 82

5.4.2.9. Enjoying while Solving 83
5.4.2.10. Being good at Teaching 84
5.4.2.11. Negative Attitudes 85
5.4.2.12. Negative Attitudes Toward Probability 85
5.4.2.13. Lack of Probabilistic Knowledge 85
5.4.2.14. Not Applicability in Daily Life 86
5.4.2.15. Uncertainty 87
5.4.2.16. Negative Past Experiences 88
5.4.2.17. Negative Attitudes Toward Teaching Probability . . . 89
5.4.2.18. Lack of Understanding 90
5.4.2.19. Uncertainty 91
5.4.2.20. Problems in Teaching 92
5.4.3. PSMTs' Perception of Relationship between Probabilistic Knowledge and Attitudes 93
6. DISCUSSION AND CONCLUSION 96
6.1. RQ1: What is the Current State of Knowledge of Preservice Mathematics Teachers about Probability and its Teaching 96
6.2. RQ1-A: Is there Any Statistically Significant Difference Among PSMTs' Current State Of Probabilistic Knowledge Based on Different Characteristics Such as Their Grade, Statistics and Probability Courses They Have Taken, and GPA? 99
6.3. RQ1-B: What are PSMTs' Common Misconceptions about Probability? 99
6.4. RQ2: What are Preservice Mathematics Teachers' Attitudes toward Probability and Its Teaching? 101
6.5. RQ2-A: What are the Reasons for PSMTs' Positive and Negative Attitudes toward Probability and Its Teaching? 103
6.6. RQ3: What is the Relationship between PSMTs' Probabilistic Knowledge and Their Attitudes toward Probability and Teaching Probability?- and RQ3-A: What is PSMTs' Perception on the Relationship Between Probabilistic Knowledge and Attitudes? 106
7. LIMITATIONS 107

7.1. Implications 108
REFERENCES 110
APPENDIX A: PROBABILITY CONCEPT INVENTORY 119
APPENDIX B: OLASILIK KAVRAM TESTİ TÜRKÇE 122
APPENDIX C: ATTITUDE SCALE TOWARDS PROBABILITY AND ITS TEACHING 124
APPENDIX D: UNIVERSITIES AND THEIR STATISTICS AND PROBABILITY COURSES 126

LIST OF FIGURES

Figure 2.1. Theoretical framework of the study. 26

Figure 5.1. General scores on PCI. 42

Figure 5.2. Common content knowledge: Boxplot. 43

Figure 5.3. Theoretical framework of the study. 44

Figure 5.4. Specialized content knowledge: Boxplot. 45

Figure 5.5. Classical meaning of probability: Distribution of scores. 46

Figure 5.6. Frequentist meaning of probability: Distribution of scores. 47

Figure 5.7. Subjective meaning of probability: Distribution of scores. 48

Figure 5.8. Normal Q-Q plot for the scores of juniors. 49

Figure 5.9. Normal Q-Q plot for the scores of seniors. 49

Figure 5.10. Normal Q-Q plot for the scores of the group who have not taken a probability course before. 51

Figure 5.11. Normal Q-Q plot for the scores of the group who took a probability course before. 51

Figure 5.12. Normal Q-Q plot for the scores of the group who took a statistics course before. 53

Figure 5.13. Normal Q-Q plot for the scores of the group who have not taken a statistics course before. 53

Figure 5.14. Distribution of scores on APT. 55

Figure 5.15. Boxplot of components of APT. 56

Figure 5.16. Normal Q-Q plot for the scores of juniors. 58

Figure 5.17. Normal Q-Q plot for the scores of seniors. 59

Figure 5.18. Normal Q-Q plot for the scores of the group who have not taken a probability course before. 61

Figure 5.19. Normal Q-Q plot for the scores of group who took a probability course before. 61

Figure 5.20. Normal Q-Q for the scores of the group who have not taken a statistics course before. 62

Figure 5.21. Normal Q-Q for the scores of the group who have not taken a statistics course before. 63

Figure 5.22. Question 8 on PCI. 66

Figure 5.23. Solution of PSMT23. 67

Figure 5.24. Revised version of question eight. 67

Figure 5.25. Question 5 on PCI. 68

Figure 5.26. Explanation of PSMT134. 68

Figure 5.27. Question 11 on PCI. 70

Figure 5.28. Solution of PSMT144 for question 5. 71

Figure 5.29. Explanation of PSMT144 for question 5. 71

Figure 5.30. Question 10 on PCI. 72

Figure 5.31. Solution of PSMT134 for question 11. 73

Figure 5.32. Explanation of PSMT10 for complexity of probability. 80

Figure 5.33. Explanation of PSMT23 for his reasons of not liking probability. . 89

Figure 5.34. Explanation of PSMT50 for his reasons of being unconfident in teaching. 90

Figure A.1. Probability concept inventory 1. 119

Figure A.2. Probability concept inventory 2. 120

Figure A.3. Probability concept inventory 3. 121

Figure B.1. Olasılık kavram testi türkçe 1. 122

Figure B.2. Olasılık kavram testi türkçe 2. 123

Figure C.1. Olasılık kavram testi türkçe 2. 124

Figure C.2. Olasılık kavram testi türkçe 2. 125

LIST OF TABLES

Table 3.1. Descriptives statistics of participants. 30

Table 3.2. The universities that PCI takers coming from. 31

Table 3.3. Items of PCI. 32

Table 3.4. Items of APT. 34

Table 3.5. Data sources. 38

Table 5.1. The groups of participants who took a probability course or not. . 52

Table 5.2. Correlation coefficient for GPA. 54

Table 5.3. Properties of the FE anchor pile models. 59

Table 5.4. Statistical results of BP and CM. 64

Table 5.5. Statistical results of BP and CCK. 65

Table D.1. Universities and their statistics and probability courses. 126

LIST OF ACRONYMS/ABBREVIATIONS

ACK	Advanced Content Knowledge
AP	Affective Component Towards Probability
APT	Attitude Toward Probability and its Teaching Scale
AT	Affective Component Towards Teaching Probability
BP	Behavioral Component Towards Probability
BT	Behavioral Component Towards Teaching Probability
CCK	Common Content Knowledge
CCP	Cognitive Competence Towards Probability
CM	Classical Meaning of Probability
CT	Teaching Probability Competence Component
FM	Frequentist Meaning of Probability
PCI	Probability Concept Inventory
PSMT	Preservice Mathematics Teacher
RMSEA	Root Mean Square Error of Approximation
SCK	Specialized Content Knowledge
SM	Subjective Meaning of Probability
VPT	Value Component Towards Probability and Its Teaching

1. INTRODUCTION

In daily life, many questions come into our minds, and we cannot have a clear answer for some of them. For instance, the questions "What is the weather forecast for tomorrow?", "Which numbers will win the lottery this year?", "Why getting 6 to 6 is more difficult than getting 1-3 when a pair of dice is rolled?" are based on an uncertain phenomenon (a future event, mostly) and are in the zone of probabilistic studies. Even if there is no exact answer, probability might be defined as a way to foresee the happening of an event using numbers based on considering various possibilities. *Chance* is the heart of unpredictable events that shape the topic of probability (Kapadia and Borovcnik, 2012). Even though probabilistic events surround the daily lives of people, it is not popular among and liked by people from different areas, such as educators, students, scientists, etc. This is because the nature of probability is constructed by ambiguity and chance (Batanero and Borovcnik, 2016). Solving a probability problem might cause conflicts in people's minds. As a result of that, the probability knowledge of the students and teachers is surrounded by some biases originating from their misunderstandings (Fischbein and Schnarch, 1997). Consequently, failures in solving a probability problem become inevitable. Although the problematic situation resulting from probability was realized in research studies, paying particular attention to and inclusion of probability in the mathematics curriculum has recently been possible. It started to be mentioned when NCTM (2000) explained principles, standards, and objectives for teaching statistics and probability. Then, The Guidelines for Assessment and Instruction in Statistics Education (GAISE) Report (Franklin *et al.*, 2007) highlighted the importance of teaching statistics and probability and how to implement it. It is stated that to be able to teach students statistics and probability, teachers should have adequate knowledge about this particular field. Teachers need to know how to handle data (collecting and analyzing data with various strategies), simulate probability experiments, use different approaches to probability, interpret the results of the experiments, and prevent students' misunderstandings. On the other hand, Estrada and Batanero (2019) claimed that in teaching probability, the attitude of the teach-

ers toward probability and its teaching is as important as their knowledge about the topic. In mathematics teaching, attitudes are considered significant (Philips, 2007). The studies showed that teachers' attitudes toward mathematics are essential, as are students' attitudes toward it (Di Martino and Zan, 2011).

1.1. Significance of the Study

In recent years, probability has gained enormous attention among education researchers worldwide, and it has started to have more emphasis and room in many countries' mathematics curricula such as in Spain and in Australia (Batanero *et al.*, 2016). In 2000, NCTM (National Council of Teachers of Mathematics) included statistics and probability as among the five main topics of mathematics education. In this report, NCTM stated the principles and standards for the students to be able to handle data and probability through K-12. The Guidelines for Assessment and Instruction in Statistics Education (GAISE) Report (Franklin *et al.*, 2007) highlights the importance of probability and probabilistic reasoning in mathematics education and its relation to other content areas in mathematics.

On the other hand, when middle school mathematics curriculums in Turkey were examined from 2005 to the present, it can be seen that the number of hours and objectives for statistics and probability decreased over the years (İlhan and Arslaner, 2019). Although statistics were included in all four middle school grades, the examination of 2005, 2009, 2013, and 2018 curriculums demonstrates that probability was eliminated after 2013. While in grade five students encountered simple probabilistic concepts in the curriculum of 2005, they continued to learn more complex probabilistic concepts in upcoming years. In the curriculum of 2009 probability was introduced in fourth grade, and theoretical, experimental and subjective approaches of probability were included for grade eight students (Doğucu, 2013). However, new curriculum reform in 2013 restricted the teaching of probability to introducing the simple probabilistic concepts in grade 8 and the emphasis on basic teaching of probability continued in the curriculum of 2018 (Ministry of National Education, 2013, 2018).

The changes and reforms in Turkey contradicts with the principals and standards of NCTM (2000), and recommendations of GAISE Report (Franklin *et al.*, 2007) and Statistics Education of Teachers (SETS) (Bargagliotti and Franklin, 2015). This research study can be important to show the necessity of inclusion of probability in the curriculum by focusing on the level of knowledge of probabilistic concepts among PSMTs.

In the literature, it is highlighted that in order to support students to learn probabilistic concepts efficiently and grasp the idea of probabilistic reasoning, teachers need to be well prepared about probabilistic concepts, and they should be aware of the potential misconceptions of the students while learning probability (Kvatinsky and Even, 2002; Stohl, 2005). Therefore, diagnosing preservice teachers' current state of knowledge and their misconceptions has a great role in the improvement of their understanding of probability concepts and related curricula in the future. This study is important to examine the knowledge of PSMTs based on different components of probability because the scale used in the study measure different probabilistic knowledge and the knowledge of different meanings of probability at once (Gómez-Torrez, 2016). Even though there are different studies using different tasks to measure probabilistic knowledge, they generally focus on one dimension and ignore others (Batanero *et al.*, 2016)

One of the important factors in learning process is attitudes toward the subject since it can influence learners' conceptual understanding (Hannigan *et al.*, 2013). Teachers' attitudes toward probability are essential, as well as teachers' pedagogical content knowledge to teach probability (Estrada and Batanero, 2019). Research showed that teachers' beliefs, attitudes, and emotions, which form their dispositions, have a significant influence on their knowledge (Philip, 2007). This means that, in addition to the factors like training or studying, the way that teachers approach the topic impacts the knowledge that they gain throughout the process. In the literature different research studies can be found about attitudes towards probability. However, most of them did not cover different attitude components and teaching probability separately. For

instance, Tat (2014) measured students' attitudes toward probability by using Probability Attitude Test developed by Bulut (1994). Although there were 28 questions, the attitudes of the participants were assessed as positive or negative without attributing different components of attitude since it was unidimensional. The same scale was used to measure attitudes toward probability by Özaytabak (2004). However, the researcher also needed to develop another scale for measuring the participants' attitudes toward teaching probability.

In accordance with the literature and the need for more research on preservice teachers' probabilistic knowledge and attitudes, the purpose of this study is two-fold: First, examining the probability content knowledge of pre-service mathematics teachers; then, examining preservice teachers' attitudes toward probability and its teaching. Also, this study aims to investigate the relationship between PSMTs' knowledge and attitude toward probability and its teaching. For this purpose, the following research questions are formulated:

- What is the current state of knowledge of preservice mathematics teachers about probability and its teaching?

  Is there any statistically significant difference among PSMTs' current state of probabilistic knowledge based on different characteristics such as their grade, statistics and probability courses they have taken, and GPA?

  What are PSMTs' common misconceptions about probability?
- What are PSMTs' attitudes toward probability and its teaching?

  What are the reasons for PSMTs' positive and negative attitudes toward probability and its teaching?
- What is the relationship between PSMTs' probabilistic knowledge and their attitudes toward probability and teaching probability?

  What is PSMTs' perception of the relationship between probabilistic knowledge and attitudes?

2. LITERATURE REVIEW

2.1. The Nature of Probability

Probability has a distinct nature that strongly deals with uncertainty. The concepts of chance, randomness, and variability are essential to understanding its nature. Although these concepts might seem strange to many people, their point of emergence is life itself. Daily examples of probabilistic situations can be found in weather forecasting, card games, lottery tickets, etc (Kapadia and Borovcnik, 2012). Probability is considered as a subtopic of mathematics, even though it is based on ambiguous events. However, it has distinct features which do not belong to mathematics. At the end of a mathematical process, certainty is achieved with a clear answer. However, the answer to a probability question is a prediction, not a precise result, which produces uncertainty (Kvatinsky and Even, 2002; Shaughnessy, 2010). The uncertainty comes from the variability of the variables in an event and the endless possible consequences of that event (Falk and Ruma, 1992). This means that there can be many factors affecting possibility of an event and it is not clear what will be the outcome even though its probability can be calculated. Thus, probability needs to be considered as a reasoning process. Franklin *et al.* (2007) described probability as an intersection of mathematics and statistics where probability offers an opportunity to compute the ambiguity of statistics. They asserted that in an experiment with a sample size of one, making an inference is not meaningful. However, when a large number of samples are acquired, making inferences and talking about the probability of the event becomes possible. Therefore, probability, compared to mathematics, is context-dependent and needs to be evaluated according to the current situation (Kapadia and Borovcnik, 2012).

There are different approaches listed by researchers when defining probability. The first categorization is formed as experimental and theoretical probability. While theoretical probability is defined as a ratio of the number of favorable outcomes to the total number of outcomes when it is possible to make this calculation, experimental

probability is found by making many trials of the experiment in sufficient numbers by using randomization (Prodromou, 2012). Therefore, the first one is attributed to the mathematical side rather than the experimental probability requiring probabilistic reasoning. According to Batanero *et al.* (2016), mainly there are seven approaches to the definition of probability as follows: classical, frequentist, subjective, intuitive, logical, propensity, and axiomatic. The probability concept inventory developed by Gomez-Torres *et al.* (2016) was used in this study, and in this concept inventory, three of the meanings were taken into account while forming questions. Therefore, in this paper, only the first three of the meanings were explained in detail.

2.1.1. Different Meanings of Probability

2.1.1.1. Classical Meaning. The classical meaning of probability addresses the ratio of favorable outcomes to all outcomes in the event. The classical approach is sometimes called the Laplace approach since Laplace (1814) defined the probability as a ratio by revising the old definitions (Kapadia and Borovcnik, 1991; Kvatinsky and Even, 2002). It is one of the leading approaches that enables calculating the probability of events without carrying out random experiments. However, this approach has some criticisms since it assumes that all the outcomes of an event are equiprobable (Batanero *et al.*, 2016). Besides the criticisms, this is the most widely taught one among the meanings of probability (Gomez-Torres *et al.*, 2016).

2.1.1.2. Frequentist Meaning. The frequentist approach is sometimes called the experimental approach (Kvatinsky and Even, 2002). As the meaning of the name implies, in order to find the probability of an event, the experiment is repeated in many trials. Since there is no calculation process like CM, the probability of an event is predicted by making an inference based on the frequency of the outcomes (Kapadia and Borovcnik, 2012). Even if this kind of meaning of probability is more beneficial while visualizing and concretizing the probability concept in teaching, it can be problematic while dealing with compound events where digital tools cannot simulate (Gomez-Torres *et al.*, 2016).

2.1.1.3. Subjective Meaning. According to the Subjective Meaning of probability, the acquired new knowledge is considered while finding the probability of an event. This approach is defined based on Bayes' theorem, in which new knowledge is used while evaluating the probability of an existing event (Gomez-Torres *et al.*, 2016). Rather than calculating the probability as an objective value, the beliefs, which are seen as subjective, play a critical role in the process of finding the probability of events, especially compound events (Kvatinsky and Even, 2002). When new information is received, the probability of an event is calculated with modifications to the primary result.

The different meanings of probability are significant to know while evaluating the chance of an uncertain event because the judgment should be made based on these approaches to achieve a meaningful result (Prodromou, 2012). Therefore, we need to think about all and choose the appropriate. However, when people lack this knowledge, some intuitive knowledge might interfere with probabilistic thinking. Hence, some biases occur while evaluating the chance of an uncertain event that are called as heuristics (Tversky and Kahneman, 1974). Tversky and Kahneman (1974) asserted that there are some heuristics (biases) that direct people's thoughts in the decision-making process of uncertain events. These three heuristics are representativeness, availability of instances, and adjustment from an anchor. Then, Falk and Konold (1992) identified that there are some biases that mislead students' probabilistic reasoning. In the following part the misconceptions or biases that interfere with probabilistic reasoning is explained, compatible with the literature.

2.1.2. Misconceptions and Biases of Probability

Misconceptions are defined as a way of understanding that misleads students systematically while learning a topic (Swan, 2001). In mathematics teaching, students' misunderstandings based on incorrect interpretations are categorized as misconceptions (Ojose, 2015). Therefore, multiplying integers using the belief that multiplication makes bigger is a kind of misconception about the multiplication of integers based on the notion of natural numbers. In probabilistic thinking, this kind of misunderstand-

ing is too common since the concept of chance is open to misinterpretation because of the use of the term chance in real life (Kapadia and Borovcnik, 2012). In other words, the nature of probability is convenient to produce biases and misconceptions. In addition to students' misconceptions, teachers' misconceptions of probability were demonstrated in research studies (Fischbein and Schnarch, 1997). To be able to teach probabilistic concepts in the right way, teachers need to overcome this issue. Kahneman *et al.* (1982) asserted that people use heuristics, which are representativeness, availability, and adjustment from anchor while making judgments under uncertainty. Even though the heuristics sometimes might be useful to make inferences, they might result in misunderstandings and systematic errors called misconceptions. Dealing with misconceptions in probability is crucial since it requires a different way of thinking compared to other mathematic topics, and new intuitions should be established for stochastic thinking (Fischbein and Schnarch, 1997).

There are various misconceptions that students and teachers perpetuate. According to the findings of Fischbein and Schnarch (1997), the most common misconceptions related to probability among preservice teachers are gambler's fallacy, the law of small numbers, compound and simple events, and the Falk phenomenon (Batanero and Sanchez, 2005; Falk and Konold, 1992; Stohl, 2005; Tversky and Kahneman, 1982).

2.1.2.1. Representativeness. When people think about the probability of an event happening, e.g "What is the most likely option for the nationality of person A?" or "What is the most and least probable option for the person B's profession?", the representativeness heuristics emerges. In these types of situations, people make judgments by relying on how much the person or object represents the given classes. While judgments are made based on representativeness, people mostly neglect prior probabilities, predictability, and sample size of the given situation. Therefore, they just consider the similarity of options rather than the current condition. This biased thinking process in probabilistic events has some consequences. The Gambler's Fallacy is the famous misconception that comes from representativeness. It is the belief that the probability of an event changes according to past occurrences of that event (Tversky and Kahneman,

1974). For instance, when a coin is flipped ten times, and the coin is landed on the head ten times in a row, sometimes it is thought that the eleventh coin needs to land on the tail because of the unlikelihood of getting 11 heads in a row. In this type of probabilistic thinking, people think that the probability of getting one of the outcomes increases when the other outcome is observed consecutively because they focuses on theoretical probability and try to equalize outstanding number of one possible outcome ,in this situation landed heads.

2.1.2.2. Availability of Instances. It is the type of thinking process that leads people to rely on the events that come to mind commonly while deciding on the probability of an event. Tversky and Kahneman (1974) argued that even if availability might work while working on the frequency of the events, it interferes with the probability of an event that is unrelated to the FM of probability. Familiarity with the objects or instances might direct people's thoughts about the probability in the wrong direction. For example, when people try to find which gender is mostly present among the shown groups with some celebrities, people tend to say the gender that includes a greater number of famous people since they focus on the known names. In addition to familiarity, finding the probability of an event according to the state of the event being possible in daily life is related to the availability heuristic. If it seems reasonable and logical in our minds, it is judged as highly probable or vice versa. However, this thinking process is not valid for all events.

2.1.2.3. Adjustment from an Anchor. Adjustment from an anchor is a kind of bias that addresses using prior knowledge while deciding the probability of an event. Most often, it results in false judgment since it overgeneralizes the current situation. Tversky and Kahneman (1974) experimented in order to explain this heuristic. Two groups of students were chosen, and while one was shown the multiplication with the following order 9x8x7x6x5x4x3x2x1, the other group saw the multiplication with the reverse order 1x2x3x4x5x6x7x8x9. The former group estimated the multiplication result as 2250, while the latter estimated it as 512. Therefore, it can be concluded that the

two groups estimated the result based on the first few numbers in the line. Hence, it is concluded that people use the first impression or prior knowledge while making a decision on the probability of an event occurring.

2.1.2.4. Gambler's Fallacy. When a person tosses a coin three times and gets three heads, s/he expects to get a tail in fourth time. This kind of probabilistic thinking serves as an equalization process where a feeling that three heads need to be equalized with tails in the next turns emerges. Therefore, the misconception of *gambler's fallacy* is also called as *negative recency effect* (Fischbein and Schnarch, 1997; Tversky *et al.* 1982).

2.1.2.5. Law of Small Numbers. The law of small numbers is another bias whose working mechanism resembles the Law of Large Numbers but in inappropriate conditions. The law of Large Numbers states that when sample size increases infinitely, experimental probability approximates theoretical probability, and the population is represented by the sample in a more accurate way. However, when the law of small numbers interferes with the probabilistic thinking of people, they overestimate their findings even if they have small a sample size (Tversky and Kahneman, 1971; Tversky *et al.*, 1982). For instance, a researcher with the law of small numbers misconception conducted a study with ten people to investigate the number of cups of tea people drink in a day in Turkey. When the findings show that the average number of cups of tea of these people is one, the researcher makes the final decision that in Turkey, people drink one cup of tea in a day. However, the sample size is insufficient to get a conclusion in this situation. Thus, the overgeneralization of the law of large numbers from a small sample interferes with stochastic thinking.

2.1.2.6. Equiprobability Bias. Compound and simple events are another source of misunderstanding in probabilistic events. It results from applying equiprobability bias to compound events. In other words, when sample space of compound events is found according to equiprobable outcomes of simple events, the misconception occurs (Batanero

and Sanchez, 2005). For instance, after two dice are rolled, the belief that the chance of getting 1 to 1 is the same as the chance of getting 3 and 5 can be an example of this type of misconception because you can get 3 and 5 on either dice (Fischbein and Schnarch, 1997).

2.1.2.7. Falk Paradox/ Time Axis Fallacy. Another form of bias is the Falk paradox or the time-axis fallacy that Falk (1979) asserted. Fischbein and Schnarch (1997) asked Falk's (1979) probability question to groups of students of different ages, and they found that the time-axis fallacy increases when the age of the students increases. It means that, in general students do not take into consideration additional information given for the later event, which retroactively affects the chance of the first event.

2.1.2.8. Outcome Approach. Outcome approach was identified as one of the biases in probabilistic reasoning by Konold (1989). In his study, students with outcome approach focused on the outcome of the experiment. A question about whether forecasting were asked as "If prediction of raining tomorrow is 70% and if it not rains, what would you consider?". Six out of sixteen undergraduate students stated that if it did not rain it means the prediction was wrong. Therefore, the decisions made were based on binary outcomes such as yes or no and evaluation of an event based on whether it will occur or not without considering its chance of happening (Falk and Konold, 1992).

2.2. Teachers' Knowledge

Teachers' knowledge has been an essential research topic for a long time. The distinction between pedagogy and content knowledge has been mentioned in research studies. While teachers' knowledge had been discussed separately on these dimensions, Shulman (1986) stated that using content knowledge separate from pedagogical knowledge is like using mere pedagogy in teaching without any sign of content knowledge. Accordingly, he proposed two types of knowledge teachers should have in teaching: Subject Matter Knowledge (SMK), which is the focus of this study, and Pedagogical

Content Knowledge (PCK), which is formed by merging pedagogical knowledge and subject matter knowledge. Later, Ball and her colleagues (2008) developed a new frame for teachers' knowledge to teach mathematics by developing Shulman's idea of SMK. They stated that the types of knowledge that are important for teachers to teach mathematics are Common Content Knowledge (CCK), Specialized Content Knowledge (SCK), and Advanced Content Knowledge (ACK). These three types of knowledge are elaborated in the following pages since the PCI used in this study measures teachers' knowledge based on this categorization.

2.2.1. Types of Content Knowledge

2.2.1.1. Common Content Knowledge. Ball *et al.* (2008) stated that CCK is a type of knowledge teachers must acquire to teach a mathematical subject. It is particular to mathematics rather than any subject like Shulman's subject matter knowledge or substantive knowledge defined by Schwab (1978). It refers to the understanding of formulas, concepts, and procedures regarding a mathematical topic, and it is one of the constructs this study examines. This knowledge does not focus on teaching; instead, CCK is general knowledge about a mathematical topic that anyone can have. For example, not only teachers but also engineers know how to solve an equation, or they use geometry too. CCK is essential for teachers because without comprehension of a mathematical topic, teaching it is not possible. For instance, the area of a triangle is found by dividing the base times its height by two, or the highest negative integer is -1. Any person who knows mathematics can provide these answers when asked.

2.2.1.2. Specialized Content Knowledge (SCK). Ball and her colleagues (2008) developed a new term to describe a type of knowledge different from general mathematical knowledge. SCK refers to the knowledge that is special to teaching mathematics. Using SCK outside of the teaching context is not applicable. Therefore, in contrast to CCK, which covers a general understanding of mathematical concepts and procedures, SCK includes mathematical knowledge necessary while designing tasks, addressing students' misconceptions, and evaluating students' learning processes. Teachers should not only

know the topics they teach the students but also know different mathematical representations or teaching methods to identify the deficiencies in the learning process and produce a solution to refine them (Hill *et al.*, 2005). For instance, anyone with mathematical knowledge knows that a half equals two-quarters or five-tenths. However, in addition to knowing the fact that two-quarters of a whole is equal to its half, a teacher should be capable of demonstrating equality with various models, conveying the meaning of simplification and expansion of fractions, and identifying any possible misconceptions that learners can have.

2.2.1.3. Advanced Content Knowledge (Horizon Knowledge). Horizon knowledge requires mathematical knowledge beyond the topic taught. That means that teachers with sufficient horizon knowledge are aware of the connection between mathematical concepts (Ball *et al.*, 2008). Therefore, as Gómez-Torres *et al.* (2016) defined, ACK implies teachers' awareness of upcoming topics. For example, a teacher should know that after teaching basic probability calculations to the students, the next topic in future grade levels will be conditional probability since it is more complicated than basic probabilistic calculations and it is built on this prior knowledge too, so there is a strong connection.

2.2.2. Probabilistic Knowledge

When teaching probability is considered in the light of the PCK by Shulman (1986) and SCK by Ball *et al.* (2008), teachers should have sufficient pedagogical and specialized content knowledge to teach probability. The reason is that to be able to support the students while dealing with data and making inferences; teachers are required to know different representations and how to explain those to students based on students' level of understanding (Kvatinsky and Even, 2002; Stohl, 2005). Teachers should have adequate knowledge about the meanings of probability, its nature, its applications in real life, and probabilistic thinking process etc. (Kvatinsky and Even, 2002). Hence, they can approach probabilistic events comprehensively so students can grasp the main idea. Even though probability has started to be included in the

curriculum, it does not ensure that teachers effectively teach the content (Estrada *et al.*, 2018). Probability requires high cognitive thinking processes where uncertainty and ambiguity are the primary sources. Therefore, its complicated nature might set boundaries for teachers' knowledge. The literature on this issue shows that PSMTs have some deficiencies in their statistics and probability knowledge, and their readiness to teach probability is inadequate (Carter, 2005; Prodromou, 2012; Stohl, 2005). Probability and statistics are two concepts that are intertwined with one another and cannot be thought of separately. Lovett (2016) asserted that when the GAISE level of the questions is increased based on the statistical investigative cycle stated by Franklin *et al.* (2007), preservice teachers' level of understanding of those concepts decreases accordingly. Hence, the study's findings displayed that preservice teachers do not have adequate knowledge to teach statistics. Their poor conceptual understanding is insufficient to handle the complicated data and make inferences with data in the teaching process.

In order to assess teachers' probability content knowledge based on different meanings of probability, Gomez-Torres *et al.* (2016) developed a questionnaire that consists of items based on classical, frequentist, and subjective meanings of probability and different types of teacher knowledge. The results of their study with preservice primary teachers demonstrated that preservice teachers have insufficient probability knowledge and they mostly use CM compared to FM and SM. When the results regarding teachers' content knowledge are examined, preservice teachers need improvement regarding SCK and ACK related to probability. As it is known, teachers should have the capabilities more than the common knowledge that mostly adults have in routine life since teaching probability demands being aware of representations, misconceptions, alternative ways, etc. (Kvatinsky and Even, 2002). Similar results were found in Turkey. The study of Kurt *et al.* (2020) with PSMTs showed that even though the level of knowledge about the CM of the probability is the highest conceptual side among the preservice teachers, their knowledge of CM at ACK is too low. Thus, preservice teachers might face challenges in effectively teaching probability lessons because they do not have an adequate understanding of the different meanings of probability

and different kinds of probabilistic knowledge. The reason for low-level knowledge of probability might be a small portion of probability in undergraduate programs and not relating probability to statistics (Kurt and Coşkuntuncel, 2020; Stohl, 2005;). Since probability is a more abstract topic and difficult to imagine compared to mathematical concepts, support from statistics might improve the conceptual understanding of probability among both students and teachers.

2.2.2.1. The Role of Technology for Improving Probabilistic Knowledge. Teachers' a knowledge of probability is crucial to support students' thinking about chance events, randomness, and risk. Therefore, improving teachers' probabilistic knowledge is compulsory to have the students with accurate understanding (Estrada and Batanero, 2019). Raising awareness among students about the presence of probabilistic events in everyday life and their relevance to real-life situations can help foster probabilistic thinking in probability education. In this way, the uncertainty of probability can seem more reasonable and connected to daily life as it is in mathematics. While improving understanding of probabilistic events, materials and tools can have a significant role in concretizing the events (Kapadia and Borovcnik, 2012). Modeling and different representations of the event can be possible with the help of various tools. In recent years, using digital tools in educational settings has become widely common. In math lessons, these tools help students understand the topic.

In addition to representing other mathematical topics with digital tools, using digital tools to simulate probabilistic events can help to overcome the difficulties of probability (Akoğlu, 2014; Biehler, 1991). Simulations offer an opportunity to work on real data and allow many trials of the event. For example, while the "sampler" property of the Common Data Analysis Platform (CODAP) enables one to see the experiment in slow motion, by using the fast mode, learners can make so many trials to see the distribution of the outcomes in a general sense where making inferences becomes easier. Therefore, the idea of chance in probability is comprehended as observable and perceivable (Batanero *et al.*, 2016; Konold and Kazak, 2009) According to Batanero *et al.* (2016), digital tools are helpful in the sense that students can have an opportunity

to try "what if" scenarios in which learners can eliminate interfering questions in their minds. Experiencing what-if scenarios can be useful in overcoming misconceptions and doubts about uncertainty.

2.3. Teachers' Attitudes towards Teaching Probability

2.3.1. Review of Teachers' Attitudes in General

Even though teachers have adequate content knowledge, it does not mean they have a positive disposition toward the topic (Wilkerson and Lang, 2007). The components of teachers' affective domain are another discussion area of research in addition to their content knowledge. In this area, there is no consensus about the definition of the terms and their hierarchy. Therefore, researchers define the terms subjectively. In the literature, there are different models for attitude from past to present. However, in general, two approaches exist: one has a simple definition, and the other has a definition of attitude with three components (Hannula, 2002). The simple definition of attitude is focusing on positive and negative feelings toward the attitude object (Hannula *et al.*,2016). As seen in this definition, emotions are emphasized where cognition is ignored.

Ajzen (1989) described two types of models for the three-dimensional definition of attitude: The Multi-component View of Attitude and the Hierarchical Model of Attitude. According to the multi-component view, attitude is formed by three components such as cognition, affect, and conation. In this model, each component is categorized by whether the response is verbal or nonverbal. If the response is based on information or an impression, it is classified as a cognitive component of the attitude. For instance, expressions of beliefs about schools like schools are prisons or teachers are incapable of teaching, can be addressed to the verbal response of the cognitive component of the attitude. On the other hand, perceptual reactions to attitude object was described as nonverbal reactions of cognitive component that are more difficult to understand than the verbal reactions. Evaluating the school with expressions such as appreciating the

school or teachers because of their manner or being angry because of the disrespectful behavior of the principal are signs of the affective component of the attitude. The difference between the cognitive and affective components of attitude is that the responses of the affective component are related to feelings toward the attitude object. As a last component, conation in the multi-component view of the attitude is related to action or behavior toward the evaluated object. In the school context above, refusing to go to school, being at school every day on time, or donating some money to school can be examples of a conative component of a positive and negative attitude. In these examples, there are positive or negative actions depending on the attitude. Ultimately, the Multi-component View defines attitude as an abstract concept. While attitude is constructed by three components, all these are considered as distinct and not related.

On the other hand, Ajzen (1989) described a Hierarchical model where cognition, affect, and conation are the first-order factors, and attitude is the second-order and more abstract factor. Compared to the Multi-component View, cognition, conation, and affect are described as independent from each other. However, it is expected to have a correlation between the factors to some extent since they all construct attitude itself at the end.

These two attitude views are called so the tripartite model of attitude. The general feature of the traditional tripartite model of attitude is that attitude is constructed by three components: affect, cognition, and behavior (Albarracin *et al.*, 2005; Rosenberg *et al.*, 1960). The contemporary view on the tripartite model is that the components described as cognition, affect, and behavior in the traditional models can be separated from attitude (Fabrigar *et al.*, 2005). It means that attitude does not consist of these elements; it is a general idea that results from evaluating the information that originates from cognition, affect, and behavior.

According to McLeod (1992) who is one of the pioneers in defining the affective components of teachers, describing the affective components is more problematic than cognitive components since it is more complicated than observable behaviors. In his

structure of the affective domain under mathematics education, there are three subdomains: beliefs, attitudes, and emotions. While beliefs are described as the most stable components of the affective domain, emotions are considered unstable forms of the affective domain. In that definition, beliefs are primarily cognitive, whereas emotions mainly include affect and a lower degree of cognition. Thus, changing someone's belief requires a longer time than changing someone's emotion (Gal *et al.*, 1997; Philip, 2007). McLeod (1992) defined the term attitude as "affective responses that involve positive or negative feelings at moderate intensity and reasonable stability" (p.581). He stated that attitudes toward mathematics can result from two reasons. The first reason is the over-repetition of an emotion toward math, and the second reason is the assigning existing attitudes toward a mathematical topic to the similar topics. Therefore, there might be a continuation of the attitude for similar subjects. As beliefs and attitudes were used interchangeably in the past literature, the distinction between the two was not explicitly described (Gal *et al.*, 1997). However, current research studies define attitudes specifically. Philip (2007) described attitudes as "a disposition or tendency or an emotion or feeling attached to an idea or object" (p.259).

Attitudes are positioned between emotions and beliefs, with a moderate level of stability and cognitive processing (Di Martino and Zan, 2015). Schwarz (2007) claimed that learners' attitudes are not something that they possess therefore they cannot have one or more attitudes. It is the consequence of the evaluation process toward a topic, and this evaluation may vary under different conditions. Attitudes of the teachers can be one of the factors that might dramatically affect the learning and teaching process (Estrada Roca and Batanero, 2020).

Therefore, teachers should enhance their attitudes toward mathematics. Raymond (1997) claimed that students bring their attitudes toward mathematics into adulthood who might be the future teachers. Therefore, diagnosing prospective teachers' attitudes toward mathematics should be considered in undergraduate programs. Even if there are many studies related to students' attitudes toward mathematics, teachers' attitudes also need to be considered widely in teaching mathematics and it

shouldn't be neglected. However, being aware of their own attitudes and monitoring them can enable teachers to convey the meaning of mathematics in the right way (Di Martino and Zan, 2011).

2.3.2. Teachers' Attitudes toward Teaching Probability

When the nature of the probability is considered, it is a complex topic to comprehend because of uncertainties (Kapadia and Borovcnik, 2012). Therefore, sometimes it might result in failures and confusion in the learning process of students. Based on that, students might develop negative affective responses toward probability rather than other math topics. Lovett (2016) found that teaching algebra and geometry is much preferred by PSMTs rather than statistics. Since statistics, like probability, requires making inferences, preservice math teachers are inclined to choose procedural statistics over conceptual ones when they have to teach statistics. Also, when the topics consist of higher Guidelines for Assessment and Instruction in Statistics Education (GAISE) level, their confidence to teach that topic decreased consecutively. León-Mantero *et al.* (2020) conducted a study with preservice teachers from different departments to assess their attitudes toward statistics. It is revealed that preservice teachers do not value the usefulness of statistics in academic life. Hence, their attitudes were negative toward statistics. Although statistics and probability are related and statistical thinking is a base for probability, their reflection on the students might be different. Because of that reason, Estrada *et al.* (2016) developed a scale to assess teachers' attitudes toward teaching probability since before that there was no attitude scale specific to teaching probability in contrast to the vast majority of scales about statistics and teaching statistics. The study was implemented with primary school preservice teachers, and the results indicated that the scores were better compared to scores in attitude scales toward teaching statistics (Estrada *et al.*, 2018; Estrada and Batanero, 2019). They argue that it might result from introducing probability in the early years of education in Spain. Even though the scores were positive, the items related to cognitive competence for teaching (CT), cognitive competence towards probability (CCP), and affect had lower scores. Preservice teachers did not feel

well prepared to teach probability even if they have an awareness of the importance of the topic (Estrada *et al.*, 2018). The same study was conducted with a different sample by Estrada and Batanero (2019). The results complemented the previous study that preservice teachers perceive the importance and difficulty of teaching probability. On the other hand, they value teaching probability with their positive attitudes. According to Fischbein and Gazit (1984), to improve students' attitudes toward probability, exposing them to the instruction does not guarantee positive change. However, the active involvement of the students in the different kinds of probability experiments can help them to develop positive attitudes by valuing probability. Accordingly, in Akoğlu's study (2018), teachers who participated in professional learning teams and MOOCs stated that their motivation increased with the courses and discussion. As a result of that, their willingness to teach statistics and to use different methods increased with improved confidence on the topic.

2.4. Theoretical Framework

This study examines two constructs mainly: knowledge and attitudes. Ernest's model (1989) for the knowledge, beliefs and attitudes of the mathematics teacher is adapted to examine these two constructs. He stated that knowledge, beliefs and attitudes of teachers are core elements for teaching. However, this study will only focus on knowledge and attitudes. According to this model, a mathematics teacher should have different kinds of knowledge such as

- knowledge of mathematics,
- knowledge of other subject matter,
- knowledge of teaching mathematics,
- knowledge of context of teaching mathematics and
- knowledge of education.

As it is seen, the model asserts that being knowledgeable about the subject is not enough for teachers. They also need to know other subjects like geography or

physics to be able to talk about the use of mathematics in other fields of our lives. Also, they need to know how to teach a mathematics topic and which materials should be used while teaching in addition to knowledge of school context. However, in the model, knowledge of mathematics was stated as a critical type of knowledge for teachers in teaching process because without having a conceptual understanding of a topic, teaching it to the students is not possible. Therefore, this study will examine the knowledge of mathematics in detail since it is a prerequisite for teaching. In this research, knowledge of mathematics takes place as knowledge of probability of PSMTs. Ernest (1989) states that examining knowledge of teachers are important to improve teacher education programs. In the light of that, this study examines knowledge of preservice teachers with a belief that diagnosing possible deficiencies before becoming teacher might be more meaningful for the process of improving it.

In the Ernest's model, attitude was considered as including two subcategories: attitudes toward mathematics and attitudes toward teaching mathematics. Attitudes to mathematics was described as valuing and liking mathematics and being confident of mathematical abilities or opposite of these dispositions. Ernest (1989) defined attitudes to teaching mathematics as valuing and liking teaching mathematics and being confident of teaching mathematics. He states that attitudes to teaching mathematics are as important as attitudes to mathematics because they can have a great impact on learning process and learners' attitudes. Accordingly, in this study both components of the attitudes of PSMTs were taken into consideration.

In this study, the scales were used designed by considering different components of knowledge and attitude by the developers. Therefore, in the following section the scales are explained to elaborate the components of knowledge and attitudes.

Gomez Torres and her colleagues (2016) developed a Probability Concept Inventory (PCI) to assess prospective teachers' probability content knowledge. In this inventory, three types of mathematical knowledge for teaching stated by Ball *et al.* (2008), and three different meanings of probability were taken into consideration. Common

Content Knowledge (CCK), Specialized Content Knowledge (SCK), and Advanced Content Knowledge (ACK) are the types of knowledge that shaped the items.

- Gomez Torres *et al.* (2016) described CCK as basic general knowledge about the topic that teachers need to teach the students. To assess that, especially for probability, the authors included primary school probability concepts in the curriculum as questions.
- SCK was identified as the teacher's knowledge related to giving explanations of the topic, representing mathematical concepts with models, and being aware of possible student answers. In order to evaluate this type of knowledge, teachers were asked to write their justifications for their solutions (e.g. which girl cheated and how can you tell?).
- ACK is not only about knowing the topic taught but also about being aware of its antecedents and precedents. Therefore, the questions that require probability concepts from middle and secondary school years and that might result in possible biases are included to measure ACK.

Also, in this inventory, three different meanings of probability, which are named classical, frequentist, and subjective, were taken into account while forming the scale. In this study, these different kinds of knowledge of probability was considered under the knowledge of mathematics component in Ernest's model (1989). Therefore, in this study only one of the components of knowledge of the model will be examined in detail. In the attitude scale used in this study, Estrada *et al.* (2016) described attitude in general according to the multi-component view with components of affect, behavior, and cognition. Then, based on the studies related to attitudes toward statistics, three general components were introduced for attitudes toward probability.

- Affective Component towards Probability (AP): Feelings of the teachers about probability,
- Cognitive Competence towards Probability (CCP): It includes teachers' self-perception about their probability skills.

- Behavioral Component towards Probability (BP): The actions of the teachers based on their decisions (e.g., whether they use it or not) related to the probability.

In addition to general components related to probability, three more were identified for teaching probability since the target of the scale is teachers.

- Affective Component towards Teaching Probability (AT): It includes the teachers' feelings toward teaching probability (e.g. enjoying probability). It might be related to AP.
- Teaching Probability Competence Component (CT): It includes teachers' self-perception by considering their performance and skills in teaching probability.
- Behavioral Component towards Teaching Probability (BT): The teachers' actions toward teaching probability. (e.g, not being willing to teach probability and postponing its teaching)

Also, one more component related to the value of teaching probability was included by the developers of the scale, called the Value Component towards Probability and its Teaching (VPT). This component was added since no other scales highlighted it before. VPT highlights the teacher's appreciation of probability in daily life and their sense of value in teaching probability. Ernest (1989) also states that attitudes to mathematics and attitudes to teaching mathematics cover valuing the topic and its teaching. Since other three components of the attitude do not meet that, the researchers, the developers of APT, were included VPT as an additional component.

The components of attitude in this study is more elaborated form of the attitudes of Ernest's (1989) model. While the model describes attitudes as attitudes to mathematics and attitudes to teaching mathematics, in this study affect, behavior and cognition were considered for two of the components of the attitudes. The model states that there can be different factors affecting attitude of teachers and they need to be questioned in teaching. For example, considering whether teacher education programs

has an influence on their attitudes might be important point (Ernest, 1989). Accordingly, possible factors which might have an influence of PSMTs' knowledge were used to form groups and compare them.

In the following part, the relationship between knowledge and attitude were discussed before introducing the framework of the study.

2.4.1. The Relationship between Knowledge of Probability and Attitudes Towards Probability and Teaching of Probability

Although skills and dispositions are recognized as distinct characteristics since one is in the cognitive domain and the other is included in the affective domain, they are interrelated (Wilkerson, 2007). When the literature is reviewed based on teaching mathematics, it can be found that many studies investigated the teachers' content knowledge about a mathematics topic. After the attitude studies, the question "Why not attitudes and knowledge are interrelated?" started to be investigated by researchers. The literature addresses the issue with a statement that beliefs and knowledge should not be separated, they need to be considered together (Philip, 2007; Thompson, 1992). The research on that topic shows that attitudes have an important effect on learning mathematics (McLeod, 1992; Philip, 2007). According to Di Martino and Zan (2011), students make a connection between their affective components toward mathematics and their success in math. If a constant failure occurs, a negative attitude toward that mathematical topic is inevitable. In León-Mantero and her colleagues' study (2020), preservice teachers who did not remember the subject of statistics very well and were not capable of solving statistics questions reflected negative attitudes toward statistics correspondingly. This might indicate that preservice teachers with adequate knowledge and understanding can be more confident on related topics, which might affect their attitudes toward the topic. In contrast to the findings above, Lovett (2016) stated that PSMTs' statistical knowledge and efficacy in teaching statistics positively correlated, yet the relationship is not strong. Since teaching probability has recently taken place in mathematics curricula, the studies that investigate whether there is a relationship

between teachers' knowledge of probability and attitudes toward teaching probability are limited.

In Türkiye, Bulut *et al.* (2002) studied attitudes and knowledge of preservice teachers in secondary school mathematics teaching programs based on gender. The results showed that both male and female PSMTs have positive attitudes toward teaching probability. The relationship between probabilistic knowledge and attitudes toward the probability of male PSMTs is significant, while there is no significant relationship between probabilistic knowledge of PSMTs and their attitude toward mathematics. Therefore, male PSMTs' attitudes toward and knowledge of probability is a predictor of each other and PSMTs who have better knowledge in probability have a more positive attitude than others (Bulut *et al.* 2002). As a result of these findings, the effect of attitudes and knowledge about probability should not be ignored in the teaching process (Gal *et al.*, 1997). On the other hand, not having a between relationship between probabilistic knowledge and attitude toward mathematics might be a sign of the difference between two subjects.

Based on the reviewed literature that elaborates the components of attitude and knowledge and highlights their relationship, this study investigates those components in the Turkish context. As it is shown in Figure 2.1 below, this research has two constructs such as knowledge and attitude. Also, the factors which might have an effect on the knowledge and attitude were taken into account as Ernest (1989) highlighted. These factors are courses taken in undergraduate years, past experience of the PSMTs during their K-12 years, their grade level and the general success of the PSMTs in the university (GPA).There is an "or more" part in the factors since more might be gained after the interviews with the PSMTs.

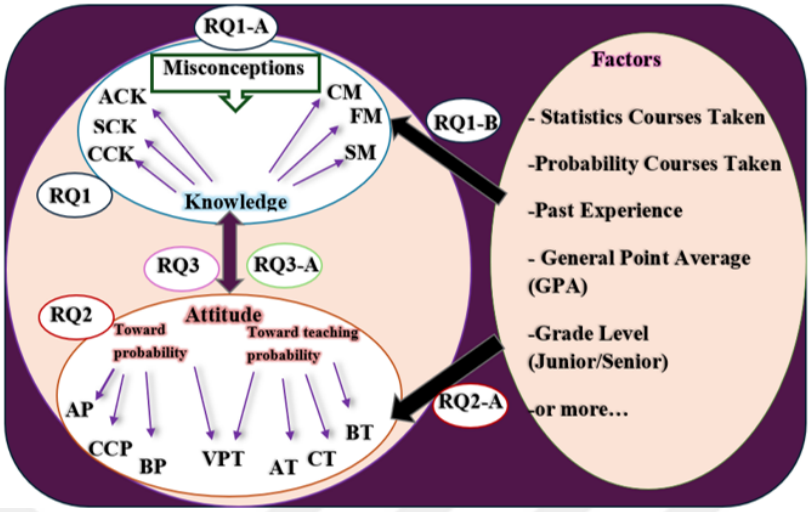


Figure 2.1. Theoretical framework of the study.

Statistics/probability course taken refers to the courses taken during the university years. While it means taking "teaching probability and statistics" courses in some of the universities, it refers to statistics or probability courses in others. More detailed description of the courses for each university was given in Appendix D. This was examined to see whether the courses taken in the universities have an effect on attitudes and knowledge of the preservice teachers or not. In Açar and his colleagues' study (2018), they designed an instruction to see whether understanding by design teaching method improves tenth grades students' achievement on probability and their attitudes toward probability. Although the difference between the pre- and post-test was not significant for the attitudes of experimental group, it was observed that their scores on achievement test improved dramatically. Accordingly, this study tries to answer a question that the general courses (probability and statistics) taken in the universities rather than special instruction have an impact on the attitudes or knowledge or not.

The second factor listed in the framework is past experience. This was especially listed to see whether the experiences related to probability and statistics before the university education influence PSMTs' attitudes or not. The studies showed that the attitudes of students toward mathematics is affected by classroom environment and teachers (Akey, 2006). Similar results might be expected to see in probability. Follow-

ing that, being junior and senior were also added as a factor to compare groups. While seniors starts to teach as intern in the schools, juniors are in the period of course taking. Therefore, the teaching environment that seniors encounter might affect their attitudes positively or in the opposite way. The "or more" part was added for the framework of the study because after interviewing with PSMTs, additional factors might be listed as a facilitator or inhibitor of attitudes.

In this framework, RQ1 "What is the current state of knowledge of preservice mathematics teachers about probability and its teaching?" was examined by considering three components of knowledge: CCK, SCK, and ACK and three meanings of probability: CM, FM, SM. The PSMT's current state of knowledge of probability were also examined according to different characteristics to see whether they can result in significant differences in their current state of knowledge, which answers RQ1A "Is there any statistically significant difference between the current state of probabilistic knowledge of PSMTs from different descriptive groups". In addition to that, the common misconceptions of PSMTs related to probability were investigated since the reviewed literature attributes that teachers also can have probabilistic misconceptions. The second construct of this study is the attitude of PSMTs. Attitudes was constructed by seven components related to probability and its teaching. In this research, the PSMTs' attitudes was studied based on these components to answer RQ2 "What are preservice mathematics teachers' attitudes toward probability and its teaching?". Under the RQ2, the possible reasons of these attitudes were also studied. The relationship between the attitudes and knowledge is what RQ3 "What is the relationship between PSMTs' probabilistic knowledge and their attitudes toward probability and teaching probability?" examines. In addition, RQ3A "What is PSMTs' perception on the relationship between probabilistic knowledge and attitudes" was examined through interviews. This study hypothesizes that the state of knowledge can have an impact on the attitude toward the probability.

2.5. Statement of the Problem

This study investigates PSMTs' probabilistic knowledge and their attitudes toward probability and its teaching. The preservice junior and senior students who enrolled in mathematics education at the university were included in the study. In this study, PSMTs' current state of probabilistic knowledge was examined, and the relationship between their knowledge and attitudes was investigated in addition to their attitudes toward probability and its teaching. Also, this study examined the factors which might have an effect on the knowledge and attitude.

3. METHODOLOGY

In this study, a mixed-method (sequential explanatory) design was used. While the primary data collection and analysis process was quantitative, qualitative findings were used to support the quantitative ones. According to Creswell and Creswell (2017), mixed method design can eliminate the disadvantages of using only one type of research design, allowing researchers to provide a more comprehensive explanation of their quantitative findings. Therefore, in this study, two questionnaires shaped the quantitative part, and the interview was the qualitative part of the study.

3.1. The Framework

The model presented by Ernest (1989) creates a base to understand PSMTs' probabilistic knowledge and their attitudes toward probability and its teaching. The model was adapted according to this study, and it provides a guideline about how data collection and analysis was done based on it. The original of the framework consists of three constructs such as knowledge, beliefs, and attitudes. However, this study focuses on knowledge and attitudes of PSMTs to find an answer to the research questions. Accordingly, knowledge and attitude was elaborated and presented in the framework of this study. As a result, Figure 2.1 summarizes what this research tries to investigate. As shown in Figure 2.1, while knowledge and attitude will be examined and compared based on different factors, the relationship of the constructs will be another focus of this thesis.

By considering that, the study hypothesizes the following:

- It is hypothesized that different factors such as GPA, being senior or junior, the courses taken, influence participants' degree of knowledge of probability. For example, a PSMT with higher GPA might have better score on PCI than a PSMT with a comparably lower GPA.

- It is hypothesized that different factors such as being senior or junior, the courses taken, influence participants' attitudes toward probability and its teaching.
- It is hypothesized that there is a relationship between PSMTs' knowledge of and attitudes toward probability and its teaching. For example, a PSMT with a higher degree of knowledge might have a strong positive attitude.

3.2. Sample

The targeted population of the study is pre-service mathematics teachers in the mathematics education program at a university in Turkey. The study sample was chosen among undergraduate students in primary mathematics education and high school mathematics education who are currently in their junior and senior years. The descriptive properties of the participants are shown in Table 3.1.

Table 3.1. Descriptives statistics of participants.

	PCI		APT	
	N	%	N	%
Juniors	49	51.6	107	54.9
Seniors	46	48.4	88	45.1
Primary Mathematics Education	80	84.2	167	85.6
High School Math Education	15	15.8	28	14.4
Females	85	89.5	167	85.6
Males	10	10.5	28	14.4

In total, 195 PSMTs who took APT participated in the study. However, only 95 PSMTs became the participants as PCI takers. All the PCI takers were also the participant of APT. On the other hand, 100 participants who completed APT did not solve PCI. Even though all universities with mathematics education program (thirty-five universities) were invited to take place in the study, the PCI takers were coming

from fifteen universities while the APT takers were coming from twenty-one different universities as shown in Table 3.2. There was a one participant whose university could not be determined.

Table 3.2. The universities that PCI takers coming from.

University Name	PCI Number of Participants	APT Number of Participants
Afyon Kocatepe University	-	3
Aksaray University	-	2
Boğazici University	31	46
Burdur University	1	3
Çukurova University	-	1
Dicle University	3	4
Gazi University	3	5
Hacettepe University	3	3
İstanbul University	6	14
İzmir Demokrasi University	1	2
Kafkas University	-	1
KastamonuUniversity	1	2
Kütahya Dumlupınar University	-	1
Mersin University	19	40
Muğla Sıtkı Koçman University	1	3
Necmettin Erbakan University	3	10
Middle East Technical University	12	26
Pamukkale University	2	4
Selçuk University	3	7
Süleyman Demirel University	-	1
Unknown	-	1
Yıldız Technical University	6	16

3.3. Instruments

3.3.1. Probability Concept Inventory (PCI)

In order to assess PSMTs' probability content knowledge, the probability concept inventory which was developed by Gómez-Torres *et al.* (2016) was used. While developing this instrument the test developers formed the questions by considering CCK, SCK, and ACK in addition to different meanings of probability such as classical, frequentist, and subjective, that are explained in detail in the previous sections. The mapping of the items with each kind of knowledge type and different meanings is demonstrated in the Table 3.3 .

Table 3.3. Items of PCI.

Type of Knowledge		Meaning of Probability	Item
Common Content Knowledge	Listing (counting) favorable/possible outcomes	Classical	1
	Comparing probabilities	Classical	2
	Joint probability (product rule): Independent experiments	Classical	3
	Joint probability (product rule): Dependent experiments	Subjective	4
	Estimating probability	Frequentist	5
	Fair game	Classical	6a
Advanced Content Knowledge	Expectation	Classical	6b
	Conditional probability	Subjective	7
	Equiprobability bias	Classical	8
	Sampling	Frequentist	9
	Perception of randomness	Frequentist/Subjective	10
	Representativeness heuristic	Frequentist/Subjective	11a
	Binomial distribution	Frequentist/Subjective	11b
	Outcome approach: Prediction	Frequentist	12a
	Outcome approach: Validity of prediction	Frequentist/Subjective	12b, c
Specialized Content Knowledge	Arguments in items 2,3,4,6a,10,11a,11b, 12b		

While developing the scale the Spanish curriculum and textbooks were taken into account. The questions were collected through different kinds of scales of different studies (Azcárate, 1995; Batanero *et al.*, 1996; Chernoff, 2011; Díaz and Batanero, 2009; Falk and Wilkening, 1998; Fischbein and Gazit, 1984; Green, 1982, 1983; Shaughnessy and Ciancetta, 2002). While different researchers reviewed the items, translation and adaptation were included when necessary. After having a pool of items, different researchers gave feedback about the compatibility of the items with different probabilistic concepts during the process. Finally, twelve items were chosen for the probability concept inventory.

The scale's content validity was measured with the help of the opinions of different researchers and having a detailed definition for measuring different kinds of knowledge and the meaning of probability. The researchers presented item difficulty and discrimination, and they calculated Cronbach's alpha = 0.768 by using the scores obtained during the development of the scale (Gómez-Torres *et al.* (2016). Since the Cronbach's alpha was higher than 0.7, it was considered acceptable (DeVellis, 2012). However, the adapted version of the original PCI was used in this study. It was adapted by Kurt and Coşkuntuncel (2020). The item difficulty and discrimination indexes were also calculated in the adapted test as it is in the original PCI. The researchers calculated Cronbach's alpha for the reliability and found as 0.82, which is higher than the original one.

In addition, the scores used in this study were examined to assess their reliability. The Cronbach's alpha was calculated as it is in both versions of the PCI, and it was found as 0.91. Therefore, the scores used in this study were acceptable since it had the evidence of high reliability (DeVellis, 2012)

3.3.2. Attitude Scale Toward Probability and Probability Teaching (APT)

In order to explore preservice teachers' attitudes toward probability and its teaching, the scale developed by Estrada et.al (2018) was used in this study. The scale is

important to assess the teachers' attitude about probability because it also includes a teaching probability component that differentiates it from other scales. The components that were considered while developing the scale and their matching with each item are listed in Table 3.4. Each component is explained in the theoretical framework part in detail.

In total, there are 28 items. While 14 of the items have positive attitude (1, 2, 4, 5, 6, 8, 9, 10, 14, 15, 19, 20, 24, and 28), other 14 have negative attitude (3, 7, 11, 12, 13, 16, 17, 18, 21, 22, 23, 25, 26, and 27). The attitude instrument was applied before the probability concept inventory.

Table 3.4. Items of APT.

Components	Scale Items
Affective component towards probability (AP)	1, 5, 16, 27
Cognitive competence towards probability (CCP)	6, 8, 17, 22
Behavioral component towards probability (BP)	2, 7, 15, 18
Affective component towards teaching probability (AT)	9, 21, 26, 28
Teaching probability competence component (CT)	3, 10, 14, 23,
Behavioral component towards teaching probability (BT)	11, 20, 24, 25
Value component towards probability and its teaching (VPT)	4, 12, 13, 19

In the process of scale development, the researchers defined each attitude component based on the literature before the creation of the items. Then, different scales were examined to form an item pool. The items in the statistics scales were used for creating new items and the name of the scales given as follows: Statistics Attitude Survey (SAS) (Roberts and Bilderback, 1980), Attitudes Towards Statistics (ATS) (Wise, 1985), Survey of Attitudes Towards Statistics (SATS) (Schau *et al.*, 1995), Multidimensional Auzmendi's Scale (Auzmendi, 1992), Scale of Attitudes Towards Statistics (EAEE) (Estrada, 2002). The selected items from the different scales were translated and adapted for assessing attitudes toward probability and its teaching. Consequently, 56 items were formed as a first phase of the process. Then, to ensure the content

validity, each item were scored from 1 to 4 by fourteen experts based on that whether the items were matching with the attitude component or not. Then, the four items with highest scores in each component were selected as a final form of the scale. The different researchers gave feedback for the improvements of the items and at the end 28 items were included in the scale.

The reliability check of the APT was done by using Cronbach's alpha (Estrada *et al.*, 2018). Firstly, the scores were collected through a pilot trial. After that, the Cronbach's alpha were calculated as 0.93, which was very high according to Nunnally (1978). The APT was implemented with a new sample to check the reliability and the Cronbach's alpha was obtained as 0.89 by the researchers.

However, in this study, the participants took the Turkish version of the scale developed by Özyıldırım-Gümüş (2021). After items were translated to Turkish by the researcher, three experts examined the translation of the items to check their meaning. Then, the experts examined the items to decide whether the translated items were compatible with the components of the attitude or not. At the end of the revision, the experts excluded the item 11 from the Turkish version of the scale. Taking expert opinion was used for the validity of the scale. Then, the test was applied to preservice teachers to check the validity and reliability. The confirmatory factor analysis was implemented for construct validity. The original seven structure model was used for the calculation. Since Root Mean Square Error of Approximation (RMSEA) value was calculated as higher than the acceptable value, the second model was used. The RMSEA value was acceptable with the second model and the calculations with the second model were confirmed the dimensionality of the scale. Hence, the scale were noted as valid according to the results. Then, the scores were used to calculate the Cronbach's alpha for reliability of the scale, and it was found as 0.92 which was higher than the original scale. According to DeVellis (2012), Cronbach's alpha higher than 0.70 is acceptable and 0.92 shows highly reliable data.

To assess the reliability of the scores of APT, Cronbach's alpha was also used. After the statistical analysis, it was found as 0.90, which was higher than the original scale and lower than the Turkish version. Since it was higher than the Cronbach's alpha of 0.70, the scores obtained from APT was reliable.

3.4. Semi-Structured Interview

Interviews forms the qualitative part of the study. The data collected via interviews were analyzed to explain the findings found in the quantitative part. Interviewees were decided according to their scores on PCI and APT. Four categories were formed as high-high, high-low, low-high, and low-low:

(i) the participants who had high scores on PCI and attitude scale,
(ii) the participants who had low scores on both scales,
(iii) the participants who had high scores on the attitude scale and low score on PCI
(iv) the participants who had low scores on the attitude scale and high score on PCI

The scores greater than the 3rd quartile were determined as high, and those lower than the 1st quartile were defined as low. The participants who met the conditions were invited. Accordingly, 10 PSMTs for the high PCI and high APT group, 11 PSMTs for the low PCI and high APT group, 10 PSMTs for the high PCI and low APT group, and 13 PSMTs for the low PCI and low APT group were invited to participate in the interviews by sending an e-mail to each. As a result, 13 participants volunteered to participate in the interview. Each category had 3 participants except one category (high PCI, low APT group). which consisted of four participants. The interviews were semi-structured, and they were done via the online meeting platform Zoom. The duration of the meetings was approximately 30-40 minutes. The interviewees were asked to explain their answers given to APT and PCI. It was held in two phases: Firstly, interviewers were asked about their attitudes toward probability and teaching probability. The underlying reasons participants explained for each answer were recorded. The same procedure was followed for PCI, and the answers to questions were asked to be explained

in addition to their current opinions. Interviews were conducted to find an answer to the following research questions: RQ1B. What are PSMTs' common misconceptions about probability? RQ 2A. What are the reasons for PSMTs' positive and negative attitudes toward probability and its teaching? RQ3A. What is PSMTs' perception on the relationship between probabilistic knowledge and attitudes?

3.5. Data Collection

The data of this study was collected as a part of Assistant Professor Kemal Akoğlu's Start-up Project at Boğaziçi University (Project Number: 18721 and Project Code: 22DSUP5). The data were collected from the two scales in three parts for two months. Therefore, the participants did not complete the scales at the same time. But each participant took the two scales within one week. First of all, the attitude scale was implemented. The APT was sent to participants as an online form, and each participant completed before participating in the project's workshops. Then, at the beginning of the project, PSMTs were asked to solve the probability concept inventory. The PCI questions were shared via online form by the researcher, and each solution was required to be uploaded as a photo. In total, 50 minutes were allocated for solving the PCI. Then, participants were sent the answers as e-mails or online form. After the participants' answers on these two scales were scored, the interviews were held. The duration between the scale implementation and interviews were four months.

3.5.1. Data Source

In this study, different types of data were used to be able to discard the disadvantages of one type of data with another. As Creswell and Creswell (2017) highlighted, qualitative data enables the study to make sense of the numerical data. The source of the data in this study came from the Start-up Project where PSMTs took place. To have greater insight, data sources were matched with the research questions in Table 3.5 and analysis of the related data was summarized.

Table 3.5. Data sources.

Research Questions	**Link to the Framework**	**Data Sources**	**Analysis Approach**
1) What is the current state of knowledge of preservice mathematics teachers about probability and its teaching?	Knowledge	Probability Concept Inventory,	Computing scores on PCI, Using an independent sample t-test for the analysis of quantitative data,
1-A) What are PSMTs' common misconceptions about probability?	Knowledge	Probability Concept Inventory, Interviews	Computing scores on PCI, Using coding for the analysis of interviews
1-B) Is there any statistically significant difference among PSMTs' current state of probabilistic knowledge based on different characteristics such as their grade, statistics and probability courses they have taken, and GPA?	Knowledge	Probability Concept Inventory,	Computing scores on PCI, Using an independent sample t-test for the analysis of quantitative data
2) What are PSMTs' attitudes toward probability and its teaching?	Attitude	Attitude Scale,	Computing scores on the Attitude Scale, Using an independent sample t-test for the analysis of quantitative data,
2-A) What are the reasons for PSMTs' positive and negative attitudes toward probability and its teaching?	Attitude	Attitude Scale, Interviews	Computing scores on the Attitude Scale, Using an independent sample t-test for the analysis of quantitative data, Using coding for the analysis of interviews

Table 3.5 Data sources. (cont.)

Research Questions	**Link to the Framework**	**Data Sources**	**Analysis Approach**
3) What is the relationship between prospective teachers' probabilistic knowledge and their attitudes toward probability and teaching probability?	Knowledge-Attitude	Probability Concept Inventory, Attitude Scale,	Using the Pearson Correlation Coefficient for the analysis of the quantitative part,
3-A)What is PSMTs' perception on the relationship between probabilistic knowledge and attitudes	Knowledge-Attitude	Interviews	Using coding for the analysis of interviews

4. DATA ANALYSIS

In the data analysis part of this research, each test distributed to the participants was scored in the light of the data analysis of the developers of the instruments. While scoring PCI, each right answer scored 6 points. Scoring as 6 points was chosen to eliminate the confusion of scoring between the questions with two and three sub-questions. Then, the total score of the test for each participant was calculated. One participant can achieve a maximum score of 72 since there are 12 questions in PCI. Also, each question's total score was determined to compare different components of probabilistic knowledge. The total number of correct answers to one question could be 95 since there were 95 participants. The questions with explanations were also examined, considering the aim of the questions and relevant interview responses. The answers in the interviews were also used to support the explanations of the questions.

The attitude scale scores toward probability and its teaching were calculated using a 5-point Likert scale. Since there are 27 questions, the maximum score a person can achieve is 135. The average score of each question was calculated to compare different scale components. The highest average score that each question can take is 5, and the lowest score is 1. The neutral score was three since it represents 'neither disagreement nor agreement'.

In order to examine the knowledge and attitudes of PSMT, descriptive statistics were used. Then, the independent sample t-test was implemented to compare the groups based on their different characteristics. The scores were compared according to the department, gender, probability, and statistics courses taken, and being senior or junior.

Pearson's product-moment correlation coefficient was calculated for the scores in the instruments and PSMTs' grade point average (GPA). Before using GPA, some of them were converted from 100-point to 4-point system for consistency. The correlation

coefficient was also calculated for the scores in the PCI and attitude scale to see whether there is a correlation between these two types of scores.

The interview participants were selected according to the results of the attitude scale instrument and probability concept inventory. Thus, high/low scores on the attitude scale and PCI were determined according to the third and first quartiles. Hence, four main categories were formed for the interviews. After the interviews were completed, they were transcribed for qualitative data analysis. Open coding was preferred as a coding method and MAXQDA was used as an analysis tool. Each transcription was coded, and while coding, similar codes were collected under a category. After reading the codes with their segments a few times, arrangements were made with the advisor to reduce the number of codes that are similar. After codes and their subcodes were formed, the advisor and the researcher coded each segment separately with a list of codes for intercoder reliability. Then, the codes were compared. When there were some disagreements on codes, coders discussed those until reaching to an agreement.

The reliability of the data obtained by the interviews were assessed by using Cohen's Kappa for intercoder reliability. The percent agreement of Cohen's Kappa was calculated. For the findings of PCI, 93% of agreement were found when the codes were compared, and it was found as 89% for the findings of APT. Creswell and Creswell (2018) states that more than 80% agreement is a sign of good reliability for qualitative studies. Although percent agreement was not 100% at the beginning, at the end the codes attributed to segments were written based on common agreement.

5. RESULTS

The scores of the participants were investigated for the quantitative part of this research study. In order to answer the following research question PSMTs' scores on probability concept inventory were examined.

(i) What is preservice mathematics teachers' current state of knowledge about probability and its teaching?

First of all, scores on the PCI were considered in general. Then, its components and different meanings of the probability were taken into account one by one.

5.1. Probability Concept Inventory (PCI)

5.1.1. General Scores of PCI

The scores on the PCI vary between 14 and 66. The highest score that one participant can reach if all the answers are correct was 72 but no one achieved this score. Figure 5.1 shows the distribution of the scores.

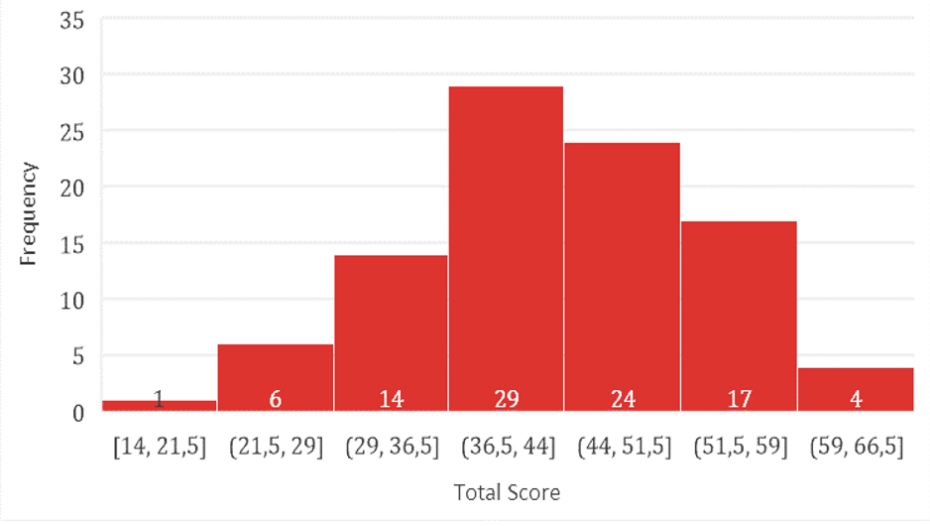


Figure 5.1. General scores on PCI.

The top score was 66 out of 72 points. If all answers are incorrect, the minimum score attainable is 0. The lowest scores among the PSMTs were 14 and 24. It is worth noting that approximately 25% of prospective teachers scored lower than 36, which is 50% of the maximum achievable score. The mean value of the PCI score was 43.7 points, with a standard deviation of 9.8 and a median of 43.5. The scores were accumulated in the range of 36.5 to 51.5.

5.1.2. Components of Probabilistic Knowledge

The PSMTs' answers were examined according to three different components, which were referred to in the PCI: Common Content Knowledge (CCK), Advanced Content Knowledge (ACK), and Specialized Content Knowledge (SCK).

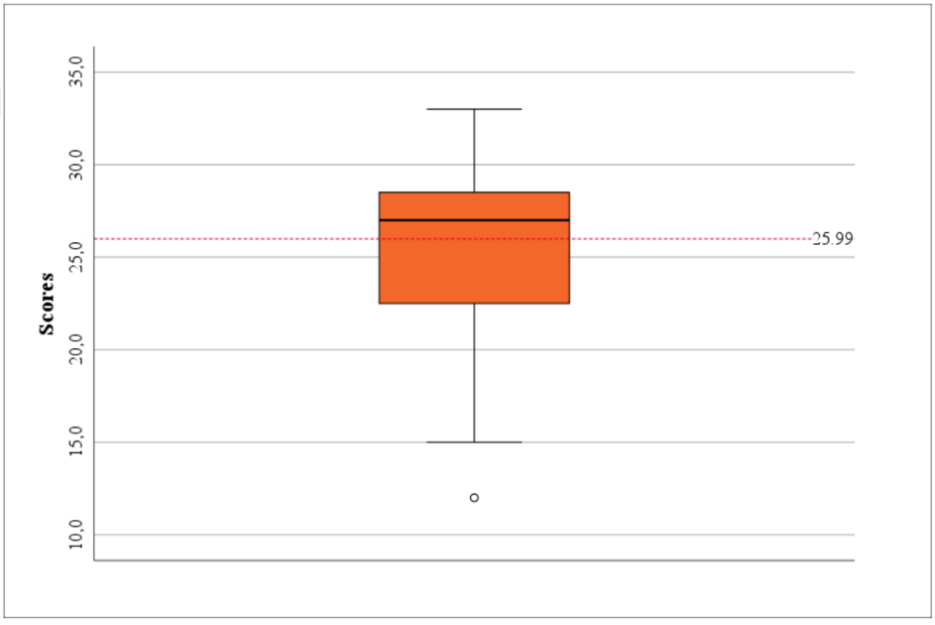


Figure 5.2. Common content knowledge: Boxplot.

5.1.2.1. Common Content Knowledge (CCK). The CCK in PCI refers to teachers' general knowledge about probability, and it is assessed by the probabilistic concepts taught in primary school. The prospective teachers' answers to the Q1, Q2, Q3, Q4, Q5, and Q6a were analyzed to assess their CCK. The maximum score for this component was 33, and 9 prospective teachers achieved it, as it is shown in Figure 5.2.

In total, there were 570 answers for the questions of CCK (considering 95 answers to each question), and 438 of them were correct, which addresses 76,8% of the total answers. The mean score of the CCK was 26.0 (SD= 4.5) out of 33, which is much higher than the scores of 50% of the maximum score. Q6a and Q1 were the questions with the highest percent of correct answers, which were 98% and 96% respectively. On the other hand, in Q5 there were the least number of correct answers. Only 19 PSMTs gave the full correct answer to the Q5, where 35 participants achieved 0 points. Q2, where 70% percent of the answers were correct, had the second lowest scores among the questions of CCK.

5.1.2.2. Advanced Content Knowledge (ACK). The ACK is knowing more than a probabilistic topic and being aware of later concepts related to it. It was assessed by including probabilistic concepts in middle and secondary school. The scores of the Q6b, Q7,Q8, Q9, Q10, Q11a, Q11b, Q12a, Q12b, Q12c were calculated for ACK. The maximum score one could achieve in this component was 39, but no one reached it. The highest scores in ACK were 34 and 33, the two PSMTs reached shown in Figure 5.3.

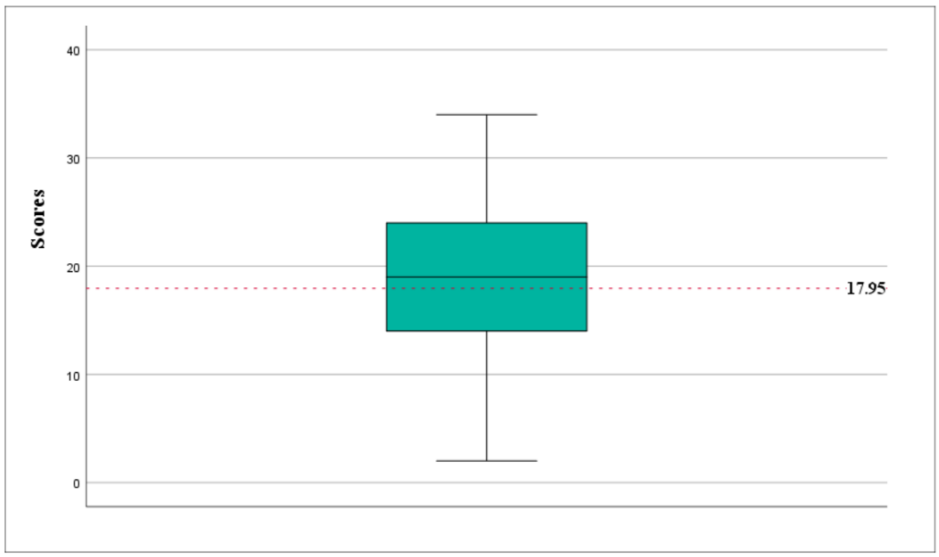


Figure 5.3. Theoretical framework of the study.

The mean score was found to be 17.97 (SD= 7.6), which is below 19.5 (50% of the highest score in the ACK). In the ACK category, 55% of the PSMTs answered

more than half of the questions incorrectly. In total, there were 950 answers, 95 for each question. 461 of the total number of answers were correct, while there were 465 incorrect answers. Thus, the percentage of the correct answers was 48,31%, ranging from 4 points to 34 points. 10.5% of the participants gave correct answers to the Q8, which is the lowest percentage among other items in ACK. Also, only half of the answers to the Q9, Q10, Q11a, Q11b were correct.

5.1.3. Specialized Content Knowledge (SCK)

The SCK is defined as knowing how to teach a probability concept or being capable of which tasks should be used to present the concept. Questions 2, 3, 4, 6a, 10, 11a, 11b and 12b were under SCK since they require justification of the answers given. The quantitative all of the items correctly was two. Also, two prospective teachers achieved the lowest points, 6 and 9 out of 35 points is shown in Figure 5.4.

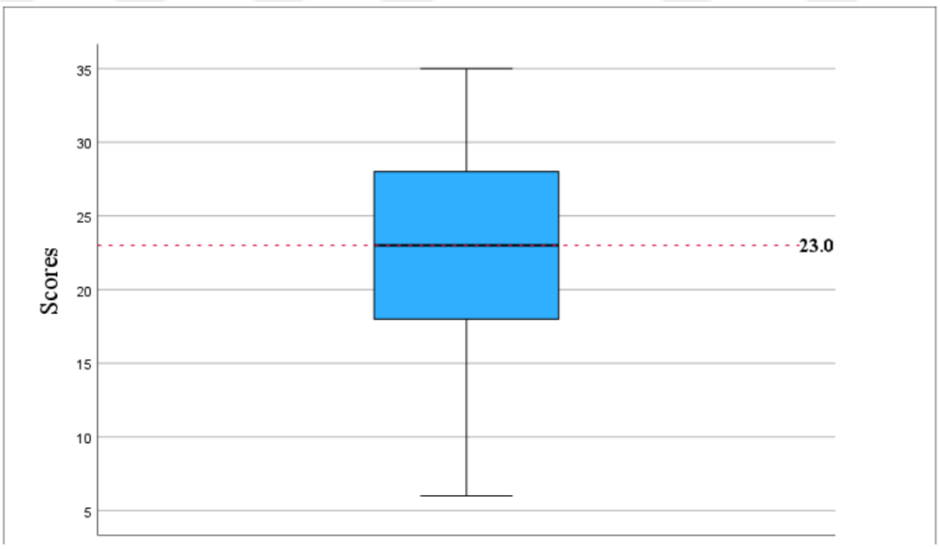


Figure 5.4. Specialized content knowledge: Boxplot.

The mean score was 22.9, with a standard deviation of 6.0, which is higher than 50% of a full score in the SCK category. 43,68% of the total number of answers to SCK was correct. Among the eight items, Q10 and Q11a, Q11b had the lowest scores. While 26% of the prospective teachers answered Q10 correctly, the percentage of the answers with a full score for Q11a was 36.8%, and for Q11b, it was 28.4%.

5.1.3.1. Classical Meaning (CM). The CM addresses finding the probability of an event as a ratio which is favorable outcomes over all possible outcomes of an event. In the PCI, the items Q1, Q2, Q3, Q6a, Q6b, and Q8 represented the CM. The scores were varying from 9 to 30. Thus, the highest achievable score was 30 for the CM of the probability. As it is shown in Figure 5.5 only 4 PSMTs reached that score. The lowest scores were 9 points and 12 points where eight prospective teachers obtained.

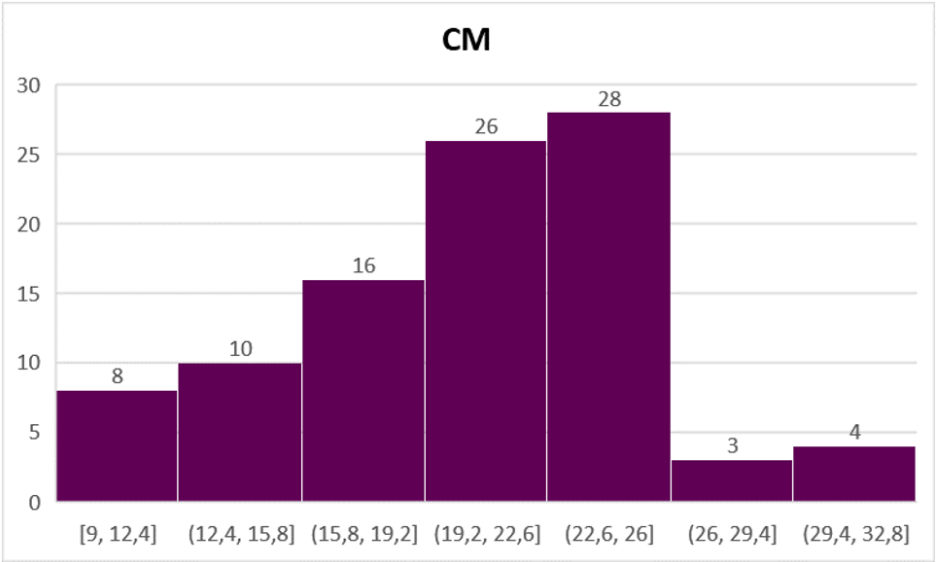


Figure 5.5. Classical meaning of probability: Distribution of scores.

The mean score was 20.5 points with a standard deviation of 1.7, which is the highest mean score among the probabilistic meanings. The scores were centered around 16 to 26 points, in which 57% of the PSMTs' scores were accumulated. Out of 6 items of the CM of probability, 68% of the answers were totally correct. Questions 6a and 1 had the highest number of correct answers not only in CM but also out of the entire test. While 96% of the answers to Q1 were totally correct, the percentage was 98 for Q6a. On the other hand, the question which had the lowest score with the least number of correct answers in this category was Q8. The percentage of the correct answers for Q8 was 6%, which is the lowest one among the twelve questions.

5.1.3.2. Frequentist Meaning (FM). The Frequentist Meaning of Probability is an experimental approach where finding the probability of an event requires many trials. In

the PCI, the questions related to FM were Q5, Q9, Q10, Q11a, Q11b, Q12a, Q12b, Q12c. The maximum score that one could get was 30. The scores varied from 0 to 27, as shown in Figure 5.6 31% of the scores were below the nine that constitute the lowest scores.

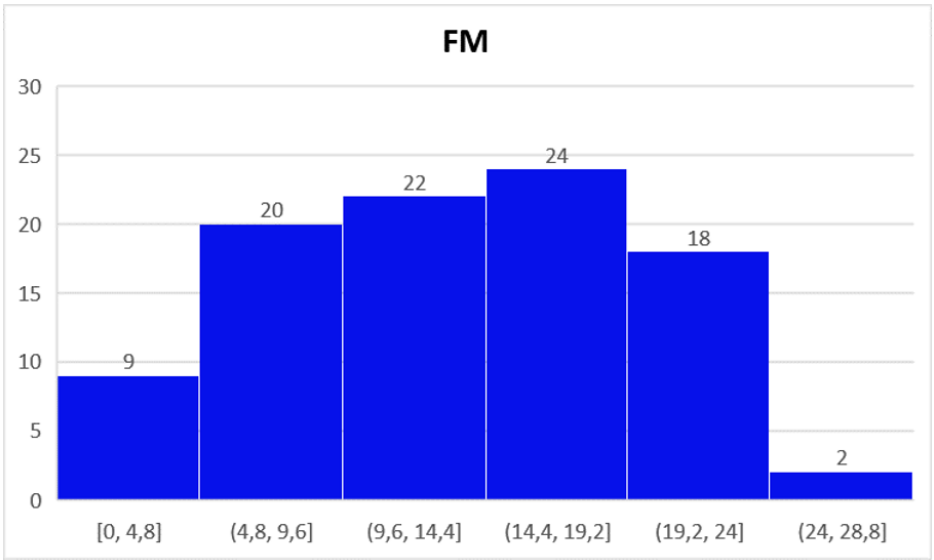


Figure 5.6. Frequentist meaning of probability: Distribution of scores.

The mean score for the FM of probability was 13.3 (SD= 6.2). Q5 obtained the lowest score (only 19 correct answers) among the items. Also, Q10 and Q11 were among the weakest since almost 75% of the solutions were incorrect. On the other hand, the percentage of the correct answers in Q12 was outscored. For instance, while Q12a had the highest number of correct answers, where 77% of them got a complete score, 74% of the PSMTs answered Q12b correctly.

5.1.3.3. Subjective Meaning (SM). The Subjective meaning of probability refers to deciding the probability of an event using beliefs and thoughts coming from past events. The questions 4, 7, 10, 11a, 11b, 12b, and 12c represent the SM. The distribution of the scores in the category is demonstrated in Figure 5.7.

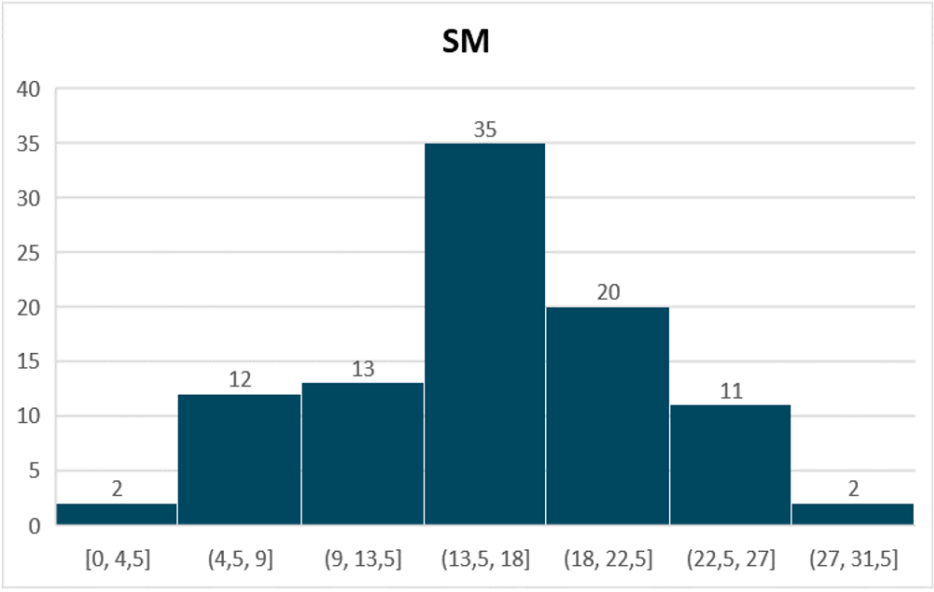


Figure 5.7. Subjective meaning of probability: Distribution of scores.

The maximum achievable score in this category was 28, which two prospective teachers gained. The scores were varying from 0 to 28 points. The mean of the PSMTs' scores was 16.3, with a standard deviation 5.9. In this type of meaning of probability, questions 10, 11a, and 11b got the lowest scores. 74% of the answers to the Q10 were incorrect, and the percentage of incorrect answers to the Q11b was 72%. Q11a follows these questions with 63% of incorrect answers. Among the seven items, Q4 had the highest score, where only 14% of the answers were given incorrectly.

5.1.4. The differences in the PCI Scores

The participants' scores were examined according to their descriptive features.

5.1.4.1. Junior vs. Senior. The first descriptive characteristic that was examined to decide whether there was a difference between the two groups was the grade level of the PSMTs. Before implementing the independent sample t-test, three assumptions was checked, as seen below.

- Normality assumption: The Kolmogorov-Smirnov test was implemented to check the normality of the scores. The distribution of the scores for both groups was found to be normal (p>.005). Thus, the assumption was met.

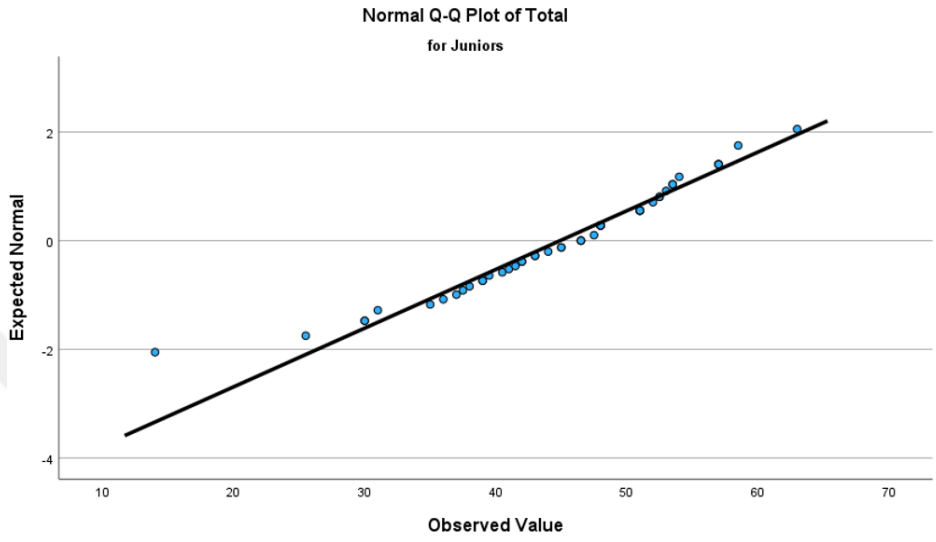


Figure 5.8. Normal Q-Q plot for the scores of juniors.

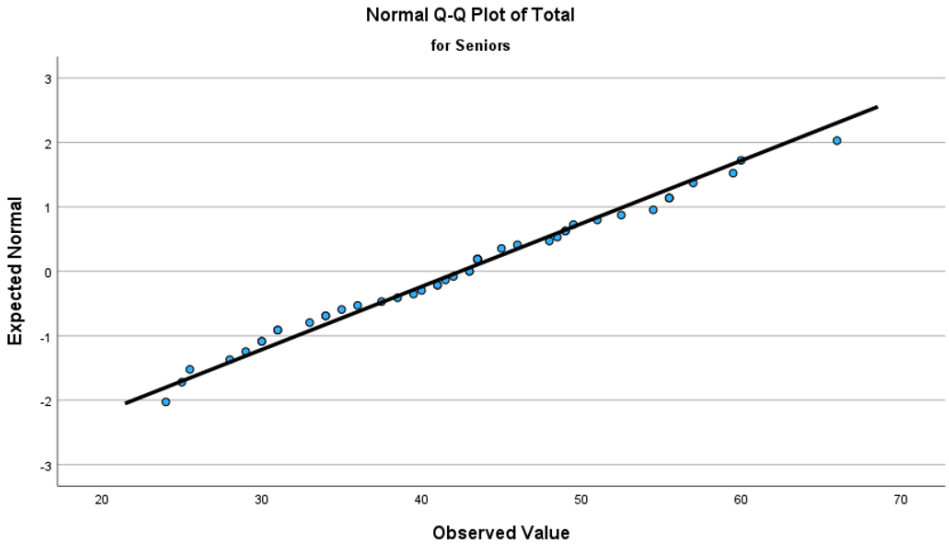


Figure 5.9. Normal Q-Q plot for the scores of seniors.

- The independence of observations: The scores in the PCI were independent from each other.
- Homogeneity of variances: The Levene's Test was implemented to check the homogeneity of variances of the groups. It was found that the variances of the

groups do not significantly differ p=0.42 0.05 (two-tailed). Thus, the homogeneity of variances was met.

There were 49 juniors and 46 seniors out of 95 preservice teachers. The mean value of the scores for junior PSMTs was 44.92, with a standard deviation of 9.3. On the other hand, the mean score of the senior PSMTs on the Probability Concept Inventory was 42.42, with a standard deviation of 10.2. The mean difference between the two groups was 2.48 (95% CI -1.49 to 6.45). However, the results showed that the scores of juniors and seniors on the PCI do not differ significantly (t (93) = 1.24, p = 0.22, two-tailed). In other words, probabilistic knowledge of the prospective teachers did not show a significant change based on their grade levels.

5.1.4.2. A Probability Course Taken or Not. The PSMTs' scores on the Probability Concept Inventory were examined based on the probability courses taken by the pre-service teachers. The number of PSMTs in the groups slightly differs. While there were 79 prospective teachers who had taken a probability course before, the number of PSMTs who had not taken it yet was 16. A t-test was implemented to compare the mean value of the scores of the two groups. However, since the number of PSMTs in the two groups differed, three assumptions were checked to ensure that the two groups' variances were homogeneous and normally distributed.

- Normality assumption: The assumption was checked by using the Kolmogorov-Smirnov test. The distribution of the scores for both groups was found to be normal (p>.005). Thus, there was no violation of the normality.
- The independence of observations: Each score in the PCI was independent, and they did not influence each other.
- Homogeneity of variances: The Levene's Test was implemented to check the homogeneity of variances. Even though the number of preservice teachers in the groups was slightly different, the variances of the groups were homogeneous p=0.08 0.05 (two-tailed). Thus, there was no violation.

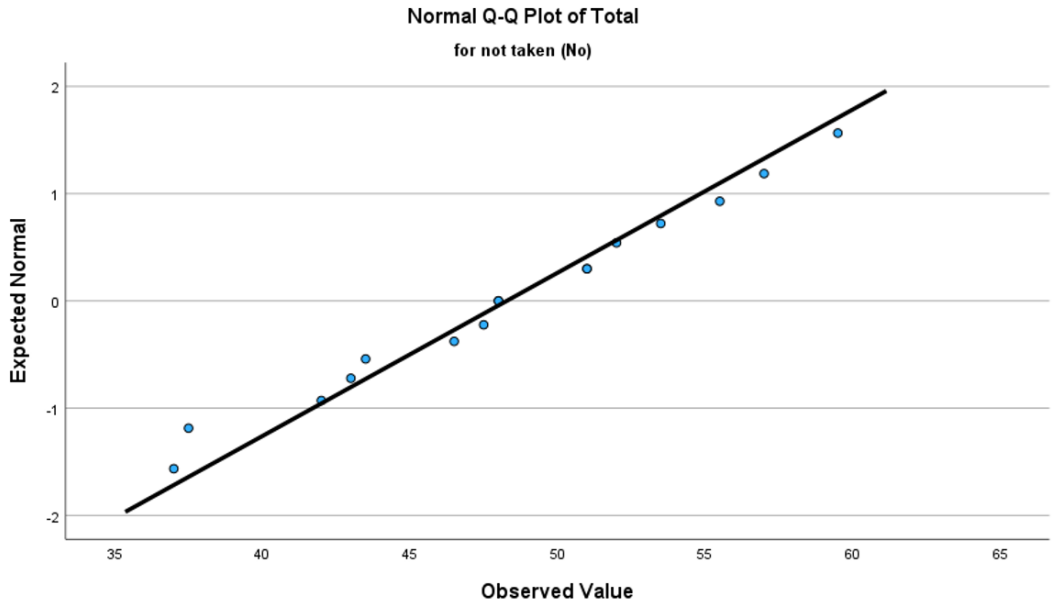


Figure 5.10. Normal Q-Q plot for the scores of the group who have not taken a probability course before.

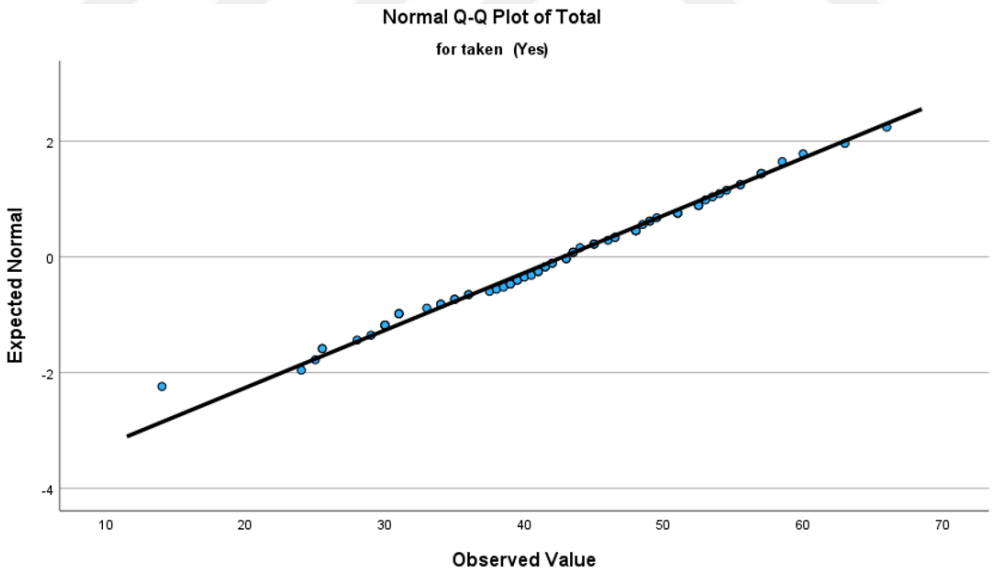


Figure 5.11. Normal Q-Q plot for the scores of the group who took a probability course before.

Table 5.1. The groups of participants who took a probability course or not.

Have you ever taken a probability course in the university?	N	Mean	Standard Deviation
yes	79	42.79	10.1
no	16	48.28	6.6

The descriptive properties of the groups were shown in Table 5.1.

The mean score of the PSMTs who took a probability course was 42.80 (SD= 10.0), while the mean value of the group who have not taken a probability course yet was 48.29, with a standard deviation of 6.6. The difference in the means for the two groups was 5.49, where 95% CI was -10.72 to -0.26. The results highlighted that the taken probability courses resulted in a difference between the scores. The difference between the scores of PSMTs was statistically significant with p =0.04 $<$ 0.05 (t (93) = -2.09, two-tailed).

5.1.4.3. A Statistics Course Taken or Not. The scores were examined under the condition of taking statistics courses. There were 89 PSMTs who had taken a statistics course before out of 95 preservice teachers. To compare the two groups independent sample t-test was used after checking the assumptions.

(i) Normality assumption: The Kolmogorov-Smirnov test was used to check the normality of the scores. As seen in the Normal Q-Q Plots in Figure 5.12 and Figure 5.13 the PSMTs' scores were normally distributed and the assumption was not violated.

(ii) The independence of observations: Each score that PSMTs gained was independent of each other. Thus, the assumption was met.

(iii) Homogeneity of variances: The homogeneity of variances of two groups of PSMTs was measured by using the Levene's Test. Although the number of PSMTs was

different in the two groups of preservice teachers, the test pointed out that the homogeneity of variances of the groups did not significantly differ.

While the mean score of preservice teachers who have not taken a statistics course before in the university was 45.50 with a standard deviation of 11.7, the mean value of the scores of PSMTs who took a statistics course was 43.60 (SD= 9.7). The difference between the mean scores was 1.90 where 95% CI -10.12 to 6.31. The results showed that the mean difference between the PSMTs who took and those who did not was not statistically significant p =0.65 > 0.05 (t (93) = -0.46, two-tailed).

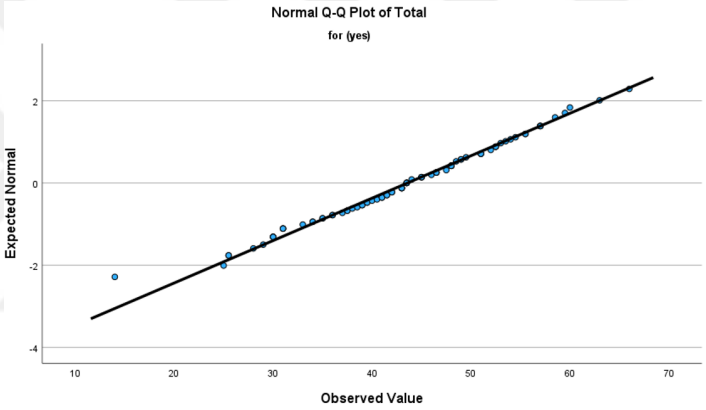


Figure 5.12. Normal Q-Q plot for the scores of the group who took a statistics course before.

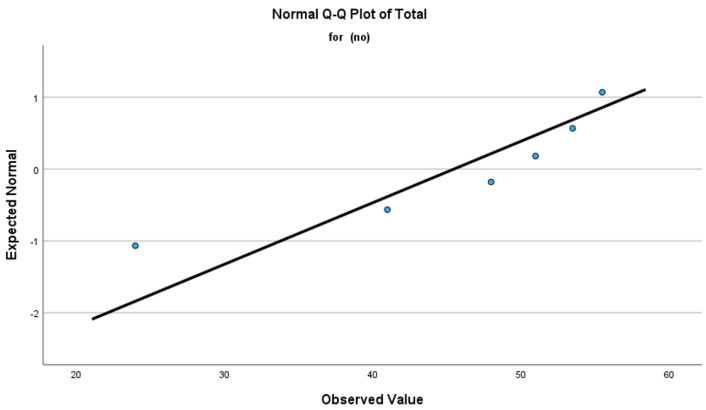


Figure 5.13. Normal Q-Q plot for the scores of the group who have not taken a statistics course before.

5.1.4.4. Grade Points Average (GPA). The preservice teachers who took PCI had different GPAs. The Pearson Correlation Coefficient was calculated to determine the association between the GPAs and scores on PCI. The normality assumption was checked using the Kolmogorov-Smirnov Test of Normality before calculating the Pearson Correlation Coefficient. While the scores on the PCI were normally distributed, the deviation of the PSMTs' GPAs from the normal distribution was statistically significant ($p=0.03< 0.05$, two-tailed). Thus, the Spearman Rank Order Correlation was calculated as an alternative to the Pearson Correlation. The results showed that preservice teachers' success based on GPA was not associated with their probabilistic knowledge, where Spearman's rho= -0.073 ($n=193$, $p>0.05$).

PSMTs' knowledge of different meanings of probability, their different types of probabilistic knowledge and their relationship with GPAs were also examined. Therefore, the components which were used to determine the relationship were CCK, ACK, SCK, CM, FM, and SM. Firstly, the Kolmogorov-Smirnov Test of Normality was implemented. CCK, CM, and SM were not met the normality assumption. Also, as it is stated above, PSMTs' GPAs were not normally distributed. Therefore, Spearman Rank Order Correlation was used rather than Pearson Product Moment Correlation Coefficient. The results indicated that there was no significant relationship between PSMTs' GPAs and their different types of probabilistic knowledge as shown in Table 5.2.

Table 5.2. Correlation coefficient for GPA.

		CCK	ACK	SCK	CM	FM	SM
GPA	Correlation Coefficient (Spearman's Rho)	-0.31	0.13	0.11	0.08	-0.01	0.16
	Significance (2-tailed)	0.76	0.22	0.29	0.41	0.90	0.13

5.2. Attitudes toward Probability and Its Teaching Scale (APT)

5.2.1. General Results

To find an answer to question 2, which is presented below, the scores on Attitudes toward Probability and its Teaching (APT) were examined.

2) What are preservice mathematics teachers' attitudes toward probability and its teaching?

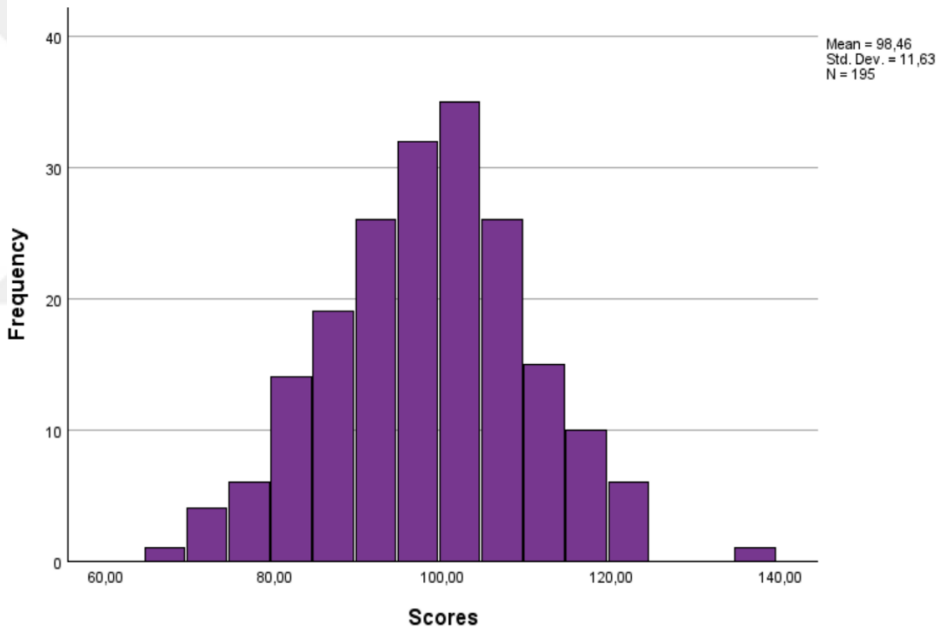


Figure 5.14. Distribution of scores on APT.

The results included 195 answers for each question, where each scored out of 5, and the average value per item varied from 2.46 to 4.48 with a mean of 3.65 (SD= 0.5). The lowest score was 67, and the highest was 135 in the total scores, where the maximum achievable value was 135. The mean value of the total scores for the scale was 98.46, with a standard deviation of 11.6. Figure 5.14 shows the distribution of the total scores. It might be argued that the PSMTs have a positive value toward probability and its teaching since the neutral value for the scale was 81 when items were scored at 3 points, but the mean score (M= 98.46) was much higher than that.

The items with the highest scores were item 11 (Probability is only useful for games of chance) with a mean score of 4.48 (SD= 0.6), item 18 (Probability knowledge helps students to reason critically) with a mean score of 4.40 (SD= 0.5), item 16 (Probability is understandable only to "science people"). with a mean score of 4.36 (SD=0.6).

On the other hand, the items that had the lowest scores were item 6 (Probability is easy) with a mean value of 2.46 (SD=0.8), item 13 (It will be easy for me to design assessment tasks related to probability) with a mean value of 2.95 (SD= 0.8), and item 20 (I feel worried about being able to reply to my students' questions about probability) with a mean value of 2.52 (SD= 1.0). The scores of items like I11, I16, and I20 were transformed before the calculations since they include negative meanings.

When the lowest and highest scores were examined accordingly, it can be concluded that PSMTs appreciate the value of probability, but they were concerned about teaching it since they do not think that probability is an easy topic to comprehend.

5.2.2. Components of APT

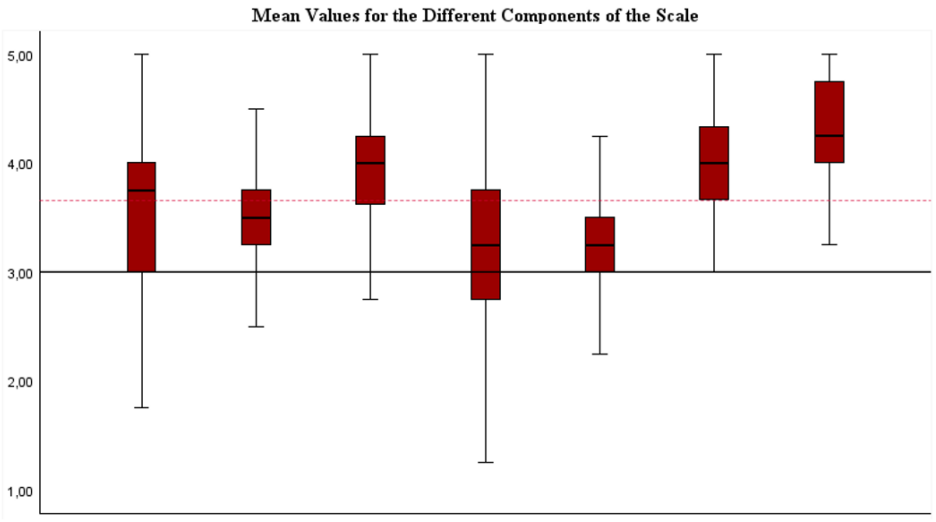


Figure 5.15. Boxplot of components of APT.

The results of the scale were also examined according to different components. The Figure 5.15 presents the mean values for each component out of 5 points where

the red dotted line represents the mean of the scores of the components of the scale (M= 3.65, SD= 0.4) and the bold black line shows the neutral score of 3.00 (average theoretical value). According to the results, almost all components were scored as much higher than the neutral score when the mean values of the components were compared. Therefore, it can be interpreted as a sign of the positive attitude of the PSMTs. However, the components where PSMTs scored as lowest and much closer to neutral value were the Teaching Probability Competence (CT) component (M=3.25, SD= 0.5) and Affective Component towards Teaching Probability (AT) with a mean value of 3.26 (SD= 0.7). Hence, PSMTs' feelings about teaching probability and their perception of self-capability to teach probability with effective tasks were closer to neutral, even if the scores were positive.

According to the results, PSMTs scored Value Component towards Probability and its Teaching (VPT) as the highest value among others (M=4.31, SD= 0.5). Its mean value is above both the mean and the median of the components in the scale. In VPT, the scores mainly accumulated over 4.00 points and above (%88 percent of the scores of the component). These findings underline that the PSMTs appreciated the usefulness of probability in daily life and its necessity for teaching. Following VPT, the scores of BP and BT were much higher than others. The mean value of BP was 3.88, with a standard deviation of 0.5, and the mean score of BT was 3.85 (SD=0.6). Based on the definition of these two components, the results might be interpreted as that the PSMTs have a positive attitude towards using probability and desire to teach probability as a preferred topic.

5.2.3. Results Based on Descriptive Properties

In order to find an answer to research question 2, first of all, an independent sample t-test was used for APT scores based on the following characteristics: being a senior or junior student, a probability course being taken or not, and a statistics course being taken or not. In addition, Pearson's correlation coefficient was calculated for PSMT's scores on the attitude scale and their GPA.

5.2.3.1. Junior vs Senior. The first characteristic that was examined to see whether there is a significant difference between the PSMTs' scores from different grade levels (junior or senior) in the university. To be able to implement the t-test, the assumptions must be met.

- Normality assumption: Even though the assumption does not need to be checked with a normality test since the sample size is not small (n>30), the Kolmogorov-Smirnov test was implemented to evidence it. The assumption was met (p>.005), the scores of the two groups did not deviate from the normality, and the t-test could be implemented.

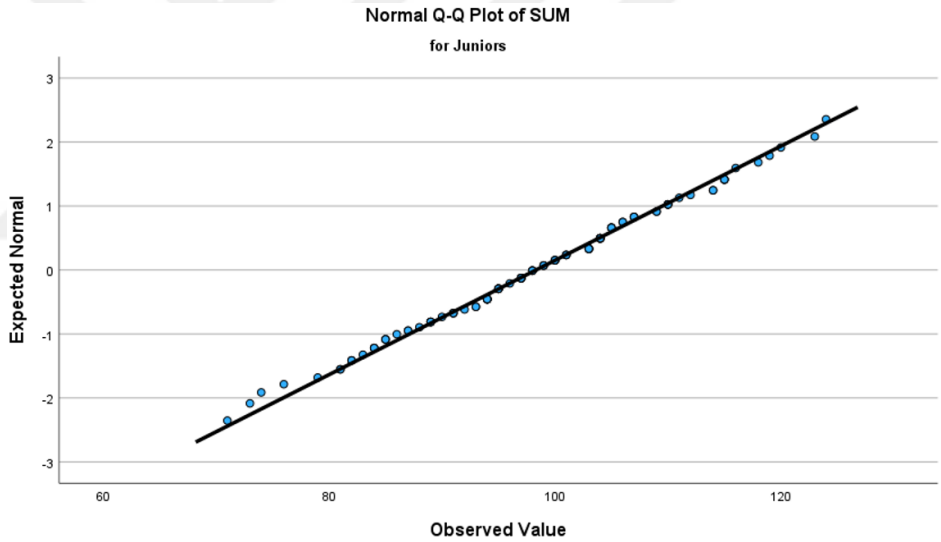


Figure 5.16. Normal Q-Q plot for the scores of juniors.

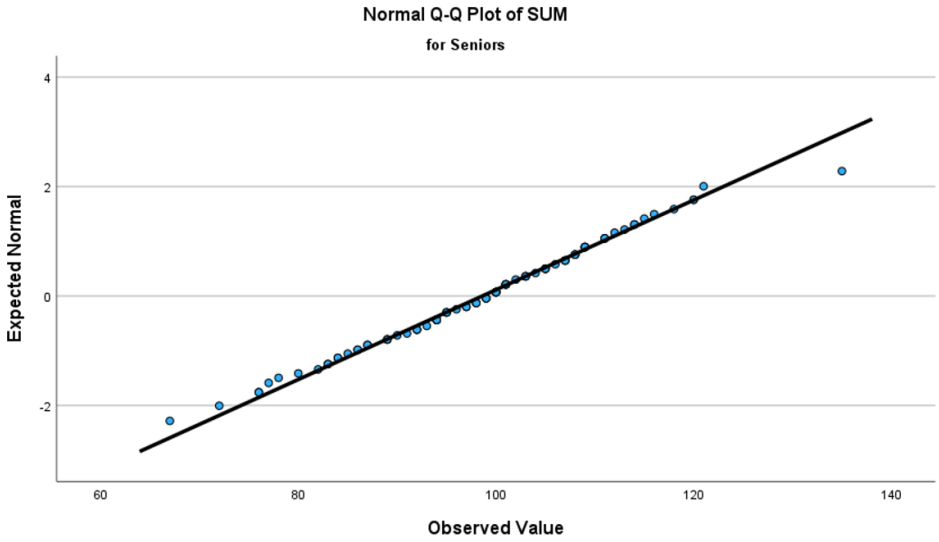


Figure 5.17. Normal Q-Q plot for the scores of seniors.

- The independence of observations: Each participant's score was independent of the other.
- Homogeneity of variances: To assess this assumption, Levene's Test was implemented. Homogeneity of variances of the two groups was met p . 05 (two-tailed).

The Table 5.3 shows descriptive statistics of total number of junior and senior students.

Table 5.3. Properties of the FE anchor pile models.

	Grade Level	N	Mean	Standard Deviation	Standard Error Mean
Scores	3 (Junior)	107	98.32	11.2	1.0
	4 (Senior)	88	98.61	12.2	1.3

While there were 88 seniors, the number of junior PSMTs was 107. The results show that the mean score in the junior PSMT was 98.32 with a standard deviation of 11.1, while the mean score in the seniors was 98.61 with a standard deviation of 12.2. The mean difference was 0.29, and there was no statistically significant difference between the scores of juniors and seniors (t (193) = -0.124, p= 0.87, two-tailed). The magnitude of the difference in the means between junior and senior groups (mean

difference = 0.29, 95% CI: -3.59 to 3.02) was too small (eta squared= 0,0002). Although there is no meaningful difference between their scores on attitude, it is shown that both of the groups have positive attitudes toward probability and its teaching since their mean value is above the theoretical/neutral mean, which is 81.

5.2.3.2. A Probability Course Taken or Not. The PSMTs' attitude scores were examined based on whether they took a probability course at the university or not. To compare two groups who took or have not taken yet, a two-sample t-test was conducted. Before the implementation of the t-test, the assumptions were checked.

- Normality assumption: The assumption was checked by applying the Kolmogorov-Smirnov test. Both groups are distributed normally shown in Figure 5.18 and Figure 5.19.
- The independence of observations: The scores of PSMTs were independent.
- Homogeneity of variances: The Levene's Test was implemented to check the homogeneity of variances for the groups where PSMTs took the probability course or not. The assumption was met p= 0.61 0.05 (two-tailed).

The number of PSMTs who did not take a probability course was 24, while 171 PSMTs took a probability course. Although the number of people in each group differs, the homogeneity of variances was checked and met, as explained before. The mean score of PSMTs who had not taken a probability course was 101.88 (SD=11.3). On the other hand, the mean value of the scores in the group, that PSMTs took a probability course, was 97.98, with a standard deviation of 11.3. The mean difference between the PSMTs who took a probability course and the PSMTs who have not taken was 3.90 (95% CI -8.88 to 1.08). However, the difference was small (eta squared=0.012) and it was not statistically significant (t (193) = -1.54, p= 0.12, two-tailed).

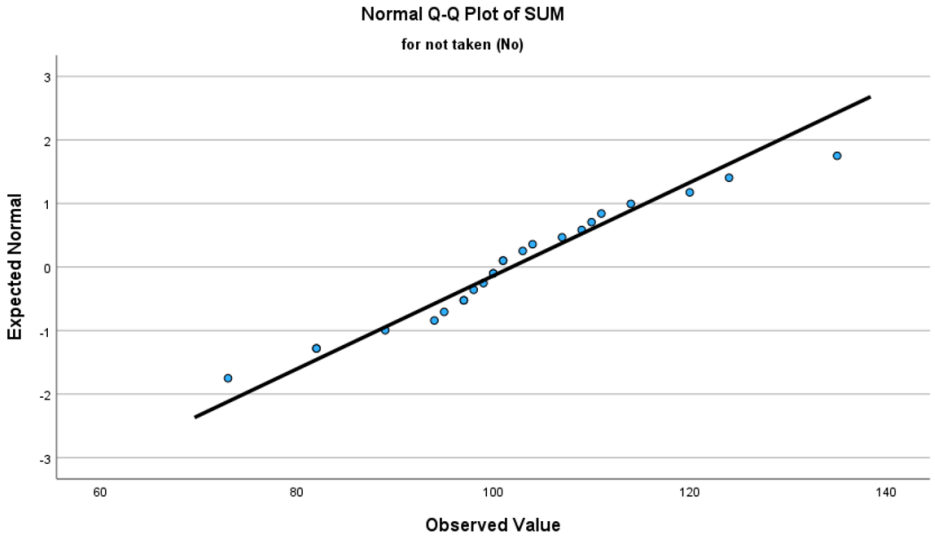


Figure 5.18. Normal Q-Q plot for the scores of the group who have not taken a probability course before.

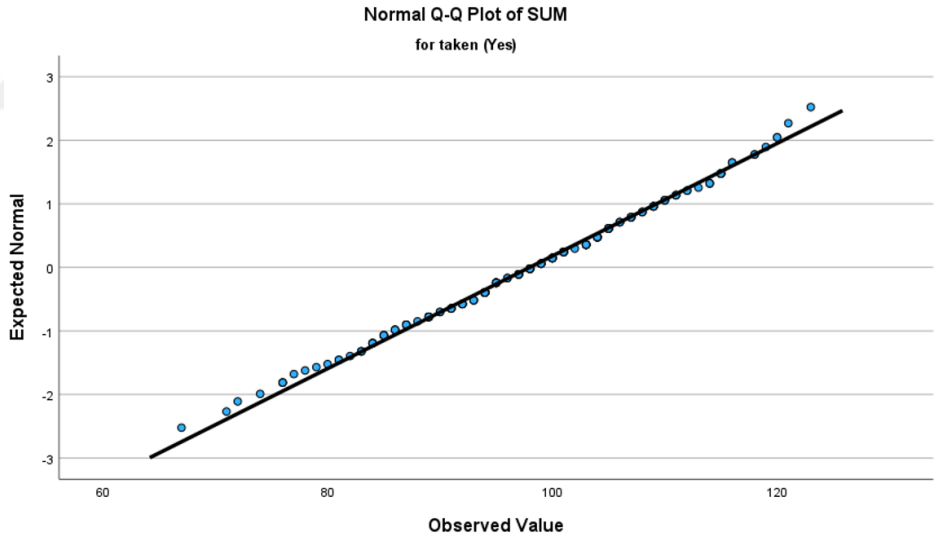


Figure 5.19. Normal Q-Q plot for the scores of group who took a probability course before.

5.2.3.3. A Statistics Course Taken or Not. The scores of the two groups were examined to find a difference between the scores of PSMTs who took a statistics course and PSMTs who have not yet taken a statistics course in the university. Two sample t-test was used for the comparison of the scores. However, before the implementation of the test, the assumptions were checked.

- Normality assumption: The assumption was checked using the Kolmogorov-Smirnov test shown in Figure 5.20 and Figure 5.21. The assumption was met (p= 0.2, p>0.05).
- The independence of observations: Each score was independent of each other.
- Homogeneity of variances: It was checked by using the Levene's Test. The assumption was met p=.66 0.05 (two tailed). Therefore, the variances of the PSMTs who took a statistics course and the PSMTs who have not taken a statistics course yet were homogeneous.

12 PSMTs have not taken a statistics course, while the number of PSMTs who took a statistics course was 183. The mean score of the PSMTs who have not taken a statistics course yet was 97.0, with a standard deviation of 12.4. The mean score of the PSMTs who took the course was 98.6 (SD= 11.6). Although the mean difference between the two group of preservice teachers was 1.55 (95% CI -5.30 to 8.40), it was not statistically significant (t (193) = 0.45, p= 0.66, two-tailed).

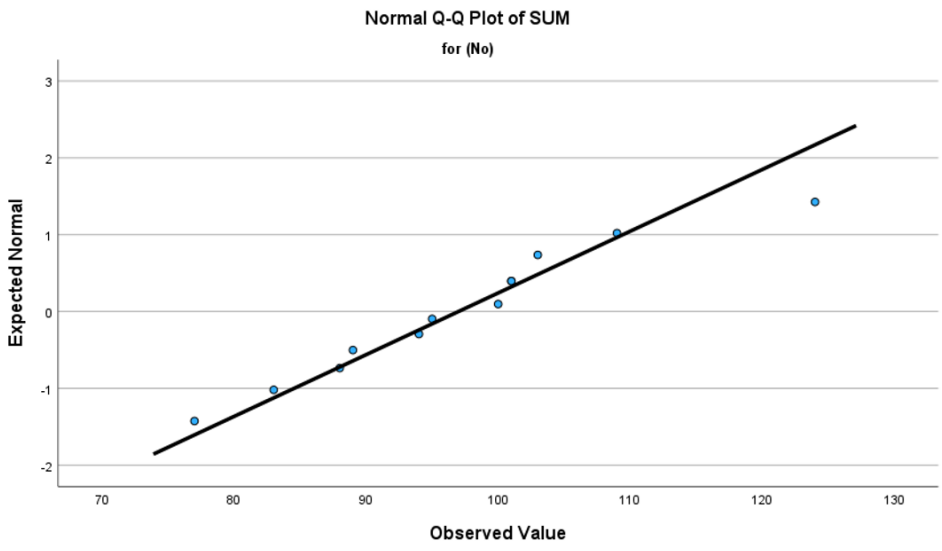


Figure 5.20. Normal Q-Q for the scores of the group who have not taken a statistics course before.

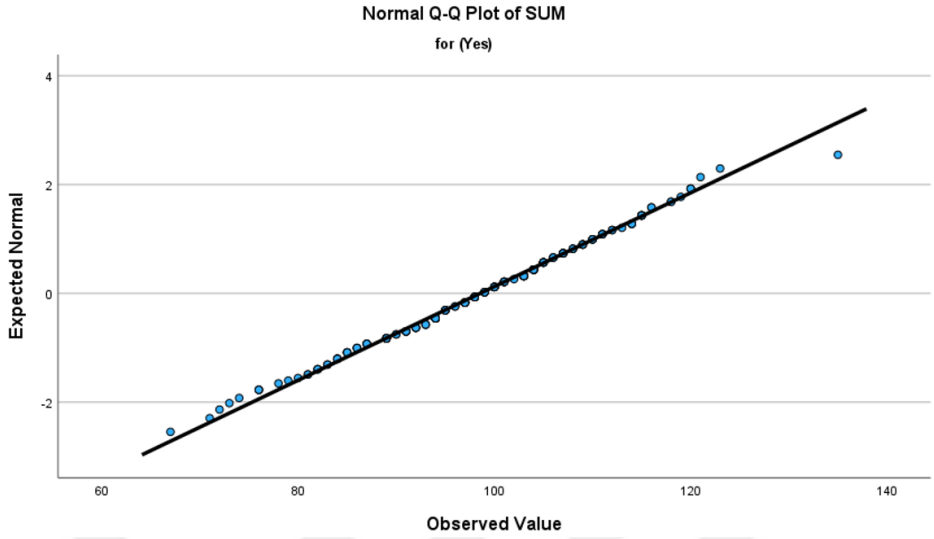


Figure 5.21. Normal Q-Q for the scores of the group who have not taken a statistics course before.

5.2.3.4. Grade Points Average (GPA). The PSMTs in the study had a wide range of different GPAs. To be able to understand whether there is a relationship between their GPAs and scores on APT, the Pearson product-moment correlation coefficient was used. However, when the normality of the GPAs and the scores on the APT were checked using the Kolmogorov-Smirnov test, the prospective teachers' GPAs were not normally distributed. (p= 0.002< 0.05). Therefore, Spearman Rank Order Correlation as a non-parametric correlation test was implemented. The results demonstrated no significant correlation between the two variables: PSMTs' GPAs and scores on APT, Spearman's rho= -0.073 (n=195, p>0.05).

5.3. Probabilistic Knowledge and Attitudes toward Probability and Teaching Probability

PSMTs' scores on APT and PCI were examined to be able to answer the third research question of this study which is given below.

Q3) What is the relationship between prospective teachers' probabilistic knowledge and their attitudes toward probability and teaching probability?

First of all, the total scores on both of the instruments were used to examine the association. Kolmogorov-Smirnov Test of Normality checked the scores' normality for the test's assumptions. As highlighted before in the results section, it was found that the scores were normally distributed ($p > 0.05$). Therefore, the Pearson Correlation Coefficient was calculated to see whether there is a relationship. The results show that preservice mathematics teachers' knowledge of probability does not have a relationship with their attitudes toward probability and teaching probability ($r(95)= 0.052$, $p > 0.05$).

After the sum of the scores of each item were investigated, the association between the components of APT and the components of PCI was explored. Spearman's rho was used to investigate the relationship since the components were not normally distributed. The results revealed a significant association between Behavioral Component Towards Probability (BP) and PCI scores. A small relationship between PCI total scores and BP were found, $r(95)= 0.22$, $p <0.05$.

The scores on the meanings of Probability and different kinds of probabilistic knowledge were explored to determine their association with the components of APT. The results indicated that the CM of probability and CCK on probability are related to BP, as shown in Table 5.4 and Table 5.5.

Table 5.4. Statistical results of BP and CM.

	Mean	Standard Deviation	N	Spearman's rho (r)	Sig. (2-tailed)
BP	3.94	0.5	95	0.22	0.033
CM	20.53	4.5	95		

Table 5.5. Statistical results of BP and CCK.

	Mean	Standard Deviation	N	Spearman's rho (r)	Sig. (2-tailed)
BP	3.94	0.5	95	0.27	0.008
CCK	25.99	4.5	95		

It was found that Preservice teachers' Behavioral Component Toward Probability as their attitude and their knowledge of the CM of probability have a relationship (p <0.05). However, this association was determined as small with r= 0.22. BP also showed a significant relationship with PSMTs' CCK. As seen in Figure 5.22, findings revealed that PSMTs' CCK had a significant relationship with their actions toward probability referring BP. However, the degree of the relationship was found to be small (Spearman's rho r= 0.27, p <0.05).

5.4. Findings from Interviews

5.4.1. Probability Concept Inventory (PCI)

PSMTs' scores on PCI were examined in order to determine what kind of misconceptions they have and what kind of difficulties they are struggling with. Then, by interviewing the participants, these results were tried to be supported. The codes were determined based on the misconceptions of probability stated in the literature. Therefore, representativeness, availability of instances, adjustment from an anchor, gambler's fallacy, law of small numbers, equiprobability bias, falk paradox and outcome approach were used as codes. Firstly, parent codes were constructed and then gathered under a code that best suits them. For example, not making sense of randomness and problems in independence were categorized as Gambler's Fallacy. Accordingly, three main misconceptions were highlighted by the participants during the interviews listed in the following part.

- Equiprobability bias
- Gambler's Fallacy
- Outcome Approach
- Representativeness heuristic: Law of small numbers

5.4.1.1. Equiprobability Bias. The equiprobability bias represents participants' misconception of attaining an equal chance of events where they are not equally likely at all. This kind of misconception was addressed in question eight by Gómez-Torres *et al.*(2016). However, this study encountered it in questions such as five and eleven in addition to question eight. The code was used 29 times during the texts.

According to the results of the PCI, Q8 was the item with the lowest score among the questions of ACK. Only 10.5% of the given answers were correct. During interviews, 10 out of 13 interviewees explained their solutions in detail. Although 5 of them were from the group of low PCI and the other five were from the group of high PCI, there was no correct answer among them. PSMT23 was one of the participants who gained a low score on PCI, where he had 37 points out of 72 and did not give a correct answer to the question. In his explanation given in Figure 5.23, he said, "3 dice... All three are independent events anyway. I thought that if one of them came with a 5, it was 1/6, and the first one got a 5, out of 6, I did the same for the other three".

Item 8. (Batanero, Garfield, & Serrano, 1996)

When three dice are rolled simultaneously:

a. Which of the following results is most likely?

- a 5, a 2, and a 3_____
- two 5s and a 3____
- three 5s____
- The chances of obtaining each of these results are the same ____

b. Is one of these results less likely than the others? Which one?

Figure 5.22. Question 8 on PCI.

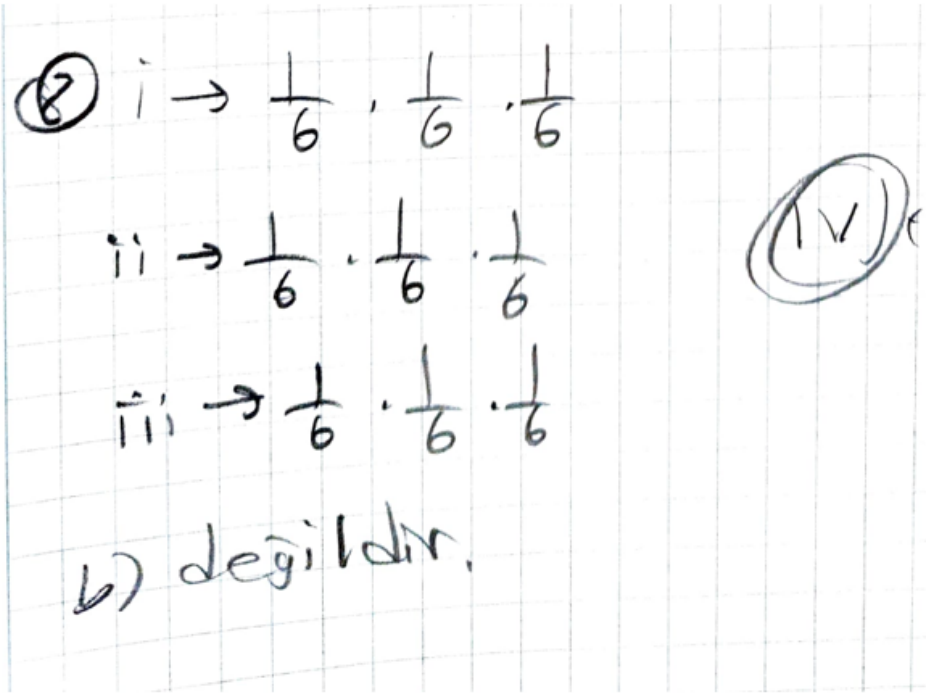


Figure 5.23. Solution of PSMT23.

Item 8. (Batanero, Garfield, & Serrano, 1996)

When three dice are rolled simultaneously:

a. Which of the following results is most likely?

- First one is 5, second one is 2, third one is 3.
- First two is 5, third one is 3.
- First, second and third one is 5.
- The chances of obtaining each of these results are the same ____

b. Is one of these results less likely than the others? Which one?

Figure 5.24. Revised version of question eight.

The same question was asked to participants by adding a second version as seen in Figure 5.25 in which the order was highlighted contrary to the original Q8.

The reason for asking for a second version was to avoid misunderstanding the question because of its translation. However, in both versions of the question, participants answered that in all cases, the probability of getting the event was the same, and mostly, the solutions were like in Figure 5.24. The PSMT23 answered the second version as follows:

"Okay, even if you specify the order. I think it is 1/216, so I would multiply 1/6, 1/6, 1/6 because these are independent events".

5.4.1.2. Outcome Approach. In question 5 which asks about the possible results of the experiment for four students, the outcome approach was observed. The results of the PCI demonstrated that only 20% of the answers obtained a complete score in question five. 12 of the interviewees were asked to elaborate on their way of thinking. Mainly, it is observed that rather than considering the physical characteristics of pins and possible outcomes, participants considered the sample space with two outcomes: landing up or not. Their thought was based on binary thinking (happening and not happening). That is why the outcome approach was addressed based on their answers.

Item 5. (Adapted from Green, 1982, 1983)

A packet of 100 drawing pins is emptied out onto a table by a teacher. Some drawing pins landed "up" and some landed "down" . The results were as follows: 68 landed up and 32 landed down. The teacher then asked four students to repeat the experiment. Each student emptied a packet of 100 drawing pins and got some landing up and some landing down. In the following table, write possible results for each student:

	Deniz	Mehmet	Duygu	Mustafa
up:				
down:				

Figure 5.25. Question 5 on PCI.

PSMT134, who scored high on PCI (53 points) but did not answer the Q5 correctly, stated as follows:

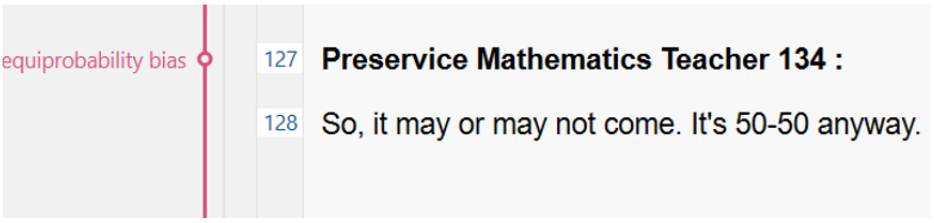


Figure 5.26. Explanation of PSMT134.

In his reasoning, the outcomes were considered without considering sample space. Thus, even if there are three different possibilities, the PSMT134 and others would give 50% as an answer since they focus on happening and not happening. However, PSMT20, who understood the FM and was aware of the sample, answered the question without misconception. PSMT20 was from the same high PCI group, and he explained the answer as follows:

> "I actually thought of the pin here, too. The pin is more likely to point downwards, that is because of physics. That is why I chose the ones close to 68-32, depending on everything. I wrote 70-71 instead of 68. Actually, I kept playing up and down".

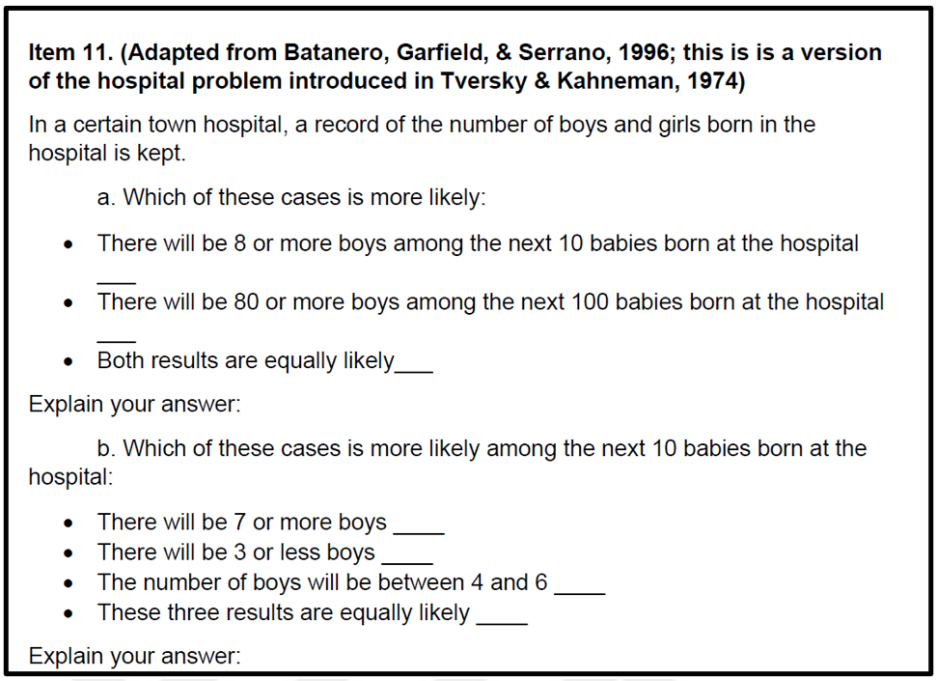
In the above explanation, PSMT20 reflects accurate reasoning by considering the previous experiment and physical characteristics for the tendency of landing up.

In question 11, there was a different scenario, but the reasoning behind the solutions was the same as the question five. More than half of the answers to Q11 were wrong. 6 participants were questioned to explain their answers. PSMT129 was in the group of low PCI. In his reasoning, there was no consideration of changing sample space and likelihood. Although his answer was short, since he focused on happening and not happening, and other conditions were ignored, the answer was considered for outcome approach as highlighted in question five. The explanation was as follows:

> "Since the probability of being both a boy and a girl is equal, I said 1/2 because of the logic that the child born will be either a boy or a girl".

The same reasoning was observed in the participants with a high PCI score. PSMT50 was one of them with a score of 52.5 points. He showed outcome approach with a statement:

> "I considered being and not being. So it may or may not happen; 3 or more males may or may not be born. Three or more male babies may or may not be born, and the number of male babies may or may not be between four and six. It is 50% - 50%; it may or may not happen".



Item 11. (Adapted from Batanero, Garfield, & Serrano, 1996; this is is a version of the hospital problem introduced in Tversky & Kahneman, 1974)

In a certain town hospital, a record of the number of boys and girls born in the hospital is kept.

a. Which of these cases is more likely:

- There will be 8 or more boys among the next 10 babies born at the hospital ___
- There will be 80 or more boys among the next 100 babies born at the hospital ___
- Both results are equally likely___

Explain your answer:

b. Which of these cases is more likely among the next 10 babies born at the hospital:

- There will be 7 or more boys ____
- There will be 3 or less boys ____
- The number of boys will be between 4 and 6 ____
- These three results are equally likely ____

Explain your answer:

Figure 5.27. Question 11 on PCI.

Thus, he meant that all the options are equally likely.

Consequently, equiprobability bias was examined in different participants and different score groups. The common logic of the participants with equiprobability bias was ignoring the sample space and possible outcomes and attaining equal chances for each event.

5.4.1.3. Gambler's Fallacy. Gambler's Fallacy is a false reasoning originating from the feeling of equalizing the deviations from the theoretical value. In questions 5 and 10, this misconception was observed based on both participants' explanations of the PCI and their answers in the interviews. According to the results of the PCI, Question 5 had the least number of correct answers in CCK, while Question 10 was among the questions with low scores in ACK. The gambler's fallacy code was repeated nine times in the transcripts. For example, PSMT144, who was in the group of high PCI with a score of 63 points, explained why he wrote the answer as it is in Figure 5.27 even though the given result of the experiment was 68 up and 32 down. Therefore, like the explanation of the PSMT144, most of the participants thought that in order to

eliminate the excess of landing up in the previous experiment, coming experiments could come vice versa. For this reason, the number of pins landing down was more than the number of pins landing down. In this situation, the probability was predicted as 50% and the number of pins landing up was tried to be equalized with the number of pins landing down.

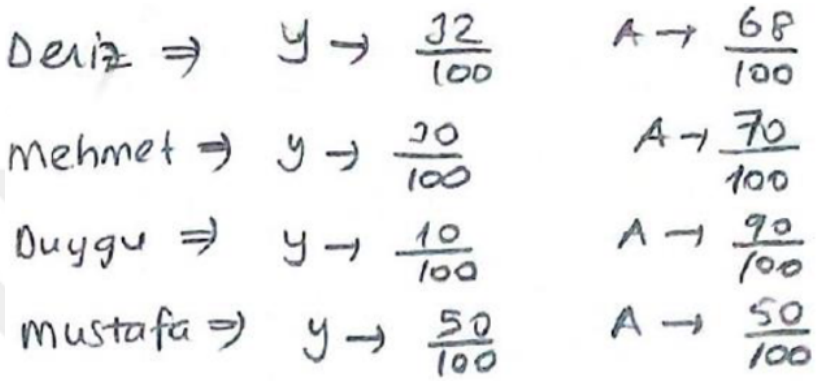


Figure 5.28. Solution of PSMT144 for question 5.

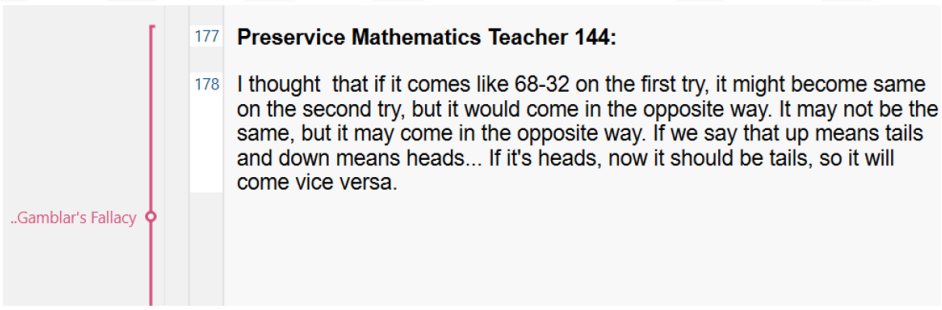
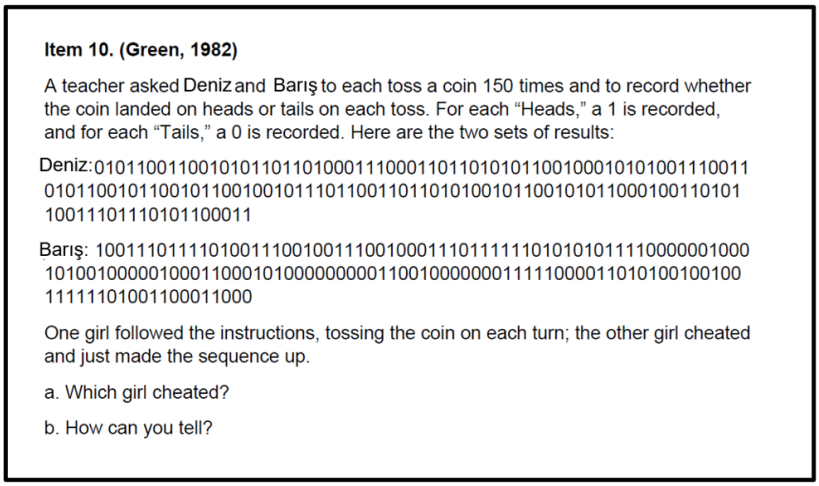


Figure 5.29. Explanation of PSMT144 for question 5.

The PSMT26, who was in the group of low PCI since he gained 37.5 points, considered the Q10 the same way as the PSMT144 in the previous example. Although the context is different in this question, when participants were asked to think about getting a tail or head contrary to pins, their reasoning was similar.

> "The zeros are very consecutive. But you know, it is more than it could possibly be. I look at the bottom line, and there are like 19 zeros there as far as I can see. And then the ones come. Okay, then the zeros come, then again, but the zeros are rampant. There are similar situations on the top line, so I say Barış is cheating".



Item 10. (Green, 1982)

A teacher asked Deniz and Barış to each toss a coin 150 times and to record whether the coin landed on heads or tails on each toss. For each "Heads," a 1 is recorded, and for each "Tails," a 0 is recorded. Here are the two sets of results:

Deniz: 010110011001010110110100011100011011010101100100010101001110011
010110010110010110010010111011001101101010010110010101100010011010 1
10011101110101100011

Barış: 100111011110100111001001110010001110111111010101011110000001000
101001000001000110001010000000001100100000001111100001101010010010 0
11111101001100011000

One girl followed the instructions, tossing the coin on each turn; the other girl cheated and just made the sequence up.

a. Which girl cheated?

b. How can you tell?

Figure 5.30. Question 10 on PCI.

This explanation indicates that the participant was not aware that each trial is independent from others and consecutive heads or tails might come. The general explanation of the participants, who fell into a trap of gambler's fallacy, to question eleven was the same as the PSMT26 on the PCI. The theoretical value interferes with the conception of independence and this results in a misconception. However, some participants, like PSMT14, demonstrated how accurate reasoning can be on the contrary of the 74% of the participants.

> "Looks like Deniz has been sly. For example, I see that he hasn't written zero more than 3 times or something like that. There are no zeros occurring more than 3 times in a row, meaning no tails. But in Barış's answer, for example, there are those zeros, zeros, ones and ones. It's like he did the experiment himself, as if he didn't guess it. But Deniz's is more like, well, let me write it down, how will the teacher understand anyway,etc. That's why I thought that the Deniz is fraudulent, Barış's is not".

5.4.1.4. Representative Heuristics: Law of Small Numbers. The law of small number misconception is an expectation to see the theoretical value in a small number of samples. This misconception was seen in the answers to question 11 in Figure 5.26 shown in the previous part. The sub-question a (Q11-A)was the main source of the problems in Q11. 10 out of 13 preservice teachers answered with the misconception, while 63% of the participants in the PCI gave incorrect answers to the Q11A. The code was repeated five times in the transcripts

PSMT19 was in the group of low PCI since his total score was 29 points. When he was asked to elaborate why he thought that having eight or more boys for the next 10 babies and having 80 or more boys for the next 100 babies are the same, the explanation of the answer was like that:

> "I probably said so because of its ratio. Here, it is 8/10 and 80% too. You know, the rates are equal to each other".

Another interviewee, PSMT20, who was in the groups of high PCI with 52 points, reasoned in the same way with different words.

> "Let us say eight or more of the last ten babies born will be boys or 80 or more of the last hundred babies born will be boys. Now, when I look at it as a percentage, it appears to be the same percentage. You know, proportionally, they are both the same".

Also, PSMT134, who was not in the interviewee group, wrote his explanation on PCI as in Figure 5.30.

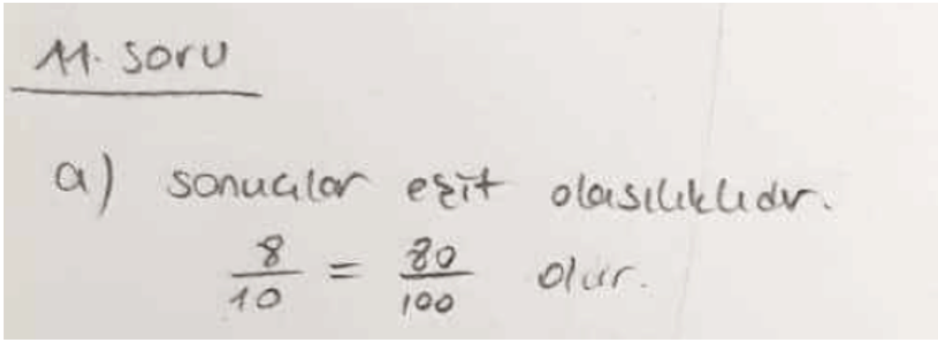


Figure 5.31. Solution of PSMT134 for question 11.

In conclusion, the answer to option a of question eleven demonstrates that most preservice teachers need to be more capable of considering the sample space of given events. They tend to evaluate the likelihood of an event by focusing on the CM of probability, which considers calculation of proportion, not consideration of frequency.

5.4.1.5. Other Problems in the Answers. There were some problems seen in the transcripts, in addition to the main misconceptions given above. They might not be put under the misconceptions heading, but they also cause mistakes while answering questions. These problems were not repeated as much as others in the transcripts but encountered on the solutions in a reasonable number. For instance, in question 6, considering only odd-even, even-even, and odd-odd outcomes rather than being aware that the order of odd-even pairs can change was a general mistake seen commonly. Mostly, PSMTs ignored changing order, resulting in the wrong answer for Q6-b.

Although Question 2 requires CCK, during the interviews, the interviewees had trouble forming equations to solve the problem. This is also found in the answers to PCI and explanations of the PSMTs. They could not comprehend how to write the equation, whether they needed to add or subtract the unknown value from black or white balls.

Ignoring order or confusion about when to use permutation and combination was the problem in question eight. In some cases, PSMTs could not decide when to use the permutation, or they used it in the wrong place. Some answers were also included using combination rather than permutation. These findings showed that PSMTs experience problems with the meaning of permutation and combination and when to use them.

The other problem was understanding conditional probability and using formulas accordingly. This problem was found in the explanations of Q7 and in the answers on the PCI. PSMTs had difficulty understanding how to use the given condition or how to decide the sample space. Lastly, PSMTs had difficulties while dealing with sampling. In Q9, they could not understand how to use the sample to guess the population. Therefore, their understanding of the FM of probability was weak. In their explanation, they lost the question during the interviews and could not explain how to use a given number of fish. The solutions on the PCI also support it. A reasonable number of answers use additive rather than proportional reasoning, resulting in failures.

5.4.1.6. The Reasons for Difficulties While Solving PCI. The interviewees were asked which questions were the most difficult ones for them and their justification for the selection. According to their answer, Q9, Q12, Q11, and Q6 were considered the most difficult questions, which were repeated at least three times in the transcripts.

PSMT144, PSMT23, PSMT144, and PSMT19 chose the 9th question as one of the most challenging. Their reason for selection focused on the multiple steps of question nine.

PSMT19 explained why he chose this question as the most difficult one, as shown below.

> I remember that this was a question that involved multiple steps, and I had a hard time answering it. Here, he fishes and marks them. He leaves it among the fish again. The next day, he takes out 250 and marks 25... It was confusing for me.

PSMT144 highlighted the ambiguity of the question with the following words:

> Yes, it was very vague, so it was a little difficult for me to visualize it. So, what path will I follow? We know the formula, but I would have trouble figuring out how to apply it. I mean, I experienced it here.

The same explanations were heard from the other three interviewees who voted question 9 as difficult. They all chose it because the steps of the question were difficult to follow. The preservice teachers also noted that applying formulas was difficult here. They also underlined how to use samples to predict the fish in the population. In "other problems" before this part, the sampling issue is noted as the problem in the solution process of Q9. Actually, these two findings support each other's accuracy.

The question 12 was repeated six times as the most difficult question in the transcripts. The reasons for that was uncertainty, and not applicability of the formula. PSMT19 explained why he chose Q12 as a difficult question by highlighting not applicability of the formula. His quote from the interview is shown below.

> This is a question that requires some further interpretation. It is a question that needs to be interpreted by making connections with daily life. That's why I can say that it is more difficult than the questions above, in terms of interpretation. Because, for example, we can reach results in other questions by using the formula above. So even if I don't remember, here is the formula, or even if I don't remember how to solve it, I can do it when we know the formula. But this question requires further interpretation by connecting it to daily life.

PSMT10' justification focused on its uncertainty as given below.

> I think he gave both a probability and a certain situation here, that's how I remember it. So, when you say a possibly definite situation at the same time, I think something was wrong. There are two possibilities: The guess might be correct and the guess might be wrong. In addition, he mentioned the possibility of rain in option c. He said it didn't rain on Monday or Tuesday, which may be confusing because he gives both a certainty and a probability at the same time.

These explanations indicates that preservice teachers hesitate when they cannot apply a formula which is a mathematical side of the probability. They experience difficulty if the question is ambiguous, which means cannot be solved with a certain procedure or formula and needs additional interpretation. The 11th question was chosen as difficult four times by the interviewees. The reasons for choosing this question as difficult were the same as it is for question nine. The uncertainty was the source of the challenge. PSMT50 explained the reasons of choosing question eleven as difficult with the following statement.

> Because I cannot do a mathematical operation on option b here rather than option A, I just think: Could it be possible or not? I cannot put this into a numerical form right now. I cannot express it numerically. There is no such thing as certainty in this. It is a possibility, I do not know, but it is not impossible. So, the last three above may not actually be equal. Just uncertainty.

The other explanations for choosing question eleven as were compatible with the justification of PSMT50. They all highlighted that finding a numerical value is not possible here and needs extra interpretation. Therefore, it can be revealed that when it is about reasoning, rather than calculation (which they can't do yet), they feel confused.

Question 6 was also chosen as a difficult question and repeated three times in the transcripts. The PSMTs especially pointed to Q6-b as the most difficult one. Their reason for not including question 6-a was that while option a can be solved with just knowledge of the multiplication of even and odd numbers, option b requires a calculation to set the fairness. The findings show that PSMTs had difficulty deciding the sample space in option b. Therefore, they experienced difficulty. The results of the PCI evidence these findings since option b of question 6 was among the questions with the lowest correct answers.

5.4.2. Attitude toward Probability and Teaching Probability (APT)

The attitudes of the preservice teachers were examined according to their APT scores. Then, possible reasons for their positive or negative attitudes toward probability and its teaching were tried to be evidenced during the interviews. Accordingly, four headings under two main concepts were formed at the end of the coding process, which are indicated below. In total, 14 codes were formed; half were positive, and the other half were under the negative attitudes part.

- Positive Attitudes
- Positive attitudes toward probability
- Positive attitudes toward teaching probability
- Negative Attitudes
- Negative attitudes toward probability
- Negative attitudes toward teaching probability

During the interviews, two of the interviewees noted that their attitudes changed positively from the time that the scale implemented to the interviews. Although they explained their positive change in the attitude as their experiences in their intern schools and their effort to get rid of the biases, they were not included in this study in detail since it is not the scope of this study.

5.4.2.1. Positive Attitudes. On APT, different items were included to examine preservice teachers' attitudes toward probability. The total scores indicate that PSMTs have positive attitudes toward probability since the average total score was 98.46 points, much higher than the neutral value of 81. The interviewees were selected from the groups with high and low APT scores. In total, 13 PSMTs participated, and the reasons for their positive and negative attitudes were elaborated based on their explanation of why they scored the items (e.g., "Probability is easy") low or high.

5.4.2.2. Positive Attitudes Toward Probability. Positive attitudes toward probability was repeated 46 times during the transcripts. According to PSMTs' justification of their positive attitudes toward probability, four reasons were determined.

PSMTs have a positive attitude toward probability because

- Probability is life itself
- Probability is a part of mathematics
- Probability includes complexity
- PSMTs have positive past experiences.

5.4.2.3. Probability is Life Itself . The "Probability is life itself" code was formed by merging different codes like using probability in daily life or connection to daily life. The name of this main code was taken from the description of one of the participants (PSMT6) in the interviews. The code was repeated twelve times on the transcripts. This code was constructed by PSMTs' answers to why they value probability, why they use probability other than chance games, or why they think that probability is easy. PSMT19, who was in the high APT group with a score of 119 points, stated that.

> "For example, we use it even when talking about a job or an assignment; can we do this, can't we do this, I guess we can. For example, even when we say, I guess we can do it, we are actually evaluating it, talking about the possibility of it happening, or using it for very basic weather forecasts, etc".

Another interviewee, PSMT50, in the group with low APT scores of 85 points, explained his positive attitude with the words below.

> "In fact, we have to think about the possibility in everything. For example, even when we accomplish something, we think about its possibility. I wonder what the probability of my success is or not. We may not be able to do this systematically in the questions, but this is the reality of life. It can be seen in everything in daily life".

Other participants who justified their positive attitude toward probability with this code used different words, but they considered it in the same way. Their explanation indicates that a positive attitude toward probability results from its general usage in daily life.

Since we encounter many possibilities in life, PSMTs appreciate the use of probability. Their answers mostly rely on BP and VPT, among the components with high mean scores on APT, and even the PSMTs with low APT scores had high VPT scores according to quantitative results of the scale.

5.4.2.4. Probability is A Part of Mathematics. The code "probability is a part of mathematics" represents the view that probability has a mathematical side, making the preservice teachers think positively about it. The code was repeated four times on the transcripts. It was formed by subcodes such as liking probability because of liking mathematics or confidence with probabilistic knowledge because of being able to do mathematics. PSMT20, who was in the group with the high score of 104 points on APT, justified his positive attitude, as given below.

> "So if we consider the mathematical part of probability, I think I am generally talented in mathematics. So I am good at that".

SMT14, with a high APT score, indicates his positive attitude with the following statement.

> "We now say that probability is a part of mathematics. Yes, it does not provide certainty in some cases, but this subject is still included in mathematics".

In conclusion, the thought that probability is mathematics results in a positive attitude toward probability. However, this conclusion was constructed by the explanation of some preservice teachers in the group of high APT. The low score group did not imply the same consideration. Therefore, this might be a particular case for the PSMTs with high attitude scores on APT.

5.4.2.5. Probability Includes Complexity. The "probability includes complexity" code was repeated nine times in the texts. Although it might be considered one of the negative characteristics of probability, some preservice teachers highlighted it as a positive. The subcodes, such as uncertainty, different perspectives, and complexity, were gathered under the main code complexity.

The explanation of the PSMT10, who had a high APT score of 103 points, demonstrates that being able to think of different perspectives makes probability exciting for him.

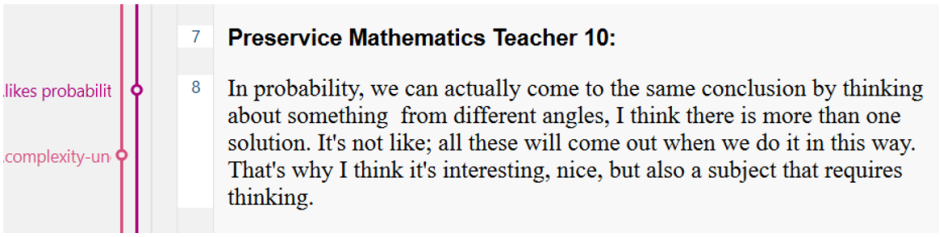


Figure 5.32. Explanation of PSMT10 for complexity of probability.

PSMT20, with 104 points on APT, explained his positive attitude toward probability by focusing on its complex nature. He stated that he likes probability since sometimes it can not be solved with mathematical procedures, and sometimes it directs our thoughts in the wrong way. His exact words are demonstrated below.

> "You know, the situation where it cannot be solved, sometimes it misleads us... In fact, what we can do with mathematical calculations does not work out with mathematical calculations in probability. I like to think of it that way".

PSMT19 in the high APT group with a score of 116 points explained why he likes probability, given below. It shows that dealing with abstract concepts interests him, and as probability has uncertainty, it produces a positive attitude toward it.

> "The subjects students have the most difficulty with are more abstract. In fact, the probability is the same, it is difficult to concretize those possibilities. That is why I like the probability".

Consequently, Complexity and uncertainty of probability attract the attention of some preservice teachers with high scores on APT. The side of the probability that can not be solved with mathematical calculations makes them enjoy, in contrast, the general perception of probability.

5.4.2.6. PSMTs have Positive Past Experiences. This code was created with the explanations of the preservice teachers who experienced probability in their past education years in a good way. Either their teachers' teaching methods of probability impacted them, or the courses they took before caused an increase in their understanding and their attitude. This code was repeated five times in the transcripts.

PSMT10 with a high score (103 points) on APT states that the courses he took before improved his understanding and he is confident about his probabilistic knowledge because of that. His justification for this confidence is as follows:

> There are still difficulties while solving problems in probability during the thinking phase. However, I took many probability courses from the mathematics department and our sced department. Of course, they have been beneficial, and we can say that I am a little clearer now, at least at the thinking stage.

PSMT20 in the high APT group explained his positive attitude with the same reason as PSMT10. He explained his positive attitude with the following words.

> I know when I do not know, or I know. Even if I pursue this, I feel better about the probability. That is why I feel like my thoughts are getting stronger with the classes I take.

In conclusion, it can be said that the preservice teacher, with a reason "past experiences" of a positive attitude toward probability, improve their thinking throughout the courses. As a result of that, they develop a positive attitude since they feel stronger on the topic.

5.4.2.7. Positive Attitudes Toward Teaching Probability. Under the positive attitudes toward teaching probability code, twenty-three segments were coded. The frequency was less than the frequency of positive attitudes toward probability. This is also evidenced in the results of APT. The mean scores of the APT's teaching components (CT, BT, VPT) were much lower than the values for probability components (AP, CCP, BP).

Finally, three parent codes were formed by bringing subcodes together. These codes were:

- Conceptual Understanding
- Enjoying while solving
- Being good at teaching

5.4.2.8. Conceptual Understanding. In this category, PSMTs explained why they feel comfortable teaching probability or designing tasks for lessons by emphasizing their high conceptual understanding of probability. Accordingly, the interviewees with this justification were from both high PCI and high APT groups. PSMT14 stated his positive attitude toward teaching probability with the following words:

> "Since I understand probability conceptually, it is now easy to teach it to someone else".

PSMT20 expressed his reasons for feeling comfortable while teaching probability with the following statement:

> "Here, the underlying factor is my knowledge. The most important thing in the teaching process is my knowledge".

The quotes from the explanations of the interviewees and the justifications throughout the interview indicated that having a conceptual understanding, which means being capable of dealing with probabilistic events, is necessary for a teacher to convey the right message to the students. The opposite of this code is given as a lack of probabilistic knowledge and preservice teachers' lack of understanding under the negative attitude part. Therefore, the findings are consistent even though the code was repeated in a limited number.

5.4.2.9. Enjoying while Solving. This code refers to having a positive attitude toward probability, thinking that dealing with probability is enjoyable. This code's frequency was the lowest, and it was repeated three times during the interviews. The interviewees were in high APT but from low and high PCI groups.

PSMT19, who was in the low PCI group, stated that

> "I think it will teach probability with fun by preparing content that is more entertaining, perhaps colorful... I know it will even challenge me, but I like to deal with probabilistic events".

PSMT14, with a high PCI score, justified his positive attitude toward teaching probability as follows:

> "Thinking about every situation while solving probability questions and trying not to forget such different situations seems fun. I think it is like a puzzle".

In conclusion, the findings revealed that PSMTs might have an interest in probability because of the nature of probabilistic events. In their opinion, when they enjoy

dealing with solving probability problems, they will be willing to teach the students, too. This results in thinking positively about teaching probability. The explanations show that the scores on PCI were not related to their enjoying solving probability because PSMTs from the low PCI group stated that they enjoy it.

5.4.2.10. Being good at Teaching. In this code group, PSMTs attributed their positive attitude toward teaching probability to their teaching abilities. They explained that the courses about teaching or their teaching experiences influenced their opinion about teaching probability. This code was repeated six times, the most repeated code in positive attitudes toward teaching probability.

PSMT6 explained why he does not have a concern while answering students' questions about probability in the following quote.

> "I think my level of awareness regarding these misconceptions or probabilistic concepts has increased. Therefore, when I see the questions, I can understand what the questions will be about or how I can explain it to the student; I have clearer ideas on this subject".

PSMT10 justified his positive attitude toward teaching probability with the following statement:

> "Since I give private lessons or at least know the market resources. I know what students need to solve at the beginner, more difficult, or intermediate levels and what kind of questions I can prepare. I think I have the resources to find beginner, intermediate, and advanced questions depending on the level of the students".

PSMT26, who was uncertain whether he is positive or negative toward teaching probability, said that his attitude shifted to the positive side with the given reasons below:

> "I did not know about CODAP when we did it. When we took the 470 course, we learned many tools. One more... I do not know, I mean, when I search the internet now when I see a site, I throw it in and look at its content. Oh, this is better, etc... Now, that indecision may have shifted to something more, to the other side. It may have shifted to the good side".

To conclude, having teaching experiences or knowledge about the teaching materials or being capable of understanding possible misconceptions and difficulties in the teaching probability tend to influence PSMTs' attitudes in a positive manner. The findings reveal that PSMTs' focus point in teaching probability is teaching rather than probability. Therefore, they feel confident about teaching probability based on their training throughout their university years.

5.4.2.11. Negative Attitudes. This part of the findings explains the reasons for negative attitudes toward probability and teaching probability in detail. Although the results of APT showed that PSMTs' attitudes are fairly positive, the interviews with 13 PSMTs revealed that they have mostly negative attitudes. While there were 46 coded segments for positive attitudes toward probability, the number for negative attitudes was 120. The attitudes toward teaching probability part also supported these findings. While the number of coded segments for positive attitudes toward teaching probability was 23, it was 64 for negative attitudes.

5.4.2.12. Negative Attitudes Toward Probability. According to the results of the APT, negative attitudes were determined when the difficulty of probability or confidence in probabilistic knowledge was asked. Parent codes were formed after the reasons for the negative attitudes were coded. Finally, four main reasons for negative attitudes toward probability were determined.

- Lack of probabilistic knowledge
- Not applicability in daily life
- Uncertainty
- Negative past experiences

5.4.2.13. Lack of Probabilistic Knowledge. During the interviews, preservice teachers talked about their difficulties while solving probability problems. Their lack of knowledge was the source of their difficulty, not liking probability or avoiding using it in

daily life. The PCI results can support their lack of knowledge because the mean value was less than half of the full score in the PCI results. However, during the interviews, preservice teachers in the high PCI group and preservice teachers in the low PCI group made their justifications based on their inadequate understanding of the probabilistic concepts. This code was repeated thirty times in the transcripts. 10 out of 13 pre-service teachers had comments on the issue.

PSMT50, in the high APT group, explained his reasoning for not liking probability with the following statement.

> I think I did not learn probability well at the time. Of course, I am working now, but I think I did not learn well in time. This does not motivate me either, I get scared when I see it.

PSMT129, in a group with low APT, stated that his negative attitude is caused by not comprehending the concepts. Since he could not understand the core of the subject, negative attitudes have grown inside. His own word is given below.

> We have a lack of understanding of fundamental concepts. After all, since we have not had it regularly since high school, we cannot improve it so much. This resulted in a negative feeling.

As PSMT50 and PSMT129 highlighted, many examples of lack of knowledge were collected throughout the interviews. The general idea behind these conversations is that PSMTs feel insecure when talking about probability.

The words of PSMT1 summarize the situation better: "If I could understand probability and if I could fully internalize it, it may not seem that difficult". The deficiencies in their conceptual understanding set barriers to their attitudes toward probability.

5.4.2.14. Not Applicability in Daily Life. Although the results of APT demonstrated that the value component of the scale was the component with the highest mean value,

during the interviews, some thoughts about the impracticability of probability in daily life appeared. Even though it was not as strong as the "lack of probabilistic knowledge" code, it was repeated five times through PSMTs' explanations.

PSMT23 in the low APT group explained why he does not care about probabilistic knowledge when encountering:

> "I might take a glance at whether there is anything new, but other than that, I don't think probability will add anything to my life".

PSMT14 in the high APT group justified his thoughts with the following statement:

> "Nowadays, technology has developed a lot. I do not look at the weather anymore. I do not look at the humidity. I look at the weather forecast like I do not use it much anymore. Yes, weather forecasters use this, okay. However, I feel like I do not use it first-hand at work because I use someone else's data. Also, life has become more planned for me after university. Then everything goes very smoothly. That is why there is not much probability involved. You know, you definitely have that class tomorrow anyway. After that, I will go to this private lesson. Everything is so obvious, so it seems like I do not use it much".

These explanations indicate whether PSMTs value probability and feel it is worth it, they might think that probability is not useful. These preservice teachers have the idea that if we do not do some calculations to find the probability of an event happening, it means that we do not use it. However, this type of consideration ignores daily life situations like thinking about winning or losing in a game or deciding on clothes based on the likelihood of rain.

5.4.2.15. Uncertainty. The uncertainty was the most repeated code after the lack of probabilistic knowledge in the negative attitudes toward probability. It appeared 18 times in the transcripts. The explanations of 9 participants formed this parent code. This code was also mentioned in the findings of PCI, where preservice teachers chose some questions as difficult because, in their opinion, these questions include uncertainty.

PSMT27, who was in the low APT group, stated probability as difficult and expressed his negative attitude in his words as follows:

> "I think probability is a very complex lesson. When we look at it as a whole, all kinds of results can arise because of uncertainty. It is a difficult lesson because it does not always give a clear result. So sometimes, for example, when I look at a question, I say, this is a simple question; I can solve it, but when I get into it a little bit, the question styles actually become difficult".

PSMT6 in the high APT group explained why he thought that probability is difficult with the following expression:

> "I cannot be sure. So, I cannot be sure that the problem I solved is correct. There was always such uncertainty. That is why the questions are like that... For example, I think I need to solve them with a clear head. Sometimes I can even solve a question differently the next time. I feel like I could even think of a different answer. That is why I think the questions are difficult".

All explanations under the uncertainty code focus on the nature of the probability. As PSMT6 and PSMT27 expressed, some probability questions require various possibilities, and these questions direct the PSMTs to have negative attitudes. This code might be linked to the lack of knowledge code because when uncertainty in the questions creates chaos in their minds, they feel a deficiency in their probabilistic knowledge. This especially happens when the FM of the probability is used. The explanations on the uncertainty code reveal that feeling insecure and negative happens when the formula does not fit in the question. This means that another perspective rather than CM needs to be considered, and at this point, PSMTs do not have adequate knowledge according to PCI results.

5.4.2.16. Negative Past Experiences. The Negative past experiences code refers to PSMTs' negative attitudes because of their history of courses taken before or in teaching. This code was repeated nine times in the transcripts. Four interviewees in the low APT group justified their negative attitudes toward liking, enjoying, or giving attention to probability with their academic background. In some cases, teachers in others their

teaching experiences were the source of negativity. PSMT23 in the low APT group expressed his negative attitude toward probability with the following statement:

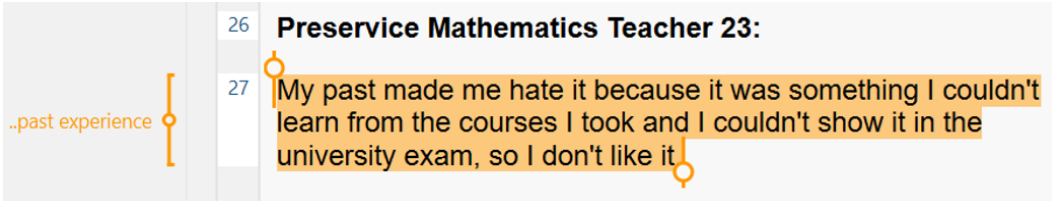


Figure 5.33. Explanation of PSMT23 for his reasons of not liking probability.

He stated that the courses were not designed to teach probability conceptually in previous years of education. Because of that reason, he does not like the probability or enjoy the lessons. On the contrary, in the next conversation, he stated, "On the other hand, there was an Atakan teacher's lecture at the university, and I loved it. If others were like that, I would like the probability". This revealed that teachers greatly influence students' future perceptions of probability.

In the explanations of PSMT144, the failure in the national exams seems to have greatly impacted his negative attitude. He explained his negative attitude with the following statement:

> "It was a subject that I often skipped during exams in my education years. I took the TEOG exam (national entrance exam for high school). I made a mistake in mathematics that I could not solve, and that was the probability question. In university, I felt like I understood probability a little bit in terms of logic, but of course, I still cannot say that I have warmed up to possibility. I think this stems from my student experiences".

In conclusion, PSMTs' explanations indicate that our experiences throughout the learning process greatly influence what we think about probability. If positive things happen, it results in positive outcomes, but if our experiences include negativity like malfunctioning teaching methods or failures, then negative attitudes gradually settle.

5.4.2.17. Negative Attitudes Toward Teaching Probability. The results of APT demonstrated that PSMTs' attitude scores on teaching probability are much lower than their

attitude scores toward probability. Especially AT and CT were among the components with the lowest mean scores. The findings from the interviews showed parallel results. It revealed that negative attitudes toward teaching probability are much higher than negative attitudes toward teaching probability. The negative attitude was seen in sixty-four coded segments. The reasons for the negative attitudes were seen in three main categories:

- Lack of Understanding
- Uncertainty
- Problems in teaching

5.4.2.18. Lack of Understanding. This code refers to negative attitudes toward probability because PSMTs did not feel having a sufficient understanding of probabilistic events. This parent code was repeated eighteen times, which was the most repeated code among the other two. The interviewees whose justifications were coded as lack of understanding were from different categories. Participants were in both low and high PCI and APT groups.

PSMT50, who was in the high PCI low APT group, stated that as he does not feel confident in his current probability knowledge, he can not help the students in the teaching process. His own words are as follows:

PSMT27 in the low PCI and low APT group explained his negative attitude with the following statement:

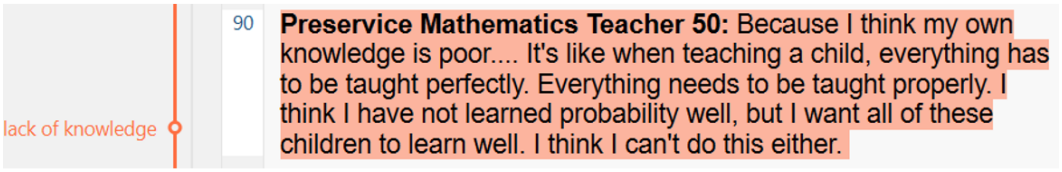


Figure 5.34. Explanation of PSMT50 for his reasons of being unconfident in teaching.

> "I always wonder if I will be able to solve it. Maybe I think with a genuine lack of self-confidence. I have never been able to adapt and fully learn about probability. Maybe because I am not good at predicting and solving probabilistic situations".

PSMT134, in the high PCI but low APT group, expressed his negative attitude with the same justification as the interviewees mentioned above. He stated that.

> "I think it is mostly because it is hard for me to understand. Maybe there was prejudice somewhere or a lack of knowledge somewhere. When I have many deficiencies, I cannot explain or convey them, primarily because of myself. When I look at it from the student's perspective, Maybe when I understand it, I will try to find a method or something. You know, maybe at some point I can handle that part, but the first problem with me is my knowledge".

In conclusion, PSMTs' causes of negative attitudes toward teaching probability in the code of lack of understanding reveal that having difficulty solving probability problems or not comprehending probabilistic events results in thinking negatively about teaching probability. They do not think they can teach probability comprehensively to the students without sufficient knowledge.

5.4.2.19. Uncertainty. The reasons for the negative attitude toward teaching probability focus on the nature of probability. Therefore, this code represents interference of uncertainty in the teaching process. Also, uncertainty was marked as one of the reasons for negative attitudes toward probability. This code was repeated seven times in the transcripts.

PSMT129 in the low PCI and low APT group stated that uncertainty can create problems for students' learning process as it is the source of an abstract nature. Because of that reason, he has some concerns about how to teach it.

> "If they come with a basic deficiency, for example, if I study at the high school level, I think there is a possibility in the ninth grade. If the student were not able to understand the fundamental concepts in eighth grade, it would be a little harder for me to explain it. Because it seems to be a more abstract topic than their way of thinking. That is why I have some concerns".

PSMT20 in high PCI and high APT groups justified why he does not want to use probability in teaching other topics with the following words:

> "I can give those examples, but I do not want to go into depth and confuse the students. This is a point that I find a bit confusing because I feel like they could be wrong. So, with another field... Of course, I cannot relate to it right now; in which subjects can I use probability? I do not associate it at the moment, but I feel like if I tried to use it with something, I would probably create confusion because ambiguity is misleading".

The explanations of the interviewees related to this code indicate that the uncertain nature of probability might cause problems in teaching. In general, PSMTs' concerns were related to creating confusion in students' minds. In their opinion, probability can mislead the students while teaching since it is an abstract topic and consists of ambiguity.

5.4.2.20. Problems in Teaching. This code was the opposite of "being good at teaching" in positive attitudes toward teaching probability. Therefore, it represents preservice teachers' attribution of their negative attitude to teaching abilities. The code was repeated eleven times in the transcripts.

PSMT50 justified his opinion for his negative attitude toward teaching probability with having concerns about what if I cannot teach it properly or they understand me in a wrong way.

> Now I'm thinking about the children; Students might not like probability or I might teach the probability in a wrong way. I mean, the responsibilities of this are very heavy, so I would like to skip it.

PSMT23 in the low PCI and low APT groups expressed his concerns about teaching probability with the following statement:

> Maybe the thought of teaching probability the way I was taught through my experiences scares me a little. How can I explain? I do not know how to explain it because I do not know how I should do it, so I would like to skip it.

PSMT6 in the high PCI and low APT groups explained his negative attitude with a lack of sufficient knowledge of teaching materials related to probability.

> I do not think I have seen much activity regarding probability. So I have insufficient background on this. I have never encountered anything like this at school when I was a student, nor have I designed such a thing while I was in university.

To conclude, PSMTs' explanations demonstrate that not having sufficient knowledge and experience in teaching methods of probability is a source of negativity toward its teaching. PSMTs have concerns about how to design lessons or tasks while trying to teach probability to students. As they do not know the proper ways of teaching it, they tend to ignore or escape from teaching probability.

5.4.3. PSMTs' Perception of Relationship between Probabilistic Knowledge and Attitudes

The relationship of knowledge of probability and attitudes toward probability and its teaching were examined through asking a question about whether PSMTs think that their knowledge and attitude are related or not. All the interviewees were answered the same question. Then, their answers were coded as positively related, negatively related, somehow related and not related. At the end, positively related, somehow related and not related were obtained as ultimate codes.

The mostly repeated code was the positive relation of knowledge and attitude. 8 participants stated that their higher level of knowledge of probability positively affects their attitudes.

PSMT14 who were in the group of high PCI and high APT, explained why he thinks in this way with the following words:

> "I think they are definitely very related to each other, a little bit like this; When you are at school, you also like the lessons of the teacher you like. For me, it's like if you don't like your mathematics teachers, you don't like mathematics either. That's why liking probability doesn't make it difficult for me in my mind.

> You know, if they asked me about a question of probability, I wouldn't be like, "Ugh." I can do it somehow, I'm like, I love it anyway, that's what I think. They definitely have a relationship".

PSMT27 who were in the low PCI and low APT group, justified how his knowledge associated with his attitude:

> "I definitely think there is because, yes, since my knowledge of probability is low, I lack self-confidence. I also have a hard time because I'm negative to probability. Because of my lack of knowledge, I don't want to work, nor do I want to care. That's why this also affects me; it creates such a paradox; the two are completely within each other. I mean, I think all fields of mathematics are like this, and extra probability is also included in this".

According to his explanation, he does not only think that just knowledge affects attitude. He stated that there is a reciprocal relationship where both of them have an influence on each other.

The second most repeated codes were somehow related and not related. Each one was repeated two times during the interviews. In somehow related code, participants stated that even though there is a relationship between knowledge and attitude, there are other factors that influence our attitude or knowledge. Therefore, they said stating a direct relationship between these two constructs might not be possible. One of the advocators of this claim was PSMT144 who were in high PCI low APT group. He justified his thought with the following words:

> "I develop a good attitude towards possibility if I can. In other words, the better my level of knowledge is, the better attitude I will develop, but I think our attitude towards probability is not affected by knowledge alone; Our experiences affect this to a great extent. I think these attitudes depend on factors other than just the curriculum knowledge taught at school".

The not related code was represented the idea that negative or positive attitude stays whatever knowledge we have. For instance, PSMT23 who were in low PCI and low APT group explained that he will keep his negative attitude even though he gains more knowledge.

> "If what I did turns out to be wrong, I will think that I really don't know, or if it turns out to be right, I will say that it means I understand something. You know, I'll be happy at least a little bit. But I guess I feel like my attitude will still be the same. It is not exactly as hate but it will remain as dislike".

Also, the not related code were seen for positive attitudes where one of the participants stated that he is aware of his lack of knowledge but he has a great interest for probability.

Although most of the PSMTs who participated in the interviews expressed that their knowledge and attitude greatly influence each other and are highly related, some interviewees talked about the stability of their attitudes no matter how they improve their state of knowledge. Also, some stated that other factors, rather than knowledge, are highly related to their attitudes.

6. DISCUSSION AND CONCLUSION

This study aimed to investigate PSMTs' probabilistic knowledge and their attitudes toward probability and teaching probability. In this scope, PSMTs' current state of knowledge, their misconceptions, and difficulties in solving probability questions were explored. Besides, the reasons for their positive and negative attitudes toward probability and teaching probability and the relationship between their knowledge and attitudes were elaborated in this study. Therefore, this research examined two constructs via three different instruments in order to achieve this aim: Probability Concept Inventory (PCI), Attitude toward Probability and Teaching Probability Scale (APT), and interviews.

6.1. RQ1: What is the Current State of Knowledge of Preservice Mathematics Teachers about Probability and its Teaching

To examine the probabilistic knowledge of PSMTs, their results on PCI were taken into account. Based on the general results of the PCI, it has been observed that PSMTs mostly have a moderate level of probabilistic knowledge since the scores were accumulated from 36.5 to 51.50, and the mean score was slightly higher than the theoretical mean (the midpoint of the highest achievable score). This was an unexpected result since the other studies found that the general probability content knowledge of the preservice teachers was insufficient (Gómez-Torres,2016; Kurt,2020). The reason for this might be the courses taken by PSMTs or their experience in solving probability problems, or their positive attitudes toward probability. As it has been seen in the results of APT, PSMTs have positive attitudes toward probability and its teaching. Their positive approach might positively affect their knowledge (Estrada *et al.*, 2018).

The components of PCI were examined to observe PSMTs' different knowledge types and to answer RQ1. The results indicated that the CCK of PSMTs is much

higher than the other two knowledge types. Therefore, it can be said that they have adequate knowledge of common probability content. The study of Gómez-Torres *et al.* (2016) indicated that preservice teachers were more successful at CCK than other types. The reason for this may be that while ACK requires a higher understanding of probabilistic concepts or SCK involves special knowledge to teach probability, CCK assesses basic probabilistic concepts. Contrary to CCK, the results demonstrated that PSMTs' ACK was weak and not enough for future mathematics teachers. Since ACK assesses the probabilistic knowledge taught in middle and high school, having problems while teaching the subject is inevitable, according to these results. This is a huge problem for the preservice teachers who will encounter students after a couple of years. As a final type of probability content knowledge, SCK was examined. The results were found to be parallel to the general results of PCI. It indicates that preservice teachers have moderate levels of SCK. The spread of the scores shows that while there are preservice teachers who have high SCK, there are some who have low-level SCK. Although the average score is higher than the theoretical mean, the scores' variability can indicate the need to improve their specialized knowledge (Gómez-Torres *et al.*, 2016). The findings of the interviews also showed that preservice teachers have problems justifying their reasoning, especially regarding the 10th and 11th questions.

Preservice teachers' knowledge of different probability meanings was examined to answer the RQ1. The results showed that PSMTs' knowledge of CM of probability was at the highest level compared to other meanings. Thus, it can be said that their ability to compute the probabilities by using the classic formula (number of favorable outcomes over the number of all possible outcomes) was high. The result was not surprising because, in schools, the CM of probability is highlighted mainly rather than the FM or SM (Batanero *et al.*, 2016). However, it should be noted that their knowledge of compound events was problematic regarding their answers to and explanations of Q8. As discussed in the misconceptions part, question eight was the source of a considerable number of misconceptions, according to the findings of interviews. The reason for this may be the type of knowledge it belongs to rather than its type of probabilistic meaning. Although the question eight is a representative of the CM of probability, among three

types of probabilistic knowledge it requires ACK which means requires higher level of probabilistic thinking. When ACK of PSMTs were examined it was found at a low level. Therefore, the difficulty of the question eight might hav resulted from requiring ACK.

PSMTs' knowledge of the FM of probability was examined based on PCI results. The results indicated that PSMTs have serious problems related to FM and are not ready to teach probabilistic concepts under FM. Generally, PSMTs tended to use the CM of probability rather than the FM As a result of that, giving solutions based on theoretical values misled them. Stohl (2005) asserts that even if FM of probability is required in the question, teachers are inclined to use CM in almost all questions. The reason for this can be the higher emphasis on CM at schools or the simplicity of CM. The findings of the interviews indicated that PSMTs chose the questions as difficult when they consisted of uncertainty and FM. That underlines their problems while applying FM of probability.

According to the results of PCI, it has been observed that PSMTs have a moderate level of knowledge of the subjective meaning of probability since the mean value was close to the theoretical mean. However, Q10, Q11, and Q12 caused misconceptions, and Q11 and Q12 were among the questions that were chosen as difficult. The findings of interviews in this study indicated that PSMTs had difficulty making interpretations based on the given information in the questions and were inclined to CM. Gómez-Torres *et al.* (2016) asserted that preservice teachers' ACK of SM was poor, and it caused difficulties and misconceptions accordingly. In her study, trying to apply CM was also a common approach among the participants.

6.2. RQ1-A: Is there Any Statistically Significant Difference Among PSMTs' Current State Of Probabilistic Knowledge Based on Different Characteristics Such as Their Grade, Statistics and Probability Courses They Have Taken, and GPA?

The scores of PSMTs were also examined according to their different descriptive features, such as grade level, courses taken, and GPA. The results indicated that being senior or junior and taking statistics courses before or not do not influence PSMTs' probabilistic knowledge. Also, there was no relationship between PSMTs' GPA and their probabilistic knowledge based on PCI results. However, it has been found that PSMTs who have not taken a probability course performed better on PCI compared to PSMTs who took some probability courses in the university. The reason might be mixing formulas or confusion about which approach to use when having more knowledge about the topic. In Fischbein and Schnarch's (1997) study, it was found that while some misconceptions decreased with age, some of them stayed stable or increased. They argued that students encounter difficulties during instruction, and they can only overcome them if they are actively involved in the learning process. Otherwise, it might lead to more confusion. In this study, PSMTs who took some probability courses might have been exposed to uncertainties and complexities that they could not handle. At the end, their existing knowledge might be disrupted. However, these results need to be examined in detail in further studies with an increased sample size.

6.3. RQ1-B: What are PSMTs' Common Misconceptions about Probability?

PSMTs' scores and solutions on PCI and their explanations in the interviews were examined to answer the RQ1-B. Consequently, it has been explored that PSMTs had difficulty with four main misconceptions: equiprobability bias, outcome approach, gambler's fallacy, and the law of small numbers, which were found as the common misconceptions in probability (Batanero *et al.*,2016; Gómez-Torres *et al.*, 2016; Kurt *et al.*, 2020; Stohl,2005). The questions Q8, Q5, Q10, and Q11 were the source of

these misconceptions. The equiprobability bias was detected in different questions. The majority of the participants could not solve Q8, which had the least number of correct answers among the questions, correctly. Although it is under the CM of probability, it requires ACK. In the study of Gómez-Torres *et al.* (2016), they found similar results. They asserted that equiprobability bias was common among the answers to the questions of ACK and FM in the solutions of PCI. In Q5 where the number of correct answers was limited, outcome approach was seen. Q5 was under the FM, and the explanations of the preservice teachers showed that they have a poor understanding of the FM. The other question was Q11, in which PSMTs ignored given information and evaluated each case based on whether the event will happen or not. The eleventh question was also one of the most difficult questions according to PSMTs. As a result of these examinations, it can be said that PSMTs have a limited understanding of how to deal with the given compound events and decide on the sample space.

The gambler's fallacy was the other misconception that was observed commonly. In Q5 and Q10, PSMTs tried to equalize the experiment based on theoretical value. Because of that reason, in question ten, most of the PSMTs showed a lack of understanding of the independence of the events, and they expected to see heads after consecutive tails. Their explanations revealed that PSMTs are not ready to teach the FM of probability. The law of small numbers also interfered with probabilistic knowledge of PSMTs in the Q11. The explanations and the solutions on PCI indicated that PSMTs tend to apply theoretical value in small numbers and use the CM of probability rather than FM. Stohl (2005) claimed that the law of small numbers emerges from the misinterpretation of the law of large numbers and the problems with FM resulting from the introduction of the concepts in teaching. She asserts that defining experimental probability as the value that will be close to the theoretical value after a large number of trials might mislead teachers, and they might think that after a precise number, they will be equal and will not diverge. Some participants in this study might be confused about the number of trials needed for the experimental approach while answering the Q11.

6.4. RQ2: What are Preservice Mathematics Teachers' Attitudes toward Probability and Its Teaching?

The results of APT were observed to examine the attitudes of PSMTs. It has been found that PSMTs have positive attitudes toward probability and its teaching since the mean value was greater than the neutral value. Some studies on attitudes toward statistics found that preservice teachers' attitudes toward statistics is negative (León-Mantero *et al.*, 2020).. However, the attitudes of the preservice teachers toward probability and its teaching were also stated as positive using the same scale in previous studies (Estrada *et al.*, 2018; Estrada *et al.*, 2020). Salifu and Dokurugu (2022) explored the attitudes of 184 preservice teachers in Ghana by implementing a multi-factorial scale of attitudes towards statistics. They stated that preservice teachers have positive attitudes toward probability and its teaching. Also, Demircioğlu and Bilgiç's study (2023) indicated positive attitudes of the PSMTs in Turkey. Although they did not examine the general attitudes of the participants toward probability and its teaching, they presented the results. Accordingly, the mean score was similar to this study and it can be interpreted as quite positive. Koparan (2022) conducted a study to explore the influence of a simulation and game-based course on PSMTs' attitudes toward probability by using APT. The results indicated that even though participants were almost neutral toward probability at the beginning of the academic term, both experimental and control grouped showed positive attitude toward probability at the end of the course. Therefore, the positive attitudes in this study were the reflection of the findings in the previous studies. These results are highly important because as Estrada *et al.* (2020) highlighted, the positive attitudes of the preservice teachers are significant since they might also influence students' attitudes toward the topic in the future.

The different components of APT were explored to make sense of PSMTs' attitudes toward probability and its teaching. The results indicated that VPT had the highest score among the components of attitude. This means that PSMTs value probability and its teaching and appreciate it. The similar results were found in different

studies (Vásquez Ortiz *et al.*, 2019). In Demircioğlu and Bilgiç's (2023) study, 212 PSMTs at primary mathematics teaching program were examined in order to compare their attitudes toward probability and its teaching with their attitudes toward uncertainty. They used APT ,as it is in the present study, to examine their attitudes toward probability and its teaching with different components of attitude. Their examination showed that VPT was the component with the highest mean score. Also, Ruz *et al.* (2021) found that PSMTs in Chile give importance to probability and its teaching since VPT had the highest score in their study. Accordingly, the results might be generalized as preservice teachers value probability and its teaching by considering the results of the previous studies discussed.

In the present study, BP and BT scores demonstrated that PSMTs have positive attitudes toward using probability and take action to teach it rather than ignore it. The results were compatible with the previous studies such as Vásquez Ortiz and her colleagues' study (2019). In their study, 124 preservice teachers in early childhood program were examined by applying the APT and they concluded that preservice teachers show a willingness to teach probability and they take actions to use it. In addition, Estrada *et al.* (2018, 2020) found similar results about willingness of teaching in their study with preservice elementary teachers. However, they stated that these students do not have a inclination to use probability in contrast to their BP. The difference of result of Estrada and her colleagues' study (2018, 2020) might result from the differences in Spanish culture or Spanish curriculum while it also might be related to the participants' departments. However, it is reasonable to attribute other differences since Vásquez Ortiz *et al.* (2019) implemented APT with preservice teachers not from the mathematics teaching department.

This study hypothesizes that PSMTs do not feel comfortable teaching probability and do not have high confidence in their teaching skills on probability since AT and CT were found to be considerably low even though all the components were positive. In the study of Estrada *et al.* (2020), it was indicated that preservice teachers scored AP, CCP and AT as low and CT as high. On the other hand, the findings of Vásquez

Ortiz *et al.* (2019) and Ruz *et al.* (2021) supports the results of the present study by indicating that preservice teachers have positive but low level of self perception about their ability and knowledge to teach probability. In addition, in Koparan's study (2022) it was shown that CCP and CT of PSMTs were negative but close to neutral value before the implementation of the instruction. To conclude that, thinking their capability of teaching probability as inadequate or perceiving that they are not good enough to teach is common among preservice teachers across studies.

The attitudes of PSMTs were compared based on different descriptive characteristics as different studies in the literature also examined. For instance, comparison of the attitudes of students and teachers based on their gender, year of education, taken courses, achievement were the source of research in many studies (Bulut et al, 2002; Estrada *et al.*, 2020; Mata, 2012). The year of education was examined based on being junior or senior student. However, there was no difference between the attitudes of juniors and seniors. Also, courses taken by PSMTs did not create a significant difference between their attitudes. Although PSMTs who took a probability course before showed a slightly negative attitude, the difference between the two participant groups was insignificant. However, in Koparan's study (2022), PSMTs both in experimental and control groups indicated improvement in their attitudes. In addition, no relationship was found between the GPA of preservice teachers and their attitudes toward probability and its teaching. Therefore, it might be concluded for this study that achievement of PSMTs in the university were not associated with their attitudes toward probability. Although Estrada *et al.* (2020) asserted that there is a difference between the attitudes of different genders, this study did not examine it since the number of males was limited in this study.

6.5. RQ2-A: What are the Reasons for PSMTs' Positive and Negative Attitudes toward Probability and Its Teaching?

PSMTs' scores on APT scores were mostly high, and they showed a positive attitude toward probability and its teaching. This study explored the reasons for their

positive attitudes toward probability. It has been claimed that the main causes of their positive attitudes might be: Probability is life itself, Probability is a part of mathematics, probability includes complexity, and PSMTs have positive past experiences. Preservice teachers with positive attitudes consider probability to be a topic that is combined in our daily lives, and they appreciate its usefulness. This result is supported by the findings of Özaytabak's study (2004) where preservice teachers stated that more emphasis should be given to the connection of probability with daily life rather than teaching it with full of formulas. In the present study, it is also observed that considering probability as a subtopic of mathematics directs them to have a positive attitude since PSMTs approach positively toward mathematics. Hannigan *et al.* (2013) explored preservice teachers' attitudes toward statistics and they asserted that preservice teachers' positive attitudes might be result from considering statistics as mathematics. They claimed that while thinking the items, they might have considered their capability and understanding in general mathematics since there is a high emphasis on mathematics in the curriculum rather than statistics itself. While their claim supports the explanations of the PSMTs in the present study, it might also have an explanation of the high scores in the APT. According to explanations of the PSMTs, it is also found that some of them like the complexity of probability and tend to have a positive attitude because of that. Others think that having positive experiences affects their attitudes toward probability.

Three main reasons were found for positive attitudes toward teaching probability based on the justifications of PSMTs. Firstly, PSMTs think that their conceptual understanding of probability is crucial to be able to teach probability. Therefore, when they feel sufficiently knowledgeable, they approach teaching probability positively. Ruz *et al.* (2021) stated that better state of probabilistic knowledge might result in positive attitudes toward the probability even though it might not be true for vice versa . Some PSMTs stated that since they enjoy solving probability problems, teaching it to the students will be enjoyable, too. Therefore, they argued that designing lessons will be fun because they like dealing with probability questions. The last reason for the positive attitude toward teaching probability found in this study is being good at

teaching. PSMTs expressed a positive attitude toward teaching probability based on their improved knowledge of the teaching methods and materials of probability learned during their university years. In Koparan's study (2022), it is clearly stated that.

Even though the mean score of the APT shows that PSMTs have positive attitudes, negative attitudes were also explored during the interviews. The findings indicated that two of PSMTs' reasons for negative attitudes toward probability and teaching probability were the same. It has been observed that uncertainty in probability and their lack of probabilistic understanding are the source of negative attitudes. In addition, PSMTs stated that their past experiences related to probability, such as failures in the exams or the structure of the lessons in the past, affected them negatively about probability. Özaytabak (2004) conducted a study with 248 preservice teachers to explore factors affecting the decisions of PSMTs on teaching probability. She concluded that teaching probability as highly abstract concept affects some preservice teachers' attitudes negatively and they states negative thoughts of teaching and learning it. Therefore, using concrete examples play important role in teaching probability. In addition, Özaytabak (2004) also found similar findings of the present study and stated that most of preservice teachers do not learn probability efficiently and their lack of understanding results in being afraid of teaching probability or believing that it is not a necessary topic to learn or teach. Also, some PSMTs noted that their negative attitudes toward the topic come from their opinion that probability does not have a place in our lives. These PSMTs underestimated the use of probability in daily life. This study indicates that PSMTs' negative attitudes toward teaching probability might result from their lack of knowledge about teaching methods, materials about probability. Since they do not have a clear idea of how to design the lessons, they tend to avoid teaching probability.

6.6. RQ3: What is the Relationship between PSMTs' Probabilistic Knowledge and Their Attitudes toward Probability and Teaching Probability? and RQ3-A: What is PSMTs' Perception on the Relationship Between Probabilistic Knowledge and Attitudes?

To explore the relationship between the two constructs, PCI scores and APT scores were examined. Then, the findings in the transcripts were used to support these results. The examination of general scores of PCI and APT indicated that there is no relationship between the two constructs. In contrary to quantitative results, explanations in the interviews showed that preservice teachers consider their knowledge as a crucial factor affecting their attitude. Compatible with this claim, the observation of components of PCI and APT demonstrated that the behavioral component toward probability (BP) is slightly associated with general PCI scores, CM, and CCK. BP refers to the actions of whether PSMTs tend to escape from using probability or not. Therefore, this study asserts that preservice teachers with a high knowledge of CM and CCK and a high understanding of probabilistic concepts tend to use probability more than others. The findings of interviews showed that positive attitudes in BP were observed in PSMTs with high APT scores. However, the PCI scores of these participants varied. Ruz *et al.* (2021) also indicated that positive attitudes toward probability were also observed among the participants with low content knowledge. Therefore, based on the findings of the present study, the relationship between BP and the components of PCI might be noted as small. A similar study was conducted with preservice teachers by Lovett (2016) in order to examine the relationship between statistics teaching efficacy and statistical knowledge. She found that the relationship exists but is not strong and asserted that the underestimation of the teaching efficacy might have happened among the preservice teachers. Therefore, a similar assumption may be made for the results of this research study. The interviewees with high scores on PCI might have underestimated their ability to teach, and they scored negatively on APT.

7. LIMITATIONS

The sample might be considered as one of the limitations of this study. PSMTs who participated in the study were chosen as volunteers from different universities. Although the invitation to participate in the study was sent to all the universities that included mathematics education programs, some of the universities did not take place. Their participation might affect the results since every university has different entrance requirements that need to be met in the national exams, and it gives a clue about the candidates' current state of knowledge. Also, PSMTs in primary math education exceed those in high school math education programs. Since the number of preservice teachers enrolled in primary math education is more than the number of preservice teachers in high school program in Turkey, the number of participants in the groups was not equally shared. Therefore, this study might be conducted under circumstances where compared groups have almost equal sizes and all the universities are represented. Also, the number of participants who took the PCI was limited. The sample size can be increased to have a broad understanding of the current probabilistic knowledge of preservice teachers. With that help, the knowledge components, which are CCK, SCK, and ACK, can also be examined better. In addition to that, teachers' specialized content knowledge of probability might be assessed with an additional instrument like designing a lesson plan, talking about misconceptions specifically, or an observation of microteaching. This is because PCI does not include teaching probability, although some questions need justification. Therefore, additional instruments might be useful to elaborate on the results and explain their way of thinking in teaching. Also, it is good to be aware of that there was a time distance between the interviews and implementation of the scale. Thus, collected data might be influenced by the experiences of the PSMTs during the process. Because of that reason, interpretation of the results from the interviews needs attention.

7.1. Implications

This study investigated PSMTs who will be future mathematics teachers in upcoming years. Therefore, their participation in this study was important to raise their awareness about their lack of probabilistic knowledge, their misconceptions, or students' possible misconceptions of probabilistic concepts. With this awareness, these PSMTs might also try to improve their knowledge before teaching probability. In addition, this study can help them be aware of their attitudes toward probability and its teaching, and they might be willing to change it to benefit students' learning.

In this study, two important constructs were tried to be examined based on two different instruments. The importance of the study comes from the instruments that were specially designed for probability by the previous researchers. A limited number of studies examined the probabilistic knowledge and attitudes toward probability and its teaching with different components and at the same time. Therefore, this study might shed light on the gap in the literature and provide a base for further studies. The studies can be conducted with different samples in Turkey by using these two scales and the results can be compared based on different probabilistic knowledge types and different components of attitudes. Also, implementing a similar study in different cultural context might increase the generalizability of the findings and undetected misconceptions and reasons of attitudes might be elaborated with increased sample. Additionally, designing a research study to develop different components of probabilistic knowledge and attitudes can be added to the literature as the next step of this study.

In order to improve the current teacher education programs at the universities and take action for the development of probabilistic knowledge of preservice teachers, the study might be a guide to their current state on the topic. The results showed that ACK and FM of probability is weak among PSMTs. Therefore, teacher education programs might give more emphasis on these concepts to improve their level of current knowledge and authorities can design additional workshops for the professional development of

this concepts in in-service teachers. Also, the reasons for PSMTs' negative attitudes can be taken into consideration while designing lessons related to probability.

REFERENCES

Açar, A., B. Ercan and S. Altun, 2019, "Teaching Probability Through Understanding by Design: An Examination on Students' Achievement, Attitude and Views", *Eğitim ve Bilim*, Vol. 44, No.198, pp.115-147.

Akey, T.M., 2006, *School Context, Student Attitudes and Behavior and Academic Achievement: An Exploratory Analysis*, Journal Article, MDRC.

Akoglu, K., 2014, *Research Based Suggestions to Overcome the Cognitive Issues in Teaching and Learning of Conditional Probability*, M.S. Thesis, NC State University.

Akoglu, K., 2017, *Blending Online Coursework and Small Learning Communities to Examine Professional Growth in Teaching Statistics: A Phenomenological Case Study*, Ph.D. Thesis, NC State University.

Albarracín, D., B.T. Johnson and M.P. Zanna, 2005, *The Handbook of Attitudes*, Psychology Press, New York.

Ajzen, I., 1989, *Attitude Structure and Behavior*, Psychology Press, New York.

Ball, D.L., M.H. Thames and G. Phelps, 2008, "Content Knowledge for Teaching: What Makes it Special?", *Journal of Teacher Education*, Vol.59, No.5, pp.389-407.

Bargagliotti, A. and C. Franklin, 2015, "The Statistical Education of Teachers: Preparing Teachers to Teach Statistics", *Chance*, Vol. 28, No. 3, pp. 19–27.

Batanero, C. and E. Sanchez, 2005, "What is the Nature of High School Students' Conceptions and Misconceptions About Probability?", *In: Jones, G.A. (eds) Exploring Probability in School, Mathematics Education Library*, pp. 241-266, Springer,

Boston.

Batanero, C. and M. Borovcnik, 2016, *Statistics and Probability in High School*, Sense Publishers, Rotterdam.

Batanero, C., E.J. Chernoff, J. Engel, H.S. Lee and E. Sánchez, 2016, *Research on Teaching and Learning Probability*, In: Research on Teaching and Learning Probability, ICME-13 Topical Surveys, Springer, Cham.

Biehler, R., 1991, "Computers in Probability Education", *Chance Encounters: Probability in Education*, Vol. 1, No. 1, pp. 169-211.

Biehler, R., 1994, Probabilistic Thinking, Statistical Reasoning and the Search for Causes-Do We Need A Probabilistic Revolution After we Have Taught Data Analysis, *Research Papers from ICOTS*, Vol.4, No.1, pp.20-37.

Bulut, S., 1994, *The Effects of Different Teaching Methods and Gender on Probability Achievement and Attitudes Toward Probability.* Ph.D. Thesis, METU.

Bulut, S., İ.E. Yetkin and S. Kazak, 2002, "Investigation of Prospective Mathematics Teachers' Probability Achievement, Attitudes Toward Probability and Mathematics with Respect to Gender", *Hacettepe University Journal of Education*, Vol. 1, No. 1, pp. 21-28.

Carter, T.A., 2005, *Knowledge and Understanding of Probability and Statistics Topics by Preservice PK–8 Teachers*, Ph.D. Thesis, Texas A and M University.

Clarke, D. and H. Hollingsworth, 2002, "Elaborating a Model of Teacher Professional Growth", *Teaching and Teacher education*, Vol.18, No.8, pp.947-967.

Creswell, J.W. and J.D. Creswell, 2017, *Research Design: Qualitative*, Quantitative

and Mixed Methods Approaches, Sage Publications.

Demircioğlu, H. and E.N.Ü. Bilgiç, 2023, "Examination of the Relationship Between Pre-Service Teachers' Attitudes Towards Uncertainty", *Probability and its Teaching, Journal of Pedagogical Sociology and Psychology*, Vol.5, No.3, pp.73-88.

Del Mas, R.C., 2004, *A Comparison of Mathematical and Statistical Reasoning, in the Challenge of Developing Statistical Literacy*, Reasoning and Thinking pp. 79-95, Dordrecht: Springer, Netherlands.

Di Martino, P. and R. Zan, 2011, "Attitude Towards Mathematics: A Bridge between Beliefs and Emotions", *Zoom Device Management (ZDM)*, Vol.43, No.4, pp. 471-482.

Doğucu M., 2013, *The Relationship Between Mathematics Teachers' Probability Approaches and Misconceptions*, M.S. Thesis, Boğaziçi University.

Ernest, P. 1989, "The Knowledge, Beliefs and Attitudes of the Mathematics Teacher: A Model", *Journal of Education for Teaching*, Vol. 15, No. 1, pp. 13-33.

Estrada Roca, M.A. and C. Batanero, 2020, "Prospective Primary School Teachers' Attitudes Towards Probability and its Teaching", *International Electronic Journal of Mathematics Education*, Vol.15, No.1

Estrada, A. and C. Batanero, 2019, "Prospective Primary School Teachers' Attitudes towards Probability and Its Teaching", *International Electronic Journal of Mathematics Education*, Vol. 15, No. 1, pp. 559-580.

Estrada, A., C. Batanero and C. Díaz, 2018, "Exploring Teachers' Attitudes Towards Probability and its Teaching", *In Teaching and Learning Stochastics*, Vol. 1, No. 1, pp. 313-332.

Fabrigar, L.R., T.K. MacDonald and D.T. Wegener, 2005, "The Structure of Attitudes", *The Handbook of Attitudes*, Vol. 1, No. 1, pp. 80-100.

Falk, R., 1979, *Revision of Probabilities and the Time Axis*, Hebrew University.

Falk, R. and C. Konold, 1992, "The Psychology of Learning Probability", *Statistics for the Twenty-First Century*, Vol. 1, No. 1, pp.151-164.

Fischbein, E. and A. Gazit, 1984, "Does the Teaching of Probability Improve Probabilistic Intuitions?", *Educational Studies in Mathematics*, Vol.15, No.1, pp.1-24.

Fischbein, E. and D. Schnarch, 1997, "Brief Report: The Evolution with Age of Probabilistic, Intuitively Based Misconceptions", *Journal for Research in Mathematics Education*, Vol. 28, No. 1, pp. 96-105.

Franklin, C., G. Kader, D. Mewborn, J. Moreno, R. Peck, M. Perry and R. Scheaffer, 2007, *Guidelines for Assessment and Instruction in Statistics Education (GAISE)*, Report.

Gal, I., L. Ginsburg and C. Schau, 1997, "Monitoring Attitudes and Beliefs in Statistics Education", *The Assessment Challenge in Statistics Education*, Vol.12, pp.37-51.

Gómez-Torres, E., C. Batanero, C. Díaz and J.M. Contreras, 2016, "Developing a Questionnaire to Assess the Probability Content Knowledge of Prospective Primary School Teachers", *Statistics Education Research Journal*, Vol.15, No.2, pp.197-215.

Hannula, M.S., 2002, "Attitude Towards Mathematics: Emotions, Expectations and Values", *Educational Studies in Mathematics*, Vol. 49, No. 1, pp. 25-46.

Hannula, M.S., P. Di Martino, M. Pantziara, Q. Zhang, F. Morselli, E. Heyd-Metzuyanim and G.A. Goldin, 2016, *Attitudes, Beliefs, Motivation and Identity in Mathematics*

Education: an Overview of the Field and Future Directions, Springer Nature.

Hill, H.C., B. Rowan and D.L. Ball, 2005, "Effects of Teachers' Mathematical Knowledge for Teaching on Student Achievement", *American Educational Research Journal*, Vol.42, No.2, pp. 371-406.

İlhan, A. ve R. Aslaner, 2019, "2005'ten 2018'e Ortaokul Matematik Dersi Öğretim Programlarının Değerlendirilmesi", *Pamukkale Üniversitesi Eğitim Fakültesi Dergisi*, Vol. 1, No. 1, pp. 46-59.

Kahneman, D., S.P. Slovic, P. Slovic and A. Tversky, 1982, *Judgment Under Uncertainty: Heuristics and Biases*, Cambridge University Press.

Kapadia, R. and M. Borovcnik, 1991, "The Educational Perspective", *In Chance Encounters: Probability in Education*, pp. 1-26, Springer, Dordrecht.

Kapadia, R. and M. Borovcnik, (Eds.), 2012,*Chance Encounters: Probability in Education* (Vol. 12), Springer Science and Business Media.

Konold, C. and S. Kazak, 2008, "Reconnecting Data and Chance", *Technology Innovations in Statistics Education*, Vol. 2, No. 1, pp. 34-60.

Koparan, T., 2022, "The Impact of a Game and Simulation-Based Probability Learning Environment on the Achievement and Attitudes of Prospective Teachers", *International Journal of Mathematical Education in Science and Technology*, Vol.53, No.9, pp.2319-2337.

Kurt, G. and O. Coşkuntuncel, 2020, "Assessment of Elementary Mathematics Teachers' Probability Content Knowledge in terms of Different Meanings of Probability", *Turkish Journal of Computer and Mathematics Education*, Vol.11, No.3, 706-732.

Kvatinsky, T. and R. Even, 2002, *Framework for Teacher Knowledge and Understanding about Probability, In Proceedings of the Sixth International Conference on Teaching Statistics (CD)*, Cape Town, South Africa: International Statistical Institute.

Laplace, P.S., 1814, *A Philosophical Essay on Probabilities*, English Edition 1951.

León-Mantero, C., J.C. Casas-Rosal, A. Maz-Machado and , M.E.V. Rico, 2020, "Analysis of Attitudinal Components towards Statistics Among Students from Different Academic Degrees", *PLoS One*, Vol. 15, No. 1, pp. 213-229.

Lovett, J.N., 2016, *The Preparedness of Preservice Secondary Mathematics Teachers to Teach Statistics: A Cross-Institutional Mixed Method Study*, Eric.

Martino, P.D. and R. Zan, 2015, "The Construct of Attitude in Mathematics Education", *In From Beliefs to Dynamic Affect Systems in Mathematics Education*, Vol. 1, No. 1, pp. 51-72.

Mata, M.D.L., V. Monteiro and F. Peixoto, 2012, *Attitudes Towards Mathematics: Effects of Individual, Motivational and Social Support Factors*, Child Development Research.

McLeod, D.B., 1992,"Research on Affect in Mathematics Education: A Reconceptualization", *Handbook of Research on Mathematics Teaching and Learning*, Vol. 1, No.1, pp. 575-596.

Ministry of National Education (MONE), 2018, "Öğretim Programlarını İzleme ve Değerlendirme Sistemi-¨Ögretim Programları", Retrieved from, 2023, http: // mufredat. meb. gov. tr/Programlar. aspx, accessed on May 10, 2023.

Moore, David S., 1990, "Uncertainty", *On the Shoulders of Giants: New Approaches to Numeracy*, Vol. 1. No. 1, pp.95-137.

Moore, D.S. and G.W. Cobb, 2000, "Statistics and Mathematics: Tension and Cooperation", *The American Mathematical Monthly*, Vol.107, No.7, pp.615-630.

National Council of Teachers of Mathematics, NCTM, 2000, *Principles and Standards for School Mathematics*, Reston, VA: Author.

Ojose, B., 2015, "Students' Misconceptions in Mathematics: Analysis of Remedies and What Research Says", *Ohio Journal of School Mathematics*, Vol. 1, No. 1, pp. 72-92.

Özaytabak, E. 2004, *Factors Affecting Preservice Mathematics Teachers' Decisions on Probability Teaching*, M.S. Thesis, Middle East Technical University

Özyildirim Gümüs, F., 2021, "Attitudes towards Probability and Probability Teaching Scale: An Adaptation Study", *International Online Journal of Education and Teaching*, Vol. 8, No. 3, pp. 1429-1438.

Pfannkuch, M. and C. Wild, 2004, *Towards an Understanding of Statistical Thinking*, In The Challenge of Developing Statistical Literacy, Reasoning and Thinking.

Philipp, R.A., 2007, "Mathematics Teachers' Beliefs and Affect", *Second Handbook of Research on Mathematics Teaching and Learning*, Vol. 1, No. 1, pp.257-315.

Prodromou, T., 2012, "Connecting Experimental Probability and Theoretical Probability", *ZDM*, Vol.44, No.7, pp.855-868.

Raymond, A.M., 1997, "Inconsistency between a Beginning Elementary School Teacher's Mathematics Beliefs and Teaching Practice", *Journal for Research in Mathematics Education*, Vol.28, pp.550–576.

Rosenberg, M.J., C.I. Hovland, W.J. McGuire, R.P. Abelson and J.W. Brehm, 1960,

"Attitude Organization And Change: An Analysis of Consistency Among Attitude Components", *Yales Studies in Attitude And Communication*, Vol.3, No. 1, pp. 124-137.

Ruz, F., B. Chance, E. Medina and J.M. Contreras, 2021, "Content Knowledge and Attitudes Towards Stochastics and its Teaching in Pre-Service Chilean Mathematics Teachers", *Statistics Education Research Journal*, Vol.20, No.1, pp.5-5.

Salifu, A.S. and M.E. Dokurugu, 2022, "The Relationship Between Pre-Service Teachers'attitude towards Statistics and Attitude towards Probability", *Infinity Journal*, Vol.11, No.1, pp.115-132.

Schwab, J.J., 1978, *Science, Curriculum and Liberal Education: Selected Essays*, University of Chicago Press, Chicago.

Schwarz, N., 2007, "Attitude Construction: Evaluation in Context, *Social Cognition*, Vol. 25, No. 5, pp. 638-656.

Shaughnessy, M., 2010, *Statistics for All: The flip side of Quantitative Reasoning*, President's Corner.

Shulman, L.S., 1986, "Knowledge Growth in Teaching", *Educational Researcher*, Vol. 15, No. 2, pp. 4-14.

Stohl, H., 2005, "Probability in Teacher Education and Development, in Exploring", *Probability in School*, Vol. 1, No. 1, pp. 345-366.

Swan, M., 2001, *Dealing with Misconceptions in Mathematics*, Issues in Mathematics Teaching.

Topbaş Tat, E., 2014, *The Effect of Conceptual Change Based Instruction on Tenth*

Grade Students' Understanding of Probability Concepts, Probability Achievement and Attitudes toward Probability, M.S Thesis, METU.

Thompson, A.G., 1992, "Teachers' Beliefs and Conceptions: A Synthesis of the Research", *Handbook of Research on Mathematics Teaching And Learning*, Vol. 1, No. 1, pp. 127-146.

Tversky, A. and Kahneman, D., 1971, "Belief in the Law of Small Numbers", *Psychological Bulletin*, Vol.76, No.2, p.105.

Tversky, A. and D. Kahneman, 1974, "Judgment Under Uncertainty: Heuristics and Biases: Biases in Judgments Reveal Some Heuristics of Thinking Under Uncertainty", *Science*, Vol.185, No.4157, pp.1124-1131.

Tversky, A., D. Kahneman and P. Slovic, 1982, "Judgment under Uncertainty: Heuristics and Biases", *Sior*, Vol. 1, No.1, pp. 3-20.

Vásquez Ortiz, C., Alvarado Martínez, H. and Ruz, F. 2019, "Attitudes of Future Teachers of Childhood Education Towards Statistics, Probability and its Teaching", *Educación Matemática*, Vol.31, No.3, pp.177-202.

Wilkerson, J.R. and W.S. Lang, 2007, *Assessing Teacher Dispositions*, Thousand Oaks, CA: Press.

APPENDIX A: PROBABILITY CONCEPT INVENTORY

The questions of PCI (Gómez-Torres *et al.*, 2016) implemented in the study was given below.

Solve the following tasks, explaining your solution in writing when required to do so:

Item 1. (Adapted from Green, 1982)

Three boys take turns playing a video game. They have to line up in a row and wait for their turn. Suppose the boys are called Andres, Benito and Carlos (A, B, C, for short). We want to write down all of the possible orders in which they could play this game: for

example, one possible order is BCA.

a. Write down all of the different orders. How many different ways are there altogether?
b. If four boys (A, B, C, D) want to play, how many different ways are there?

Item 2. (Adapted from Falk & Wilkening, 1998)

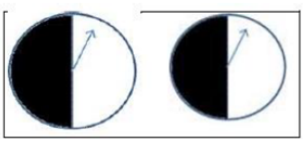
Pablo puts 5 white balls and 7 black balls into an urn. Miguel puts 3 white balls and 5 black balls into another urn. How many black or white balls should be moved from one urn to another if we want both children to have the same chance of drawing a black ball? Why?

Item 3.(Adapted from Shaughnessy & Ciancetta, 2002)

These two fair spinners are part of a carnaval game. A player wins a prize only when both arrows land on black after each spinner has been spun once.

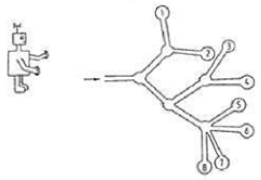
a. What is the probability of winning the game?

b. Why?



Item 4. (Adapted from Green, 1982)

A robot is put into a maze, which it begins to explore. At each junction, the robot is as likely to follow any one path as any other (however, it will not go back the way it came). There are eight traps at the end of each of the eight paths (see the picture).



a. In which trap (or traps) is the robot most likely to finish up? Why?

Figure A.1. Probability concept inventory 1.

b. In which traps or traps is the robot least likely to finish up? Why?

Item 5. (Adapted from Green, 1982, 1983)

A packet of 100 drawing pins is emptied out onto a table by a teacher. Some drawing pins landed "up"  and some landed "down"  . The results were as follows: 68 landed up and 32 landed down. The teacher then asked four students to repeat the experiment. Each student emptied a packet of 100 drawing pins and got some landing up and some landing down. In the following table, write possible results for each student:

Daniel	Martin	Diana	Maria
up:	up:	up:	up:
down:	down:	down:	down:

Item 6. (Adapted from Azcárate, 1995)

Miguel and Luis play a game that involves rolling two fair dice (each numbered from 1 to 6). They roll both dice and multiply their numbers. Miguel receives 1 euro if the product is an even number Luis receives 1 euro if the product is an odd number

a. Is this game fair? Why?

b. If Miguel receives 1 euro every time the product of both dice is even, how many euros should Luis receive every time the product is odd if the game is to be fair?

Item 7. (Adapted from Díaz & Batanero, 2009)

Two dice (one red and one blue die) are rolled, and the product of the two resulting numbers is 12. What is the probability that one of the two numbers is a six? (We take the order of the numbers into account.)

Item 8. (Batanero, Garfield, & Serrano, 1996)

When three dice are rolled simultaneously:

a. Which of the following results is most likely?

- a 5, a 2, and a 3_____
- two 5s and a 3____
- three 5s____
- The chances of obtaining each of these results are the same ____

b. Is one of these results less likely than the others? Which one?

Item 9. (Adapted from Fischbein & Gazit, 1984)

On a farm, there is a fishing pool. The owner wants to know how many fish there are in the pool. He took out 200 fish and marked each of them with a coloured sign. He released the marked fish back into the pool and let them get mixed in with the others. On the second day, he took out 250 fish in a random fashion and found that, among them, 25 were marked.

Figure A.2. Probability concept inventory 2.

a. What is the approximate number of fish in the pool?

b. If owner randomly takes 100 more fish, approximately how many will be marked?

Item 10. (Green, 1982)

A teacher asked Clara and Luisa to each toss a coin 150 times and to record whether the coin landed on heads or tails on each toss. For each "Heads," a 1 is recorded, and for each "Tails," a 0 is recorded. Here are the two sets of results:

Clara:010110011001010110110100011100011011010101100100010101001110011010110010110011001010111011001101101010010110010101100010011010110011101110101100011

Luisa:100111011110100111001001110010001110111111010101011110000001000101001000001000110001010000000001100100000001111100001101010010010011111101001100011000

One girl followed the instructions, tossing the coin on each turn; the other girl cheated and just made the sequence up.

a. Which girl cheated?

b. How can you tell?

Item 11. (Adapted from Batanero, Garfield, & Serrano, 1996; this is is a version of the hospital problem introduced in Tversky & Kahneman, 1974)

In a certain town hospital, a record of the number of boys and girls born in the hospital is kept.

a. Which of these cases is more likely:

- There will be 8 or more boys among the next 10 babies born at the hospital ___
- There will be 80 or more boys among the next 100 babies born at the hospital ___
- Both results are equally likely___

Explain your answer:

b. Which of these cases is more likely among the next 10 babies born at the hospital:

- There will be 7 or more boys ____
- There will be 3 or less boys ____
- The number of boys will be between 4 and 6 ____
- These three results are equally likely ____

Explain your answer:

Item 12. (Adapted from Batanero, Garfield, & Serrano, 1996; the problem is inspired by a situation described by Konold, 1989)

A weather forecaster says that this year, there is a 70% chance of rain in Santiago de Compostela.

a. If this forecaster is right, how many rainy days would you expect this year in Santiago de Compostela?

b. Suppose that the forecaster said there was an 80% chance of rain this week and that it did not rain on Monday. What would you conclude about the statement that there was a 80% chance of rain?

c. If the prediction was 80% chance of rain, but it did not rain on Monday or Tuesday, what would you conclude?

Figure A.3. Probability concept inventory 3.

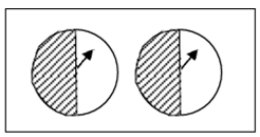
APPENDIX B: OLASILIK KAVRAM TESTİ TÜRKÇE

The questions of Turkish version of PCI (Kurt *et al.*, 2020) implemented in the study was given below.

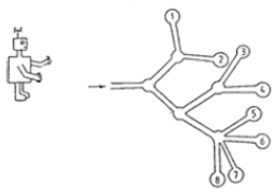
1. Ali, Barış ve Cenk (kısaca A, B, C) isimli üç erkek çocuk sırayla bir video oyunu oynuyorlar. Bunun için sıraya girerek oyun sırasının kendilerine gelmesini beklemeleri gerekiyor. Bu oyunu oynayabilecekleri tümolası sıralamayı yazmak istiyoruz: Örneğin sıralamalardan biri B-C-A'dır.
   a. Farklı sıralanmaların tümünü yazın. Kaç farklı şekilde sıralanabilirler?
   b. Dört erkek (A, B, C, D) oynamak istiyorsa kaç farklı sıralanma mümkün olur?

2. Deniz, bir torbanın içine 5 beyaz ve 7 siyah top koyuyor. Mustafa ise başka bir torbaya 3 beyaz ve 5 siyahtop koyuyor. Eğer Deniz ve Mustafa'nın torbalarından siyah top çekme şanslarının eşit olmasını istiyorsakbir torbadan diğerine kaç tane siyah veya beyaz top aktarmamız gereklidir? Neden?

3. Yandaki ikili çarkıfelek bir karnaval oyununun bir parçasıdır. Biroyuncu, çarkıfelekler durduğunda her iki ok da siyah bölümde duruyorsa oyunu kazanmaktadır.
   a. Oyunu kazanma olasılığı nedir?
   b. Neden?



4. Bir robot şekildeki gibi bir labirente bırakılıyor. Labirentin 8 çıkışının her birinde şekildeki gibi numaralandırılmış birer tuzakvardır. Kavşaklardaki yollardan herhangi birini izleyerek ve girdiği yoldan da geri dönmemek kaydıyla;
   a. Yakalanma olasılığı en yüksek olan tuzak veyatuzaklar hangileridir? Neden?
   b. Yakalanma olasılığı en düşük olan tuzak veyatuzaklar hangileridir? Neden?



5. Bir kutudaki 100 raptiye öğretmen tarafından masaya boşaltılıyor. Bunların 68 tanesinin uçları yukarıyı ve32 tanesi de aşağıyı göstermiştir. Daha sonra öğretmen 4 öğrenciye (A, B, C, D) deneyi tekrarlamalarını söylüyor. Aşağıdaki tabloda her bir öğrencinin yaptığı deneyin olası sonuçlarını yazınız.

	Deniz	Mehmet	Duygu	Mustafa
Yukarı				
Aşağı				

6. Deniz ve Barış hilesiz iki zarla oynanan bir oyun oynuyorlar. Zarları yuvarlıyorlar ve üstte gelen yüzdekisayıları çarpıyorlar. Eğer,
   i. Çarpım çift sayı ise Deniz 1 Lira alıyor
   ii. Çarpım tek sayı ise Barış 1 Lira alıyor
   a. Oyun adil midir? Neden?

Figure B.1. Olasılık kavram testi türkçe 1.

7. Eğer çarpım sonucu çift sayı olduğunda Deniz 1 Lira alıyorsa, oyunun adil olabilmesi için,çarpım sonucu tek sayı olduğunda Barış kaç Lira almalıdır Biri sarı diğeri mavi iki hilesiz zar atılıyor. Üstlerinde gelen noktaların çarpımı 12 elde ediliyor. Zarlardanbirinin 6 gelmesi olasılığı nedir? (Sayıların sırası göz önüne alınmaktadır.)

8. 3 zar aynı anda atılıyor. Sonuçlar (a, b, c) şeklinde sıralı üçlü olarak kaydediliyor.
   a. Aşağıdaki sonuçlardan elde edilme olasılığı en yüksek olan hangisidir?i. (5, 2, 3)…….
      ii. (5, 5, 3)……
      iii. (5, 5, 5)……
      iv. Bu sonuçların her birinin elde edilme şansı eşittir.....
   b. Bu sonuçlardan herhangi biri diğerlerinden daha az olasılığa sahip midir? Hangisi?.....

9. Bir balık çiftliğinin sahibi havuzdaki balıkların sayısını öğrenmek istiyor. Havuzdan 200 balık çıkararak bunları işaretliyor ve tekrar diğer balıkların arasına bırakıyor. Sonraki gün 250 balığı havuzdan çıkarıyor ve25 tanesinin işaretlenmiş balıklar olduğunu görüyor.
   a. Havuzdaki balık sayısı yaklaşık olarak nedir?
   b. Eğer havuzdan rasgele 100 balık daha çıkarılırsa yaklaşık kaç tanesi işaretli olur?

10. Bir öğretmen 2 öğrencisine (Deniz ve Barış) düzgün bir parayı 150 kez atmalarını ve tura gelirse 1, yazı gelirse 0 ile kaydetmelerini istiyor. Aşağıdaki sonuçlar elde ediliyor. Öğrencilerden biri talimatlara uygun şekilde parayı atarak kayıt oluşturmuş ancak diğeri talimatlara uymamış, hile yaparak sadece bir sayı serisiyazmıştır.
    a. Hile yapan hangi öğrencidir?
    b. Bunu nasıl söyleyebiliyorsunuz?

    Deniz:
    010110011001010110110100011100011011010101100100010101001110011010110010110010110010
    010111011001101010100101100101011000100110101100111011101011000011
    Barış: 100111011110100111001001110010001110111111010101011110000001000101001000 0
    010001100010100000000011001000000011111000011010100100100111111010011000110 00

11. Bir şehir hastanesinde doğan erkek ve kız çocuklarının kaydı tutulmaktadır. Buna göre;
    a. Aşağıdakilerden hangisinin olasılığı daha yüksektir? Açıklayınız.
       i. Son doğan 10 bebeğin 8 veya daha fazlası erkek olacaktır.
       ii. Son doğan 100 bebeğin 80 veya daha fazlası erkek olacaktır.
       iii. Yukarıdaki iki sonuç da eşit olasılıklıdır.
    b. Son doğan 10 bebek için aşağıdakilerden hangisi en yüksek olasılığa sahiptir? Açıklayınız.
       i. 7 veya daha fazla erkek bebek doğacaktır.
       ii. 3 veya daha az erkek bebek doğacaktır
       iii. Erkek bebek sayısı 4 ile 6 arasında olacaktır.
       iv. Yukarıdaki 3 sonuç da eşit olasılıklıdır.

12. Bir meteoroloji uzmanı bu yıl Adana'da yağmur ihtimalinin %70 olduğunu söylemektedir.
    a. Eğer tahmini doğru ise, bu yıl Adana'da kaç günün yağmurlu olmasını beklersiniz?
    b. Meteoroloji uzmanının bu hafta yağmur yağma olasılığının %80 olduğunu söylediğini ve Pazartesi günü de yağmur yağmadığını düşünelim. %80 yağmurlu olması tahmini hakkında nedüşünürsünüz?
    c. Eğer yağmur olasılığı %80 ise, ama Pazartesi veya Salı yağmur yağmadıysa nasıl bir sonuccıkarırsınız?

Figure B.2. Olasılık kavram testi türkçe 2.

APPENDIX C: ATTITUDE SCALE TOWARDS PROBABILITY AND ITS TEACHING

The questions of APT(Estrada *et al.*, 2018) used in the study was given below.

Note: Each item below was followed by a 5-point scale ranging from 1 (strongly disagree) to 5 (strongly agree). Scores in items with negative wordings were rescaled before computing the mean.

Item
I1. I enjoy the lessons in which probability is explained.
I2. I use probability information when making decisions.
I3. It will be hard for me to teach probability.
I4. Probability helps you understand today's world.
I5. I like probability; it is a subject that always interested me.
I6. Probability is easy.
I7. I never used probability outside mathematics.
I8. I have mastered the main contents of probability.
I9. I am sure I will enjoy teaching probability in school.
I10. I think I can notice and correct students' errors and difficulties with probability.
I11. I will only teach probability if there is time available after teaching the other topics.
I12. Probability is only useful for games of chance.
I13. Probability is not as valuable as other areas of mathematics.
I14. It will be easy for me to design assessment tasks related to probability.
I15. I use probability in everyday life.
I16. I feel scared when faced with probability information.

Figure C.1. Olasılık kavram testi türkçe 2.

I17. Probability is understandable only to "science people."

I18. I avoid reading information that contains probability terms (e.g. in drugs prospects).

I19. Probability knowledge helps students to reason critically.

I20. Probability should be taught since the first teaching levels.

I21. I feel worried about being able to reply to my students' questions about probability.

I22. I do not feel well enough prepared to solve any basic probability problem.

I23. I would not be able to prepare suitable didactic resources for the probability lesson.

I24. I will use probability when needed in other topics I teach.

I25. If I could skip a topic, I would skip probability.

I26. I am not interested in teaching probability, even if it appears in the curriculum.

I27. I do not enjoy solving probability problems.

I28. As a teacher, I would feel comfortable when teaching probability.

Figure C.2. Olasılık kavram testi türkçe 2.

APPENDIX D: UNIVERSITIES AND THEIR STATISTICS AND PROBABILITY COURSES

The universities that participants were coming from and their probability and statistics courses for mathematics education departments was shown on the table.

Table D.1. Universities and their statistics and probability courses.

Universities	**Primary School Mathematics Education**	**Secondary School Mathematics Education**
Afyon Kocatepe University (Afyon)	3rd term İMAE213: Olasılık 4th term İMAE210:İstatistik 6th term İMAE304: Olasılık ve İstatistik Eğitimi	-
Aksaray University (Aksaray)	5th term İMÖ 305: İstatistik ve Olasılık I 6th term İMÖ 306: İstatistik ve Olasılık II Elective: İMÖA429: Eğitim İstatistiği ve Alan Uygulamaları	-
Boğaziçi University (İstanbul)	5th term SCED301: Inferential Statistics 1 6th term SCED354: Teaching Probability and Statistics Elective: SCED302: Inferential Statistics 2	5th term SCED301: Inferential Statistics 1 Elective: SCED302: Inferential Statistics 2

Table D.1 Universities and their statistics and probability courses. (cont.)

Universities	**Primary School Mathematics Education**	**Secondary School Mathematics Education**
Burdur Mehmet Akif Ersoy University (Burdur)	4th term 18AEZ2244:Olasılık 5th term 18AEZ2253: İstatistik 6th term 22AEZ2262: Olasılık ve İstatistik Öğretimi	-
Çukurova University (Adana)	4th term MATZ208: Olasılık 5th term MATHZ305: İstatistik 6th term MATHZ304: Olasılık ve İstatistik Öğretimi	-
Dicle University (Diyarbakır)	5th term İMP303: Olasılık 6th term İMP304: İstatistik	5th term MTÖ303: Olasılık 6th term MTÖ304: İstatistik
Gazi University (Ankara)	5th term İMTÖ301: Olasılık İMTÖ302: İstatistik	5th term MATÖ307: Olasılık MATÖ308: İstatistik
Hacettepe (Ankara)	3rd year MAE301: İstatistik 2022-23 2nd year MAE208: Olasılık 3rd year MAE304: Olasılık ve İstatistik Öğretimi	3rd year MTE303: Olasılık Elective: MKÖ361: Olasılık

Table D.1 Universities and their statistics and probability courses. (cont.)

Universities	**Primary School Mathematics Education**	**Secondary School Mathematics Education**
İstanbul-Cerrahpaşa University (İstanbul)	4th term AEMO2010: Olasılık 5th term AEMO3003: İstatistik 6th term AEMO3006: Olasılık ve İstatistik Öğretimi	-
İzmir Demokrasi University (İzmir)	4th term İMÖ208: Olasılık 5th term İMÖ305: İstatistik 6th term İMÖ304: Olasılık ve İstatistik Öğretimi	-
Kafkas University (Kars)	4th term Olasılık 5th term İstatistik 6th term Olasılık ve İstatistik Öğretimi	-
Kastamonu University (Kastamonu)	4th term İMZ208: Olasılık 5th term İMZ305: İstatistik 6th term İMZ304: Olasılık ve İstatistik Öğretimi	-

Table D.1 Universities and their statistics and probability courses. (cont.)

Universities	**Primary School Mathematics Education**	**Secondary School Mathematics Education**
Kütahya Dumlupınar University (Kütahya)	4th term 191714230: Olasılık 5th term 191715233: İstatistik 6th term 191716238: Olasılık ve İstatistik Öğretimi	-
Mersin University (Mersin)	4th term İMA202: Olasılık 5th term İMA303: İstatistik 6th term İMA304: Olasılık ve İstatistik Öğretimi	-
Muğla Sıtkı Koçman University (Muğla)	4th term İME20008: Olasılık 5th term İME30005: İstatistik 6th term İME30004: Olasılık ve İstatistik Öğretimi	-
Necmettin Erbakan (Konya)	4th term AEZ310250015: Olasılık 5th term AEZ310250021: İstatistik 6th term AEZ310250024: Olasılık ve İstatistik Öğretimi	5th term AEZ310600019: Olasılık 6th term AEZ310600023: İstatistik

Table D.1 Universities and their statistics and probability courses. (cont.)

Universities	**Primary School Mathematics Education**	**Secondary School Mathematics Education**
Middle East Technical University (Ankara)	3rd term STAT201: Introduction to Probability and Statistics 1 4th term STAT202: Introduction to Probability and Statistics 2	5th term STAT201: Introduction to Probability and Statistics 1 6th term STAT202: Introduction to Probability and Statistics 2
Pamukkale University (Denizli)	4th term IMO2008: Olasılık 5th term IMO3005: İstatistik 6th term IMO3004: Olasılık ve İstatistik Öğretimi	-
Selçuk University (Konya)	4th term Olasılık 5th term İstatistik 6th term Olasılık ve İstatistik Öğretimi	-
Süleyman Demirel University (Isparta)	4th term İMO200: Olasılık 5th term İMO305: İstatistik 6th term İMO304: Olasılık ve İstatistik Öğretimi	-

Table D.1 Universities and their statistics and probability courses. (cont.)

Universities	**Primary School Mathematics Education**	**Secondary School Mathematics Education**
Yıldız Technical University	4th term IMO2242: Olasılık 5th term IMO3331: İstatistik 6th term IMO3322: Olasılık ve İstatistik Öğretimi	-