NON-ROUTINE PROBLEM SOLVING PROCESSES OF TURKISH MATHEMATICALLY GIFTED STUDENTS

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ABSTRACT

NON-ROUTINE PROBLEM SOLVING PROCESSES OF TURKISH MATHEMATICALLY GIFTED STUDENTS

This study aimed to investigate the problem solving processes of Turkish mathematically gifted students when they engaged in solving non-routine mathematical problems. In accordance with the nature of the study, case study were chosen as research design. The sample of the study was five mathematically gifted students who were members of Mathematics Olympiads Group organized by National Ministry of Education, and at the top ten degree of in National Mathematic Olympiad regulated by TUBITAK, the National Science Foundation (NSF) in Turkey. Data was collected by two-hour long paper and pencil test containing eight non-routine problems and one-hour long structured task-based interviews. Data was analyzed by within and cross case data analysis techniques.

Results of the study showed that Polya's four-phase problem solving processes were attainable by all these mathematically gifted students. Particular to especially the looking back and the carry out the plan stages, results showed that for mathematically gifted students problem solving processes include justification within the looking back stage. In addition, carry out the plan phase of the problem solving process of mathematically gifted students involve three common characteristics: focusing on the quantities, associating numerical and symbolic results with the givens in the problem context, being aware of all parts/steps of the solution processes. These results in-juxtaposition to each other implied that while solving non-routine problems mathematically gifted students engage in quantitative reasoning. Implications for teaching suggests extricating the mathematically gifted students' reasoning processes during classroom discussions to foster average students' reasoning quantitatively and to learn from each other.

ÖZET

MATEMATİKTE ÜSTÜN YETENEKLİ TÜRK ÖĞRENCİLERİN RUTİN OLMAYAN PROBLEM ÇÖZME SÜREÇLERİ

Bu çalışma, matematiksel olarak üstün yetenekli öğrencilerin rutin olmayan matematiksel problemleri çözme sürecindeyken problem çözme süreçlerini incelemeyi amaçlamıştır. Çalışmanın doğasına uygun olarak, araştırma deseni olarak durum çalışması seçilmiştir. Çalışmanın örneklemini Milli Eğitim Bakanlığı tarafından organize edilen Matematik Olimpiyatları Grubu üyesi olan ve TÜBİTAK tarafından düzenlenen Ulusal Matematik Olimpiyatı'nın ilk ona girmiş olan matematiksel olarak üstün yetenekli beş öğrenci oluşturmaktadır. Araştırma verisi iki saatlik sekiz rutin olmayan problemden oluşan yazılı test ve bir saatlik yapılandırılmış görüşmelerden elde edilmiştir. Veri analizi için iç durum ve çapraz durum analizi teknikleri kullanılmıştır.

Çalışmanın sonuçları, bütün üstün yetenekli öğrencilerin Polya'nın dört aşamalı problem çözme süreçlerinden geçtiğini göstermiştir. Özellikle geri dönme ve planı uygulama aşamalarına özgü olarak, sonuçlar matematikte üstün yetenekli öğrencilerin problem çözme süreçlerinden geriye bakma aşamasının doğrulama içerdiğini göstermiştir. Buna ek olarak, matematikte üstün yetenekli öğrencilerin problem çözme süreçlerinden planı uygulama aşaması üç ortak özellik içermektedir: niceliğe odaklanma, sayısal ve sembolik sonuçları problemin içeriğindeki verilenlerle ilişkilendirme, çözüm süreçlerinin tüm bölümlerinin/ adımlarının farkında olma. Bu sonuçların yan yana koyulmasıyla matematikte üstün yetenekli öğrencilerin rutin olmayan problem çözerken nicel akıl yürütme yaptıkları ortaya çıkmaktadır. Öğretim için önerilerde, matematikte üstün yetenekli öğrencilerin sınıf içi tartışmalar sırasındaki akıl yürütme süreçlerinin ortalama öğrencilerin nicel akıl yürütme süreçlerini geliştirmek ve birbirlerinden öğrenmeyi sağlamak için açığa çıkarılması önerilmektedir.

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1. INTRODUCTION

One of the famous sayings of Atatürk who is the founder of Turkish Republic is that "Teachers: the new generation will be your devotion." The saying emphasizes the importance of education. Development of a country depends on educated people, especially the gifted ones. In the history, Einstein, Ramanujan, Newton, etc. found an opportunity to reveal their giftedness, and they utilize this opportunity to contribute to the development of humankind. Unfortunately, there are many gifted people who passed away without even realizing their abilities. Accordingly, it is important to identify gifted people, and educate them appropriate to their giftedness in order to extend the evolution of science.

Mathematics is the core requirement of all the other sciences because mathematics cannot be separated from any kind of science and scientific research. So, the identification and education of mathematically gifted people are among the most essential parts of education to support the developments in real life and in science. Mathematically gifted people are excellent problem solvers, good at reasoning, recognizing patterns, and logically analyzing problems (Gagne, 1995; Gardner, 2011 ; Sternberg, 1993). To improve their strength in problem solving, their education should be designed and applied carefully. Concordantly, there is a current movement to develop mathematics curriculum for the gifted and talented students in Turkey.

Problem solving is the cornerstone of mathematics education. As one of the NCTM standards problem solving is described as developing "*a deep understanding of concepts and methods by trying of problematic tasks where the mathematics to be learned is embedded*" (p.270). In other words, problem solving is a crucial part of mathematics learning because it helps to understand the mathematical content, to link the previous knowledge to new one, and to improve mathematical skills (Van De Walle, 2004; Umay ,2003). There are plenty of problem solving models consisting of the steps to point the problem solver a logical direction, to create awareness even if s/he is able to solve, and to improve their problem solving skills

(Polya,1957; Garofalo & Lester,1985; Mayer,1985; Schoenfeld, 1985; Artzt &Thomas,1992; Montague & Applegate, 1993; Goos, Galbraith & Renshaw, 2000).

In addition to effective contribution of problem solving models, the problem types are also an important component to lead to thinking and to improve problem solving skills. Non-routine problems, called also as unusual or real-life problems, are the ones that cannot be solved immediately by selecting one or several processes correctly or just applying a rule or formula (Altun,2015). This property of non-routine type problems supports flexible thinking, data organization, recognition of pattern and relationships and the use of previous knowledge extensively (Souviney, 1989). Simply, non-routine problems force the problem solver to think in an unusual way, and improve their skills. Gifted students' problem solving and non-routine problem solving has been studied mostly in two approaches: comparison of gifted and average students and changes in problem solving skills of gifted students after amount of instructions.

The purpose of this study is to investigate non-routine problem solving process of mathematically gifted students. In this regard, six mathematically gifted students will be asked to attend to a written problem solving session involving eight non-routine problems without time limitation. The non-routine problems are categorized as number and operation problems, algebra problems and real life problems. Upon completion of the written problem solving sessions, one hour long task-based interviews will be conducted with each student. Using Polya's problem solving model, data from both the written sessions and the task-based interviews will be analyzed by within and cross-case analysis. With the results of this study, I expect to contribute to the field by providing a detailed description of the mathematically gifted students' problem solving process on non-routine problems. Also, the results might shed light on mathematically gifted students' education through the use of non-routine problems support flexible thinking, data organization, recognition of pattern and relationships and the use of previous knowledge extensively (Souviney, 1989).

2. LITERATURE REVIEW

In this chapter, there will be three main parts: definition of problem and models of problem solving, mathematical giftedness and, previous research pointing to the significance of the study. In the first part, definitions of problem in the literature will be explained by focusing on two problem types: routine and non-routine. Then, problem solving models will be described in chronological order, and Polya's four phase framework will be explained explicitly. In the third part, mathematical giftedness and previous research on mathematical giftedness will be explained to have an overview about problem solving of mathematically gifted students. In the last part, the significance of the study will be explained.

2.1. Definition of Problem

The word "problem" emerges in daily life and mathematics. These two poles are elaborately given as two definitions in Oxford dictionary:

Definition 1: "*A matter or situation regarded as unwelcome or harmful and needing to be dealt with and overcome.*"

Definition 2: "*In mathematics and physics, an inquiry starting from given conditions to investigate or demonstrate a fact, result, or law*."

Along with the definition 2, the four arithmetical operations in the mathematics class are usually the first thing that comes to mind. However, the definition of problem is a broader concept in related literature, and it comprises both definitions.

Polya (1962) scrutinize all processes of problem solving in the book *"How to Solve It?"*. The name of the book is not "*The Only Way to Solve It*" to emphasize that the solution is not immediately reached with the use of a formula or a constant outline for the solution (p.xxix). In accordance with the main idea of Polya's book, it can be stated that in order for a situation

to be a problem, it has to be seeking a clear and understandable action which is appropriate for a purpose but this action should not be immediately reached. In other words, if a situation can be handled without any struggle, it cannot be called as a problem. Dewey defines a problem as "disturbance" or "imbalance in organic-environmental interaction"(1938). Disturbance challenges the human mind and creates a tension which takes the attention of the person (Vuchinich,1999). But this challenge is only a problem when it is difficult to solve, and the individual has a structure that encourages research and exploration (Van De Walle, 1980; Orton & Wain, 1994). One of the widely accepted definitions of the term "problem" is supplementary of Dewey's definition as follows: When an individual encounter a situation which cannot not be solved immediately, he decides to accomplish and thinks about it. We call this situation as a problem (Foong, 2002). Along with these, Bingham (1998) argues that a problem has three characteristics: an aim/s, the way to solutions is not easy, and an intention to reach the aim. Therefore, problem definitions in literature can be summarized in three main points: it arouses the solution request, it does not have solution procedures at present, and it requires high-level thinking (Krulick & Rudnick, 1987; Schoenfeld, 2016; NCTM, 2000).

In the literature, problems can be classified by different approaches. Given the differences in teaching, problems can be divided into two classes: Routine and Non-routine Problems (Altun, 2015). Routine problems are also called as the four arithmetical operations problems. Routine problems can be found generally in mathematics text books. They are solved by one or more operations, but there is a single solution. (Altun, 2015). If the solution of a problem is already explained and sampled, then solving it by following the previous example step by step is named as a routine problem (Altun, 2015). Although, routine problems are important to improve math skills needed in everyday life and, to polish problem solving steps up as well as to began problem solving; they make a little contribution to the learning of mathematics and thinking in a higher level (Altun,2015). To the contrary, non-routine problems, called also as unusual or real-life problems, are different from routine problems such that they cannot be solved immediately by selecting one or several processes correctly or just applying a rule or formula (Altun,2015; Ulu,2011). This is because, beyond transactional skills, the solution requires an ability to think flexibly, to organize and classify the data, to see relationships among them, and to expand the previous knowledge by exploring

the relations (Souviney, 1989, p.66). Also, non-routine problem solving processes can be used to develop openness to other environmental events which is why it is called real- life problems (Altun,2015). Though, if similar examples are not resolved, the example, “In a meeting of 10 people if everyone shakes hands with each other how many handshakes occur?” might be thought as an example for a non-routine problem (Altun & Arslan, 2007). But when the solution method is taught to the students and similar samples are solved, this problem is turned into a routine problem because it is only for practical purposes which do not require higher thinking.

Furthermore, a non-routine problem for a primary school student can be a routine problem or just a practice for a high school student (Altun,2015). In addition, in reform-based teaching of mathematics, generalizations and formulas are expected not to be memorized but to be discovered by students. According to this perspective, students’ advance flexible thinking and discovering new solutions satisfies the non-routine problem solving steps. Therefore the characteristics of non-routine problems might be considered as follows:

(i) They require a higher level of cognitive thinking than routine problems require,
(ii) The solution or goal is not immediately attainable,
(iii) Beyond the application of learned rules, it requires improvement of solution strategies and thinking ,
(iv) There should not be past experiences with the exact problem or similar ones (Mullis et al, 2009; McLeod, 1988; Nancarrow,2004).

In along with the characteristics of non-routine problems, students learn how to manage their own metacognitive processes in order to solve a non-routine problem. They learn to think unfamiliar solutions by recalling previous knowledge. These are also the base objectives of mathematics teaching which is stated in NCTM (2000) and in the National Mathematics Curriculum of Turkey (MEB, 2018). All these explanations suggest that non-routine problems are important for not only to reach the goals of mathematics teaching but also to develop students' own thinking to solve a range of problems encountered in everyday life (Umay, 2003).

As well, the problem is important for mathematics teaching, problem solving is one of the cornerstones of mathematics (NCTM,2000; Van De Walle, 2004; Umay ,2003) . NCTM states that problem solving is to "develop a deep understanding of concepts and methods by trying of problematic tasks where the mathematics to be learned is embedded" (p.270). Also, five process standards, which are problem solving, reasoning and proof, communication, connections and representation, are the mathematical processes of students while attaining and using mathematical knowledge (Van De Walle, 2004). Similarly, in Turkish National Primary Mathematics Program (2018), the aspect of view to problem solving can be summarized as follows: A problem should not be perceived as a practice or a question the way of solution of which is already known. Students should be able to express their own thoughts and reasoning in the problem-solving process, to relate problems to real life, and to see the gaps in others' mathematical reasoning.

Umay (2003) also states a parallel view that mathematics education takes on far more important functions than just gaining four arithmetical operation skills (addition, subtraction, multiplication, division). Teaching mathematics should involve thinking, linking events, reasoning, making predictions, and problem solving. Therefore, problem solving takes an important place in mathematics teaching.

2.2. Models of Problem Solving

Regarding problem solving strategies students use, there are several problem solving models provided to the field (Polya,1957; Garofalo & Lester,1985; Mayer,1985; Schoenfeld, 1985; Artzt &Thomas,1992; Montague & Applegate,1993; Goos, Galbraith & Renshaw , 2000). Though all these models are mainly based on Polya's problem solving model. In this regard, in the following paragraphs, I first explain Polya's model.

As the hallmark of problem solving strategies, leading all the other models, George Polya (1957) has a four phases model. He put emphasis on the importance of following the phases by saying "*Trying to find solution, we may repeatedly change our point of view, our*

way of looking at the problem" (p.5). That is by paying attention to the phases, students might gain time for a solution; they might overcome many mistakes and they might answer the questions: "*Where should I start? What can I do? What can I gain by doing so?, etc".* more easily. The four phases are given in the following Table 2.1 with the prompting questions to accomplish each phase.

Table 2.1. Summary of Polya's Four Phases Framework

HOW TO SOLVE IT	
1. UNDERSTANDING THE PROBLEM	
First **You have to understand the problem.**	What is the unknown?
	What is the condition?
	Is it possible to satisfy the condition?
2. DEVISING A PLAN	
Second **Find the connection between the data and the unknown. You may be obligated to consider auxiliary problems if an immediate connection cannot be found. You should obtain eventually a plan for the solution.**	Do you know a related problem?
	If you know, Could you use it? Could you use its results? Could you use its method?
	If you do not know, Could you imagine a more accessible related problem? A more general problem? A more special problem? An analogous problem?
	Could you restate the problem?
	Could you solve the parts of problem? Could you derive something useful from the data? Could you think of other data appropriate to determine the unknown? Could you change the unknown or the data or both if necessary?
	Did you use all data? Did you use whole conditions? Have you taken into account all essential notions involved in the problem?
3. CARRYING OUT THE PLAN	
Third **Carry out your plan. Check each step.**	Can you see clearly that the step is correct?
	Can you prove that it is correct?
4. LOOKING BACK	
Fourth **Examine the solution obtained.**	Can you check the result? Can you check the argument?
	Can you derive the result differently? Can you see it at a glance?
	Can you use the result, or the method, for some other problem?

(Table 2.1. is adapted from Polya, G. 1957 p.xvi-xvii)

Understand the Problem: This phase represents the understanding of the verbal statement of the problem. "Principal parts of the problem attentively, repeatedly, and from various sides" should be considered. Also, if the problem requires suitable notations are introduced; and an auxiliary figure is drawn since if a problem is started to be solved without understanding, it may lead to an unwanted end. Therefore, the unknown, condition and the data of the problem is questioned until it is understood well. To ensure the understanding of the problem the questions: "*What is the unknown? What are the data? What is the condition?*" are also asked repeatedly until a satisfying answer is obtained.

Devise a Plan: In this phase, calculations, computations or constructions to obtain the unknown are planned at least as an outline. Polya (1957) stated that "*The main achievement in the solution of a problem is to conceive the idea of a plan*" (p.8). The crucial part for this phase is to have required knowledge of the subject related to the problem. Otherwise, devising a plan phase cannot be applied. It is the same as the analogy: a house cannot be constructed without collecting the necessary materials. Another essential part of this phase is to find a helpful previous knowledge as a similar problem or a somehow related one. As a trigger, the questions "*Do you know a related problem*?", and "*Can you use it?* " might be asked. These questions are expected to reveal at least little knowledge related to the problem or to arouse an idea for the solution. In the case of interaction to previous knowledge, the question "*Could you restate the problem?*" might be asked to give another point of view and explore different aspects of the problem. With the use of generalization, specialization and analogy, an outline for the solution of a problem is prepared in this phase.

Carry Out the Plan: This phase is much more easier than first two phases, just need patience to apply the devised plan. The plan gives an outline for the solution, but details in each step have to be examined to check if there is a hidden error. To ensure the step continued correctly, the questions "*Can you see clearly that step is correct?* "and " *Can you also prove that the step is correct?*"might be asked. Also, the reasoning process in this phase can be intuitive or formal.

Look Back: This phase is used as double check the solution. There is always a possibility of mistake even in the solution of a fairly good mathematician. Proving the solution or finding another solution method is not only required for the check of the current problem, but also for the improvement of problem solving skills. Looking back to the result and the path which led to it arouses the consciousness of the problem solver. To ensure the phase is applied, the questions: "*Can you check the result?* ", "*Can you check the argument?*", "*Can you derive the result differently?* " , and "*Can you use the result, or the method, for some other problem?* " might be asked.

Building on the Polya's model, within the same year, there came out different problem solving models from different fields (Garofalo & Lester,1985; Mayer,1985; Schoenfeld, 1985). Garofalo and Lester used the model of Polya's four-phase description of problem solving as a prototype; though, integrating it with Sternberg, Schoenfeld models such that they added these models to Polya's steps regarding the explanations related to cognitive and metacognitive components. So, Garofalo and Lester's model consists of four categories as Polya: These steps are orientation, organization, execution and verification. The first step orientation is a "*strategic action to understand and evaluate the problem*" which fits to Polya's first step "understanding the problem". Orientation is defined as "*planning of behavior and choice of actions*" which is called as "devising a plan" in Polya. The third step, the execution, represents "carrying out the plan" and is defined as " *regulation of behavior or to conform to plan*". Finally, verification is the last step in accordance with "looking back", and is defined as "*evaluation of orientation and organization, evaluation of execution*". The difference between Garofalo & Lester's model and Polya's model is broadly the definitions of steps sampling metacognitive behaviors associated with each step.

Mayer (1985) also developed a four-step process similar to Polya's. These steps are mainly translating (structuring a mental representation of each statement in the problem), integrating (associating actions given in each sentence with the actions given in the other sentences), planning (devising a plan for how to solve the problem); and executing (reaching to the result by doing the required mathematical operations). The first two steps of

the model created by Mayer (1985) are aimed to determine the relationship between the problem text and the problem state and it can be called meaning-based; and the last two steps are the stage at which action is taken to solve the problem. Mayer (1985) model differs from Polya's model by putting emphasis on the understanding of the problem; and also there is no looking back step to check the result and to find a second way of solution.

Among these models, Schoenfeld's (1985) framework for problem solving included seven steps. Using Schoenfeld 's framework in their research with 27 students as a sample, Artzt and Thomas (1992) stated that problem solving requires metacognitive skills more than cognitive skills. In their work, they appended Schoenfeld 's framework with the cognitive level explanations of steps. These seven steps were read, understand, analyze, explore, plan, implement and verify . The first step "read" is in cognitive level and refers to reading the problem text. The step "understand" is to state the important parts of the text, interpret the problem situation, remember similar problems, determine the critic knowledge and set the purpose. "Analyze" step is choosing appropriate point of view, reformulating the problem, putting the appropriate rules and techniques, associating the purpose with givens. The fourth step "explore" is searching knowledge related to problem and using heuristics. The step "Plan" is defining solution steps, using strategies to associate solution steps. "Implement" step is checking the model whether logical or consistent, following the plan. And the last step "Verify" refers to checking the calculations, solution process and its logic. These steps are mainly breaking the version of Polya into pieces. The categorization of these seven steps might be considered as: cognitive (e.g., reading step), metacognitive (e.g., understanding, analyzing and planning steps); and both cognitive and metacognitive (exploring and verifying steps).

Despite the differences in terminology, seven steps of problem solving processes used by Montague and Applegate (1993) is the same with Schoenfeld 's framework. Their steps are read (comprehension), paraphrase (translation), visualize (transformation), hypothesize (planning), estimate (prediction), compute (calculation), check (evaluation). In addition, Goos, Galbraith and Renshaw (2000) added monitoring and editing to Schoenfeld’s model. They pointed out that monitoring and regulation must proceed in the problem solving process

as a control process in which the process is being progressed and how difficulties can be resolved since ,from the point of view, monitoring makes the reasoning of proceeding while editing makes the reasoning of concrete.

In sum, from different fields, different researchers pointed to the similar steps for problem solving. While Garofalo & Lester's and Mayer's models have four steps as Polya, they used different names for the steps. Schoenfeld' s and Montague & Applegate's models have seven steps which are splintered version of four steps of others. For example, understand, analyze, explore and plan steps of Schoenfeld can be found in devising a plan step of Polya. Therefore, all of the models have been built on the grounds of Polya's four phase model: understanding the problem, making a plan, applying the plan and controlling all of the steps. Because it is the base of all others and contains all the steps of them and also the main focus of this study was not students' metacognitive abilities, in this study I used Polya's model to investigate mathematically gifted student' non-routine problem solving processes. In the following, I explicate mathematical giftedness.

2.3. Mathematical Giftedness

The identification of gifted or talented students varies in too many ways in the literature (Callahan, Hunsaker, Adams, Moore, & Bland, 1995). Unfortunately, there is not a universally accepted definition of giftedness in the literature (Gagne, 1985 ; Sternberg, 1993). In the past, giftedness was initially connoted high IQ. This connotation was based on the assumption that giftedness comes with the high intelligence and high test score ability. However, with the development of cognitive science, psychology and understanding of learning's importance, giftedness is defined again (Gagne ,1985; Renzulli, 1986; Sternberg & Zhang, 1995).

Particularly, Sternberg (1986) defined intelligence as flexible and effective adaptation ability to the environment and argued that IQ tests, as a traditional measurement of intelligence, capture just a part of the meaning of intelligence. Similarly, Gagne (2005)

considered, giftedness including two parts as individual's existing potential and abilities that could be improved. He defined a gifted individual having extraordinary ability in at least one area and potential to be among the top 10% of the society. He also asserted that one might have the ability but there should be a catalyst to make the ability come into existence: he called it as excellence criterion as better performance relative to peers. So for him, environmental factors (family, school, etc.) are important for giftedness. Following Gagne, Renzulli (1986) also defined giftedness with three ring conceptions: above-average ability, task- commitment, and creativity. Above-average ability is used to explain and discriminate general and special abilities. Task-commitment is thought as being motivated to do the task as well as completing the task and not leaving it incomplete. Lastly, creativity is considered as the combination of flexible thinking, originality, continuity, being open to experiences, and taking risks. In pursuit of Renzulli, Sternberg and Zhang (1995) came up with five components of giftedness: excellence, rarity, demonstrability, productivity and value criteria. They defined excellence criterion as better performance relative to peers. For example, if an 8 year-old can do mathematics that a 10 year-old could perform, it is thought as a part of giftedness expression. Secondly, they thought of rarity criterion as a high level of attribute relative to peers. They stated that to see rarity, the results of a given exam should discriminate and a rare result should appear given that an exam in which none has excellent results. Third, they expressed productivity criterion as being open to be productive in the field of giftedness. That is, once there is no potential productivity, one should demonstrate that s/he can do something. Moreover, they defined demonstrability criterion as the proof of giftedness through one or more tests which are valid assessments. Finally, they defined value as one showing an outstanding performance valued by the society. Sternberg and Zhang (1995) argued that the gifted individual is the one who carries all five criteria. Though, Gagne (1985), Renzulli (1986) and Sternberg and Zhang (1995) all had a commonality that they emphasized ability, creativity and the effects of environment for the giftedness.

Regarding the definition of mathematical giftedness, a similar evolution could be observed. In fact, the earliest definitions of mathematical giftedness centered around showing mathematical distinction, high achievement in mathematics exams and to be born superior to

other individuals in general intelligence (Broody&Stanley,2005). Similar to giftedness, to define mathematical giftedness, achievement in mathematics is also measured. According to NCTM, mathematical giftedness is defined as being %95 or more successful in standardized achievement tests. By the same token, to consider a student as mathematically gifted, he should be in the level of doing mathematics at least two years over his own age (Miller,1990).

On the other hand, Polya and Krutetskii focused on psychological factors of mathematical giftedness rather than the success on mathematics exams. In particular, Krutetskii claimed that computational ability, memorization of formulas, numbers or symbols is not crucially important to have a high mathematical aptitude. Krutetskii (1976), defined mathematical giftedness with three components: reception, processing, and retention of mathematical information. Reception is the ability to comprehend the formal structure of the problem and to formalize the material's perception. Processing is the ability to generalize, question, think logically and work hard on the problem inherently to reach a clear solution. And, retention is the ability to generalize mathematical relationships, to use schemas for proof, argument and problem solving strategies. Similarly, Polya (1962) defined mathematical giftedness as ability to solve problems with creativity, originality, judgment and independence and to form analogy (Mann, 2006).

Similarly, Gardner (2011) identified logical-mathematical intelligence as referring to high mathematical ability. He defined mathematical intelligence as the ability of exploring mathematical numbers, focusing on the structures which can be generalized more than just computation.

In sum, mathematical giftedness can be summarized as: being motivated to study mathematics, having an extraordinary ability in the field, having better performance relative to peers and being at the top 10% of the society, and being creative for the originality and to think flexibly. In this study, acknowledging all the other definitions, I embrace both Polya's

and Krutetskii's definitions for the mathematical giftedness since both definitions' emphasis is on problem solving pointing to the person's comprehension and processing of the problem structure creatively.

2.3.1. Previous Research on Problem Solving of Mathematically Gifted Students

Gifted students are thought as better problem solvers than other students, so the process of their problem solving is intriguing for researchers. In the literature, most research studies focused on problem solving of mathematically gifted students compared to the average students (Garofalo, 1993; Heinze, 2005; Koçyiğit, 2015; Montague, 1991; Sriraman ,2003; Threlfall & Hargreaves, 2008; Span and Overtoom-Corsmit ,1986). Although scarce, there is also some research investigating the gifted students' problem solving process while solving non-routine problems. (Durmaz, 2014; Karabey, 2010; Tjoe, 2015).

Particularly, Span and Overtoom-Corsmit 's study (1986) compared gifted and average students in order to understand their differences of information processing in problem solving. 14 gifted and 14 average eight grade students were selected for their research. They selected participants with the results of three standardized tests (a test for general information from a Dutch research project, the Raven Progressive Matrices, and a creativity test) which were administered to 399 students. Seven math problems were asked to the participants individually. These problems were from the areas of combinatory, transformation, permutation, and chance calculation. Once the participant finished solving each problem, then he or she was interviewed to describe the solution. All of the answers were videotaped and transcribed for analysis. In this study, orientation, execution and evaluation processes are taken into consideration. The results of the study showed that gifted participants solved more problems, and they were faster than average ones in finding the correct solution. Their information processes were different than the average participants. In the orientation step, gifted ones spent more time to carry out the problem thoroughly and were able to orient themselves. They planned their approaches for what to do. On the other hand, average students

started to solve problems right away. They did not plan what to do and did not understand the requirements of the problem. In the execution step, gifted students used more efficient strategies to solve the problem whereas average students solves problems by trial and error. In the evaluation step, gifted students were able to talk and discuss about their strategies to solve, while average students could not. These results suggested that gifted students' orientation was towards understanding the problem clearly which lead to succeed in solving them more easily and appropriately.

Garofalo (1993) compared problem solving of students who are meaning-oriented and number-oriented. 11 participants, who were seventh graders, were selected. Three of them were identified as mathematically gifted by their school districts. Their percentile rank on the Iowa Test of Basic Skills (ITBS) was 99, and they were attending to math program for gifted students in University of Virginia. Another three students were in an advanced math course, and their percentile rank on ITBS was between 87-99. These six students were called as meaning-oriented. Last five students who were called as number-oriented were in a regular seventh grade math course, and their percentile rank on ITBS was between 36-82. Eight problems consisting of routine, multi-step, and non-routine math problems were asked to students. During 45 minutes long interview, each participant were asked to solve 4-8 problems. After they finished the solving process, they were asked to explain their strategies of problem solving. These interviews were videotaped and transcribed for analysis. The framework of Garofalo and Lester's model was used for analysis. Meaning-oriented students tried to interpret the quantities and conditions of the problem meaningfully. They also constructed relationships between quantities to develop a plan for solution. On the other hand, number-oriented students focused on numbers' magnitude to decide what to do. They also used many calculations to understand the problem. The results of this study showed that both groups chose the easier routine problems once graded. However, when it was not graded, the meaning-oriented students chose a non-routine problem or a multi-step problem because it was a challenge for them to try to solve a problem which they did not solve previously. In contrast, the number-oriented students still chose an easier routine problem or a one step problem because it was easy, short, and simple. They did not choose multi-step and non-routine

problems because they had difficulty in understanding them. The meaning-oriented students considered problem solving as an opportunity for understanding, figuring, and success. They spent more time for rereading the problem many times, thinking about the solution, and checking again when finished to solve. They also tried to generalize solution of the problems. As a conclusion, researcher recommended that there should be not time limitation for a gifted problem solver since they spend more time during problem solving process.

Sriraman (2003) studied on problem solving, abstracting, and generalizing of mathematical concepts of students who are nine graders. 9 students participated in the study. Four of them were gifted in terms of the IQ scores (higher than 124), the Stanford Achievement Test (95 percentile), and recommendations of teacher and counselor. Other five students were average, non-gifted students. Participants attended to accelerated Algebra class which was taught by the researcher. They were assigned a problem every week and, asked to solve it in their journals. For this study, five non-routine combinatorial problems were asked them to solve in their journals. Students had 7-10 days for solving of each problem. In their solutions, they were asked to follow three steps which were the beginning of four stages of Garofalo and Lester's model.: "What is the problem asking?", "How would you begin solving the problem?" and "Solve the problem and write a summary of what worked and what did not work.". Researcher wrote possible questions for the interview after reading participants' solutions. Participants were asked to verbalize their problem solving processes. Gifted students started to solve the problems by paraphrasing them in their own words. They spent time in understanding the problem and identifying the requirements clearly. They made a plan to solve the problem, applied it and made generalizations. When they finished the solution process, they also checked all process for accuracy and consistency. The findings showed that gifted students achieved to discover and verbalize the generalization of the all five problems, while non-gifted ones were unable to discover the hidden generalization.

Heinze (2005) compared problem solving process of gifted and non-gifted children who are 6 to 10 years old. Gifted students were determined by nomination of parents and teachers

and high scores in intelligence test. After the observation of students during courses, they were selected. Four problems were asked to participants on the purpose of revealing the differences between gifted and non-gifted students. Students' problem solving processes was videotaped and transcribed for analysis. Results indicated that gifted students worked more systematically and quickly. They got an insight into the mathematical structure of the problems. They were also able to verbalize and explain their solutions.

Koçyiğit (2015) compared problem solving approaches of 5 gifted and 5 non-gifted middle school students. Students' thinking processes were examined to understand similarities and differences among them. A written exam with 10 non-routine problems was given to participants. In the pursuit of written exam, semi-structured interviews were made. The accuracy of the solutions, relevance to real life, approaches and strategies to solve the problem, expression of the solutions were compared. The results indicated that gifted students outperformed in all categories than non-gifted students. They used problem solving strategies and approaches which requires cognitive thinking more than non-gifted one, were also more capable of producing authentic solutions for each problem. On the other hand, non-gifted students tended to use routine strategies as writing algebraic equations and using trial-error to reach the solution.

Threlfall & Hargreaves (2008) compared problem solving of gifted and older average-attaining students. Gifted students, who were 9 years old, were selected with the nomination of teacher appropriate to the criteria "in the top 10 percent ability range in mathematics". Older average-attaining students were 13 years old and defined as average ability in mathematics by their teachers. Ten mathematics problems in World Class Tests (WCT) were asked to students. WCT was used since it consisted of everyday mathematics problems. Booklets were sent to schools and administrated under test conditions. Students' marks and answers were recorded and analyzed. The study focused on three of the problems related to the case. The results showed that the number of correct problems of gifted students were similar to older average-

attaining students. It revealed that 9 year-old gifted students could perform as an average 13 year-old average-ability student.

Similarly, Montague (1991) conducted a study with three gifted and three learning-disabled gifted 8th grade students to examine their problem solving strategies. The identification of giftedness in this study was being superior in an IQ test, a standard check list of giftedness characteristics and the documentation of the need for a special program. Two problems are asked to the participants. Firstly, participants watched a videotape which demonstrated think aloud process while problem solving. Then problems were given to the students, and they were asked to solve problems by thinking aloud of their solving processes. These interviews were audio taped and transcribed for analysis. The results showed that gifted students read the entire problem, and paraphrased it while reading. They reread the problem part-by-part while also solving it. They represented the problem in different ways to make it easier. They were completely aware of the strategies they used. They made a plan for solution after determining the goal statements. Then, they solved the problem quickly and systematically. They checked their solution by rereading, redoing calculations and checking the steps and process. On the other hand, learning-disabled gifted students read the problem, then made a plan for the solution and computed the answer. They also used representation of the problem to help the difficulty in solving it. Distinctly, there was no evidence of actual paraphrasing the problem. One of them used estimation for solution but all of them reported that they used estimation to predict the answer. As a conclusion, this study showed that gifted students use paraphrasing as a process of translation which facilitated solving the problem and they were completely aware of the planning, and solutions they executed.

Durmaz (2014) examined changes on different criteria of gifted students to reveal the extent they could learn problem solving strategies to solve non-routine problems. 121 gifted students from Science and Art Centers (Bilsem) were selected. Their scores on the scales containing several criteria (*achievement in mathematical problem solving, attitudes towards mathematics, problem solving skills and strategies, attitudes towards mathematical problem*

solving, mathematical self efficacy, mathematical academic self concept and self regulation strategies) were compared before and after instructions based on the understanding by design to improve their problem solving skills. The results indicated that the number of problem strategies students used was significantly different, and increased compared to the problem solving strategies they used prior to the instructions. Also, simplifying the problem, drawing a diagram and reasoning strategies were the most powerful differentiation among gifted students. Researcher suggested that to improve problem solving skills and the number of different strategies of gifted students, non-routine problems should take a part in their education.

Karabey (2010) also examined the level of creative problem solving and critical thinking skills of highly gifted students. 32 sixth grade and 32 seventh grade highly gifted students were selected for the research. Students were given four instruments: a mathematics test including problems two grade higher than their levels, a criterion test for determination of mathematical giftedness, a scale about creative problem solving and Watson-Glaser Critical Thinking Appraisal Form. The results showed that they had high scores on mathematics tests and creative problem solving scale. So, it was suggested that gifted students should get education to improve their creative problem solving strategies.

Tjoe (2015) examined perception of their own preferred solution and others of mathematically gifted students and expert mathematicians. Expert mathematicians were selected from the list of tenured mathematics faculty which was identified from top 10 research universities. Three of 50 experts were volunteered. 54 mathematically gifted students were selected from one of the nine specialized high schools. Students were at 5% of 30000 applicants in the entrance examination. They were also recommended by their calculus teachers. Three nonstandard problems and 15 approaches were developed by the researcher. Three experts were asked to choose their most preferred approaches for these three problems, and to rank order the approaches from the most preferred to the least one. Nine of fifty four students volunteered to take the paper-and-pencil test which had three problems. Students

were asked to solve problems in as many different approaches as they could without a time limitation and a calculator. Firstly, the correctness of the problems were checked. Then interviews were done with students to explain how they solve the problems. Findings showed that expert mathematicians found gifted students' approaches the least "beautiful" and not aesthetic, and approaches selected by expert mathematician's were not chosen by the students. The study also demonstrated that mathematically gifted students were unfamiliar with the looking-back, which was the fourth step of Polya's problem solving processes, to solution to find different approaches. They also were uninterested in aesthetic values in problem solving.

In sum, some researchers compared gifted students to average students while others investigated their problem solving processes. Results suggested that gifted students are better problem solvers when compared to their average peers. Furthermore, results of the study comparing gifted students to older average students overlap the Miller's gifted definition: being capable of solving problems which were taken from the mathematics curriculum at least 3 grades above their levels. Also, results regarding problem solving processes of gifted students after amount of instructions indicate that gifted students should get a special education since their problem solving levels increase significantly after instruction. The purpose of this study is to investigate non-routine problem solving process of mathematically gifted students without any instruction. With the results of this study, it is expected to contribute to the field by providing a detailed description of the mathematically gifted students' problem solving process on non-routine problems. Also, the results might shed light on mathematically gifted students' education through the use of non-routine problems support flexible thinking, data organization, recognition of pattern and relationships and the use of previous knowledge extensively (Souviney, 1989).

2.4. Significance of the Study

Problem solving is the most important component of mathematics education (MEB, 2018; NCTM ,2000). It is expected that problem solving in mathematics education contains the four steps discussed previously: understanding, thinking, planning and reasoning. Due to its nature, non-routine problems are one of the best way to make students think (Souviney, 1989; Altun & Arslan, 2007; Mullis et al, 2009; McLeod, 1988; Nancarrow, 2004). The use of steps also arouse the awareness of the problem solvers about the process they are going through. Polya stated that the last step "looking back" of his problem solving model can be found interesting by the students and; they might *have the consciousness of having done well* (1957). Since the motivation is a crucial component of being gifted (Gagne, 2005), the four phase model of Polya should be studied with mathematically gifted students to investigate their consciousness and to improve their motivation towards problem solving.

In addition, in Singapore and Korea, ranked having the highest level of mathematics achievement in TIMSS 1999 and TIMSS 2007, mathematics education focus on non-routine problems and strategy instructions from the first years of primary education (Ulu, 2011). Regarding the results of TIMSS 1999 and 2007, the rank of Turkey in mathematics exam is 31th over 38 countries, and 31st among 48 countries respectively. Therefore, with the results of this study, explicating the characteristics of mathematically gifted students' reasoning processes on non-routine problems, I aim at improving both gifted and average students' problem solving processes: various research has shown that mathematically gifted students show unique problem solving behaviors which cannot be observed in problem solving of regular students (Garofalo, 1993; Heinze, 2005; Koçyiğit, 2015; Montague, 1991; Sriraman ,2003; Threlfall & Hargreaves, 2008; Span and Overtoom-Corsmit ,1986). This study might not only provide cultural knowledge about the current situation by examining Turkish mathematically gifted students' non-routine problem solving processes (Tjoe, 2015), but also might shed light on the goals of special education needed by gifted students.

Furthermore, by the National Ministry of Education and National Science Foundation (TUBITAK), there is currently an effort for the development of mathematics curriculum for

the gifted and talented students in Turkey. It is clear that gifted students should get a special education to reveal their abilities (Karabey ,2010; Durmaz ,2014).Therefore, the findings of this study might extend the current research on Turkish gifted students, and help to plan and adjust classroom instruction of problem solving to give a better support to the education of gifted students.

3. STATEMENT OF THE PROBLEM

The aim of this study was to examine the Turkish mathematically gifted students processes of solving non-routine problems.

3.1. Research Questions

(i) What is the nature of problem solving processes that Turkish mathematically gifted students use as they engage in solving non-routine mathematical problems?

(ii) What steps of Polya's problem solving model do Turkish gifted students use when they engage in non-routine problem solving?

4. METHODOLOGY

In this chapter, the sample, the research design, the instrument, the procedure and the data analysis of the study will be explained. Sample section will include sampling method and description about the participants. In research design section, method of the study will be introduced. Instrument section will include the process of instrument construction and the characteristics of selected problems. Afterwards, the implementation of the study will be explained in detail with the procedure section. Lastly, the data analysis will be described.

4.1. Sample

In this study purposive sampling is used. In purposive sampling, the researcher selects a sample based on her purpose and experience (Gay, Mills, & Airasian, 2009). In this study, five mathematically gifted students were purposefully chosen based on the following common characteristics: being a member of Mathematics Olympiads Group organized by National Ministry of Education and their success in National Mathematic Olympiad regulated by TUBITAK, the National Science Foundation (NSF) in Turkey. Two of them attend to preparation class of high school, two of them are eighth grade students and one of them is seventh grade student.

As the first common characteristic, all participants were chosen to study upper level mathematics as a member of Mathematics Olympiads Group. Selection process of Mathematics Olympiads Group contains five stages in accordance with Miller's (1990) discovery techniques of mathematical talent: nomination, pre-evaluation, pre-selection exam, individual examination and observation, group examination and last-evaluation. The first stage is the nomination by school teachers with observation forms of students in school mathematics lessons. Then, these observation forms are examined as pre-evaluation stage. If the student passes these two parts, s/he is invited to participate in the pre-selection examination. This exam measures students' mathematical knowledge at their own level and their reasoning skills.

If they pass through this exam, students take courses from one Mathematics Olympiad teacher for two weeks. Three expert Mathematics Olympiad teachers attend to courses for observation of students' attention, motivation, answers, estimations and interpretations, or basically their situations during the course. Each teacher takes notes about all of the students individually. At the end of the courses, students take an exam related to the course schedule of two weeks duration. This group examination evaluates learning skills, problem solving skills, and whether students follow homework and content of courses which is an indication of their motivation to learning mathematics (Gagne, 2005). After the last exam, teachers have a meeting and report their opinions based on their analysis on each student's observations as well as their prior examinations. With the accord of all teachers, students are registered to Mathematics Olympiads Group. In this Mathematics Olympiads group, they learn higher school mathematics curriculum at 6th and 7th grades; and the level of university mathematics at 8th grade. Passing through the mentioned selection processes shows that participants are able to do mathematics at upper level relative to their peers. Their already being successful at their level's mathematics achievement tests and, their engagement in concepts and problems at least 3 grades above their mathematics curriculum are indicators for being mathematically gifted (Miller, 1990; Sternberg and Zhang, 1995).

As the second common characteristic, the participants in this study were at the top ten degree of TUBITAK Mathematics Olympiads. TUBITAK regulates national Olympiads every year. From one school, at most ten students who have high mathematical ability can enter Mathematics Olympiads. 5000 students as average enter the first phase of this Mathematics Olympiads every year. The most mathematically talented 50 students are determined in the first phase. After the first phase, those 50 students are invited to TUBITAK courses which are offered by academicians who are expert in mathematics. Then the second phase of TUBITAK Mathematics Olympiads is applied and 24 students get medals. Participants in this study passed first phase two years in a row. They are at top of 2-3% of the population as mathematical ability, suggesting that they have a special gift in mathematics (Miller, 1990; Gagne, 2005). Additionally, three participants got medals by passing the second phase of the exam.

4.2. Research Design

The design of this study is a case study, specifically the multiple case study. The case study is an in-depth description and analysis of a bounded system (Merriam, 2009). The number of gifted students is at the top %2 of the population. This group of students can be counted as a bounded system since the bounded system might be just one person or a group of people who have similar properties.

Also, among types of qualitative case study, multiple case fits the study because there are more than one gifted person and more than one non-routine problem in this study. Even if there is one problem, there will be data which is suitable for multiple comparison from six gifted students (Yin,2014; Şimşek & Yıldırım, 2011).

4.3. Instrument

Problems should be selected carefully to allow different solution approaches which are not immediately apparent, yet it should be accessible with mathematics knowledge consisting of algebra and arithmetic (Tjoe,2015). With this perspective, ten non-routine problems are selected from a Non-Routine Problems Book for 12th grade high school students (Duş et al., 2017). The writers of the book are mathematicians, and also some of the writers are academicians working on problem solving and non-routine problems. There are five chapters in the book. Ten problems were chosen from the first four chapters: Number and Transactional Skills, Real Life Problems, Algebraic Problems, Counting and Probability Problems. The last chapter was geometry. Because the geometry problems requires pre-knowledge to solve, no problems are chosen from this chapter. Ten problems are chosen from the ones not requiring formulas and promoting thinking. So, selecting ten problems at the beginning was a precaution for that not every solution will give data to describe the process and to form a pattern among obtained data.

The content validity of the problems were proceeded as follows: Firstly, ten non-routine problems were chosen by the researcher from the Non-Routine Problems Question Bank for

12th grade high school students. Then the researcher and a mathematics educator came together to have a consensus on the chosen problems as a non-routine problems. Then, three experts, who have spent more than ten years on Mathematics Olympiads, examined the problems. These experts had a consensus on eight of these problems based on the following criteria: First, the nature of non-routine problems requires not to have past experience with the exact problem or similar ones (Mullis et al, 2009; McLeod, 1988; Nancarrow,2004). In order to check this characteristic of non-routine problems, one of the experts, the teacher who has been teaching the participants for the last three years and, also the team leader of Mathematics Olympiads Group, provided feedback. Second, beyond transactional skills, the solution requires an ability to think flexibly, to organize and classify the data, to see relationships among them, and to expand the previous knowledge by exploring the relations (Souviney, 1989, p.66). In order to check this characteristic, three experts provided feedback. Based on these criteria, three experts had a consensus on two of the problems (Problem 5 and 8) to be taken out of the instrument, leaving eight problems to be used in total (See Table 4.1). Problems are given in Appendix A.

Table 4.1. Chapters of Selected Problems

Number	**Chapter**
Problem 1	Number and Transactional Skills
Problem 2	Number and Transactional Skills
Problem 3	Number and Transactional Skills
Problem 4	Real Life Problems
Problem 6	Real Life Problems
Problem 7	Algebraic Problems
Problem 9	Counting and Probability Problems
Problem 10	Counting and Probability Problems

4.4. Procedure

Two different data sources were used for gathering data for this study: paper and pencil test including eight problems and the structured task-based interviews (Yin, 2014; Goldin,2000). Problems were given to the students in a classroom with no time limitation (Tjoe,2015). Time should not be considered as an important component of a gifted problem solver since they spent more time in problem solving process (Garofalo,1993). The prompts in the protocol of paper and pencil test required participants to provide solutions in at least two different ways if possible. Paper-pencil test lasted approximately one and half hour.

Following the paper-and-pencil test, structured-task based interview were conducted with participants since the questions in the interviews were predetermined for making a systematic observation (Goldin, 2000). Interview questions are given in Appendix D. According to Polya's problem solving steps, the researcher interviewed each student individually to examine their problem solving processes. Using Polya's problem solving steps, the goal of the interview was to determine the similarities and differences in the participants' problem solving processes. During interviews, audiotape which is the most common recording was recorded. Audiotape is chosen instead of videotape for two main reasons. Non-verbal behaviors were not needed in the process, and interview were done by just one researcher and one participant in the room, not a crowded environment (Asan,& Montague, 2014). Interview sessions with each participant lasted approximately one hour in the classroom where they took the paper-pencil test.

4.5. Data Analysis

In this section, the written responses to paper and pencil test and the transcripts of interviews were used for analysis. Since each participant is a different case, within and cross case data analysis techniques fit to the nature of the study (Merriam, 2009). Within case technique was used to describe the processes of each participant presented while solving non-routine problems. Following the within case analysis, cross case technique was used to build a

general explanations of non-routine problem solving processes that fits each problem for all participants.

First of all, the researcher checked paper and pencil test results after the interview. The goal of the analysis of written data was to check if the problem is solved or not, and to gain an insight into what kinds of strategies and solutions participants used. It was important that problem can be correctly solved by the participant. Otherwise, the data cannot be used to make an inference for the problem solving process. Before the within and cross case analysis, unanswered problems were eliminated to clear the misleading data.

Following the elimination of unanswered problems, the researcher listened to all of the interviews. She transcribed the data by herself to gain more insight into participants' answers. Then the researcher read through all of the transcripts sentence-by-sentence with the goal of identifying whether problems are non-routine or not and examining the nature of non-routine problem solving process of mathematically gifted students. Based on the data, four problems were eliminated because of the incorrectness and being routine. After this elimination, four problems left, which equal to 20 sets of transcripts from 4 problems and 5 participants in order to make inferences and to find a pattern consisting of Polya's Four Phase Framework. Then, tables containing interview questions and the answers of each participants were constructed for each problem to see if there is a pattern. Analysis of each problem were done with respect to Polya's Four Phase Framework. Similarities and differences in participants' responses were written in details. Solutions of each participants were given in both paper and pencil test and interview transcripts. Their solutions were compared to find out strategy and approaches used by participants.

After the results of each problem were given, the researcher read the transcripts of each participant for all problems. The aim was to understand if the results were consistent with the findings coming from each problem analysis; and to check if there is a different pattern individually. Then, all of the transcripts were read with the advisor of the thesis to check the consistency. This part of the analysis supported first findings coming from cross-case analysis.

5. RESULTS

In this chapter, first I explain the results pertaining to the correctness of problems and point to the routine problems from the participants' perspectives. Then, I report on each problem by examining participants' problem solving processes in terms of Polya's Four Phase Framework. Lastly, I report on each participant's problem solving processes for a possible pattern within.

5.1. Correctness of Problems and Routine Problems

As already mentioned, eight problems were given to the participants during the paper-pencil test. I first examined the answers to these problems to check for their correctness. As shown in Table 5.1., right answers were marked as ✓, while wrong answers are marked as ×. After I checked for the correctness of the answers, I read transcripts of the interviews line by line to check if participants had seen the same problem or a similar one which might have been solved with the same solution method. Therefore, in Table 5.1., I also marked the problems already seen by the participants as R to indicate those problems became routine problems for the participants.

Table 5.1. Correctness of Problems & Routine Problems

	B	**E**	**İ**	**M**	**S**
Problem 1	✓	×	R	R	R
Problem 2	✓	✓	✓	✓	✓
Problem 3	✓	×	R	×	×
Problem 4	✓	×	✓	✓	✓
Problem 5	R	R	R	R	R
Problem 6	×	✓	✓	✓	✓
Problem 7	✓	×	✓	✓	✓
Problem 8	✓	R	×	✓	✓

As seen in Table 5.1., Problem 1 is solved just by participant B. Also, three of the participants indicated that they had past experiences with similar problems. In addition, one of the participants could not find the correct answer for Problem 1. Regarding Problem 2, all participants were able to correctly solve the problems, while Problem 3 is solved just by participant B again. In addition, three of the participants could not solve this problem correctly; and one participant stated that he solved a similar problem. Moreover, Problem 4 is correctly solved by four of the participants while Problem 5 turned out to be a random problem for these participants since they have seen similar ones solved in their classroom. Also, for Problem 6 and 7, four participants were able to correctly solve these problems while one participant could not reach the correct answer. Finally, Problem 8 is solved by three participants while one participant could not reach the correct answer and one participant stated that he solved a similar problem.

Therefore, as seen in the Table 5.1, participant B solved 6 problems correctly and one problem incorrectly while reporting one problem as a routine problem. Participant E was able to solve only two problems correctly while incorrectly solving four of the other problems and reporting two problems as routine ones. Participant I was able to solve four problems correctly and one problem incorrectly although stating that three of the problems were routine problems for him. Finally, Participant M and S both solved five problems correctly and one problem incorrectly. They also reported that two problems were random problems for them.

Based on the analysis above, to avoid the mislead of wrong solutions and routine problems, for further examination, I took into consideration of the problems that were solved correctly by *at least four* participants. Therefore, I did cross-case analysis on Problem 2, Problem 4, Problem 6 and Problem 7.

5.2.Results for Each Problem

In this section, I report on the analysis on Problem 2, Problem 4, Problem 6, and Problem 7 since these problems were solved correctly by at least four participants and determined to be non-routine problems for each participant. For the analysis, I first describe each participant's responses to the problems during both the paper and pencil test and the interview. Then, I provide my analysis on each participant's responses in terms of the problem solving stages of Polya.

5.2.1.Problem 2

Problem 2 was as follows:

> *"The first one of the 20 cows in a farm milks 1kg per day, the second 2kg, the third 3kg, ..., the 20th milks 20kg. Five partners of this farm decide to leave the partnership and want to share the cows as each of them gets an equal amount of milk. So, at most how many cows can any of the partners get?".*

As mentioned earlier, Problem 2 was solved correctly by all participants during the paper and pencil test. The results from the paper and pencil test and the interview transcripts were shown in Table 5.2.

Table 5.2. Interview and Paper-Pencil Results of Problem 2

Phases		B	E	İ	M	S
Understand the Problem	Unknown	✓	✓	✓	✓	✓
	Data	✓	✓	✓	✓	✓
	Is it possible to satisfy the condition?	✓	✓	✓	✓	✓
Devise a Plan	Same Problem	NO	NO	NO	NO	NO
	Similar Problem	NO	NO	NO	NO	NO
	Restate the problem	✓	✓	NO	✓	✓
	Did you use all givens of problem?	✓	✓	✓	✓	✓
Carry Out the Plan	Did you check all steps of your solution?	✓	✓	✓	✓	✓
	Can you prove that it is correct?	✓	✓	✓	✓	✓
Look Back	Did you check your result?	✓	✓	✓	✓	✓
	Can you derive the result differently	NO	✓	✓	✓	✓
	Can you use the result, or the method, for some other problem?	✓	✓	✓	✓	✓

As seen in Table 5.2., the participants did not have any difficulty of understanding the verbal statement of the problem. The unknown, the condition and the data given in the problem were questioned during the interview. Participants explained all of them correctly and clearly. Also, since they did not see any similar problem, Problem 2 was a non-routine problem for all of the participants. In addition, four participants restated the problem using different numbers, while one participant stated that there was no need to change the problem. Moreover, all of the givens in the Problem 2 were used by the participants; and it was possible to satisfy the condition for all the participants. They further stated that they did not find any unnecessary data which should be removed from the problem statement. They also checked their results both in paper and pencil test and during the interview. Lastly, four participants were able to find another solution for Problem 2 while one participant stated that it was not possible to solve the problem in a different way.

Participants' answers particular to the "Understanding the Problem" phase were provided in Table 5.3.

Table 5.3. Answers of Participants for Understand The Problem Phase of Problem 2

	Participant B	**Participant E**	**Participant I**	**Participant M**	**Participant S**
Unknown	How many cows one of the partners can get, in other words the number of cows that can be taken.	The proper distribution of cows when maximum number of cows can be taken by any of the partners. In other words, the maximum number of cows that any partner can get.	The unknown in this question is what is asked, that is, how many cows a person can get.	The number of cows taken by one who gets the most.	We will try to keep the number of unknown cows more. So, the question want what the maximum number of cows is.
Data	It is told us there are 20 cows in a farm. Then it is said that how much milk is given by cows respectively with the examples as the first cow milks 1kg, second one milks 2kg and so on twentieth cow milks 20 kg. Then it is told that there are five partners in this farm; and each of them gets equal amount of milk.	Now, we have the following information, there are 5 people, they break the partnership, and each person wants to get an equal amount of milk and the cows give milk in proportion to the sequence numbers. There are 20 cows.	There are 5 people who will get equal amount of milk.. Total amount of milk is 20 times 21 divided by 2.	Each person receives the cows which milks equal amount as total per person. And there are 1 to 20 cows.	The number of cows is given, which is 20. The amount of milks for each cow is given. And then, the number of partners is given, which is 5 people. There are 20 cows and each one milks proportional to its number.
Satisfying the condition	Yes	Yes	Yes	Yes	Yes

Particularly, data in Table 5.3 showed that all participants stated that "*the number of cows*" were the unknown in the problem context. For instance, participant M stated that the unknown was the number of cows taken by one who got the most. Similarly, all participants were able to state that data in the problem context included "*the number of cows, the number of partners, the amount of milk for each cow and the equality of milk amount in sharing*". They also stated that it was possible to satisfy the condition and solve the problem with the given data in the problem. Thus, the data showed that all participants were able to understand the verbal text of the problem. This showed that they were able to explain the main steps in Polya's first phase: the unknown, the condition and the data of the problem.

For the second phase "Devise a Plan", participants' answers in the interview were given in Table 5.4.

Table 5.4. Answers of Participants for Devise a Plan Phase of Problem 2

	Participant B	**Participant E**	**Participant I**	**Participant M**	**Participant S**
Same Problem	No	No	No	No	No
Similar Problem	No	No	No	No	No
Restate the problem	I can change the number of people. Or I can change the amount of milk that cows give. I can also change the number of cows.	The logic of this question is actually completely related to regulation. For example, instead of cow and milk, the feet numbers of animals in this farm can be used. Or we can write a money problem, for example we can try to get 20 TL with 5TL, 5Kr and 25Kr . This is a distribution problem. For example, it can be the number of feet, money or sugar, or the pocket money given in proportion to age or inversely proportional instead of the number of cows.	There is no need to rewrite this problem. It is ok like this.	For example, we cannot put 21 kg here. We can remove one cow which milks 15 kg and ask the same question again I mean, we can remove any cow which milks the number can be divided by 5.	Let's think a child who read a book 1 page on the first day, then 2 pages on the second day and continues with this pattern for 50 days. The number of pages... It doesn't seem like restating. Or, Let's think about a person who collects apple, he puts each apple into a different bucket every day, and at the end of the collecting he gives buckets to five friends. If everyone has the same number of apples, What is the most number of buckets given to one person?"
Use of all givens	Yes	Yes	Yes	Yes	Yes

As the data in Table 5.4 showed, all participants stated that they did not know the problem earlier. Similarly, they all stated that all of the givens in the problem were required in order to solve the problem. Furthermore, Participant I did not want to change the original problem, while three of the participants used the same verbal statement changing the numerical values. However, participant S restated the problem creatively changing the verbal statement of the original problem. Her restatement of the problem was as follows:

" Let's think about a person who collects apple, he puts each apple into a different bucket every day, and at the end of the collecting he gives buckets to five friends. If everyone has the same number of apples, What is the most number of buckets given to one person?"

As the data showed, Participant S restated the problem without changing the given numbers though she wrote a new problem that required the same solution as the original one. Thus, the data showed that all the participants had no previous experiences with the problem and they used all of the givens in the problem text. Though, only one participant was able to explore different aspects of the problem by creatively restating the problem while the others suggested simple numerical changes as restatement. This suggested that "Devise a Plan" phase of the problem solving process was attainable by all the participants.

In the following paragraphs, I depict in three tables, the participants' answers in the interview and paper-pencil test particular to "Carry Out the Plan" phase of the problem solving. Particularly, Table 5.5 shows data for participant B and S; Table 5.6 shows data for participant I and M; and Table 5.7 shows data for participant E. Furthermore, for the last phase "Looking Back", participants' answers in the interview were given in Table 5.8. Afterwards, "Carry Out the Plan" and "Looking Back" phases were explained together.

Table 5.5. Solutions of Participant B and S for Problem 2

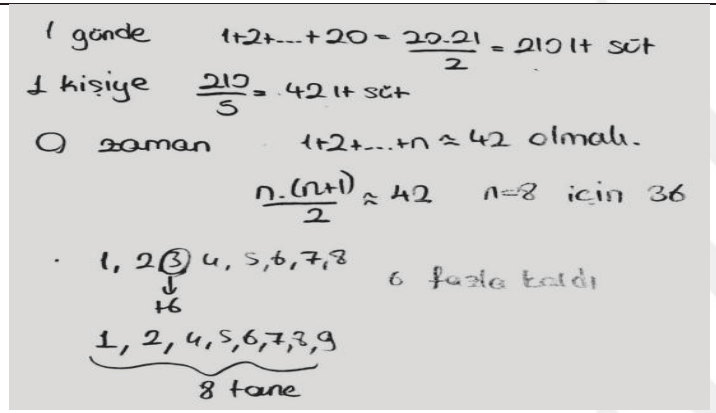
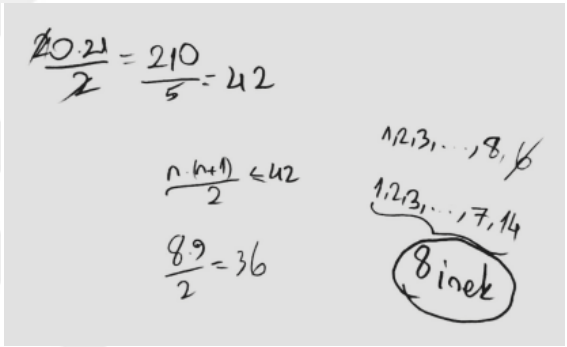
	Participant B	**Participant S**
Paper Pencil Test		
Interview	I've found the total milk produced in one day. 210 liters of milk is produced. Here, since five people will get equal amount of milk, each of them gets 42 liters of milk. Now, I need to bring at least the milking cows together to get the most cows. Then let me get all the cows starting from one to any number. If I take one to eight, I have 36 liters of milk. I got 6 liters of extra milk left. I need to share it. Now I will add the remaining 6 liters of milk to one of them and the number of cows will remain the same. In other words, the number of cows will remain 8. Or I have to combine two of the cows and get 9. But it didn't turn nine cows.	I found total milk first. I found total milk because each person will get an equal amount of milk. I found the sum with the formula " n times n + 1 divided by 2" . I found 42 with 20 times 21 divided by 2. I tried to select the cows with the least milk in order to increase the number of cows. I tried to get the lesser ones like 1,2,3. Then I found 42 like this: if the sum of numbers from 1 to 10 is equal to and less than 42, that number will be the answer I want. I found n times n + 1 divided by 2 is less than 42 or equal to it. The biggest number that provides this condition is 8. Here we have 36 from 8 and we still need 6 to add. But there's just one 6. So I took 7 cows instead of 8. This is the first 7 cows. I've added 8 and 6, in other words I added extra 6kg milk because I needed to reach 42. So I found 14. And there were 8 cows.

Table 5.6. Solutions of Participant I and M for Problem 2

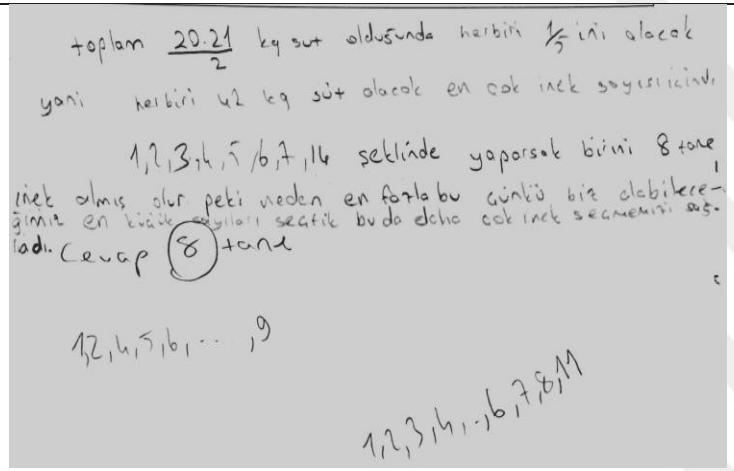
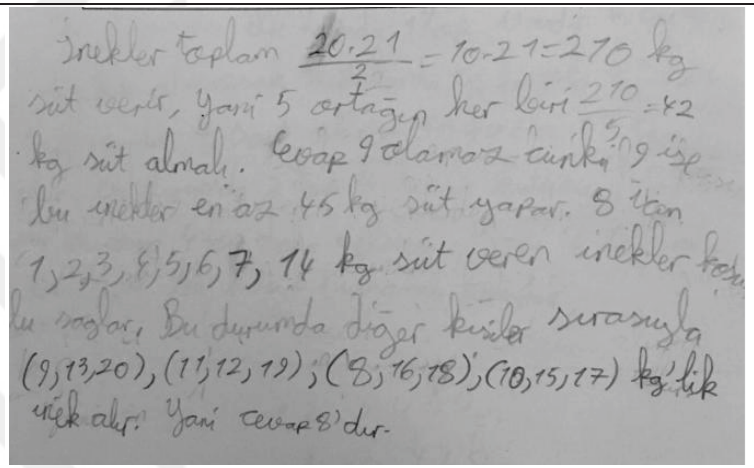
	Participant I	**Participant M**
Paper-Pencil Test		
Interview	Total amount of milk is 20 times 21 divided by 2. In the problem, it is stated that everyone will get equal amount of milk. Because there are 5 partners, everyone will get the one fifth of the total milk. When divided by 5, everyone gets 42kg of milk. It goes on as 1, 2, 3, 4,.., 20. Even the first 9 cows are taken, it is 45kg milk. So one person cannot 9 cows. If we give an example for eight cows, we will see how many cows a person will have. My example is 1 2 3 4 5 6 7 14. There are a few more examples. I started to go sequentially, so I started 1 2 3 4 5 6 7 but here we could not put 8 or 9. It cannot be 9. I choose a number which completes the sum to 42.	First of all, everybody will get an equal amount of milk. Let's find out the total amount of milk. Total amount of milk is 210 kg from the summation formula. If we divide it into 5, it should be 42kg per person. Then I found a sample of 8 cows first, 42. Then I thought I can show that 9 cows cannot be taken, and I showed it. In order for a person to get the most number of cows, he should take the cows which milks least like 1, 2, 3, 4, etc. If the cows from 1 to 8 are taken, its sum it 36kg milk. Then I reach 42 with all numbers 1 to 7 and 14. If I choose 8, the sum is 36 which is smaller than 42. If I choose 9, the it is greater. So, I chose the numbers from 1 to 7 and I took 14 instead of 8 and 9.

Table 5.7. Solutions of Participant E for Problem 2

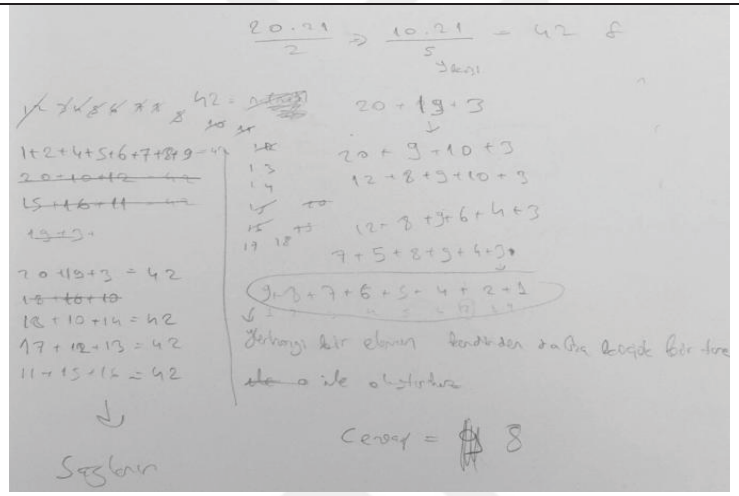
	Participant E
Paper-pencil Test	
Interview	Let me tell my solution. If there are 20 cows, it is 42 liters per person. Then I can get 42 with 20 + 19 + 3. Then, for example, I tried to obtain each term here with smaller numbers. For example, I wrote 9 + 10 instead of writing 19 that increased the number of cows. Then I wrote 20 as 12 + 8. Then I wrote 10 as 4 + 6. I wrote 12 as 5 + 7. The reason for that I chose close and small numbers, for example, we can write 2 as 1 + 1 but there is only one number 1. So, if I use 1 and 2, then I can use 3 when I have to use it. I tried to reduce my alternatives. Then I found that I could get 42 by subtracting 3 from the sum of the numbers 1 to 9. Then, when we look at the summation of numbers from 1 to 9, which is already 45. Then I tried to find 42. I knew that I could find it by subtracting 3 from the sum of the numbers 1 to 9, in other words I can fin 42 with the sum of 8 numbers. However, the num of number 1 to 8 is 36 which is less than the number which I try to find. I changed the cows to make it 42. Then I share remaining cows to other four partners to check my result.

Table 5.8. Answers of Participants for Looking Back Phase of Problem 2

	Participant B	**Participant E**	**Participant I**	**Participant M**	**Participant S**
Did you check your result?	Yes	Yes	Yes	Yes	Yes
Can you derive the result differently?	No	Instead of distributing it one by one, I could start to write directly from 1 to 9. When I reach 9, I subtract 3 and get 42. This solution could have been shortened.	Can we do it before we find out how many a person get? I think we can't do it. To show that there are 8 cows, we should show that it cannot be 9. Then we should give an example for 8 cows. Examples for 8 cows can vary, there are a lot of examples.	When we say another solution method, we need to show that it cannot be 9 in order to show that 8 will be exact.	I couldn't use another method. I can't do anything else because I have to get the least milking cows to increase the number of cows.
Can you use the results, or the method, for some other problem?	Yes I do.	Yes, absolutely.	When the maximum of something is asked, we showed what was not, and we gave an example to a small one, and we could use it in other questions. So when it is asked for the highest value of something, we can prove it cannot be higher than the answer we found by giving example or we give an example to our answer.	Yes, I use it for the problems require distribution.	As I told before, if there is a problem with the same logic, I can use it.

Data pointed to important results: First, as the data in tables (Table 5.5, Table 5.6 , Table 5.7, Table 5.8.) showed, all participants *focused on the quantities* given in the problem text, which was the total amount of milk. They all used the formula "n times n + 1 divided by 2" to

find out the total amount of milk as 210kg. They also knew why they used the formula: They had 1,2,3,… 20 kg of milk and to find out the total amount they had to add them. Then, they all calculated the milk amount per person, 42 kg, by dividing the total amount of milk with the number of people, 5. Therefore, they were all aware of the reasons behind the use of calculations. That is, all participants were able to *make sense of the procedures* they used such that they were *aware of the meaning of each step in their solutions*. In addition, data also showed that all participants were able to *associate numerical results with the givens* in the problem context. For instance, in order to reach more number of cows, three of the participants started to get cows which milked the least. For that ,first, they got the cows producing milk one to eight kg . However, they found out that the total amount would be 36kg which was less than the amount one person should get. So, they tried to get the cows from one to nine. They realized that this did not work either since the sum was 45kg. Therefore, they found the solution by changing one of the cows, producing milk from one to eight kg. For example, participant I and M changed the amount "eight" to "fourteen" while participant B changed the amount "three" to "nine". All these suggested that these participants *associated the numerical results with the givens* in the problem context. Though, participant E used a different method from these three participants. Still, results also suggested that he was able to *associate numerical results with the givens* in the problem context. He separated 42 into smaller numbers one by one. Particularly, he separated 42 firstly into 20+19+3, then continued as follows: 20+9+10+3 = 12+8+9+10+3 = 12+8+9+6+4+3 = 7+5+8+9+6+4+3 = 7+5+8+9+6+4+2+1

At the end, he also reached eight cows. In addition, even their methods differed, the numbers participant E reached was the same with the numbers participant B had chosen. Moreover, participant S used the sum formula to find out maximum cow numbers. At the end, the cows with different amount of milk she has chosen was also the same with participant I and M. This further indicated that all participants were able to *make sense of the procedures* they used such that they were *aware of the meaning of each step in their solutions*. Therefore, during the interview, they stated that they did not need to recheck the correctness of the steps. This is also an important result indicating to the following: First of all, all the participants

stated that they checked their solutions step by step during the paper and pencil test. Still, during the interview, these participants' explaining their reasoning by focusing on 1) *the quantities* and 2) what the numerical results refers to in terms of the quantities suggested that they *made sense of the procedures* they utilized. In addition, they were able to *associate the numerical results with the givens* in the problem context such as the most number of cows one can get and the amount of milk per person. All these suggested that these participants were able to *justify* their solutions since they were aware of the meaning of each step in their solutions. That is, they did not need to recheck their solution process because they were already aware of their solution process. This claim will be supported further with data from the solution processes of the participants in the other problems.

In sum, all these suggested that all the participants focused on the quantities in the problem context; were able to associate the numerical results with the givens; and were able to make sense of the procedures within each of the solution steps.

5.2.2. Problem 4

Problem 4 was as follows:

> *"Sevinç and Dilay takes sugar from a jar for10 days by the following rules given below.*
>
> *• On the first day, Sevinç takes candy, on the second day Dilay takes candy and they continue in this order.*
>
> *• For the first time Sevinç takes x candies, Dilay receives y candies.*
>
> *• Then the person who takes the candy, takes the number of candies which is taken for the last two days.*
>
> *• Dilay takes 66 candies more than Sevinç for 10 days.*
>
> *Since the total number of candies which is taken from the jar can be divided by 4, what is the number of candies Sevinç takes from the jar for her third time?"*

As mentioned earlier, Problem 4 was solved correctly by four participants during the paper and pencil test. Participant E's answer is not taken into consideration because it was incorrect. The results from the paper and pencil test and the interview transcripts were shown in Table 5.9.

Table 5.9. Interview and Paper-Pencil Results of Problem 4

Phases		**B**	**I**	**M**	**S**
Understand the Problem	Unknown	✓	✓	✓	✓
	Data	✓	✓	✓	✓
	Is it possible to satisfy the condition?	✓	✓	✓	✓
Devise a Plan	Same Problem	NO	NO	NO	NO
	Similar Problem	NO	NO	NO	NO
	Restate the problem	NO	✓	✓	✓
	Did you use all givens of problem?	✓	✓	✓	✓
Carry Out the Plan	Did you check all steps of your solution?	✓	✓	✓	✓
	Can you prove that it is correct?	✓	✓	✓	✓
Look Back	Did you check your result?	✓	✓	✓	✓
	Can you derive the result differently?	NO	NO	NO	NO
	Can you use the result, or the method, for some other problem?	✓	✓	✓	✓

As seen in Table 5.9., the participants did not have any difficulty of understanding the verbal statement of the problem. The unknown, condition and the data given in the problem were questioned during the interview. Participants explained all of them correctly and clearly. Also, since they did not see any similar problem, Problem 4 was also a non-routine problem for four participants. Moreover, three participants restated the problem using different numbers, while one participant could not restate it. In addition, all of the givens in the Problem 4 were used by the participants; and it was possible to satisfy the condition for all the participants. One participant further stated that there was unnecessary data in the problem statement which might have been removed. In addition, none of the participants determined a different solution for the problem.

Participants' answers during the interview particular to the first phase "Understand the Problem" were given in Table 5.10.

Table 5.10. Answers of Participants for Understand The Problem Phase of Problem 4

	Participant B	**Participant I**	**Participant M**	**Participant S**
Unknown	The number of candies taken by Sevinç in the third time is unknown.	In the problem, it is not given the number of candies for the first day. The numbers are given as x and y. So, we don't know them. Problem asks the number of candies taken at the third time. This is also unknown.	The number of candies that Sevinç takes in the third time.	Here, two people get candies, Sevinç and Dilay. It is asked that how many candies Sevinç takes for third time.
Data	Candies are taken for 10 days, they are taken in a certain order, first day Sevinç takes, second day Dilay takes, then Sevinç and go on like this. From the third day, the person who takes the candy takes as much as the number of candies taken in the previous 2 days. As a result of 10 days in total, Dilay gets 66 candies more than Sevinç. And the total number of candies is divided by 4.	It is said that Sevinç takes x number candies for the first time, and Dilay gets y number candies. The sequence for taking candy is given. Firstly Sevinç, then Dilay, then again Sevinç, they get candies in a determined order. For 10 days Dilay gets 66 candies more than Sevinç. And it is also given that the total number of candies is divided by 4.	The number of candies that Sevinç and Dilay takes at the first time. In other words, conditions of the problems are the data. There are five conditions. 4 conditions are in the beginning and there is also one condition at the end.	There is a jar and there are candies in the jar, Sevinç and Dilay take candies in order. They're getting candy for 10 days. The first time they get candies x and y. And at the end, total number of Dilay's candies are 66 more than Sevinç's. And, the total number of candies is divided by 4.
Satisfying the condition	Yes	Yes	Yes	Yes

As the data in Table 5.10 showed, all participants stated that the unknown in the problem context was "*the number of candies taken by Sevinç in the third time*". Similarly, they were able to state that data included the conditions given in the problem. For example,

Participant S stated data as "*There is a jar and there are candies in the jar, Sevinç and Dilay take candies in order. They're getting candy for 10 days. The first time they get candies x and y. And at the end, total number of Dilay's candies are 66 more than Sevinç's. And, the total number of candies is divided by 4*.". They also stated that it was possible to satisfy the condition and solve the problem with the given data in the problem. Participant S even stated that there was unnecessary information which was " *For the first time Sevinç takes x candies, Dilay receives y candies*.". She stated that even if "x and y" were not stated in the problem statement, she could have named them as "a and b". In addition, all participants constructed a table to organize the data (See Table 5.11 and Table 5.12). Thus, the data showed that all participants were able to understand the verbal text of the problem. This showed that they all were able to explain the main steps in Polya's first phase: the unknown, the condition and the data given in the problem.

Participants' answers during the interview particular to the second phase "Devise a Plan" were given in Table 5.11.

Table 5.11. Answers of Participants for Devise a Plan Phase of Problem 4

	Participant B	**Participant I**	**Participant M**	**Participant S**
Same Problem	No	No	No	No
Similar Problem	No	No	No	No
Restate the problem	I cannot restate this problem.	If I want to restate, I give the order of taking candies. It is said that they take candies for 10 days. I can make it any number can be divided by 2, such as 12 or 20. It is said that Dilay gets 66 candies more. I can change this number.	For example, Sevinç and Dilay put balls into a jar for 10 days according to the rule below. The first day, the next day, Dilay puts the ball. I give the total number of ball which are put in the jar. I change the names ,Dilay and Sevinç into another names.	I' change the names. Or maybe there are three people. I change 10 day to 25 days. I don't need to call x or y at first. I say that after the first three people takes candy, on the 4th day, the first person gets the total number of candies in the other 3 days. Then I tell the difference, and the total number of candies is divided by the number that will lead to solution.
Use of all givens in the problem	Yes	Yes	Yes	Yes

As the data in Table 5.11 showed, all participants stated that they did not know the problem earlier. Similarly, they all stated that all the givens in the problem were required in order to solve the problem. Also, three of the participants restated the problem except the Participant B who stated that she could not restate the problem. Though, similar to Problem 2, three of the restatements in this problem also included only the change of numbers or names given in the original problem. Participant S suggested to change the number of people and wrote a new rule by stating "*I say that after the first three people takes candy, on the 4th day, the first person gets the total number of candies in the other 3 days.*". Furthermore, Participant I focused on the association between the number of people and total number of candies. He stated that "*It is said that they take candies for 10 days. I can make it any number can be divided by 2, such as 12 or 20.*". In sum, data showed that participants had no previous experiences with the problem; used all of the givens in the problem text; and, were all tend to

restate the problem by changing names or numerical values. Even they could not restate the problem, they were all aware of the associations between quantities such that they offered the numerical changes without changing the solution method.

In the following paragraphs, I depict in two tables, the participants' answers particular to the third phase "Carry Out the Plan" during the interview and the paper and pencil test. Table 5.12 shows data for participant B and participant I, Table 5.13 shows data for participant M and participant S. Furthermore, for the last phase "Looking Back", participants' answers in the interview were given in Table 5.14. Afterwards, "Carry Out the Plan" and "Looking Back" phases were explained together.

Table 5.12. Solutions of Participant B and I for Problem 4

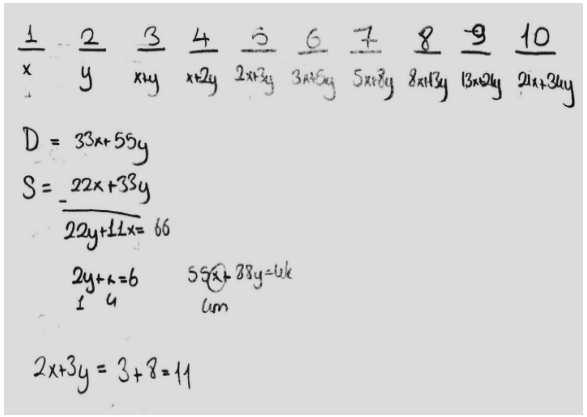
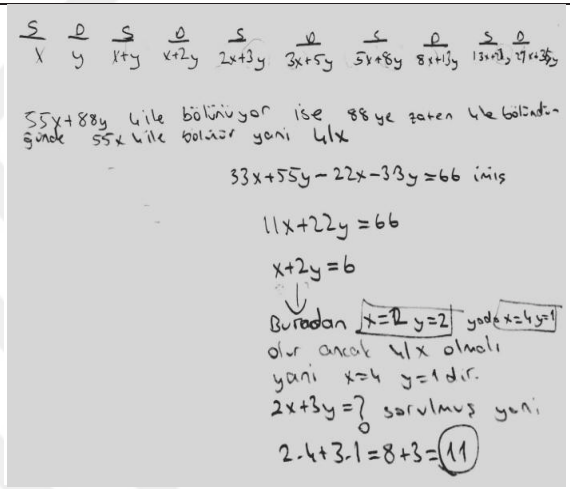
	Participant B	**Participant I**
Paper-Pencil Test		
Interview	I first found it one by one according to the days. I constructed a table, and write in it. Then I wrote the total candies received by Dilay and Sevinç. Their difference was 66. I found the difference. Then we have a very limited chance to try. It is stated that the total number of sugars can be divided by 4. Then I found that x should be divided by four. There was only one value for y. I found the values of x and y. Then the 5th day was the third day of Sevinç gets candy. And I found it.	It is stated that Sevinç gets x candies for the first time, Dilay gets y candies. After that, I wrote how many candies they take one by one to the table. It is stated that everybody took as much as the sum of the last two days. I wrote like x, y, x + y, x + 2y, etc. I've written everything for 10 days. For 10 days, Dilay gets 66 candies more than Sevinç. I found out how many candies Dilay took, then I found out how many candies Sevinç took. Then I found the difference and equalized it to 66. Also, the total number of candies is divided by 4. I found the total number of candies again from the table. 55x + 88y can be divided by 4. 88 can be divided by 4. So I said 55x must be divided by 4 because the sum is already divided by 4. 11x + 22y equals to 66. x + 2y equals to 6. Here x=2 and y=2 or x = 4 y = 1. But 55x has to be divided by 4. So it has to be x=4 and y=1.. There were two condition, and I reduced it to one condition. Then how many candies Sevinç takes for the third time is asked. It is 2x + 3y candies. I used the values of x and y, found 11.

Table 5.13. Solutions of Participant M and S for Problem 4

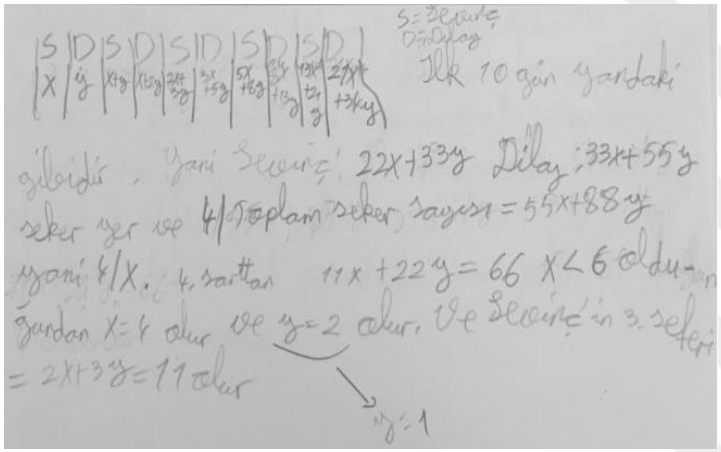
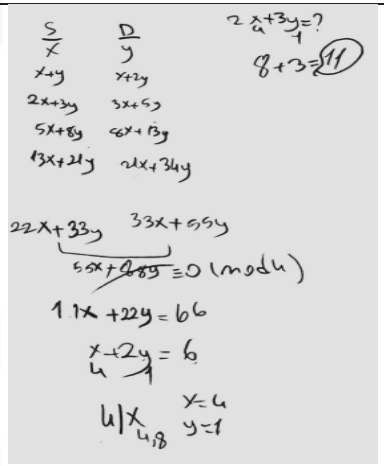
	Participant M	**Participant S**
Paper-Pencil Test		
Interview	Firstly, I wrote the number of candies they received for 10 days in the order Sevinç, Dilay, Sevinç, Dilay, etc. Then I wrote the number of candies they took for 10 days. Then I wrote the total number of candies received by Sevinç and Dilay. The total number of candies was 55x + 88y. It can be divided by 4. 88 is already divided by 4. We don't need to look at this. Since 55 is not divided by 4, x has to be. Because it is our condition that 55x can be divided by 4. That would be 4 divisions. By taking fourth condition, which is Dilay gets 66 candies more than Sevinç for 10 days, it is 11x + 22y = 66. x is divided by 4 and x is smaller than 6. So x = 4. For x = 4, y =1. So the third time of Sevinç is 2x + 3y = 11.	Then I found their total candies. The sum of Sevinç's candies is 22x + 33y, and Dilay's is 33x + 55y. I got the difference. I have found that the difference is 11x + 22y = 66 by using the condition Dilay gets 66 candies more than Sevinç for 10 days. I've divided it by 11. I found x + 2y = 6. Then I used the condition that the total number of candies taken from the jar can be divided by 4. Total sugar 55x + 88y is divided by 4. This is equal to 0 in mod4. 88y was already divided by 4. Because 55 cannot be divided by 4, x should be divided. I wrote 4 divides x. Because x cannot be zero, x must be 4. x + 2y has to be equal to 6. If x is 8, y must be minus 1. There is no such thing. Then I found that x has to be 4 and y equals to 1. The number of candies taken by Sevinç was 2x + 3y. I put 4 instead of x and 1 instead of y. Then I found the answer.

Table 5.14. Answers of Participants for Looking Back Phase of Problem 4

	Participant B	**Participant I**	**Participant M**	**Participant S**
Did you check your result?	Yes	Yes	Yes	Yes
Can you derive the result differently?	No, I don't think that there is another way than constructing a table. Because the solution might be confusing without a table.	No	No	No
Can you use the results, or the method, for some other problem?	Yes, absolutely I do.	Like this solution, I can use the method which is writing one by one according to the rules. Thanks to two given conditions of the problem , the possibilities reduced to one.	For a similar problems, I can say that I should construct a table to make the solution easier.	I can use this question as follows: the next number is always the sum of the previous 2 numbers, for example, I can do this in the form of a number pattern. Or if it is in the form of a reading page, for example, on the first day a pages are read, on the second day b pages are read, the other days always read the sum of the previous two days.

Data again pointed to important results: First, as the data in tables (Table 5.12, Table 5.13, Table 5.14) showed, all participants *focused on the quantities* given in the problem text, candies taken by Sevinç and Dilay. They constructed a table to organize their solutions and to show the givens systematically such that they all wrote the number of candies taken per day and per person. Then they found the total amount of candies for Sevinç "22x+33y", for Dilay "33x+55y" and for both of them"55x+88y". This showed that knew what those algebraic expressions referred to. Then they subtracted Sevinç's total candy for 10 days from Dilay's total candy and found out "11x+22y". Therefore, they were all aware of the reasons behind the use of calculations. This showed that all participants were able to *make sense of the procedures* they used such that hey were *aware of the meaning of each step in their solutions*. In addition, data showed that all participants were able to *associate numerical and* symbolic results with the given condition, "Dilay takes 66 candies more than Sevinç for 10 days", in the

problem context. That is how they found 11x+22y=66. They all abbreviated the equation with 11, and found out x+2y=6. Furthermore, they *associated the total number of candies with the given condition* " such that they knew that "total number of candies which is taken from the jar can be divided by 4" in the problem text. So, this condition led them to decide that 55x+88y should be divided by 4. This way they all focused on 55x to make, 55x+88y, divided by 4 since they knew that 88y was divisible by 4. They also were able to *associate the symbolic results* such as x+2y=6 and 55x to be divided by 4, so that they were left with just one possibility for x=4 and y=1.Therefore, they plugged 4 in x, 1 in y; and found the number of Sevinç's candies in the third time. This further indicated that all participants were able to *make sense of the procedures* they used such that they were *aware of the meaning of each step in their solutions*.

Furthermore, they all again stated that they checked their solution step by step during the paper and pencil test. They did not need to recheck it during the interview. Though again, I make the following claim: During the interview, these participants' explaining their reasoning by focusing on 1) the quantities and 2) what both the numerical and the symbolic results refers to in terms of the quantities suggested that they made sense of the procedures they utilized. In addition, they were able to associate the numerical and the symbolic results with the givens in the problem context such as x+2y=6 and 55x to be divided by 4. All these suggested that these participants were able to *justify* their solutions since they were aware of the meaning of each step in their solutions. That is, they did not need to recheck their solution process because they were already aware of their solution process. This claim again will be supported further with data from the solution processes of the participants in the following problems.

In addition, regarding the "Looking Back" phase, Participant S mentioned that examples of similar problems can be solved by the same solution method: "*The next number is always the sum of the previous 2 numbers, for example, I can do this in the form of a number pattern. Or if it is in the form of a reading page, for example, on the first day a pages are read, on the second day b pages are read, the other days always read the sum of the previous two days*.".

In sum, all these suggested that all the participants focused on the quantities in the problem context; were able to associate both the numerical and the symbolic results with the givens; and were able to make sense of the procedures within each of the solution steps.

5.2.3. Problem 6

Problem 6 was as follows:

"• a and b are integers.

• The first two numbers of a number pattern are a and b respectively.

• The third number in the pattern is obtained by subtracting the first number from the second number. Accordingly, the third number is b - a.

• Each number in the pattern is obtained by subtracting the previous number from the former number. For example, the seventh number = the sixth number - the fifth number

What is the sum of the first 700 numbers in this number pattern?"

As mentioned earlier, Problem 6 was solved correctly by four participants during the paper and pencil test. Participant B's answer was not taken into consideration because it was incorrect. The results from the paper and pencil test and the interview transcripts were shown in Table 5.15.

Table 5.15. Interview and Paper-Pencil Results of Problem 6

Phases		E	I	M	S
Understand the Problem	Unknown	✓	✓	✓	✓
	Data	✓	✓	✓	✓
	Is it possible to satisfy the condition?	✓	✓	✓	✓
Devise a Plan	Same Problem	NO	NO	NO	NO
	Similar Problem	✓	✓	✓	✓
	Restate the problem	✓	✓	✓	✓
	Did you use all givens of problem?	NO	NO	NO	NO
Carry Out the Plan	Did you check all steps of your solution?	✓	✓	✓	✓
	Can you prove that it is correct?	✓	✓	✓	✓
Look Back	Did you check your result?	✓	✓	✓	✓
	Can you derive the result differently	NO	NO	NO	NO
	Can you use the result, or the method, for some other problem?	✓	✓	✓	✓

As seen in Table 5.15, the participants did not have any difficulty of understanding the verbal statement of the problem. The unknown, the condition and the data in the problem context were questioned during the interview. Participants again were able to explain all of them correctly and clearly. Since they had not come across a similar problem, problem 6 was also a non-routine problem for all of the participants. In addition, all participants restated the problem. All givens in the problem were used by the participants; and it was possible to satisfy the condition for all participants. In addition, they all stated that there were unnecessary data that might be have been removed from the problem context.

Participants' answers particular to the "Understanding the Problem" phase were provided in Table 5.16.

Table 5.16. Answers of Participants for Understand The Problem Phase of Problem 6

	Participant E	**Participant I**	**Participant M**	**Participant S**
Unknown	The unknown as it is told in the problem, summation of first 700 numbers.	It is asked that the sum of the first 700 numbers is equal to what . We don't know it.	First and second number. Numbers of pattern.	There are unknown numbers, we need to find the sum of the first 700 numbers.
Data	First two terms and how to order them is given. The rule is given. the other terms. He gave me the rule, then an example is given to explain the rule. Then it is asked to solve the problem by using those given. In other words, it is asked to find unknown.	First two numbers are told. There is a rule by using them. The third number is obtained by subtracting first number from the second number. So there is a rule given.	First and second number, and a condition is given. The third number is obtained by subtracting the second number from the first number. Also an example is given.	First two numbers are given. Then it is said that each number in the pattern is obtained by subtracting the previous number from the former number. For example, we find third number b-a, forth number -a, fifth number -b, etc according to the rule.
Unneces-sary Data	Instead of giving the example " the third number is obtained by subtracting first number from the second number, just the rule could be given. Or just one example was enough to understand.	In fact, there are extra information here. because the rule is given like four times. Firstly it is told that third number is obtained by subtracting first number from the second number. Then it is said that each number in the pattern is obtained by subtracting the previous number from the former number. It was enough to give one of them. For example, the 7th number is obtained by subtracting the 5th number from the 6th number and the third number is obtained by subtracting the first number from the 2nd. it was enough to give one of them.	It was not necessary to give fourth condition and example.	Each number in the pattern was obtained by subtracting the number from the previous number was sufficient. So there is no need to tell about the third number. Only the 4th condition was enough. I didn't need an example
Satisfying the condition	Yes	Yes	Yes	Yes

Particularly, data in Table 5.16 showed that all participants stated that "*the sum of first 700 numbers*" were the unknown. Similarly, all participants were able to state that data in the problem context included "*the first two numbers of the pattern, the rule of the pattern which is each number in the pattern is obtained by subtracting the previous number from the previous number, and two examples for the application of the rule*". They also stated that it was possible to satisfy the condition and solve the problem with the given data in the problem. In addition, participants stated there was unnecessary data in the problem which are two examples following the rule of the pattern. They all stated that just the rule or one example might have been enough to write the following items of the pattern. For instance, Participant I stated that "*The rule is given like four times. Firstly it is told that third number is obtained by subtracting first number from the second number. Then it is said that each number in the pattern is obtained by subtracting the previous number from the former number. It was enough to give one of them. For example, the 7th number is obtained by subtracting the 5th number from the 6th number and the third number is obtained by subtracting the first number from the 2nd. It was enough to give one of them.*". In addition, all participants constructed a table to organize the data (See Table 5.18 and Table 5.19).

In sum, the data showed that all participants were able to understand the verbal text of the problem. This showed that they all were able to explain the main steps in Polya's first phase: the unknown, the condition and the data given in the problem.

Participants' answers during the interview particular to the second phase "Devise a Plan" were given in Table 5.17.

Table 5.17. Answers of Participants for Devise a Plan Phase of Problem 6

	Participant E	**Participant I**	**Participant M**	**Participant S**
Same Problem	No	No	No	No
Similar Problem	No	No	No	No
Restate the problem	As I said, I would not give examples. In this problem, there is a rule that we can easily see the difference. I would write a rule which doesn't immediately lead to write systematically. I would make the rule for the pattern harder to notice.	Instead of first 700 numbers, we can write another number after finding a repeat in 6. In is said that each number in the pattern is obtained by subtracting the previous number from the previous number. instead of this rule, it can be said that each number in the pattern is obtained by subtracting the two previous number from the former number. Or we'll find another rule.	Instead of a number, a ball or candy can be taken. And it could have been the first 600 instead of the first 700.	How would I do it ... I'd give 3 numbers at the beginning. I'd start with a, b, c. The fourth number can be found by subtracting c from the sum of a and be. I don't know if it continues and repeats at some point, but I would write a series with has repetitions like the original problem.
Use of all givens in the problem	Yes	Yes	Yes	Yes

As the data in Table 5.17 showed, all participants stated that they did not know the same or similar problem earlier. Similarly, they all stated that all the givens in the problem were required in order to solve the problem. Also, three of the participants restated the problem. Though Participant I stated that some parts of the problem could have been changed. However, he did not change any of them. In addition, Participant S restated the problem as follows: " *I'd give three numbers at the beginning. I'd start with a, b, c. The fourth number can be found by subtracting c from the sum of a and b. I don't know if it continues and repeats at some point, but I would write a series with has repetitions like the original problem.*". Even it is a numerical change, Participant S suggested to change the number of starting figures and to write a new rule by associating it with the original rule.

In sum, data showed that participants had no previous experiences with the problem; they used all of the givens in the problem text; Participant E, I and M were all tend to restate the problem by changing names or numerical values while Participant S restated the problem. Even they could not write a different problem, they were aware of the associations between quantities because they offered that numerical changes without changing the solution method.

Also, participants' answers particular to the third phase "Carry Out the Plan" during the interview and the paper and pencil test were given in two tables. Table 5.18 shows data for participant E and participant I, Table 5.19 shows data for participant M and participant S. Furthermore, for the last phase "Looking Back", participants' answers in the interview were given in Table 5.20. Afterwards, "Carry Out the Plan" and "Looking Back" phases were explained together.

Table 5.18. Solutions of Participant E and I for Problem 6

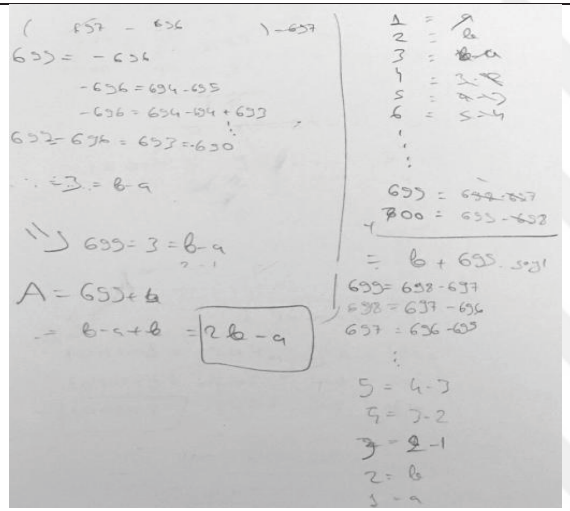
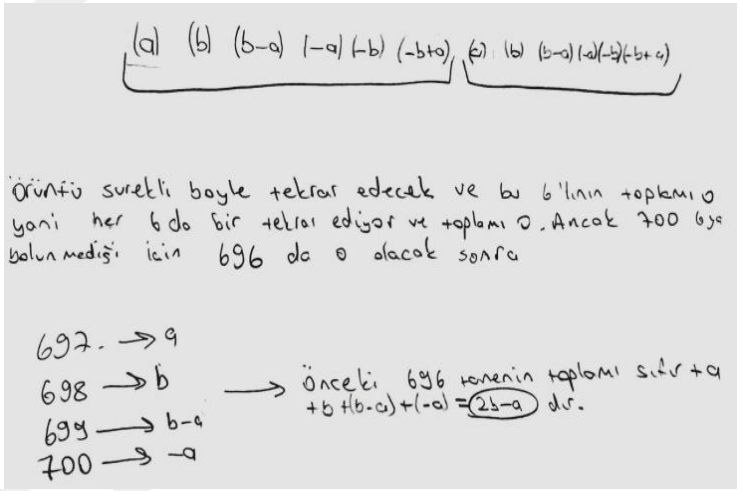
	Participant E	**Participant I**
Paper-Pencil Test		
Interview	First of all, I wrote every number systematically to simplify the addition and subtraction. I thought that I may find a nice pattern if I write one under the other. Then I named the sum of first 700 numbers as A. Then I noticed that A equals to the second number and 669th number. Second number was b. I had to find 699th number. Then I wrote the numbers backwards from 699th to one. While writing it, I noticed that 669 equals to minus 696, 696 equals to +693, 693 equals to minus 690. In other words, if we carefully look at it, we find that the odd numbers can be divided by 3 are equal to even numbers can be divided by three. So 699 is an odd number can by divided by 3, and it equals to other odd numbers divided by three. It is b-a. So 699 equals to +b. And the sum is b-a+b =2b-a .	696 numbers are repeating and their sum is zero. So I just wrote 697, 698, 699 and 700 to find the sum of first 700 numbers. They're repeating a, b, b-a, -a, -b, -b + a. Actually I divided 100 into 6, and the remaining was 4. So I've written the last four numbers. Then I added the last four numbers and found the answer.

Table 5.19. Solutions of Participant M and S for Problem 6

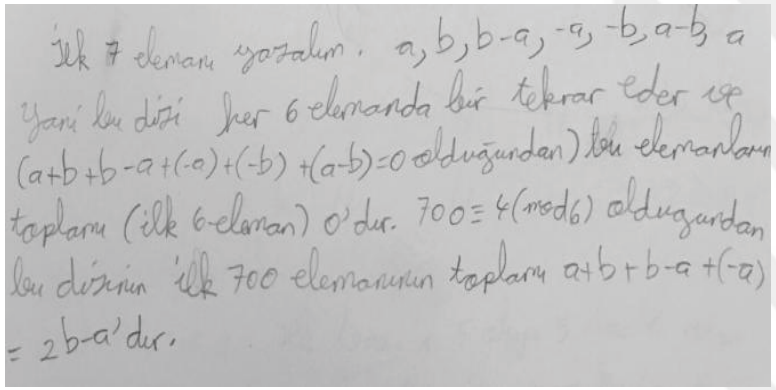
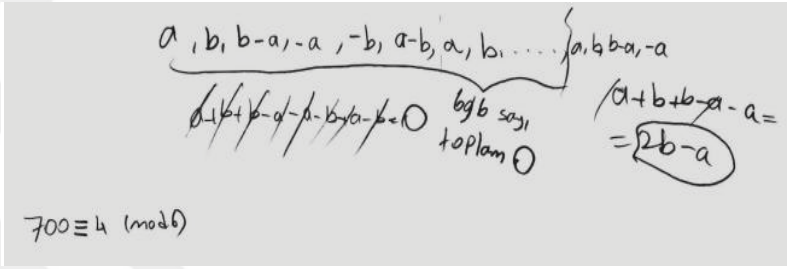
	Participant M	**Participant S**
Paper-Pencil Test		
Interview	I started writing terms first. According to the given condition, they are a, b, b - a, - a, - b, a-b. And after the first six terms, I saw that the 1st, the 7th and 13th numbers were the same. And I found that the sum of numbers 1 to 6 was zero. If this is zero, the remaining of 700 divided by 6 is 4. So we have the first 4 terms to look at. Its sum is 2b-a.	It's repeating for every six numbers. Therefore I found that the value of 700 in mode 6 is 4. So, 4 numbers remained at the end. I found the sum of first 6 numbers is zero. So up to 696 the sum was always zero. Then I wrote 4 more numbers to reach 700 numbers. Those numbers were a, b, b-a, -a. And I added them, found the result.

Table 5.20. Answers of Participants for Looking Back Phase of Problem 6

	Participant E	**Participant I**	**Participant M**	**Participant S**
Did you check your result?	Yes	Yes	Yes	Yes
Can you derive the result differently?	Instead of writing systematically, hmmm, it asks for first 700 numbers. Therefore, as in the previous question, I would have noticed a equality here. For instance, if I had examined the numbers first rather than directly calculating, I could reach 699 numbers which is + b. As soon as I reached 699th number, I could write the answer more quickly. But in this way, systematic writing made me see the answer.	No	No	I cannot recall it. It was supposed to find the pattern. We're not going to try to add all numbers one by one without finding the pattern. It would be an absurd way to solve the problem. If I had four and a half hours, and I could not find my own method, I would think of writing one by one.
Can you use the results, or the method, for some other problem?	There is a high probability that I use it. There will be a series of patterns, and it will require a similar method.	Because the sum of first 700 numbers is asked, there has to be a pattern. So if I face with a similar problem, I use the same method and try to find a rule.	I cannot say that I will use.	I may use it.

Data one more time pointed to important results: First, as the data in tables (Table 5.18, Table 5.19, Table 5.20) showed, three participants began solving the problem with writing terms. They all *focused on repeating quantities* (e.g. numbers) to find a pattern. They were aware of the fact that they should not write all first 700 terms to find the sum. Then, they found out terms repeating in every six numbers. In other words, first and seventh terms were the same such that the sum of every six numbers was zero. This way they *associated this result with the sum of first 700 numbers* such that they decided to divide 700 by 6. Therefore, they were all *aware of the reasons behind the use of calculation*. The remainder in the division was 4. They also interpreted the remainder 4 as the first four terms. All these showed that they knew what 6 and 4 referred to. That is, since the sum of every six number was zero, the

answer was the sum of first 4 numbers, which was 2b-a. All this showed that all participants were able to *make sense of the procedures* they used such that they were *aware of the meaning of each step in their solutions*. Though participant E used a different solution method, *associating the terms with each other* by finding equality among them. For example, he stated that "*669 equals to minus 696, 696 equals to +693, 693 equals to minus 690*". Then the terms canceled each other. Just two terms left which were the second term "b" and 699th term “b-a” making the sum again 2b-a. Although differing in solution process, this still indicated that he was able to *make sense of the procedures* he used such that he was *aware of the meaning of each step in the solution.*

Again, they all stated that since they checked their solutions during the paper and pencil test, they did not need to recheck it during the interview.

Still I make the following claim: During the interview, these participants’ explaining their reasoning by focusing on 1) the quantities (the numbers in this case) and 2) what the numerical results such as 4 and the symbolic results such as 2b-a refers to in terms of the quantities in the problem context suggested that they made sense of the procedures they utilized. In addition, they were able to *associate the numerical results with the givens* in the problem context such as the sum of every six number being zero and 4 referring to answer of the sum of the first four numbers. All these suggested that these participants were able to *justify* their solutions since they were aware of the meaning of each step in their solutions. That is, they did not need to recheck their solution process because they were already aware of their solution process.

In sum, all these suggested that all the participants focused on the quantities in the problem context; were able to associate both the numerical and the symbolic results with the givens; and were able to make sense of the procedures within each of the solution steps.

5.2.4. Problem 7

Problem 7 was as follows:

> *"Positive integers that can be written using only the numbers 1, 5 and 8 are ordered from small to large shown as below.*
>
> *1, 5, 8, 11, 15, 18, 51, 55, 58, 81, ...*
>
> *What is the 364 element of the number sequence obtained in this way?"*

As mentioned earlier, Problem 7 was solved correctly by four participants during the paper and pencil test. Participant E's answer was not taken into consideration because it was incorrect. The results from the paper and pencil test and the interview transcripts were shown in Table 5.21.

Table 5.21. Interview and Paper-Pencil Results of Problem 7

Phases		B	I	M	S
Understand the Problem	Unknown	✓	✓	✓	✓
	Data	✓	✓	✓	✓
	Is it possible to satisfy the condition?	✓	✓	✓	✓
Devise a Plan	Same Problem	NO	NO	NO	NO
	Similar Problem	NO	NO	NO	NO
	Restate the problem	✓	✓	✓	✓
	Did you use all givens of problem?	✓	✓	✓	✓
Carry Out the Plan	Did you check all steps of your solution?	✓	✓	✓	✓
	Can you prove that it is correct?	✓	✓	✓	✓
Look Back	Did you check your result?	✓	✓	✓	✓
	Can you derive the result differently	NO	NO	NO	NO
	Can you use the result, or the method, for some other problem?	✓	✓	✓	✓

As seen in Table 5.21, the participants did not have any difficulty of understanding the verbal statement of the problem. The unknown, the condition and the data of the problem was questioned during the interview. Participants explained all of them correctly and clearly. Since they did not see a similar or the same problem, Problem 7 was a non-routine problem for all of the participants. Also, four participants restated the problem. In addition, all of the givens in the Problem 7 were used by the participants; and it was possible to satisfy the condition for all the participants. They further stated that they did not find any unnecessary data which should be removed from the problem statement. In addition, none of the participants determined a different solution for the problem.

Participants' answers particular to the "Understanding the Problem" phase were provided in Table 5.22.

Table 5.22. Answers of Participants for Understand The Problem Phase of Problem 7

	Participant B	**Participant I**	**Participant M**	**Participant S**
Unknown	364th element of the number sequence.	The unknown is the 364th element.	The unknown is the 364th element.	364th number is asked
Data	We are creating numbers and we use only numbers 1-5-8 in this issue. These numbers are sorted from small to large.	We can only use the numbers 1, 5, 8. it is also important to be sorted from small to large. We're going to sort all the numbers.	The numbers are written by numbers using only 1,5,8 figures, and they are sorted from small to large.	Here, we can sort the numbers from small to large by using numbers 1,5, and 8.
Satisfying the condition	Yes	Yes	Yes	Yes

As the data in Table 5.22 showed, all participants stated that the unknown in the problem context was "*the 364th element*". Similarly, all participants were able to state that data in the problem context included "*the numbers are written by numbers using only 1,5,8*

figures; and these numbers are sorted from small to large". They also stated that it was possible to satisfy the condition and solve the problem with the given data in the problem context. Three participants stated that there was not any unnecessary data given in the problem while Participant S stated that "*It is unnecessary to give the examples beginning with 51 because it was enough to show 11 in order to understand that I can use the same number twice*.". In addition, all participants constructed a table to organize the data (See Table 5.24 and Table 5.25).

In sum, the data showed that all participants were able to understand the verbal text of the problem. This showed that they all were able to explain the main steps in Polya's first phase: the unknown, the condition and the data given in the problem.

Participants' answers during the interview particular to the second phase "Devise a Plan" were given in Table 5.23.

Table 5.23. Answers of Participants for Devise a Plan Phase of Problem 7

	Participant B	**Participant I**	**Participant M**	**Participant S**
Same Problem	No	No	No	No
Similar Problem	No	No	No	No
Restate the problem	I change the given digits and the asked number.	Here, the digits 1,5,8 is not important unless it is zero. Even if it says 1,4,7 instead of 1,5,8, the result wouldn't change.	I could say the numbers doesn't have the same digits twice. Also, I can add or remove some of the information.	I increase the number of digits. I can make four, for example, again. Again, I choose something special the first number of the 5 or 6 digit numbers. Then I ask that number.
Use of all givens in the problem	Yes	Yes	Yes	Yes

As the data in Table 5.23 shows, all participants stated that they did not know the problem earlier. Similarly, they all stated that all the givens in the problem were required in order to solve the problem. Also, they restated the problem by changing the numerical values. For instance, Participant I emphasized the importance of zero given as a starting digit by stating that "*Digits 1,5,8 are not important unless it is zero. Even if it says 1,4,7 instead of 1,5,8, the result wouldn't change.*". Furthermore, Participant S restated the problem by changing more than one numerical values. She stated that "*I increase the number of digits. I can make four, for example, again. Again, I choose something special the first number of the 5 or 6 digit numbers. Then I ask that number.*".

In sum, data showed that participants had no previous experiences with the problem; used all of the givens in the problem text; and, were all tend to restate the problem by changing numerical values. Even they could not paraphrase the problem in their own words, they were aware of the associations between quantities because they offered that numerical changes without changing the solution method.

Participants' answers particular to the third phase "Carry Out the Plan" in the interview and the paper and pencil test were given in two tables. Table 5.24 shows data for participant B and I; and Table 5.25 shows data for participant M and S. Furthermore, for the last phase "Looking Back", participants' answers in the interview were given in Table 5.26. Afterwards, "Carry Out the Plan" and "Looking Back" phases were explained together.

Table 5.24. Solutions of Participant B and I for Problem 7

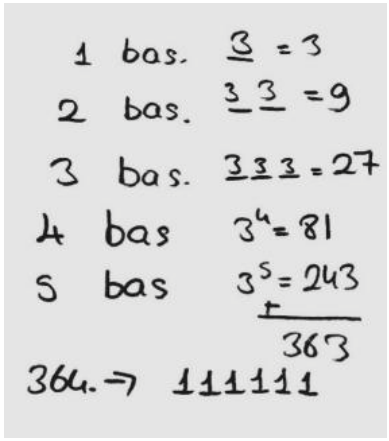
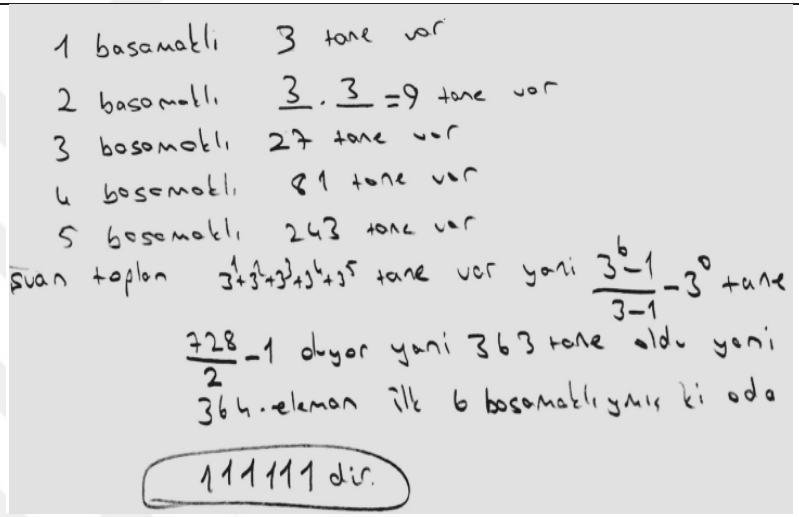
	Participant B	**Participant I**
Paper-Pencil Test		
Interview	I counted one by one. I tried to approach 363. I counted one digit numbers and two digit numbers. So I continued up to five digit numbers. Now I found 3 numbers for one digits, 3 times 3 or 9 numbers for two digits. With the same pattern, I continues till five digits. There was 243 five digit numbers. If I had passed six digit numbers, I would have passed the number 364. I added one digit to five. It was 363. And I realized that this number is the smallest number I can get in 6 digits. I wrote it by myself.	There was 3 one digit numbers. Then 9, 27, 81,.. I added them. When I calculate five digit numbers, I found 243. I realized that I approached to 364. I stopped and added all of them. It was 363. If I continued, the number was going to be greater than 364. 364th number was the first six digit number. Under these conditions, it is 111111.

Table 5.25. Solutions of Participant M and S for Problem 7

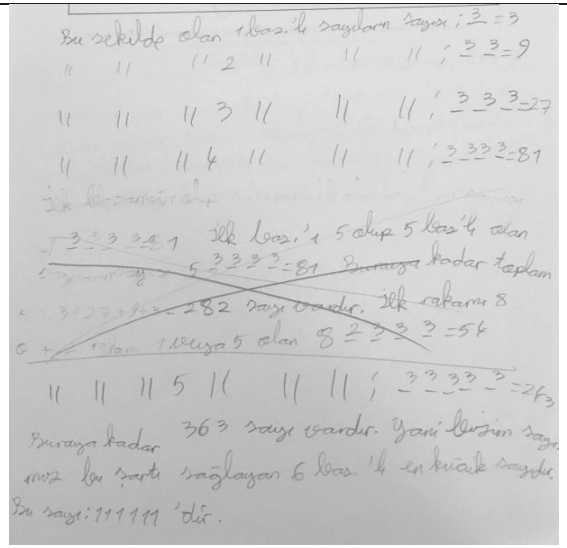
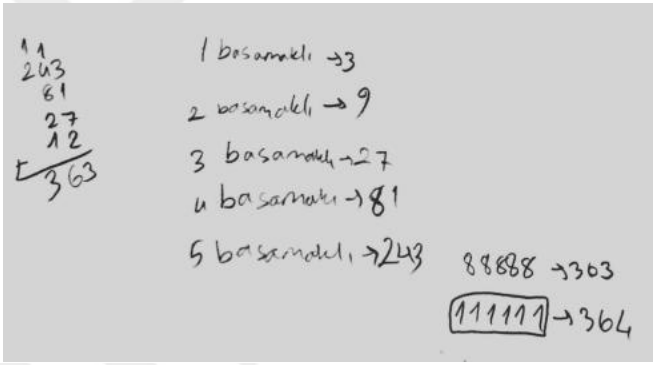
	Participant M	**Participant S**
Paper-Pencil Test		
Interview	First I looked at the numbers that had digits 1 to 5. If it is a one digit number, it is 3 numbers. Then for two digit number, it is 3 numbers for the first digit and 3 numbers for the second digit, and equals to 9. I continued in this way, 3 9 27 81 243, their sum was 363. 364th number was the first six digit number, 111111.	I thought that there could be three numbers with one digit. It's 9 for two-digit, and that's always going on as the forces of 3, because I put a line in my mind for each digit and there were three numbers that could come to the line. 2 digits has 2 lines which had 3 possibilities for each digit. 3 times 3 equals to 9. 3 digits has 3 lines which had 3 possibilities for each digit. 3 times 3 times 3 equals to 27. I continues like that, and I wanted to have something close to 364. I stopped when it came to 5 digits because 364th was in 6 digits. The 4 digits did not come too close to the 364, so I wrote and added it up to 5 digits and found sum. The biggest number to be written in 5 digits is 363. 364th is asked, and 364th will be the smallest number that can be written in 6 digits.

Table 5.26. Answers of Participants for Looking Back Phase of Problem 7

	Participant B	**Participant I**	**Participant M**	**Participant S**
Did you check your result?	Yes	Yes	Yes	Yes
Can you derive the result differently?	No	No	No	If I didn't think of writing a two-digit three-digit, it would be a way of writing one by one, but I probably wouldn't do that.
Can you use the results, or the method, for some other problem?	Absolutely, I use.	I use it again for the solution of similar problems.	If I face with a problem with digits, I use it.	I can use it in difficult questions, repetitive or not. You can use the table again.

One more time, data pointed to important results: First, as the data in tables (Table 5.24, Table 5.25, Table 5.26) showed, all participants again *focused on the quantities* given in the problem text, digits 1, 5, 8. They constructed a table to organize their solutions and to show the givens systematically such that they all wrote how many 1 digit numbers, 2 digit numbers, etc. They were *all aware of the meaning of each step in their solutions*. Thus, they continued calculating up to 5 digit numbers. They all stopped calculating after finding 5 digit numbers; and found the sum of all findings to check that the result was close to 364th number or not. This showed that all participants were able to *make sense of the procedures* they used such that they were *aware of the meaning of each step in their solutions*. The sum of one to five digit numbers was 363. Therefore, they all found out that the 364th number was the smallest number in 6 digit numbers. Furthermore, they *associated this finding with the given condition* such that "positive integers that can be written using only the numbers 1, 5 and 8 are ordered from small to large" in the problem text. The condition led them to find out 111111 as the 364th number. This further indicated that all participants were able to *associate numerical results with the givens* in the problem context.

Furthermore, they all again stated that they checked their solution step by step during the paper and pencil test. They did not need to recheck it during the interview. Though again, I make the following claim: During the interview, these participants' explaining their reasoning by focusing on 1) the quantities and 2) what both the numerical refers to in terms of the quantities suggested that they made sense of the procedures they utilized. In addition, they were able to associate the numerical results with the givens in the problem context such as the smallest number could be written in 6 digit numbers was 111111. All these suggested that these participants were able to *justify* their solutions since they were aware of the meaning of each step in their solutions. That is, they did not need to recheck their solution process because they were already aware of their solution process.

In sum, data suggested that all participants firstly focused on the quantities given in the problem. They were all able to associate the numerical results with the givens; and they were able to make sense of the procedures while passing through the solution steps.

6. CONCLUSION AND DISCUSSION

In this study, I investigated the nature of problem solving processes Turkish mathematically gifted students used as they engage in solving non-routine mathematical problems with respect to Polya's Four Phase Problem Solving Framework. Thus, I discussed the conclusions regarding the first and the second research questions under each phases of Polya. The differences and participants' problem solving patterns were retrospectively determined by focusing on their written solutions from the paper and pencil test and their explanations during the interview. Results demonstrated that five mathematically gifted participants of this study moved through all phases of Polya while solving non-routine problems even if there were some differences.

6.1.Polya's Four Phase Framework

Polya (1957) stated that "Understanding the Problem" phase represents the unknown, the condition and the data given in the verbal statement of the problem. Therefore, from this perspective, the answers to the questions: "*What is the unknown? What are the data? What is the condition?*" should be satisfying to solve a problem (1957). He further argued that if the problem requires the use of tables, shapes or diagrams, they also should be used to organize the data for the problem to be well understood. Similarly, researchers claimed that gifted students tend to simplify the problems by drawing a diagram or constructing a table to work systematically and quickly (Heinze, 2005; Durmaz, 2014). In this study, participants' solutions in written work and their uttering during the interview also supported previous claims such that all participants constructed tables when needed. In particular, as the data showed all participants were able to explain the verbal statement of the problem correctly and clearly. They all constructed a table, when necessary, to organize the data given in such as Problem 4, Problem 6 and Problem 7.

Previous research suggested that gifted students spend more time in understanding the problem and identifying the requirements (Garofalo, 1993; Span and Overtoom-Corsmit, 1986; Sriraman, 2003). However, the participants of this study did not need extra time during the interview such that, they were all able to start talking about their answers to the questions related to the understanding phase. Still, this might have been because of the time interval between the paper and pencil test and the interview, which was two days. That is, the participants might have remembered the problems with relative ease enabling them not to spend much time on understanding the problem.

Polya (1957) stated that devising a plan is the most important component of problem solving since it includes calculations, computations or constructions to obtain the unknown in the problem context. Therefore, he suggested to lead students during the problem solving process with the questions such as, "*Do you know a related problem*?", "*Can you use it?* ", "*Have you seen it before? Or have you seen the same problem in a slightly different form?*", in order to trigger solution processes on the part of students. In this study, I used these questions during the interview to determine whether the problems were routine or non-routine problems for the participants. Researchers argued that for problems to be non-routine, participants should not have past experiences with the exact problem or similar ones which can be solved with the same solution techniques (Mullis et al, 2009; McLeod, 1988; Nancarrow, 2004). Altun (2015) further stated that previous knowledge related to problems were required to solve even the non-routine problems. Heid (1983) argued that, mathematically gifted could solve unfamiliar problems based on what is known before. That is, for them the solution of the problem is not a learned ability before, but the information the gifted uses to solve the given problem has its bases on the information gifted already has. She further claimed that among the most seemingly necessary links in a logical train of thought, mathematically gifted can choose the best information which would lead to the solution of the problem (Heid, 1983). As the results showed (see Table 5.1) all the participants had previous knowledge to solve the problems but they did not have any past experiences with the same or similar problems. In addition, results showed that participants used all the givens in the problems and were also able to differentiate necessary or unnecessary information given in the problems. For example,

in Problem 6, they were all able to determine and analyze the extra information pointing to the unnecessity of the information provided in the problem context. In addition, as part of the "Devise a Plan" phase during the interview, all participants also were asked to restate the problem in their own words. Polya (1957) argued that restating the problem might provide opportunities for students to explore different aspects of the problem; and arouses the awareness of associations in the problem context. Also, previous research has shown that gifted students are able to paraphrase the problem in their own words (Sriraman,2003). However, results of the study showed that all but one of the participants tended to change superficial aspects of the problem such as numerical values or the names used within the problem context instead of constructing an original problem. Even they could not restate the problem, they were all aware of the associations between quantities such that they offered the numerical changes without changing the solution method. Distinctly, Participant S was able to write a completely new problem without changing the solution method; suggesting that she was aware of the associations between the quantities given in the problem context. Previous research has shown that mathematically gifted could reverse their thinking quickly and could solve problems in more than one ways in need of finding the most elegant solution. That is, while searching for the alternative solutions to the problem, mathematically gifted finds the most elegant solution (Heid, 1983). Therefore, the fact that these gifted students did not need to find other solutions might be because of their realization that the solutions were already elegant from their point of view. Aligning with earlier research results, these suggested that "Devise a Plan" phase of the problem solving process was attainable by all the participants despite the slight differences (Schoenfeld,1985).

In addition, previous research has shown that gifted students are able to verbalize and explain their solutions when asked during the interview (Heinze, 2005). Similarly, in this study all participants were able to apply their devised plan and check for their solution processes during the written sessions. However, regarding the proof of the solution during the interviews all participants repeatedly pointed to the unnecessity for checking the correctness of their solution processes. Yet, they repeated their solutions by expressing the meanings of quantities in the problem context and the relations between them as a proof of their solution.

In addition, participants had a chance to double check their solutions during the paper and pencil test and the interview in this study. However, all participants stated that they did not need to recheck the correctness of their solution steps during the interview since they checked it step by step during the paper and pencil test. Again, this might have been because of the time interval between the paper and pencil test and the interview, which was two days. Still, these participants' explanations during the interview suggested that they were able to justify their solutions which enabled them to be sure that the answers were correct.

In particular, while repeating their solutions for all the problems, all of the participants explained their reasoning by focusing on the quantities and what the numerical and the symbolic representations refer to in terms of the quantities in the problem context. This suggested that they not only made sense of the procedures they utilized but also were able to associate the numerical and the symbolic results with the givens in the problem context. Therefore, these participants were able to justify their solutions since they were aware of the meaning of each step in their solutions. In other words, they did not need to recheck their solution process because they were already aware of their solution processes. Thus, I propose that for gifted students, problem solving processes might include justification within the looking back stage in Polya's problem solving phases.

Most importantly , results of this study showed that carry out the plan phase of the problem solving process of these gifted students involved three common characteristics: 1) focusing on the quantities, 2) associating numerical and symbolic results with the givens in the problem context, 3) being aware of all parts/steps of the solution processes. In particular, for all the problems, explanations regarding the participants' solutions began with the meanings of the quantities given in the problem context. The quantities in the problems and the participants' solutions were both numerical and symbolic such as x, y. Given the quantities, participants were both able to explain what those numerical/symbolic expressions referred but also relate them through their calculations. That is, when asked, they were all able to explain the meaning of newly constructed quantities obtained through calculations; and associate the

numerical/symbolic results with the givens in the problem context. This further indicated that all participants were able to make sense of the procedures they used such that they were aware of the meaning of each step in their solutions. All these results align with previous research focusing on problem solving behaviors of students engaging in quantitative reasoning (Moore, Carlson and Oerthman, 2009; Moore, 2010).

Smith III and Thompson (2008) refers to quantitative reasoning as the mental actions involved in conceiving of a situation such that it is composed of measurable attributes (quantities) and relationships between these quantities. Previous research has shown that there is a link between problem solving behaviors of students and the quantitative reasoning such that while solving the problem "students established quantitative meaning for the result of a calculation previous to performing the calculation" (Moore, 2010, p. 308). Similarly, they were able to discuss the "various relationships between quantities…"(p. 308). Moore (2010) further suggested that "the findings from this study imply that understanding rooted in quantitative relationships support a student's ability to solve novel problems" (p. 311). Therefore, based on the results of this study, I propose that gifted students might be utilizing quantitative reasoning during the carry out phase of problem solving processes while solving non routine problems.

6.2. Limitations of the Study

First limitation of this study was the small number of participants who were members of Mathematics Olympiads Group organized by National Ministry of Education, and at the top ten degree in National Mathematic Olympiad regulated by TUBITAK, the National Science Foundation (NSF) in Turkey. Because of the uniqueness in common characteristics of selected participants, purposive sampling method was used. Since there was not a similar group of participants, a pilot study could not be applied.

Second limitation was the non-routine criteria of problems. The main characteristics of non-routine problems was that students were not familiar with the same problems or similar ones which could be solved with the same solution method. To ensure this characteristic, problems were selected carefully, and the validity was checked. Notwithstanding, some of the problems turned out to be routine problems. Although, the number of selected problems were ten at the beginning, four problems were taken into account for the analysis of this study.

Third limitation was the time and the availability of participants. In this study, grades and schools of participants were different because the participant selection process depended on the characteristics of giftedness, not being at the same grade . Different schools lead to difficulties on adjusting a timeline for the interview sessions. Time interval between the paper and pencil test and the interview was two days. Therefore, participants remembered problems with relative ease; they were unwilling to check their results and answer the related questions in the interview. Their unwillingness to answer interview questions limited the study with regard to understand how much time participants spend for Polya's phases.

6.3. Implications and Suggestions for Further Studies

The findings of this study indicated possible suggestions for further research to improve the problem solving processes of mathematically gifted students. Previous research was mainly focused on the comparison of average and gifted students' problem solving processes. Therefore, this study might serve as an initial attempt for investigating Turkish mathematically gifted students' problem solving processes more profoundly. Particularly, results of this study showed that while solving non-routine problems, three characteristics were involved within the carry out phase of the problem solving. Specifically gifted students focused on quantities, associated the numerical and the symbolic results with the givens in the problem context and made sense of the procedures and the calculations used within the solution. In addition, the gifted students were able to justify their solution processes enabling them to conclude on the unnecessity of checking the correctness of their solutions. Though, this study involved the

participation of five Turkish mathematically gifted students. Therefore, further research might be done with more gifted students to investigate their problem solving processes.

In addition, results of this study has shown that non-routine problem solving processes of mathematically gifted students involves quantitative reasoning. Further research could be conducted to scrutinize gifted students' quantitative reasoning processes deeply on different subjects with different types of problems. For example, problems with missing knowledge could be chosen to determine how much the students understand it, or to lead them making assumptions for missing parts. In this study, problems were chosen from four chapters: Number and Transactional Skills, Real Life Problems, Algebraic Problems, Counting and Probability Problems. The problems from the last chapter, Geometry Problems, were not chosen to avoid requirement of pre-knowledge as formulas. Geometry problems could also be studied with mathematically gifted students in the further studies. Besides, along with reasoning, participants were able to construct quantitative relationships among concepts by focusing on quantities and associations among numerical or symbolic values. So, further research focusing on gifted students' reasoning processes might shed light on the relations among different mathematical concepts.

Although there is an attempt, there is not a special education for all gifted students in Turkey. Therefore, they mostly attend to regular classrooms. Taking that as an opportunity, the results of this study suggest that teachers might create classroom discussions extricating the problem solving processes of gifted to foster average students' problem solving skills. Particularly, the interview questions used in this study might be used as triggers to create such classroom environments (see Appendix D). This is important because Shoenfeld (1992) argued that successful problem solvers often monitor their problem solving processes while unsuccessful problem solvers do that less frequently. Similarly, researchers emphasized that self-management of the problem solving behavior differentiates gifted from an average performer (Sternberg & Davidson, 1986). Similarly, for the education of the gifted, results of this study suggest the exposition of the gifted students' problem solving processes during

teaching. In particular, once asked to explain how and why they executed the solution as they did, all gifted students in this study were able to explain their reasoning very clearly and succinctly. They were all aware of the known and unknown quantities within the problem context and were all able to associate them to the numerical/symbolic results they came up with. Therefore, it is important for the gifted to share such processes within classroom environments to not only learn from each other but also to consciously reflect on them.

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APPENDIX A: PAPER-PENCIL TEST OF INSTRUMENT

Tarih:

Yer ve zaman:

İsim-Soyisim:

Öncelikle geldiğiniz için teşekkür ederim. Bu çalışmanın başlığı "Matematikte Üstün Yetenekli Öğrencilerin Rutin Olmayan Problem Çözüm Süreçleri". Çalışmanın amacı sizin rutin olmayan problemleri çözerken kullandığınız stratejileri, bakış açınızı ve sürecinizi anlamak.Yazılı olarak aldığınız bu soruları çözdükten sonra, sizinle belirlediğimiz zamanda çözüme nasıl ulaştığınızı anlamak için bir görüşme yapacağım. Bu görüşmeyi birebir olarak yine burada gerçekleştireceğiz.

Bu testteki sorularda çözümünüzün doğru olup olmadığı benim için önemli değil. Bu konuda rahat olabilirsiniz. Cevaplarınız sadece kullandığınız stratejileri, bakış açınızı ve rutin olmayan bir problemi çözerkenki sürecinizi anlamak amacıyla kullanılacak. Hiçbir şekilde not verme amacı yoktur.

Testin çözümü için bir zaman kısıtlamanız yok. İstediğiniz ve çözümün gerektirdiği kadar zaman harcayabilirsiniz. Eğer mümkünse birden fazla çözüm yöntemi kullanmanızı rica ediyorum. Anlaşılır basamaklar şeklinde yazmanız görüşme sürecinde çözümünüze bakarak açıklama yapmanızı kolaylaştıracaktır.

Katılımınız ve bu çalışmaya zaman ayırdığınız için tekrar teşekkür ederim. Bu çalışma ve sizin çalışmaya olan katkınız benim için çok değerli. Soruları çözmeye başlayabilirsiniz. Bitirdiğiniz zaman lütfen bana bildirin.

SORU 1

Elif, çapı bir cm olan küre biçiminde özdeş boncuklarla her bir telin uzunluğu 20 cm olan bir abaküs yapıyor. Elif, abaküsün en üst teline 1, alttaki tele 2, onun altındaki tele 3 boncuk yerleştiriyor. Bu biçimde devam ederek daha sonra gelen her bir tel için boncuk sayısını birer artırıyor. Bu şekilde ilerleyerek bir tel tamamen dolunca işlemini tamamlıyor. Ancak en alttaki telin boş kaldığını görünce önceki bir telden itibaren her tele birer tane fazla boncuk dizdiğini anlıyor.

197 tane boncuk dizen Elif, ilk hatayı kaçıncı sırada yapmıştır?

SORU 2

Bir çiftlikte bulunan 20 inekten birincisi günde 1kg, ikincisi 2kg, üçüncüsü 3kg, ... , yirmincisi 20kg süt vermektedir. Bu çiftliğin ortağı olan 5 kişi ortaklıktan ayrılmaya karar veriyor ve her biri eşit miktarda süt alacak şekilde inekleri paylaşmak istiyor.

Buna göre, ortaklardan herhangi biri en çok kaç inek alabilir?

SORU 3

- 1'den 6'ya kadar numaralandırılmış altı kutu vardır.
- Her kutuda en az bir top vardır.
- Her kutuda en çok numarası kadar top vardır.
- Kutulardan sadece birinde 2 top vardır.

Bu koşullarda top sayısı en çok 17 olabildiğine göre, kaç numaralı kutuda 2 top vardır?

SORU 4

Sevinç ile Dilay, 10 gün boyunca bir kavanozdan aşağıdaki kurala göre şeker alıyorlar.

- İlk gün Sevinç, ikinci gün Dilay şeker alıyor ve bu sırayla devam ediyorlar.
- Sevinç ilk seferde x tane, Dilay ilk seferde y tane şeker alıyor.
- Daha sonra şeker alan kişi, kendinden önceki iki gün boyunca alınan şekerlerin toplamı kadar şeker alıyor.
- 10 gün boyunca Dilay, Sevinç'ten 66 şeker fazla alıyor.

Kavanozdan alınan toplam şeker sayısı 4 ile tam bölünebildiğine göre, Sevinç'in üçüncü seferde kavanozdan aldığı şeker sayısı kaçtır?

SORU 5

Bir otelde 12 tane tek kişilik, 18 tane çift kişilik ve 25 tane üç kişilik olmak üzere toplam 55 oda bulunmaktadır. Tam kapasite dolu olan bu otelle ilgili aşağıdakiler bilinmektedir.

- Otelin 30 odasında yerli müşteriler kalmaktadır.
- Yerli müşteriler çift kişilik odalarda kalmamaktadır.
- Yabancı müşteriler tek veya çift kişilik odalarda kalmaktadır.

Buna göre, bu oteldeki yabancı müşteri sayısı kaçtır?

SORU 6

- a ve b birer tam sayıdır.
- Bir sayı örüntüsünün ilk iki sayısı sırasıyla a ve b 'dir.
- Örüntüdeki üçüncü sayı, ikinci sayıdan birinci sayı çıkarılarak elde edilir. Buna göre, üçüncü sayı b - a 'dır.
- Örüntüdeki her sayı, kendinden önceki sayıdan bir önceki sayı çıkarılarak elde edilir. Örneğin yedinci sayı = altıncı sayı - beşinci sayı

Buna göre, bu sayı örüntüsündeki ilk 700 sayının toplamı neye eşittir?

SORU 7

Sadece 1, 5 ve 8 rakamları kullanılarak yazılabilecek pozitif tam sayılar aşağıdaki gibi küçükten büyüğe doğru sıralanmıştır.

1, 5, 8, 11, 15, 18, 51, 55, 58, 81, ...

Bu şekilde elde edilen sayı dizisinin 364. elemanı kaçtır?

SORU 8

Bir kutunun içinde 4 kırmızı, 3 yeşil ve 2 mavi zar bulunmaktadır.

Kırmızı zarların yüzeylerine 1, 2, 3, 4, 5, 6;

Yeşil zarların yüzeylerinde 1, 1, 2, 2, 3, 3;

Mavi zarların yüzeylerinde 1, 1, 1, 5, 5, 5 rakamları yazılıdır. Kutudan rastgele çekilen bir zar bir defa atılıyor.

Zarın üst yüzüne 1 sayısı geldiğine göre, atılan zarın kırmızı renkli olma olasılığı yüzde kaçtır?

APPENDIX B: PAPER-PENCIL TEST WITH CHAPTERS AND ANSWERS

SORU 1

Elif, çapı bir cm olan küre biçiminde özdeş boncuklarla her bir telin uzunluğu 20 cm olan bir abaküs yapıyor. Elif, abaküsün en üst teline 1, alttaki tele 2, onun altındaki tele 3 boncuk yerleştiriyor. Bu biçimde devam ederek daha sonra gelen her bir tel için boncuk sayısını birer artırıyor. Bu şekilde ilerleyerek bir tel tamamen dolunca işlemini tamamlıyor. Ancak en alttaki telin boş kaldığını görünce önceki bir telden itibaren her tele birer tane fazla boncuk dizdiğini anlıyor.

197 tane boncuk dizen Elif, <u>ilk hatayı</u> kaçıncı sırada yapmıştır? (Sayı ve İşlem Becerisi)

Çözüm: Bir telin uzunluğu 20cm olduğu için en çok 20/1=20 tane boncuk alabilir. Demek ki abaküs 20 sıra telden oluşacaktır. Elif doğru dizseydi 19. sıra tamamlandığında 1 + 2 + 3 + ... + 19= 19.20/2= 190 boncuk dizmiş olacaktı. 197-190 = 7 boncuk fazla dizmiş. Hatayı yaptığı ilk telden itibaren her sırada 1 fazla boncuk dizmiştir. 19. sıradan itibaren 7 geri gidersek ilk hatayı 13. sırada yapmıştır.

SORU 2

Bir çiftlikte bulunan 20 inekten birincisi günde 1kg, ikincisi 2kg, üçüncüsü 3kg, ... , yirmincisi 20kg süt vermektedir. Bu çiftliğin ortağı olan 5 kişi ortaklıktan ayrılmaya karar veriyor ve her biri eşit miktarda süt alacak şekilde inekleri paylaşmak istiyor.

Buna göre, ortaklardan herhangi biri <u>en çok</u> kaç inek alabilir? (Sayı ve İşlem Becerisi)

Çözüm: Bütün ineklerden alınabilecek toplam süt miktarı 20.21/2= 210kg 'dır. Buna göre kişi başına 210:5=42 kg süt düşer. Herhangi bir ortağın çok inek alabilmesi için az süt veren inekleri seçmesi gerekir. İlk 8 inekten 8.9/2= 36kg süt alınabilir. 6kg eksik kaldığı için 6 ile 12 yer değiştirilirse 42 kg elde edilir. Kalan inekler ise şu şekilde paylaşılır.

15,16,11

19,17,6

10,14,18

13,9,20 daha farklı paylaşımların olabileceğini kontrol edebiliriz.

SORU 3

- 1'den 6'ya kadar numaralandırılmış altı kutu vardır.
- Her kutuda en az bir top vardır.
- Her kutuda en çok numarası kadar top vardır.
- Kutulardan sadece birinde 2 top vardır.

Bu koşullarda top sayısı en çok 17 olabildiğine göre, kaç numaralı kutuda 2 top vardır? (Sayı ve İşlem Becerisi)

Çözüm: Kutuların sadece birinde 2 top olduğu belirtilmiştir. Eğer 2 numaralı kutuda 2 top olsaydı kutulardaki toplam top sayısı en çok 1 + 2 + 3 + 4 + 5 + 6= 21 olabilirdi. Demek ki kutuda 2 top değil 1 top vardır. Maksimum top sayısı 20'ye düştü. Maksimum top sayısının 17'ye düşmesi için 5 numaralı kutuda 2 top olmalıdır. Kutulardaki top sayısı sırasıyla 1, 1 ,3 ,4 , 2 , 6 olur.

SORU 4

Sevinç ile Dilay, 10 gün boyunca bir kavanozdan aşağıdaki kurala göre şeker alıyorlar.

- İlk gün Sevinç, ikinci gün Dilay şeker alıyor ve bu sırayla devam ediyorlar.
- Sevinç ilk seferde x tane, Dilay ilk seferde y tane şeker alıyor.
- Daha sonra şeker alan kişi, kendinden önceki iki gün boyunca alınan şekerlerin toplamı kadar şeker alıyor.
- 10 gün boyunca Dilay, Sevinç'ten 66 şeker fazla alıyor.

Kavanozdan alınan toplam şeker sayısı 4 ile tam bölünebildiğine göre, Sevinç'in üçüncü seferde kavanozdan aldığı şeker sayısı kaçtır? (Gerçekçi Hayat Problemleri)

Çözüm: Aşağıdaki tabloya göre denklemleri kuralım:

	1	2	3	4	5	6	7	8	9	10
Sevinç	x	-	x+y	-	2x+3y	-	5x+8y	-	13x+21y	-
Dilay	-	y	-	x+2y	-	3x+5y	-	8x+13y	-	21x+34y

Terimlerin katsayılarına dikkat edilirse Fibonacci sayıları olduğu görülür. Her ikisinin de aldıkları şeker sayıları bu sayılar kullanılarak elde edilebilir. 10 günde Dilay, Sevinç'ten 66 şeker fazla aldığına göre, 22y+11x=66 olur. Bu da 2y+x=6 demektir. Kavanozdan alınan toplam şeker sayısı 4'ün katı olduğuna göre; 55x+88y= 4k (k bir pozitif tamsayı olmak üzere) x, 4'ün katı olmalıdır. 2y+x=6 eşitliğinde x=4 ve y=1 bulunur. Sevinç, 3. seferde (5.günde) 2x+3y=2.4 +3.1=11 şeker almıştır.

SORU 5

Bir otelde 12 tane tek kişilik, 18 tane çift kişilik ve 25 tane üç kişilik olmak üzere toplam 55 oda bulunmaktadır. Tam kapasite dolu olan bu otelle ilgili aşağıdakiler bilinmektedir.

- Otelin 30 odasında yerli müşteriler kalmaktadır.
- Yerli müşteriler çift kişilik odalarda kalmamaktadır.
- Yabancı müşteriler tek veya çift kişilik odalarda kalmaktadır.

Buna göre, bu oteldeki yabancı müşteri sayısı kaçtır? (Gerçekçi Hayat Problemleri)

Çözüm: Yerli müşteriler toplam 30 oda kullandığına göre bunların 25'i üç kişilik, 5'i tek kişiliktir. Yabancı müşteriler geriye kalan 7 tek kişilik ve 18 çift kişilik odalarda kaldığına göre; 7.1 +18.2=43 yabancı müşteri vardır.

SORU 6

- a ve b birer tam sayıdır.
- Bir sayı örüntüsünün ilk iki sayısı sırasıyla a ve b 'dir.
- Örüntüdeki üçüncü sayı, ikinci sayıdan birinci sayı çıkarılarak elde edilir. Buna göre, üçüncü sayı b - a 'dır.
- Örüntüdeki her sayı, kendinden önceki sayıdan bir önceki sayı çıkarılarak elde edilir. Örneğin yedinci sayı = altıncı sayı - beşinci sayı

Buna göre, bu sayı örüntüsündeki ilk 700 sayının toplamı neye eşittir? (Cebir Problemleri)

Çözüm: Sayı örüntüsü a,b, b-a, -a, -b, a-b, a,b,b-a,-a,-b,a-b,.... biçiminde oluşur. Her altı terimde kendini tekrar eden bir örüntüdür. Kendini tekrar eden ilk altı terimin toplamı

a+b+(b-a)+(-a)+(-b)+(a-b) = 0'dır. 700:6= 116 ve kalan 4'tür. İlk 700 terimin toplamı a+b+(b-a)+(-a)= 2b-a'dır.

SORU 7

Sadece 1, 5 ve 8 rakamları kullanılarak yazılabilecek pozitif tam sayılar aşağıdaki gibi küçükten büyüğe doğru sıralanmıştır.

1, 5, 8, 11, 15, 18, 51, 55, 58, 81, ...

Bu şekilde elde edilen sayı dizisinin 364. elemanı kaçtır? (Sayma ve Olasılık Problemleri)

Çözüm:

1 basamaklı:3

2 basamaklı:3.3=9

3 basamaklı: 3.3.3=27

4 basamaklı: 3.3.3.3=81

5 basamaklı: 3.3.3.3.3=243 şeklinde en çok 5 basamaklı 3+9+27+81+243=363 tane sayı oluşturulur. O halde 364. sayı 6 basamaklı en küçük sayıdır. Bu durumda aradığımız sayı 111111'dır.

SORU 8

Bir kutunun içinde 4 kırmızı, 3 yeşil ve 2 mavi zar bulunmaktadır.

Kırmızı zarların yüzeylerine 1, 2, 3, 4, 5, 6;

Yeşil zarların yüzeylerinde 1, 1, 2, 2, 3, 3;

Mavi zarların yüzeylerinde 1, 1, 1, 5, 5, 5 rakamları yazılıdır. Kutudan rastgele çekilen bir zar bir defa atılıyor.

Zarın üst yüzüne 1 sayısı geldiğine göre, atılan zarın kırmızı renkli olma olasılığı yüzde kaçtır? (Sayma ve Olasılık Problemleri)

Çözüm: Çekilen zarın üstünde 1 sayısı göründüğüne göre, mümkün olan durumlar: 4/9.1/6 + 3/9.2/6 +2/9.3/6 = 16/54 olur. Bu durumda istenen olasılık 4/54 : 16/54 =1/4 yani %25 bulunur.

APPENDIX C: INTERVIEW PROTOCOLS IN TURKISH AND ENGLISH

Mülakat Protokolü

Tarih:

Yer ve zaman:

Görüşmeyi yapan:

Katılımcı:

Öncelikle geldiğiniz için teşekkür ederim. Yazılı test aşamasında söylediğim gibi bu çalışmanın başlığı "Matematikte Üstün Yetenekli Öğrencilerin Rutin Olmayan Problem Çözüm Süreçleri". Sizin bir probleme nasıl yaklaştığınızı ve problemi çözerken nasıl düşündüğünüzü anlamam mümkün değil. Bu yüzden, sizden problemi nasıl çözdüğünüzü anlatmanı isteyeceğim. Siz çözümünüzü açıklarken düşüncelerinizi daha iyi anlayabilmek ve bir detay kaçırmamak için bazı sorular sorabilirim. Eğer sizi rahatsız eden bir soru olursa, lütfen bana bildirin. Ya da bir sorunun cevabını bilmiyorsanız, yine bu durumu bana bildirin lütfen. Böylece bir sonraki soruya daha çabuk geçebiliriz.

Çözümünüzün doğru olup olmadığı benim için önemli değil. Bu konuda rahat olabilirsiniz. Bu çalışma sadece sizin kullandığınız starejileri, bakış açınızı ve rutin olmayan bir problemi çözerkenki sürecinizi anlamayı amaçlıyor. Cevaplarınızda notlandırma olmayacak.

Bu görüşmede ses kaydı yapılacak. Çünkü tüm cevapları not almam ya da hatırlamam mümkün değil. Ses kaydı sadece benim tarafımdan dinlenecek ve bu çalışma için kullanılacak. Görüşemeyi istediğiniz zaman bitirme hakkına sahipsiniz. Tekrar geldiğiniz için teşekkür ederim. Bana ayırdığınız zaman için minnettarım. Bu çalışma ve sizin çalışmadaki yeriniz benim için çok önemli. Katkınızdan dolayı çok mutluyum.

Interview Protocol

Date:

Place and Time:

Researcher:

Participant:

First of all, thank you for coming. As I informed you during the paper-pencil test, the title of this study is "Non-Routine Problem Solving Processes of Turkish Mathematically Gifted Students." I am interested in the processes you use when solving problems. I cannot understand how you approach the problem or how you think while solving the problem. So, I will firstly ask you to tell me how you solve the problem. While you are explaining your solution, I may ask some questions to help myself understand your thoughts while solving problems and not to miss any detail about it. If any of the questions disturbs you, please let me know. Or, if you do not have an answer for the question, again please let me know. Thus, we can easily get through to another question.

The correctness of your solution is not important for me. Please be relaxed about this. This study aims just to understand the strategies you use, your point of view and processes you had been through while solving a non-routine problem. There will be no evaluation of results as grading.

This interview will be audio-taped because it is not possible to note all of your answers or to remember them. The audio tape will be just listened by me and used for this study. You have a right to end this interview whenever you want. Again thank you for coming. I appreciate the time you spare for me. This study and your part in it is too important for me. I am too happy for your contribution.

APPENDIX D: INTERVIEW QUESTIONS IN TURKISH AND ENGLISH VERSIONS

MULAKAT SORULARI	INTERVIEW QUESTIONS
PROBLEMİ ANLAMA	**UNDERSTANDING THE PROBLEM**
Sence problemdeki bilinmeyenler neler?	What is the unknown?
Problemin verilenleri neler?	What are the data?
Sence bu problemdeki koşullar neler?	What is the condition?
Bu soruyu çözebildin mi?	Do you solve the problem?
Bu problemde isteneni bulmak mümkün müydü?	Is it possible to satisfy the condition?
Problemi çözmek için verilenler yeterli miydi?	Is the condition sufficient to determine the unknown?
Fazla bilgi var mıydı?	Is it redundant?
Soruyu anlamak için bir şekil çizdin mi?	Do you draw a figure to understand the problem?
Soruyu anlamak için bir tablo oluşturdun mu?	Do you make a table to understand the problem?
PLAN HAZIRLAMA	**DEVISING A PLAN**
Bu problemin aynısını daha önce görmüş müydün?	Have you seen it before?
Aynısı değil ama benzeri dediğin bir problemle daha önce karşılaştın mı?	Or have you seen the same problem in a slightly different form?
Bu problemin çözümünde sana yardımcı olan başka bir problem biliyor musun?	Do you know a related problem that could be useful?
Bu problemle aynı bilinmeyenlere sahip olan ya da benzer olan bir problem yazabilir misin?	Look at the unknown! Could you think of a familiar problem having the same or a similar unknown?
Bu problemin sonuçlarını ya da çözüm yöntemini başka bir problemin çözümünde kullanabilir misin?	Could you use the results or methods of this problem to solve another problem?
Bu problemi farklı kelimelerle yeniden yazabilir misin?	Could you restate the problem?
Bu problemi çözmek için sana yardımcı olacak daha kolay ya da daha genel olan benzer bir problem düşünebiliyor musun?	Could you imagine a more accessible or general problem to solve this problem?
Eğer çözememişse: Problemi parçalara ayırsak, bu parçaları çözebilir misin?	If the problem is divided into parts, could you solve the parts?
Koşulları tek tek çözmeye çalışırsan, çözüme ne kadar yaklaşabilirsin? Bu şekilde bilinmeyeni bulabilir misin?	If you try to solve the parts, how far is the unknown determined? Could you determine the unknown like this?
Bilinmeyeni bulmak için verilenler başka ne olabilirdi?	Could you think of another data to determine the unknown?
Eğer çözememişse:Bilinmeyeni ya da verilenleri değiştirip daha yakın sayılar kullanarak problemi çözebilir misin?	Could you change the unknown or the data appropriate to determine the unknown?
Problemi çözerken tüm verileri kullandın mı?	Did you use all the data?
Problemi çözerken bütün koşulları kullandın mı?	Did you use the whole conditions?
Bu problemin kapsadığı bütün fikirleri göz önüne aldın mı?	Have you taken into account all essential notions involved in the problem?

PLANI UYGULAMA	**CARRIYING OUT THE PLAN**
Çözümünüzdeki her bir basamağı kontrol ettin mi?	Did you check each step?
Her adımın doğru olduğunu açıkça görebildin mi?	Can you see clearly that the step is correct?
Bu çözümün ve adımlarının doğruluğunu bana kanıtlayabilirmisin?	Can you prove that it is correct?
GERİYE BAKMA	**LOOKING BACK**
Sonucu kontrol edebilir misiniz?	Can you check the result?
Çözümünü kontrol edebilir misin?	Can you check the solution?
Bu sonucu daha farklı bir şekilde çıkarabilir miydin? Başka bir yöntemle çözebilir miydin?	Can you derive the result differently? Can you solve it with another method?
Sence bu problemin sonucunu ya da çözüm yöntemini başka bir problemde kullanabilir misin?	Can you use the result or the method for some other problem?