THE EFFECTS OF CAPACITY PROBLEM IN AN ERA OF DECREASING NON-RENEWABLE RESOURCES

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THE EFFECTS OF CAPACITY PROBLEM IN AN ERA OF DECREASING NON-RENEWABLE RESOURCES

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The Effects of Capacity Problem in an Era of

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Tez Özeti

Erdal Karahan, "Azalan Tükenebilir Enerji Kaynakları Döneminde Kapasite Probleminin Ekonomik Bireylerin Optimal Problemlerine Etkileri "

Bu tezin amacı azalan tükenebilir enerji kaynakları döneminde ekonomik bireylerin optimal davranışlarına kapasite probleminin nasıl bir etki yapacağını analiz edebileceğimiz bir model oluşturmaktır. Model, Tahvonen ve Salo'nun (2001) makalesinin referans modelinde değişiklikler yapılıp kesik zamanlı modele uyarlanmasıyla oluşturulmuştur. Ayrıca bizim modelimizde enerji bir aramalı olarak üretilip tüketim malı üretiminde kullanılmaktadır.

Enerji marketlerinde tekelci eğilimler de gözlemlendiği için modelimizi bir kez de enerji üreticisini tekel olarak dizayn edip analiz ettik. İki modelden ilki, başlangıç şartlarına bağlı olarak Tahvonen ve Salo'nun da ikinci modelinde tahmin ettiği gibi önce azalıp daha sonra artan enerji fiyatları veya sürekli artan enerji fiyatları verirken. Enerji şirketini tekel olarak yeniden dizayn ettiğimiz model de enerji fiyatları Tahvonen ve Salo'nun referans modelinde bulduğu gibi sürekli artan bir eğilim sergilemektedir.

Daha sonraki bölümlerde modelde kullanılan parameterelere duyarlılık testi uyguladık. Buradan çıkan ilginç bir sonuç da tekelci model de yenilenebilinir enerji giderlerinin artmasına rağmen enerji üretiminin artabileceği oldu. Bunun sebebi kısaca şöyle özetlenebilir. Model de ekonomik bireyler birbirlerine bağlı olduğu için modelin kararlı durumunda tekel'in fiyatları kontrol etmesi olanıksızlaşıyor ve bu duruma tekelin tepkisi üretimi arttırmakşeklinde oluyor.

Thesis Abstract

Erdal Karahan, "The Effects of Capacity Problem in an Era of Decreasing Non-Renewable Resources"

The aim of this thesis is to establish a model where we can analyze the effect of capacity investments on the optimal choices of economic agents in an era of decreasing non-renewable energy resources. We employed a discrete time model which is a modified version of Tahvonen and Salo's (2001) benchmark model. In our model, we have energy as an intermediate good which is used in the final good production.

Since the firms in the energy market tend to behave monopolistically we also offer a second model where the energy producer is a monopolist. Our first model, depending on the initial conditions, either predicts a U-shaped time path for prices as in Tahvonen and Salo's second model, or ever increasing prices as in benchmark model. In all cases of our second model, we found ever increasing prices for energy as our benchmark model in Tahvonen and Salo (2001).

We also carry out sensitivity analysis for the parameters employed in the model. A suprising finding is that the level of energy produced icreases with the unit cost of energy in the decentralized model. This finding is a little bit controversial since we expect monopolist to produce less when it faces higher costs of production. The reason behind this unusual result is briefly as follows: energy producer's problem is interdependent with households decisions therefore energy firm is unable to control energy prices at the steady state.

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CHAPTER 1

INTRODUCTION

Production of energy and its utilization in various activities have always been one of the most important factors in the economy. However, the emergence of industrial revolution has changed energy's role in the economy and in our life style. The source of primary energy consumption prior to industrialization was almost entirely renewable resources in which the highest share was owned by biomasses. With industrial revolution the composition of energy sources used in our economic activities shifted from renewable to non-renewable energy sources, mostly fossil fuels. In 2008 energy supply distribution by source was; oil 33 %, coal 27%, gas 21% (fossil 81%), renewable (hydro, solar, wind, geothermal power and biofuels) 13% and nuclear 6%. Oil was the most popular energy fuel.

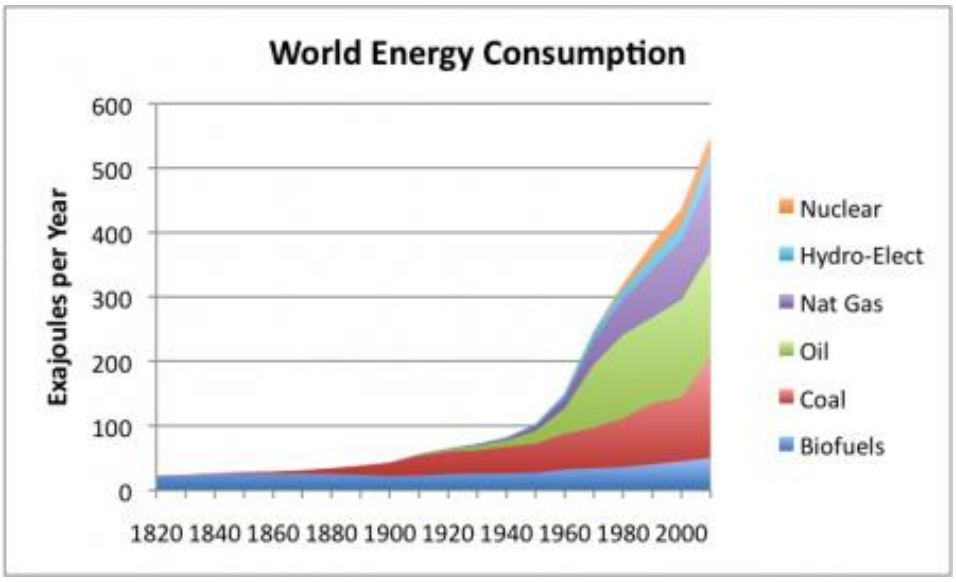


Figure 1: World Energy Consumption by Source. Based on Vaclav Smil estimates from Energy Transitions: History, Requirements and Prospects (2010) together with BP Statistical Data for 1965 and subsequent (Source: http://ourfiniteworld.com)

Non-renewable energy sources (including nuclear energy) combined represented over 85% of the world energy supply in 2008 (IEA/2011). A non-renewable resource is a natural resource which cannot be reproduced, grown, generated, once depleted there is no more available for future needs. Throughout this thesis, resources which can be replenished but are consumed much faster than nature can create them are also going to be considered as non-renewable energy sources.

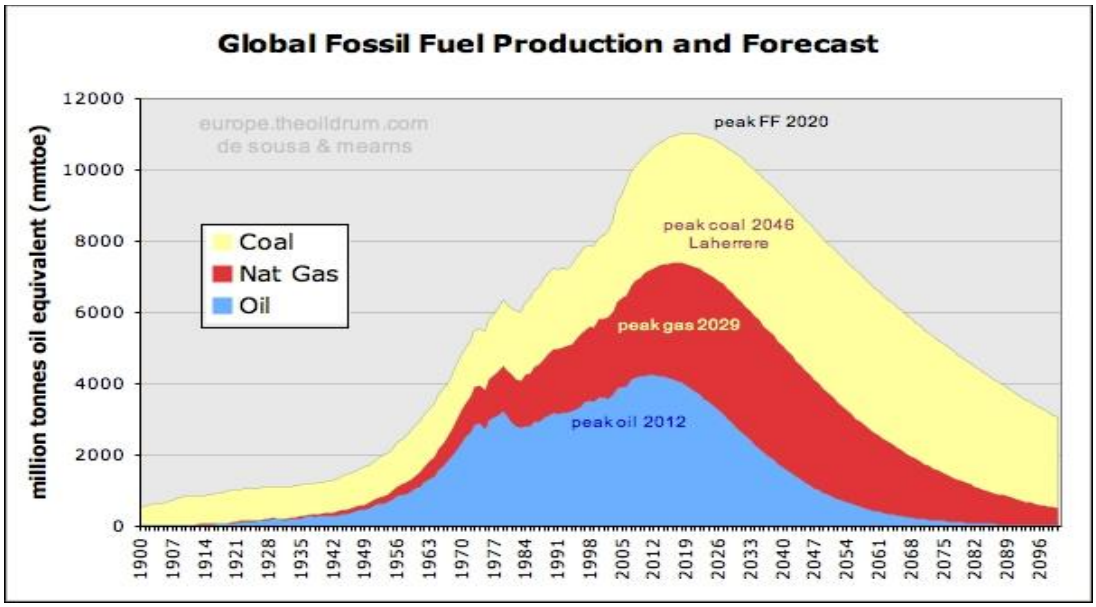


Figure 2: Global fossil fuel production and forecast, 1900-2096. Source: Mearns and de Sousa (2008)

Historical trends in the use of energy sources show that the share of renewable energy is in decline since the beginning of industrial revolution (see Figure 1). The trend shows that first coal then oil and gas usage has increased their shares in the primary energy consumption. Even though new resources are found during the last two decades it is almost certain that resource stocks are limited and in decline According to the BP's report (2010) if we continue to extract the fossil resources at the current rate, remaining

reserves of these resources will last for another 45, 60 and 115 years for oil, gas and coal respectively.

Nuclear energy seems more sustainable in the long run. Remaining uranium and thorium reserves, if carefully harvested and used, have an expected lifetime of sun's lifespan (5 billion years). Although they are subjected to extraction and eventually be exhausted, there is also a tendency in the literature to consider the nuclear fission in breeder reactors as sustainable and renewable. The reason behind this is, breeder reactors create more fissile isotopes then they consume. However the problem with nuclear power is generally about the waste disposal, safety issues and practicability. Another handicap about nuclear energy is the future extraction levels are not guaranteed. Apart from direct mining the largest amount of uranium can be extracted exists in seawater. Seawater extraction is too costly and its sustainability is questionable. In any case the question about the status of nuclear energy among other energy resources is not within the scope of this paper. Therefore we will accept the conventional explanation about it which is nuclear energy is non-renewable.

Renewable energy is defined as the energy which can be replenished naturally. Natural resources such as sunlight, wind, rain, tides, and geothermal heat are examples of the renewable energy. About 13% of global primary energy consumption comes from renewables, with 7% coming from traditional biomass[1], and 3% from hydroelectricity. New renewables (small hydro, modern biomass, wind, solar, geothermal, and biofuels) are accounted for around 3% and growing rapidly. The share of renewable resources in

[1] REN21's reports adjust Biomass energy findings as 10% at 2011 and 8% at 2012 due to the fact that primary energy consumption researches are likely to undercut the share of traditional biomass

electricity generation is around 19%, with 16% of global electricity coming from hydroelectricity and 3% from new renewable energy sources.

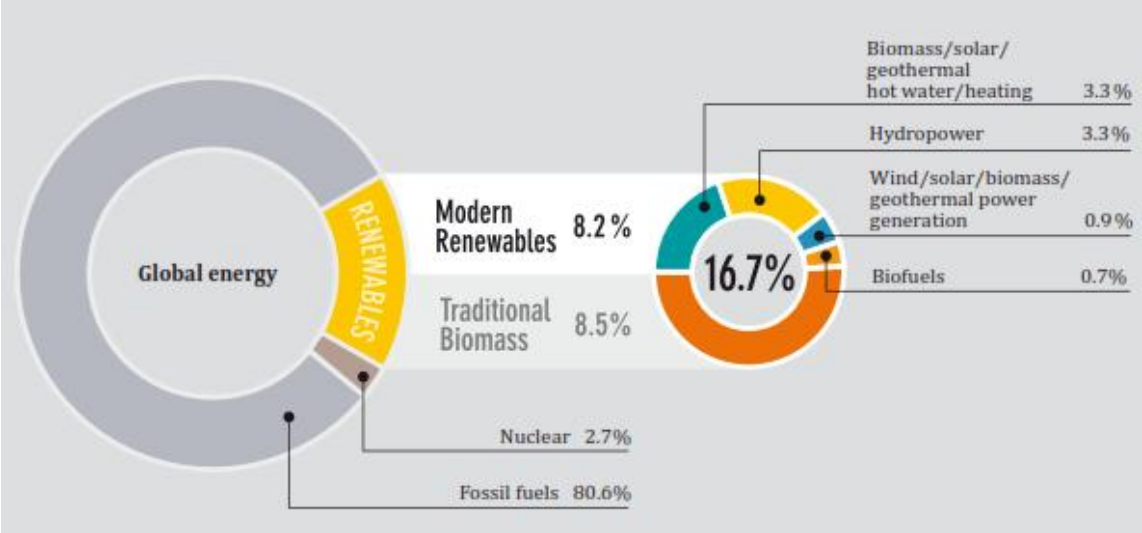


Figure 3: Share of renewable energy within the whole global final energy consumption Source: REN21 Global status Report (2012) see also footnote 1 about the discrepancies with previous findings

Although they estimated different shares for renewable energy almost all projections about the future of energy sources predicts that the share of renewable energy is going to increase in the future and finally dominates the total energy production. In EIA (2011), estimations predict that around 20% of primary energy consumption with renewable by the year 2030 and 30% by 2050. Proponents of renewable energy predict this number could be 50% by 2050 if policies for transition are carefully planned and applied.

Many OECD countries have roadmaps to convert a huge part of the composition of their primary energy sources into renewable. Today around a hundred countries have policy targets for transition to renewable energy. Most targets are for electricity and aim to reach 10-30% within a decade. Countries like Brazil, Finland and Netherlands also

have targets for biofuels to replace oil usage in the transportation sector. These targets are usually set for wind and solar power since most of the OECD countries (except Canada and Turkey) have already used their hydropower capacities.

European Union's 2010 target for wind and solar power has already been reached. Furthermore, if the growth of renewable energy continue like this, the target of having 20% of EU's total energy to come from renewable resources by the year 2020 is very much likely to be surpassed (REN21, 2011). Like EU, China and U.S. also have renewable energy policies and are the two largest investors in renewable energy capacity by the year 2010. U.S. has recently announced a new incentive package for investment in renewable energy. And in some states of U.S. (like Texas and Washington) the share of non-renewable energy is already over 20% and increasing.

One last concern about the nature of energy sector is the market power of the producers. The final report of European Union's energy commission (EC, 2007) points out that too much concentration at national markets and customers being tied to suppliers through long-term downstream contracts are serious problems within EU. Empirical analyses show that except U.K. and Scandinavia all national markets are dominated by 1 to 3 energy firms (Matthes, Grashof & Gores, 2007). Furthermore recent trends about mergers and accusations in this market implies that if not regulated the market concentration of European energy market will reach critically high levels in a few years.

The Market power of energy producer is not just dependent on market share. According to some market analyses even though the individual shares of companies in

this sector is small their ability to affect the prices and supply might be higher due to the structure and lack of regulations in the energy market (Borenstein, Bushnell & Knittel, 1999). For example in energy crisis experienced in California, even though there were several energy suppliers, Enron with its vast resources managed to control energy supply and manipulate the prices. At the time the energy crisis happened in California installed capacity was around 45GW whereas the total demand was 28GW.

Although there are successful privatization examples, like U.K. and Scandinavia, most of the energy markets around the world are inclined to form oligopolistic market structures, if they are not already dominated by the state. Energy crisis experienced in Argentina is one of the biggest hunchbacks of the proponents of deregulation and privatization of the energy sector. In Argentina after the 2002 economic crisis, the energy producers had problems to cope with the increasing demand of economic recovery. One reason behind that was installed capacity to produce electricity was not enough. The other reason was the price of other fossil fuels were adjusted upwards after devaluation of Argentinean peso, however the price of natural gas was not. That created an excess demand where the gas producing companies were reluctant to supply it.

In light of the facts I mentioned above I wanted to build a model in which I can compare different market structures for energy market. Furthermore I wanted to analyze the effect of capacity limitations on the production of energy. To better understand the effect of capacity problem I incorporated it into a model where a monopolist produces the intermediate good. In the model monopolist tries to maximize its lifetime profits. Therefore he has to choose between investing capacity today or tomorrow. Capacity

investments decrease this period's profit, however it enables monopolist to produce a larger quantity of energy in the next period. A larger quantity of energy doesn't always mean a higher revenue since it may cause a decrease in prices. Therefore monopolist has to choose the level of capacities and the extraction rate of non-renewable resources carefully. Namely, he chooses where marginal revenue of an extra unit of capacity equals to marginal cost of it. For the extraction rate, the key optimality rule is obtained by equating the marginal revenue of an extra unit of resources to the discounted marginal revenue of that extra unit of resource at the next period. This optimality conditions are different than the optimality conditions of a social planner who faces the same problem.

CHAPTER 2

LITERATURE REVIEW

In recent years, during which fossil fuels have become more and more a scarce commodity, economists have shown an increasing interest in transition from non-renewable energy resources to the renewable ones. Several main questions are studied about this transition process. The most important of them are the following. Is growth (or a sustainable economy) possible with depleting natural resources and what are the characteristics of an optimal path during the transition to the equilibrium?

Non-Renewable Resource Literature

Some economists argue that technological progress is enough for a sustainable economy and even economic growth is possible even if resources are depletable (Krautkraemer, 1998; Schou, 1996 and 2000). Others believe that the economy will eventually experience a transition from non-renewable to renewable resources (Chakravorty, Roumasset & Tse, 1997; Tahvonen and Salo, 2001; D'Alesandro, Luzatti & Moroni, 2009). All of these studies focus on the improvement in technology in order to ensure the productivity increases in final good production which in turn create the growth in economic activities.

My paper differs from these studies in two different aspects. First, those papers do not consider the effects of a monopolistic intermediate good market. Secondly, rather than the possibility of growth, I examine the effect of capacity problem on the behavior of optimal path during the transition period. Therefore I included neither technological

growth nor population growth. As a consequence, our model predicts no growth like Hartwick (1989) who also claims that economic growth is not possible without technological growth.

My result about the growth contrasts with the claim of Stiglitz (1974) and Dasgupta and Heal (1974). These authors argue that, even without technical progress, capital accumulation can offset the constraint on production possibilities that occurs due to the limited amount of non-renewable resources. They argue that in most of the cases it is possible to invent a reproducible input that can be substituted with the essential input, thereby preventing the economic collapse. In our paper we have a similar assumption we assumed that energy produced from renewable resources and non-renewable resources are perfect substitutes. However energy is assumed to be vital for production and we defined the substitutability between energy and physical capital to be low. In fact we assumed the two factor of production to be complement to each other.

Optimal Depletion Literature

The transition between renewable and non-renewable energy is a long-time debated topic. This topic starts with Harold Hotelling's well-known idea about the optimal depletion of a non- renewable resource (1931). In that study rather than the transition period, optimal pricing method for a non-renewable resource was studied. However its general attitude towards non-renewable resources is an important step in this literature. By introducing the idea of depletable resources that study opens the problem of optimality in the presence of non-renewable resources into discussion. Beginning from 60's and 70's the growth literature started to flourish and even, offered models to deal

with sustainability/growth problem in a world where lack of resources harms the economy. The most general result obtained from the study of this problem is about the optimality condition about the depletion of the non-renewable resource. Optimal depletion suggests that at the margin, the benefit of the use of the resource stock should be equal to the present value of its cost. In Hotelling's paper (1931) this cost is the loss of future profits that the economic agents could gain if they chose to not extract the resource today. In other papers such as Stiglitz (1974) and Dasgupta and Heal (1974) the cost is the present value of the foregone future consumption of this resource.

As a result of the energy crisis in 1970's and the Club of Rome's report on sustainability, economists focused on the sustainable growth and Environmental degradation problem again. Among them, Krautkramer (1998) and Schou (1996) suggested that economic growth is possible even in the presence of depletion of non-renewable resources. These models argue that if there is technologic growth then we can experience a continuous economic growth. However their model assumes that technology growth is exogenously given and this growth is just the continuation of historical trend.

Later on, many other authors (see for example, Schou, 2000; Grimaud and Rouge 2005; Grimaud 2000; Tahvonen and Salo 2001) solved the exogeneity problem and showed that economic growth is still possible for different types of production functions and technological growth assumptions. For example Grimaud and Rouge (2005) assumes only one type of technological change. Smulders and de Nooij (2003) allows for directed technological change. But in their setup the supply of energy is given

exogenously. In their work, Bovenberg and Smulders (1996) considers an endogenous improvement in technology which ensures the productivity increases of non-renewable energy resources this in turn creates the desired sustainable economic activities and in some cases even a sustainable per capita growth.

Backstop Technology Literature

Even the earliest studies of which focus was the depletion of non-renewable resources (especially fossil fuels), pointed out the significance of alternative resources to ensure sustainability. What is sustainability, why is it important, how can we achieve it? Economists and ecologists have already studied those questions within the last fifty years. In my paper I will not discuss these topics again since they are out of the scope of this paper[2]. Alternative resources and substitute technologies are generally discussed as solutions to the depletion problem

As I mentioned before intertemporal choice of resource extraction is mainly discussed within the optimal depletion theory's literature. The alternative resource topic however employs more interesting and various solutions. A solution to the sustainability problem can be the discovery of a new alternative energy source. In Tsur and Zemel (2005), Chakravorty et al. (1997) and many other papers the limits imposed by non-renewable resources are overcome by R&D efforts and improvements on a backstop technology.

[2] The importance of sustainability and how to achieve it can be better understood within the environmental economics literature. For a detailed discussion see Part 2 of The handbook of environmental economics. 1995, edited by Bromley, D. W.

In these studies (Tsur and Zemel, 2003; Heal, 1976) this alternative energy resource is generally referred as a backstop technology and it is assumed to be abundant (or renewable) and cheaper than fossil fuels. Models with backstop technology mainly focus on the sustainability of current economic activities till this backstop technology is discovered and can be considered as an alteration of the classic optimal depletion problem with infinite time to finite time.

Even though it is very likely to invent a backstop technology, it is still a speculative approach to the energy problem which employs too many assumptions. A third solution to the energy problem is more realistic in the sense that rather than assuming a backstop technology with lower costs than current non-renewable resources this approach considers a transition between non-renewable resources to current renewable energy resources while recognizing the reality of high costs for the latter. The transition period and style change from one model to another. For example Tahvonen and Salo (2001) offers a model in which relative usage of both energy types change over time whereas in D'Alessandro et al's model (2009) this transition occurs at a once where the prices of non-renewable resources exceeds that of renewable energy resources.

Among these two approaches the former one has a more realistic model whereas the latter one has some important implications if we want to examine a policy adoption. Regardless of their approach to the transition these models lack two important elements. The energy in these models is used as an input but its producers and their relationship with the economy are rarely questioned. In all those models energy is supplied by a

perfectly competitive market where its marginal product is equal to its marginal cost. Secondly even when the cost of producing energy (in both types) is examined (such as Tahvonen and Salo, 2001) the limits to energy production are rarely considered in this literature.

Substitution between Factors of Production Literature

The introduction of energy into the production process is important since the usage of non-renewable resources creates some complexities. Among them intertemporal choice of resource use and sustainability of economic activities are the two main topics discussed by optimal depletion literature. However the relationship between energy consumption and economic growth is also another controversial topic in the growth literature.

Even though it is a well established topic in the literature, the direction of causation of the relationship is not clear. For example Kraft and Kraft (1978) found a unidirectional causality from GNP to Energy consumption for U.S. whereas Soytas et al. (2001) found other way around for Turkey. If we look at early studies on capital –energy substitution we can see similar controversial results. For instance an early study by Berndt and Wood (1975) found that capital and energy were complements. On the other hand Griffin and Gregory (1976) concluded that they were substitutes.

In Apostolakis (1990), author surveyed the early works in the literature and concluded that most studies based on time-series data classify the two inputs as complements, whereas studies based on pooled cross-section of countries or regions find that energy and capital are substitutes in the production process. According to him the

best explanation for this difference is that time-series data reflects short-term relationships, while cross-section analyses capture long-term effects. A similar result is also mentioned in Griffin and Gregory (1976).

Empirical studies suggest that energy usage and output has a positive relationship. This relationship is close to being one to one in the short and medium run, whereas in the long run there seems to be a degree of substitutability between energy and capital (Koetse, de Groot, & Florax, 2007). Since we have to use energy to produce anything, many economists conclude that, the substitution between capital and energy must be limited (Stern, 1997; Smulders, 1995). Although the exact relationship between energy and output is not clear; for us what important is that, resource economics literature accepts energy as an input in the production process and we will employ this assumption in our model too.

Motivating Points of Our Study

To our best knowledge there is no paper in energy and resource literature where a capacity investment is required for production of energy. This topic might have been neglected due to the difficulties in separation of capital and capacity investment or for practical purposes. For example if an author assumes that energy is produced in a perfectly competitive market and capacity investments can be scrapped and sold like capital then there is no practical gain in defining capacity as different type of capital[3].

[3] We can have a similar result by just adjusting the technology parameters. However even now, introducing limitations to energy production can't be done without adjusting capital and energy substitution rate, which is a very tricky thing to do and needs too many assumptions as well as empirical evidence and consistent parameters.

Nevertheless limitations in energy production are a reality and it needs to be incorporated in energy-resource models. One thing missing in this literature is, none of those studies emphasize the capacity needed to utilize these resources. The other thing is that the energy market may behave differently than a competitive market. In this study I examined the effect of these two missing elements on the optimal path during the transition process.

There are several reasons that explain why I choose to focus on these two missing elements. First of all, energy transition issue is examined in many other papers but very few of them actually focus on the energy market. And those who did employ a social planners' approach rather than a decentralized market approach. However we know that energy producers may behave monopolistically. The cases like energy crisis in California and Argentina are examples to this kind of behavior. The existence of cartels like OPEC and a global market dominated by huge multinational energy companies are also other motivations why I chose to model like this. At the very least most of the local energy producers and even the state itself may employ a monopolistic price setting mechanism. Therefore in most parts of the world energy market is far from being a competitive one. Secondly for short periods energy production is limited with installed capacity. Producing energy over the capacity is either very costly or dangerous. However in the literature energy is assumed to be ready in the nature and can be supplied at simultaneously at demand. However a model allowing its' economy to produce whatever energy level s desired without considering capacity limitations is far from realistic and I wanted introduce that limitation in my model.

CHAPTER 3

THE MODEL WITH PERFECTLY COMPETITIVE MARKETS

This project focuses on the energy problem of the economy in an era of decreasing non-renewable resources. In this chapter a dynamic macro model will be developed to understand how a social planner would determine its production and capacity levels. I have a two sector model with production of final goods and an energy sector. Energy (E_t) is used as an intermediate good in the production of final goods (Y_t).

Final goods can be used as consumption goods (C_t) by households or as investment goods (I_t^i). There are three types of Investment in the model. Capital investment (I_t), capacity investment in renewable energy (I_t^R) and capacity investment in nonrenewable energy(I_t^N). To study the effects of capacity problem and a monopolistic energy producer I will compare my two models with each other. The first model, which will be presented in this chapter, is a version of Tahvonen and Salo (2001) where we only introduced capacity problem. Since markets are assumed to be competitive social planner decides the levels of three types of investments along with the amount of nonrenewable resource (Z_t) to be extracted, and consumption level. In the second model, which is going to be presented in next chapter, representative household chooses the capital investment and consumption levels and energy producer chooses capacity investment levels and the resource extraction rate.

Definition of the Environment

In both models households and firms are assumed to be homogenous, therefore we do all calculations by solving the problems of the representative household and the firm. In this benchmark case, we assumed that the economy is perfectly competitive. Therefore we are dealing with Social Planner's Problem (SPP). We assumed that the production function for final good is of constant elasticity of substitution type where the elasticity of substitution, γ, assumed to be negative.

In my model social planner chooses how much energy will be produced and the composition of it. Social planner can make the economy to produce renewable (E_t^R) or non-renewable energy (E_t^N) or both. The two types of energy are assumed to be perfect substitutes: $E_t = E_t^R + E_t^N$. We assumed that energy is produced by an existing capacity which is accumulated only for this purpose and a technology parameter. For simplicity we assumed no technology growth. In addition to these, non-renewable energy production requires use of non-renewable resources. This resource flow is defined as: $Z_t = X_t - X_{t+1}$, to produce energy. Non-renewable resources are depletable and we assume that the amount of nonrenewable resources or resource flow cannot be lower than zero. Capacity is subject to depreciation and it cannot be sold or rented to another party. However energy producer can freely choose next periods capacity and if it chooses to liquidate some of its capacity it can be sold without penalty.

Cost of producing energy is defined in terms of final goods. Cost of producing renewable energy R_t is a function of the amount of energy produced and a technology parameter, $R_t = (A, E_t^R)$. The productivity of renewable energy capital (K_t^R) is assumed

to be decreasing, since the less costly ones (or the ones with highest productivity) will be used at first. Producing nonrenewable energy is assumed to become more expensive as stock of the non-renewable resources approaches to the zero. In line with the literature I assumed that marginal cost of nonrenewable energy approaches infinity as resource stock (X_t) goes to zero. This assumption, along with the other assumptions on energy costs, secures a positive amount of stock to be left at the steady state. We can define the function for the total cost of producing non-renewable energy as $N_t = \left(X^t, E_t^N(K_t^N, Z_t)\right)$. Here K_t^N is the capacity stock for producing non-renewable energy. All other parameters are already defined. For simplification, we assumed that all of the existing capacity at time t has to be used for producing energy.

SPP with Capacity Problem

Social planner's problem can be written as

$\max_{C_t} \sum_{t=0}^{\infty} \beta^t U(C_t)$ where; $0 < \beta < 1$

s.t. $C_t + I_t^R + I_t^N + I_t = F(K^t, E^t) - R_t - N_t$,

$X_t > 0$, $Z_t > 0$, $C_t > 0$

The Lagrangian for this problem can be written as

$$L = \sum_{t=0}^{\infty} \beta^t \{U(C_t) + \lambda_t [F(K_t, E_t) - C_t - K_{t+1}^i + (1-\delta)K_t^i - R\left(E^R(K_t^R)\right) - N\left(X_t, E^N(K_t^N, Z_t)\right)] + \mu_t X_t + \vartheta_t Z_t\}$$

Where $i = R, N, \{\}$ and Since we assumed that two types of energy are perfect substitutes, we should have:

$E_t = E_t^R + E_t^N$ and μ_t, ϑ_t and $\lambda_t$are the Lagrange multipliers associated with the constraints. The necessary conditions for optimal solution includes F.O.C. obtained at below;

$$[C_t]: U'(C_t) = \lambda_t \tag{1}$$

$$[K_{t+1}]: \lambda_t = \lambda_{t+1}\beta[F_K(K_{t+1}E_{t+1}) + 1 - \delta] \tag{2}$$

$$[\lambda_t]: 0 = F(K_t, E_t) - C_t - K_{t+1}^i + (1-\delta)K_t^i - R_t - N_t \tag{3}$$

$$[K_{t+1}^R]: \lambda_t = \beta\left[\frac{\partial F_{t+1}}{\partial E_{t+1}}\frac{\partial E_{t+1}}{\partial E_{t+1}^R}\frac{\partial E_{t+1}^R}{\partial K_{t+1}^R} - \frac{\partial R_{t+1}}{\partial E_{t+1}^R}\frac{\partial E_{t+1}^R}{\partial K_{t+1}^R} + 1 - \delta\right]\lambda_{t+1} \tag{4}$$

$$[K_{t+1}^N]: \lambda_t = \beta\left[\frac{\partial F_{t+1}}{\partial E_{t+1}}\frac{\partial E_{t+1}}{\partial E_{t+1}^N}\frac{\partial E_{t+1}^N}{\partial K_{t+1}^N} - \frac{\partial N_{t+1}}{\partial E_{t+1}^N}\frac{\partial E_{t+1}^N}{\partial K_{t+1}^N} + 1 - \delta\right]\lambda_{t+1} \tag{5}$$

$$[X_{t+1}]: \lambda_t\left[\frac{\partial F_t}{\partial E_t}\frac{\partial E_t^N}{\partial Z_t} - \frac{\partial N_t}{\partial E_t^N}\frac{\partial E_t^N}{\partial Z_t}\right] + \vartheta_t = \beta\{\left[\frac{\partial F_{t+1}}{\partial E_{t+1}}\frac{\partial E_{t+1}^N}{\partial Z_{t+1}} - \frac{\partial N_{t+1}}{\partial E_{t+1}^N}\frac{\partial E_{t+1}^N}{\partial Z_{t+1}} - \frac{\partial N_{t+1}}{\partial X_{t+1}}\right]\lambda_{t+1} + \mu_{t+1} + \vartheta_{t+1}\} \tag{6}$$

By using above equation we can characterize some optimality conditions that can help us to solve the transitional dynamics. From equations (4) and (5) we have

$$\frac{\partial E_t^R}{\partial K_t^R}\left[\frac{\partial F_t}{\partial E_t} - \frac{\partial R_t}{\partial E_t^R}\right] = \frac{\partial E_t^N}{\partial K_t^N}\left[\frac{\partial F_t}{\partial E_t} - \frac{\partial N_t}{\partial E_t^N}\right] \tag{7}$$

From equations (3) and (4) we have

$$\frac{\partial E_t^R}{\partial K_t^R}\left[\frac{\partial F_t}{\partial E_t} - \frac{\partial R_t}{\partial E_t^R}\right] = F_K(K_t, E_t) \tag{8}$$

$$\frac{\partial E_t^N}{\partial K_t^N}\left[\frac{\partial F_t}{\partial E_t} - \frac{\partial N_t}{\partial E_t^N}\right] = F_K(K_t, E_t) \tag{8'}$$

If we rewrite 6 by assuming $\mu_t = \vartheta_t = 0$ we get

$$\lambda_t \left[\frac{\partial F_t}{\partial E_t} - \frac{\partial N_t}{\partial E_t^N}\right] \frac{\partial E_t^N}{\partial Z_t} = \beta \lambda_{t+1} \left\{\left[\frac{\partial F_{t+1}}{\partial E_{t+1}} - \frac{\partial N_{t+1}}{\partial E_{t+1}^N}\right] \frac{\partial E_{t+1}^N}{\partial Z_{t+1}} - \frac{\partial N_{t+1}}{\partial X_{t+1}}\right\} \quad (9)$$

Model Specification and Transitional Dynamics for Centralized Model

To analyze the model better, we specify some functional forms in it. We chose the functional models as follows, in order to simplify further analyses.

1) Households' utility function is log utility $U(C_t) = \log C_t$

2) Production function is CES production function $F(K_t, E_t) = \left(K_t^\gamma + E_t^\gamma\right)^{1/\gamma}$. Stern (1997) and Uzawa (1962) point out that when factors of production are equal to two CES type of production function can be effectively used. Furthermore, CES type of production function with elasticity of substitution smaller than 1, satisfies certain properties like; $F_E > 0, F_K > 0, F_{KK} < 0, F_{EE} < 0, F_{KE} > 0$ which are crucial to have bounded average products.

3) Cost of producing renewable energy is assumed to be linear in the amount of Energy produced $R_t = AE_t^R$. Where energy production is assumed to be a function of the amount of renewable energy capital $E_t^R = (K_T^R)^\mu$ and A is the unit cost for producing energy

4) Cost of producing non-renewable energy is assumed to be increasing as resource stock decreases. In our model specification we chose the following type of cost function in order to meet the criteria we presented and have a model as simple as possible.

$N_t = \left(B + \frac{1}{X_t^2}\right) E_t^N$, furthermore D'Alessandro et al. (2009) uses a similar cost function. At above specification *B* is the constant cost parameter independent of the amount of non-renewable resource stock left. The amount of non-renewable energy produced is a function non-renewable energy capital and resource flow $E_t^N = (K_t^N)^{\alpha}(Z_t)^{1-\alpha}$

Transitional dynamics of the system will be governed by the following equations:

$$\Delta K_t = F(K_t, E_t) - \delta(K_t + K_t^R + K_t^N) - R_t - N_t - \Delta K_t^R - \Delta K_t^N \tag{10}$$

$$\frac{\Delta C_t}{C_t} = \beta[F_K(K_t, E_t) + 1 - \delta] - 1 \tag{11}$$

$$\Delta K_t^R = \left[\frac{F_K(K_{t+1}, E_{t+1})}{F_E(K_{t+1}, E_{t+1}) - A}\right]^{1/\mu - 1} - \left[\frac{F_K(K_t, E_t)}{F_E(K_t, E_t) - A}\right]^{1/\mu - 1} \tag{12}$$

$$\Delta K_t^N = \left[\frac{F_E(K_{t+1}, E_{t+1}) - \left(B + \frac{1}{X_{t+1}^2}\right)}{F_K(K_{t+1}, E_{t+1})}\right]^{\frac{1}{1-\alpha}} Z_{t+1} - \left[\frac{F_E(K_t, E_t) - \left(B + \frac{1}{X_t^2}\right)}{F_K(K_t, E_t)}\right]^{\frac{1}{1-\alpha}} Z_t \tag{13}$$

$$Z_t = -\left[\frac{F_K(K_t, E_t)(K_t^N)^{1-\alpha}}{\left\{F_E(K_{t+1}, E_{t+1}) - \left(B + \frac{1}{X_{t+1}^2}\right)\right\}\alpha}\right] \tag{14}$$

These dynamics are obtained from solving the equations from equations (4) to (9).

<u>Proposition 1:</u> At any optimal path K_t^N and Z_t goes to zero. First consider the marginal cost of non-renewable energy: $\frac{\partial N_t}{\partial E_t^N} = \left(B + \frac{1}{X_{t+1}^2}\right)$. As resource stock (X_t) goes to zero marginal cost of producing non-renewable energy becomes positive infinity; $\lim_{X_t \to 0} \frac{\partial N_t}{\partial E_t^N} = +\infty$. Then equation 8' will look like; $\frac{\partial E_t^N}{\partial K_t^N}[F_E(K_t, E_t) - \infty] = F_K(K_t, E_t)$

since we assumed a CES production function with $\gamma = -1$, $F_E(K_t, E_t)$ is bounded. Therefore As resource stock goes to zero is left hand side (L.H.S.) of equations 8' becomes negative. That means, marginal productivity of capital needs to be negative if this happens. Since marginal productivity of capital and energy has to be some positive values, at any optimal path we need X_t to converge some positive real number. If X_t converges some positive real number, then flow of resources has to converge to zero at any optimal path. Then by using equation (14) and the property $Z_t \to 0$ we can conclude that at any optimal path as $t \to \infty$, K_t^N should approach to zero. Therefore at any optimal path K_t^N and Z_t goes to zero.

Proposition 2: Above system has a steady state. Proof involves careful examination of the system and eliminating all other possible paths except the one that system has constant positive physical and energy capital in the long run. Formal proof is going to be presented at appendices. Here we present the intuition of it. An optimal path for this system should imply convergence to a steady state where both physical and energy capital have positive values.

Note that CES production function implies that marginal productivity of the capacity stocks in the production function goes to zero and -γ for the capital stock, as the ratio between capital and energy goes to zero. This creates an incentive to keep the ratio between 0 and 1. Another important factor is the return from investment to Energy capital declines faster as it goes to infinity. This creates a natural barrier to the amount of energy capital agents want to hold. Even though physical capital does not have such an increasing cost, the fact that CES production function used implies that the productivity

of the capital stock declines when we produce less energy. This fact forces the system to converge to a steady state. Another characteristic of the model which is, increasing capital investment needs to produce more energy, also affects the steady state ratio between the two capital stocks. Agents want to hold more physical capital than energy capital at the steady state, due to the fact that accumulating the same amount of physical capital has higher returns than energy capital.

Calculation of the Steady State value for SPP

We already showed that optimal path suggests $Z_t \to 0$ which implies $K_t^N \to 0$ in any case. At the appendices we will present the proof for the steady state and optimality conditions imply for any optimal path K_t^R and K_t converge a positive steady state. At previous section we already presented intuition for it. Therefore we are going to continue our analyses by assuming steady state exists. If we rewrite equations (7) and (8) by assuming we are at steady state, they become;

$$\left(K_{ss}^{\gamma} + (K_{ss}^{R})^{\mu\gamma}\right)^{1-\gamma/\gamma} K_{ss}^{\gamma-1} = \beta^{-1} + \delta - 1 = F_K(K_{ss}, E_{ss}) \tag{15}$$

By using equation (2) and evaluating it at steady state we get;

$$K_{ss}^{R} = {K_{ss}}^{1/\mu} \left[(\beta^{-1} + \delta - 1)^{\gamma/1-\gamma} - 1\right]^{1/\mu\gamma} = \left(K_{ss} H^{1/\gamma}\right)^{1/\mu} \tag{16}$$

where $H = (\beta^{-1} + \delta - 1)^{\gamma/1-\gamma} - 1$

$$\mu\left[\left(K_{ss}^{\gamma} + (K_{ss}^{R})^{\mu\gamma}\right)^{1-\gamma/\gamma} K_{ss}^{\gamma-1}\right] - A\mu\left({K_{ss}}^{R}\right)^{\mu-1} = \beta^{-1} + \delta - 1 \tag{17}$$

$$\frac{\beta^{-1}+\delta-1}{\mu} H^{\frac{1-\mu}{\mu\gamma}} = K_{ss}{}^{\frac{\mu-1}{\mu}} \left[(\beta^{-1}+\delta-1)H^{\frac{\gamma-1}{\gamma}} - A\right] \quad (17')$$

Comparative analysis and robustness tests findings will be represented in the coming parts. Note that by using (17') we can calibrate and solve for the steady state values of K and K^R in centralized economy. Here I'm going to offer some values for those parameters in order to establish a baseline to compare with decentralized economy's case. Let's assume β^{-1}=1.06; α=0.5; γ=-1; δ=0.1; μ=0.5 and A=0.1 then we will have $K = 1{,}6875 - 1.5A$

Time Path for Prices in SPP

In line with the general economic analysis the prices for factors of production are given by their energy marginal productivity in our model. Therefore we will examine the time path of the marginal productivity of energy in order to analyze the time path for energy. First of all let's focus on this marginal productivity in our model and what it depends on. If we define the ratio $K_t/E_t = y_t$ than for $\gamma = -1$, $F_E(t)$ becomes $\left(y/1+y\right)^2$, By using this we can reach that, price is an increasing function of the ratio y. Actually for every $\gamma < 0$ CES production function will give us prices as an increasing function of the ratio between capital and energy. To analyze the evolution of energy prices in the model we are going to use this property with equations (7) and (9). Since $\frac{\partial N_t}{\partial Z_{t+1}} = 0$ and Lagrange multipliers μ and ν are zero unless we have boundary solutions, equations (7) & (9) give us

$$F_K(K_t, E_t) + 1 - \delta = \frac{F_E(t+1)\frac{\partial E^N_{t+1}}{\partial Z_{t+1}} - \frac{\partial N_{t+1}}{\partial E^N_{t+1}}\frac{\partial E^N_{t+1}}{\partial Z_{t+1}} - \frac{\partial N_{t+1}}{\partial X_{t+1}}}{F_E(t)\frac{\partial E^N_t}{\partial Z_t} - \frac{\partial N_t}{\partial E^N_t}\frac{\partial E^N_t}{\partial Z_t}} \quad (9')$$

Proof for the decreasing prices at the beginning of the system will be give at appendices. In this section we will present the intuition behind it. First of all note that we assumed extraction costs due to resource stock are insignificant at the beginning. A similar result is reached by Tahvonen and Salo (2001). Without that assumption the results about decreasing costs may or may not hold. Depending on the initial conditions we may also experience increasing prices. If we have relatively high capital stock at the beginning then system starts with a decline of prices. Otherwise we have increasing prices.

Now let's try to characterize the system around the steady state. The answer we are searching for is, should we have decreasing or increasing prices while we approach to the steady state? While we approach the steady state F_K approaches to a constant and the term in the brackets at equation 8' goes to zero this implies that we should have $\frac{\partial E^N_{t+1}}{\partial K^N_{t+1}} \to \infty$ this implies $\frac{\partial E^N_t}{\partial Z_t} > \frac{\partial E^N_{t+1}}{\partial Z_{t+1}}$. From here we should have right hand side of the above equation to be lower than one while we approach to steady state. However optimality condition at equation 2 implies that right hand side to be larger than one which is a contradiction. Therefore we need F_K to be decreasing over time. That means F_K should converge to the constant from above. In conclusion while we converge to the steady state F_E should be increasing. Hence we should have increasing prices

CHAPTER 4

MODEL WITH ENERGY FIRMS

In this chapter I will present another model which is an extension of my model in chapter 3. In this new model (Decentralized Model) I assumed that Energy good producer is a monopolist therefore I designed the intermediate good market as a monopolistic market. Apart from that all other assumptions, including model specifications, I made in previous chapters about the environment are untouched. Since markets are not perfect we have to deal with the problems of economic agents separately.

Households

Households own the capital and energy firms therefore they receive an interest income plus the profits of energy producers. However they cannot collaborate with energy firms. Even though energy firms are owned by households they are assumed to act independently without thinking the best interest of the households which is in fact a very plausible assumption. With their available income households choose the level of next periods' capital and this periods' consumption in order to maximize their overall utility. As usual household has a concave utility function which exhibits diminishing marginal utility and disables the possibility of zero or negative consumption.

In the model, in line with the literature, I assumed that households do not consume energy as an independent agent. The reason behind this assumption is, we modeled energy as an input to final good production. Therefore households do not have

any demand for energy. This assumption might be oversimplified since households consume 30% of total energy. However introducing household demand separately will offer no new intuition within the scope of this paper. Furthermore to our best knowledge none of the models in the literature has considered household demand as well as our benchmark model (Tahvonen and Salo, 2001). Therefore we choose to not incorporate household demand in our model. Household's problem can be written as;

$\max_{C_t} \sum_{t=0}^{\infty} \beta^t U(C_t)$ where; $0 < \beta < 1$

s.t. $C_t + I_t = r_t K_t + \pi_t$, $I_t = K_{t+1} - (1-\delta)K_t$

$K_0 > 0$

It is assumed that there are no population and technology growth. π_t is the profits of the intermediate good firms at period t and K_0 is the initial capital stock and it is given

Lagrangian of the HH problem can be written as;

$$L = \sum_{t=0}^{\infty} \beta^t \{U(C_t) + \lambda_t [r_t K_t + \pi_t - C_t - K_{t+1} + (1-\delta)K_t]\}$$

where Households take π_t and r_t as given and $K_0 > 0$,

then F.O.C. of the household problem becomes

$$[C_t]: U'(C_t) = \lambda_t \quad (18)$$

$$[K_{t+1}]: \lambda_t = \lambda_{t+1}\beta[F_K(K_t, E_t) + 1 - \delta] \quad (19)$$

$$[\lambda_t]: r_t K_t + \pi_t - C_t - K_{t+1} + (1-\delta)K_t = 0 \quad (20)$$

Final Good Producer's Problem

Firms in final good sector behave competitively. They maximize their profit at every period thus their problem is simple. Final good sector is perfectly competitive and as inputs this sector uses physical capital and energy. Final good firm produces one type of good which can be either used as consumption or investment good. Her problem is;

$\max_{K_t,E_t} Y_t - r_t K_t - p_t E_t$ where; $Y_t = F(K_t, E_t)$

From final good producer's problem factor prices are solved as;

$r_t = F_K(K_t, E_t)$ (21) $p_t = F_E(K_t, E_t)$ (22)

Intermediate (Energy) Good Producer's Problem

The two type of energy are considered as perfect substitutes .Capacity is a tangible asset of the energy firm. Unlike the capital rented by Final good producer it is owned by the firm. In order to build capacity, energy firm has to purchase final goods. Thus the capacity problem of the energy producer is, either invests in capacity for anticipated increase in demand for next turn or lets it depreciate or liquidates capacity. However energy firm cannot change current period's capacity level.

Both type of capacity are assumed to be fully-utilized. Renewable energy is produced through an AK type technology. Non-renewable energy is produced via using both capacity and a depletable resource. Since all capacity will be used to produce energy cost of producing clean energy is a function of the capacity. Cost of non-renewable energy is a function of non-renewable energy produced and the stock of resources left at the beginning of the period. Thus it can be defined as a function of the

capacity and the stock and flow of non-renewable resources. In the literature we can observe that the cost of producing energy is either identified in terms of final goods or in terms of energy. In our setup we identified the costs in terms of final goods and assumed that all existing capacity is used to produce energy.

Thus the energy producer chooses next periods' capacity levels and this period's fossil fuel extraction in order to maximize its' profits for all periods. It behaves like a monopolist. Its' problem is;

$\max_{\dot{X}_t K_t^N K_t^R} \sum_{t=0}^{\infty} \beta^t \pi_t$ s.t:

$E_t = E_t^R + E_t^N \; and \; C_t > 0, \; Z_t > 0, \; X_t > 0$

where π_t is defined as;

$$\pi_t = p_t E_t - R_t[E_t^R(K_t^R)] - N_t[E_t^N(Z_t, K_t^N)X_t] - K_{t+1}^N + (1-\delta)K_t^N - K_{t+1}^R + (1-\delta)K_t^R$$

The energy producer's problem can be written in Lagrangian form as;

$$L = \sum_{t=0}^{\infty} \beta^t \{p_t E_t - R_t[E_t^R(K_t^R)] - N_t[E_t^N(Z_t, K_t^N), X_t] - K_{t+1}^N + (1-\delta)K_t^N - K_{t+1}^R + (1-\delta)K_t^R - \mu_t X_t + \vartheta_t Z_t\}$$

The necessary conditions for optimal solution includes F.O.C. obtained at below;

$$[K_{t+1}^R]: \beta^{-1} + 1 - \delta = \frac{\partial^2 F_{t+1}}{\partial E_{t+1}^2}\frac{\partial E_{t+1}^R}{\partial K_{t+1}^R} E_{t+1} + \frac{\partial F_{t+1}}{\partial E_{t+1}}\frac{\partial E_{t+1}^R}{\partial K_{t+1}^R} - \frac{\partial R_{t+1}}{\partial E_{t+1}^R}\frac{\partial E_{t+1}^R}{\partial K_{t+1}^R} \quad (23)$$

$$[K_{t+1}^N]: \beta^{-1} + 1 - \delta = \frac{\partial^2 F_{t+1}}{\partial E_{t+1}^2}\frac{\partial E_{t+1}^N}{\partial K_{t+1}^N} E_{t+1} + \frac{\partial F_{t+1}}{\partial E_{t+1}}\frac{\partial E_{t+1}^N}{\partial K_{t+1}^N} - \frac{\partial N_{t+1}}{\partial E_{t+1}^N}\frac{\partial E_{t+1}^N}{\partial K_{t+1}^N} \quad (24)$$

$$[X_{t+1}]: = \beta\left[\frac{\partial^2 F_{t+1}}{\partial E_{t+1}^2}\frac{\partial E_{t+1}^N}{\partial Z_{t+1}}E_{t+1} + \frac{\partial F_{t+1}}{\partial E_{t+1}}\frac{\partial E_{t+1}^N}{\partial Z_{t+1}} - \frac{\partial N_{t+1}}{\partial E_{t+1}^N}\frac{\partial E_{t+1}^N}{\partial Z_{t+1}} - \frac{\partial N_{t+1}}{\partial X_{t+1}} - \mu_{t+1} + \vartheta_{t+1}\right] =$$

$$\frac{\partial^2 F_t}{\partial E_t^2}\frac{\partial E_t^N}{\partial Z_t}E_t + \frac{\partial F_t}{\partial E_t}\frac{\partial E_t^N}{\partial Z_t} - \frac{\partial N_t}{\partial E_t^N}\frac{\partial E_t^N}{\partial Z_t} + \vartheta_t \qquad (25)$$

Model Specification and Transitional Dynamics for Decentralized Model

In decentralized model we have two types of producer; an energy firm who behaves like a monopolist and a final good producer who operates in a perfectly competitive environment. HH takes profits as given. Note that there are no changes in the functional forms of the equations and goods market clears since profits obtained by firms are in the budget of households. Therefore making household budget constraint to look like;

$$C_t + I_t = r_t K_t + p_t E_t - R_t[E_t^R(K_t^R)] - N_t[E_t^N(Z_t, K_t^N)X_t] - K_{t+1}^N + (1-\delta)K_t^N - K_{t+1}^R + (1-\delta)K_t^R$$

Above equation is identical to the resource constraint we have in the centralized model. Therefore we conclude that final good market and Energy market clear. By using the equations 18-25 we can find the following equations that can help us to characterize the transitional dynamics of decentralized model.

$$\Delta K_t = r_t K_t + \pi_t - C_t + \delta K_t \qquad (26)$$

$$\pi_t = p_t E_t - R_t[E_t^R(K_t^R)] - N_t[E_t^N(Z_t, K_t^N)X_t] - K_{t+1}^N + (1-\delta)K_t^N - K_{t+1}^R + (1-\delta)K_t^R \qquad (26')$$

$$\frac{\Delta C_t}{C_t} = \beta[F_K(K_t, E_t) + 1 - \delta] - 1 \qquad (27)$$

$$\Delta K_t^R = \left[\frac{\beta^{-1}+\delta-1}{\mu}\right]^{1/\mu-1} \{[F_{EE}(t+1)E_{t+1} + F_E(t+1) - A]^{1/1-\mu} - [F_{EE}(t)E_t +$$

$$F_E(t) - A]^{1/1-\mu}\} \quad (28)$$

$$\Delta K_t^N = \left[\frac{\beta^{-1}+\delta-1}{\alpha}\right]^{1/\alpha-1} \{Z_{t+1}\left[F_{EE}(t+1)E_{t+1} + F_E(t+1) - B - \frac{1}{X_{t+1}^2}\right]^{1/1-\alpha} -$$

$$Z_t\left[F_{EE}(t)E_t + F_E(t) - B - \frac{1}{X_t^2}\right]^{1/1-\alpha}\} \quad (29)$$

$$Z_t = -\left[\frac{K_t^N(\beta^{-1}+\delta-1)^{1/1-\alpha}}{\left\{F_{EE}(t)E_t+F_E(t)-B-\frac{1}{X_t^2}\right\}^{1/1-\alpha}}\right] \quad (30)$$

By using equations (23) and (24) and the specified functional forms we can find the following equation that describes the relationship between the production of the two energy types

$$\mu(K_t^R)^{\mu-1}[F_{EE}(t)E_t + F_E(t) - A] = \alpha\left(\frac{\dot{X}_t}{K_t^N}\right)^{1-\alpha}\left[F_{EE}(t)E_t + F_E(t) - B - \frac{1}{X_t^2}\right] \quad (31)$$

Proposition 3: Above system has a steady state, in which at the optimal path we have $K_t^N$and $Z_t$converge to zero. Check that on the optimal path for (31) to hold we need X_t to converge to some positive number rather than zero, otherwise right hand side of (31) becomes negative. That means energy firm makes negative profits at that point. This contradicts with energy firms' problem, therefore $X_t = 0$ is not going to be chosen on the optimal path. Instead we'll have $X_t \to constant$, hence $Z_t \to 0$ then (30) implies $K_t^N \to 0$. Thus at the optimal path we have $K_t^N$and Z_t converges to zero.

The rest of the proof very much resembles the proof for the steady state in the centralized model, where we examined the system and eliminated all other possible paths except the one that system has constant positive physical and energy capital in the long run. Formal proof is going to be presented at appendices.

Calculation of the Steady State Value for Decentralized Economy

We showed that steady state values of K_t^N and $\dot{X}_t$ are equal to zero by using optimality conditions. By using equation (19) we find

$$K^R = K^{1/\mu}\left[(\beta^{-1}+\delta-1)^{\gamma/1-\gamma}-1\right]^{1/\mu\gamma} = \left(KH^{1/\gamma}\right)^{1/\mu} \tag{32}$$

where $H = (\beta^{-1}+\delta-1)^{\gamma/1-\gamma}-1$

At the steady state, equation (23) becomes

$$\left\{\left[\gamma K_{ss}^{\gamma}+(K_{ss}^R)^{\mu\gamma}\right]\left[K_{ss}^{\gamma}+(K_{ss}^R)^{\mu\gamma}\right]^{\frac{1-2\gamma}{\gamma}}-A\right\}(K_{ss}^R)^{\mu-1}\mu = \beta^{-1}+\delta-1 \tag{23'}$$

If we replace (32) in (23') we get

$$\left\{[\gamma+H][\beta^{-1}+\delta-1]^{\frac{1-2\gamma}{1-\gamma}}{K_{ss}}^{1-\gamma}-A\right\}(K_{ss})^{\frac{\mu-1}{\mu}} = \frac{\beta^{-1}+\delta-1}{\mu}H^{\frac{1-\mu}{\mu\gamma}} \tag{33}$$

If we use the same parameter values, we used to calculate the steady state values of the centralized model we find $0{,}096K^2-A-0{,}48K=0$. We can rewrite this as $6K^2-30K-62{,}5A=0$. This equation has two roots and we are interested with the positive one which is; $\frac{30+\sqrt{900+1500A}}{12}$. Note that optimal capital stock for steady state grows as A increases, whereas optimal capital stock for centeralized model decreases as

the parameter A increases. The reason this difference lies under the optimality condition that at the steady state the ratio between renewable energy and physical capital stocks will be determined by the equation 19. The interdependency between households' problem and monopolist's will lead to a constant price for SS no matter how high or low the parameter *A* is. Hence only equation that will determine the renewable capital stock (also the physical capital stock) is equation (33). Note that right hand side of this equation is constant. Therefore if *A* increases to keep the right hand side constant we should increase capital stock.

An important problem in here is as *A* increases π in (26') becomes negative. To see that, just remember the equations (26') and (33). From these two we know that profits are dependent on the $F_E(t)$, in other words, price of the energy. However price is dependent on the parameter γ in the long run. For example if $\gamma = -1$ then price is between zero and one which means in any case parameter *A* should be smaller than 1 otherwise energy firm will make a loss and to avoid this, he would not operate in the market. Note that Social planner has higher tolerance for the unit cost of producing energy, parameter A. For example, for the same parameter values, we used to calculate the steady state values of the decentralized model, Energy producer stops producing energy for a value higher than (roughly) 0.3 of the parameter *A*. However Social planner still operates even, when the unit energy cost is higher than 1.

Time Path for Prices in the Decentralized Model

Again let's define the ratio between capital stock and energy as y. Now if we rewrite equation (19) we will have;

$$\beta\left[\frac{\partial^2 F_{t+1}}{\partial E_{t+1}^2}E_{t+1}+\frac{\partial F_{t+1}}{\partial E_{t+1}}-\frac{\partial N_{t+1}}{\partial E_{t+1}^N}\right]\frac{\partial E_{t+1}^N}{\partial Z_{t+1}}-\frac{\partial N_{t+1}}{\partial X_{t+1}}=\left[\frac{\partial^2 F_t}{\partial E_t^2}E_t+\frac{\partial F_t}{\partial E_t}-\frac{\partial N_t}{\partial E_t^N}\right]\frac{\partial E_t^N}{\partial Z_t}$$

Now let's focus on the first two terms in both brackets and rewrite them in terms of y then we will have

$$\left[\frac{y_{t+1}^2}{1+y_{t+1}^2}\frac{y_{t+1}-1}{1+y_{t+1}}-\frac{\partial N_{t+1}}{\partial E_{t+1}^N}\right]\frac{\partial E_{t+1}^N}{\partial Z_{t+1}}-\frac{\partial N_{t+1}}{\partial X_{t+1}}=\beta^{-1}\left[\frac{y_t^2}{1+y_t^2}\frac{y_t-1}{1+y_t}-\frac{\partial N_t}{\partial E_t^N}\right]\frac{\partial E_t^N}{\partial Z_t}$$

Even though they are very small at the beginning resource extraction implies that $\frac{\partial N_{t+1}}{\partial E_{t+1}^N}>\frac{\partial N_t}{\partial E_t^N}$, note that we also have $\frac{\partial N_{t+1}}{\partial X_{t+1}}>0$ and $\beta<1$. If $y_{t+1}\le y_t$ then we should have $\frac{\partial E_{t+1}^N}{\partial Z_{t+1}}>\frac{\partial E_t^N}{\partial Z_t}$. However this is a cotradiction with optimality condition 24,

$$[K_{t+1}^N]\colon \beta^{-1}+1-\delta=\frac{\partial^2 F_{t+1}}{\partial E_{t+1}^2}\frac{\partial E_{t+1}^N}{\partial K_{t+1}^N}E_{t+1}+\frac{\partial F_{t+1}}{\partial E_{t+1}}\frac{\partial E_{t+1}^N}{\partial K_{t+1}^N}-\frac{\partial N_{t+1}}{\partial E_{t+1}^N}\frac{\partial E_{t+1}^N}{\partial K_{t+1}^N}$$

Because $y_{t+1}\le y_t$ implies $\frac{\partial E_t^N}{\partial K_t^N}<\frac{\partial E_{t+1}^N}{\partial K_{t+1}^N}$ and from here we should have $\frac{\partial E_{t+1}^N}{\partial Z_{t+1}}<\frac{\partial E_t^N}{\partial Z_t}$. Therefore in the case of monopolists only price path compatible with both optimality conditions is $y_{t+1}>y_t$. Note that $y_{t+1}>y_t$ does not put any restrictions on the relationship between $\frac{\partial E_t^N}{\partial K_t^N}$ and $\frac{\partial E_{t+1}^N}{\partial K_{t+1}^N}$ as well as $\frac{\partial E_{t+1}^N}{\partial Z_{t+1}}$ and $\frac{\partial E_t^N}{\partial Z_t}$. Since other path is prooved to be infeasible we must have ever increasing prices for energy at the case of monopolist.

Steady State Levels of Non-renewable Resources Left in the Model

Another important finding in the model is that steady state levels of non-renewable resources left in the two model differ from each other. The model with monopolist suggests a higher level of extraction. The reason for this is, since monopolist can increase the energy prices by cutting down the supply of energy he can operate at levels of productivity lower than social planner simply because he can still make profits.

I calculated the steady state level of resources left by using a simple assumption. We know that at steady state non-renewable energy production should be zero. This also suggests that non-renewable energy capital stock should go to zero too. Hence for social planner the term in the brackets at equation (8') should go to zero as we approach steady state. Otherwise producing positive amount of non renewable energy will be profitable. Therefore at the steady state we should have;

$$F_E(K_t, E_t) - \frac{\partial N_t}{\partial E_t^N} = 0$$

If we use steady state values for the variables, the steady state level of non-renewable resources left in social planner's problem can be found as

$$X_{SS}^{SPP} = \left[\underbrace{(1+H)^{\frac{1-\gamma}{\gamma}} H^{\frac{\gamma-1}{\gamma}}}_{a} - B\right]^{-\frac{1}{2}} \tag{34}$$

where H is equal to $(\beta^{-1} + \delta - 1)^{\gamma/1-\gamma} - 1$. For the energy monopolist, we can apply the same rule to equation (24) when we rearrange the equation we will get

$$\frac{\partial E_t^N}{\partial K_t^N}\left[F_{EE}(t)E_t + F_E(t) - \frac{\partial N_t}{\partial E_t^N}\right]$$

We should let the term in the brackets go to zero as we approach the steady state otherwise optimality for energy producers problem will imply positive amounts of non-renewable capital and resource extraction. Since this is not feasible due to high costs (and also resources will deplete unless X→0) we know that the term in the brackets goes to zero as we approach the steady state. When we equate the term in the brackets to zero and solve for the steady state levels of variables we will get;

$$X_{SS}^{DE} = \left[\underbrace{(\gamma + H)(1+H)^{\frac{1-2\gamma}{\gamma}} H^{\frac{\gamma-1}{\gamma}}}_{b} - B\right]^{-\frac{1}{2}} \tag{35}$$

Comparing the term a in equation (34) and the term b in equation (35) we observe that a>b. Therefore the steady state level of resource stock in SPP is smaller than the decentralized model. Another important finding in here is that the decentralized model is much more responsive to the parameter γ than social planner's model. Note that if γ decreases too much than the first term in the brackets in equation (35) will become negative. This implies that the steady state level for resource stock left is negative. This is quite troublesome because that means we have a corner solution. We have this same term $(\gamma + H)$ in (33) that determines the steady state level for capital stock for decentralized model too. Check that a negative $(\gamma + H)$ implies a negative capital stock too!

Comparative Statics

The changes in the steady state values for physical capital and energy capital stocks in social planner's problem is given as;

$$\frac{\partial K_{SS}}{\partial \gamma} > 0, \frac{\partial K_{SS}^{R}}{\partial \gamma} > 0, \frac{\partial X_{SS}}{\partial \gamma} < 0$$

Proof will be given in appendices. For now we provide the intuition of comparative statics for capital stocks around the steady state. An increase in the elasticity of substitution between capital and energy increases steady state levels of physical and renewable energy capital but decreases the stock of non-renewable resources left. First two relationships are straightforward. Note that for the same amount of capital stocks, if elasticity of substitution is negative, an increase in elasticity will increase the marginal productivity rates of both, therefore social planner will be inclined to hold higher levels of capital stocks for both of them. The third one is a little harder to understand but not impossible. Remember that the reason we use resource stock is that it facilitates the production of non-renewable energy, in a way it increases the productivity of non-renewable energy's capital. When γ increases it increases the productivity of both types of energy capital however the increase in the productivity of renewable energy capital makes the further use of fossil resources unnecessary.

$$\frac{\partial K_{SS}}{\partial A} < 0, \frac{\partial K_{SS}^{R}}{\partial A} < 0$$

An increase in unit cost of renewable energy (parameter *A*) decreases steady state levels of both capital stocks which is straightforward. Since a higher marginal cost

for energy will imply a lower level of investment. In the same way a higher level of fix marginal cost for non-renewable energy will have the same effect on resource stock. fossil fuels will become less productive therefore social planner will stop extracting resources earlier if he faces a higher fix cost for non-renewable energy.

$$\frac{\partial X_{SS}}{\partial B} > 0, \frac{\partial K_{SS}}{\partial \beta} > 0, \frac{\partial K_{SS}^{R}}{\partial \beta} > 0, \frac{\partial X_{SS}}{\partial \beta} > 0, \frac{\partial K_{SS}}{\partial \delta} < 0, \frac{\partial K_{SS}^{R}}{\partial \delta} < 0, \frac{\partial X_{SS}}{\partial \delta} < 0$$

An increase in δ or β^{-1} decreases the steady state level of physical capital stock but increases the level of energy capital. This is somehow confusing since we expect a parameter which would lead to increase of one of the capital stocks should also affect the other one in the same way. However note that unlike γ a change in δ or β^{-1} has opposite effects on the levels of these two capital stocks social planner desires to hold at steady state. For example an increase in depreciation would decrease the amount of physical capital that social planner wants to hold at the steady state, but increases the ratio between capital and energy Since the parameter *H* is decreasing with δ or β^{-1}. Their effect on steady state level of non-renewable resource left would be same with the parameter B since they basically do the same thing.

$$\frac{\partial X_{SS}}{\partial \mu} = 0, \frac{\partial K_{SS}}{\partial \mu} > 0, \frac{\partial K_{SS}^{R}}{\partial \mu} > 0$$

Finally an increase in the productivity parameter for renewable capital stock μ will have a positive effect on capital stock levels. Since it increases the productivity of renewable energy capital, it creates an incentive to hold more energy capital. A higher

level of energy capital makes physical capital more productive, therefore increases the steady state level of physical capital too. On the other hand it will have no effect on the level of resource stock left since it is dependent on the ratio between capital and energy and unit costs for non-renewable energy. Since parameter μ does not alter those variables at the steady state it does not change the level of resource stock left either.

In decentralized model the changes in the steady state values for physical and energy capital stocks with respect to changes in the parameters is dependent on the magnitude of the parameters, except the parameters *A* and *B*. These two parameters suggest a higher level of both capital stock and steady state level of resource stock left. The effects of all other parameters that we have analyzed before, on the levels of capital stock and resource stock left are ambiguous. The reason for that is these parameters have two conflicting effects on capital stocks. On the one hand a change in those parameters changes the productivity of capital stocks which creates an incentive for both households and energy producer to hold more capital stocks. On the other hand it increases the responsiveness of prices to the changes in the ratio between capital stock and energy which creates an incentive for energy producer to cut energy supply.

We can still analyze the effects of the all other parameters in decentralized economy around the specific values that we used to calibrate our case. However these findings will not be global.

CHAPTER 5

CONCLUSION

The aim of this thesis was to analytically explore the effects of capacity problem on the optimal choices of economic agents in an era of decreasing non-renewable energy resources by using two models. In order to analyze the effect of capacity problem we introduced capacity requirement as essential to produce one of the factor of production namely, energy. Both models are extensions of the basic models in energy transition literature in which we used two different assumptions about the market structure in order to cover different scenarios. In one of these scenarios we assumed all markets to be perfectly competitive this assumption enabled us to solve the social planner's problem during the transition. In the second scenario we assumed energy market to have a monopolistic structure. Then we solved a decentralized market where the intermediate good producer behaves like a monopolist.

Analytical examination of the two models gave us different time paths for energy prices and sensitivities to some essential parameters of the model. In the case of monopolistic energy market we found energy prices to be ever increasing, on the other hand perfectly competitive markets suggest that energy prices should be decreasing whenever the extraction costs for non-renewable resources is insignificantly small and we have a high capital to energy ratio. In that case, we have a U shape price curve, otherwise our findings for ever increasing prices does not change. Historical price path of energy suggest a similar time path for energy prices, where in the beginning we have

decreasing real prices for non-renewable energy then a stable and low level of prices and for the last two decades an increasing pattern for prices.

Regarding to the sensitivity of model to the different essential parameters, the most curious case was the impact of an increase in the cost of renewable energy. In the centralized case an increase in the unit cost of renewable energy produced forces social planner to choose a smaller amount of production capacity as well as capital stock at steady state. Whereas in decentralized market an increase in the unit cost of renewable energy increases the steady state level of capacity stock chosen by energy producer. The reason behind this is the steady state price of energy is determined by intertemporal Euler equation of households and a change in the unit cost of renewable energy does not affect this condition. Therefore energy producer responds an increase in costs by increasing production levels in order to earn higher profits.

Another important finding was about the sensitivity of the models to the elasticity of substitution between capital and energy. In centralized model a decrease in elasticity of substitution implies a decrease for both capital stock and energy level to be chosen, however for our benchmark parameters we found that a decrease in elasticity of substitution implies an increase for the levels of both capital stock. The effects of other parameters were found to be same for both cases.

Though it offers some insights on historical price pattern and effect of capacity problem on the usual energy transition models, our model lacks certain improvements regarding growth possibilities. In our model we didn't introduce growth; the reason behind this was our primary concerns was to build a model where we can examine the

effect of capacity problem and a monopolistic intermediate good market. Problems like sustainable growth, endogenous growth and time path of energy prices and calibration of the model were assumed to be handled better, once the model is established. Indeed as future extensions we are planning to deal with those topics.

We also assumed a CES production function for final goods where the elasticity of substitution between capital and energy is negative. This assumption about the production function can be improved by adding labor to the production function and allowing energy and composite of capital and labor to be complements to each other. Our initial assumption about the final good production function altered our model to be basically an AK type of model. In general AK type of models offers growth or convergence to zero economic activity at the SS.

Since our model assumed decreasing productivity for energy production our model was able to capture a steady state. However the assumption about decreasing productivity for renewable energy capacity investments may not be true. The R&D efforts on renewable energy production should also be a desirable expansion for he model.

Last but not least we built our model in a discrete time setting. This setting let us examine the model easily with an analytical framework. However a continuous time setting might present more insights about the time paths for energy prices and capital stock and capacity levels as well as total energy production.

APPENDIX A

Proof that SPP Has a Steady State

Condition 1: It isn't possible to have only one of K_t^R or K_t go to zero

If K_t goes to zero but not K_t^R that means $F_K(t) \to +\infty$. Note that, $F_E(t)$ decreases and approaches to some constant (dependent on the parameter γ) too, therefore we will have $F_K(t+1) > F_K(t)$ and $F_E(t+1) < F_E(t)$. From equation (12) we know that $\frac{F_K(t+1)}{F_K(t)} > \frac{F_E(t+1)-A}{F_E(t)-A}$ which implies $\Delta K^R < 0$. This dynamics is not sustainable since equation (11) implies consumption to be increasing with an increasing rate whereas capital stocks and production decreases even further.

If K_t^R goes to zero but not K_t, then equation (22) implies ΔK^R to be positive. That is a contradiction too therefore condition 1 is satisfied.

Condition 2: It isn't possible to have both capital stocks go to zero.

For both of them to go to zero we need $\Delta K^R < 0$ this means the ratio K^R/K to be decreasing. In order to satisfy equation (12) from here we have $F_K(t+1) < F_K(t)$. If we define the ratio $K_t/E_t = y_t$ then that means we need to have y_t to be increasing.

However with an increasing y_t we need to have $\partial E_t^R / \partial K_t^R$ to be decreasing. This implies an increasing renewable energy capital stock, therefore we have a contradiction.

Condition 3: It isn't possible to have both capital stocks to go to infinity.

Assume that $\Delta K^R > 0$ this means the ratio K^R/K is increasing. First let's define the growth rate of these two capital stocks as R and M respectively. Then (12) implies R > M, and equation 8 implies

$$\frac{(K_t^R)^{\mu-1}}{(K_t)^{\gamma-1}} = \frac{\left[K_t^\gamma + (K_t^R)^{\gamma\mu}\right]^{\frac{1-\gamma}{\gamma}}}{\mu\left\{\left[K_t^\gamma + (K_t^R)^{\gamma\mu}\right]^{\frac{1-\gamma}{\gamma}} (K_t^R)^{\mu(\gamma-1)} - A\right\}} \tag{36}$$

If we iterate this for one period towards with growth rates R and M then 36 becomes

$$\frac{(K_t^R)^{\mu-1}}{(K_t)^{\gamma-1}} = \frac{(1+M)^{\gamma-1}\left[(1+M)^\gamma K_t^\gamma + (1+R)^{\mu\gamma}(K_t^R)^{\gamma\mu}\right]^{\frac{1-\gamma}{\gamma}}}{(1+R)^{\mu-1}\mu\left\{\left[(1+M)^\gamma K_t^\gamma + (1+R)^{\mu\gamma}(K_t^R)^{\gamma\mu}\right]^{\frac{1-\gamma}{\gamma}} (1+R)^{\mu(\gamma-1)}(K_t^R)^{\mu(\gamma-1)} - A\right\}} \tag{36'}$$

For (36') to hold we need either M > R or M = R = 0. Therefore only possible choice compatible with optimality conditions is R=M=0 at the SS. To complete the proof we should check $\partial K_{t+1}/\partial K_t$ around the steady state which will be done at next part

The Proof of Monotone Dynamics around the Steady State

To calculate $\partial K_{t+1}/\partial K_t$ first we should calculate $\partial K_t^R/\partial K_t$ if we take the total derivative of equation (8) we will get

$$F_{KK}(t+1)dK_{t+1} + F_{KE}(t+1)\frac{\partial E_{t+1}}{\partial K_{t+1}^R}dK_{t+1}^R = \frac{\partial^2 E_{t+1}}{(\partial K_{t+1}^R)^2}\{\dots\} + \frac{\partial E_{t+1}}{\partial K_{t+1}^R}\{F_{EK}(t+1)dK_{t+1} +$$

$$F_{EE}(t+1)\frac{\partial E_{t+1}}{\partial K_{t+1}^R}dK_{t+1}^R\}$$

If we rearrange the above equation, it will look like;

$$\frac{dK_{t+1}^R}{dK_{t+1}} = \frac{F_{KK}(t+1) - \frac{\partial E_{t+1}}{\partial K_{t+1}^R} F_{EK}(t+1)}{\frac{\partial^2 E_{t+1}}{\left(\partial K_{t+1}^R\right)^2}\{\dots\} + F_{EE}(t+1)\left(\frac{\partial E_{t+1}}{\partial K_{t+1}^R}\right)^2 - F_{KE}(t+1)\frac{\partial E_{t+1}}{\partial K_{t+1}^R} dK_{t+1}^R}$$

From here we know that $\frac{\partial K_t^R}{\partial K_t} > 0$. Now to retrieve the relationship between $K_{t+1}$and K_t we are going to use equations 1 and 2

$$C_{t+1} = C_t \beta [F_K(K_t, E_t) + 1 - \delta]$$

we can write C_t as a function of K_t and K_t^R then we can define a new equation; $G(K_t, K_{t+1}^R) = C_t \beta [F_K(K_t, E_t) + 1 - \delta] - C_{t+1}$. Then by using implicit function theorem we can get $\frac{dK_{t+1}}{dK_t} = -\frac{G_{K_t}(.)}{G_{K_{t+1}}(.)}$

$$G_{K_t}(.) = \beta [F_K(K_t, E_t) + 1 - \delta] \left\{ F_K(t) \frac{\partial R_t}{\partial E_t^R} \frac{\partial E_t^R}{\partial K_t^R} \frac{\partial K_t^R}{\partial K_t} + (1 - \delta) \left(1 + \frac{\partial K_t^R}{\partial K_t}\right) \right\}$$

$$G_{K_{t+1}}(.) = -\left\{ F_K(t+1) \frac{\partial R_{t+1}}{\partial E_{t+1}^R} \frac{\partial E_{t+1}^R}{\partial K_{t+1}^R} \frac{\partial K_{t+1}^R}{\partial K_{t+1}} + (1 - \delta) \left(1 + \frac{\partial K_{t+1}^R}{\partial K_{t+1}}\right) \right\} + \beta C_{t+1} F_{KK}(t+1) -$$

$$\beta([F_K(t+1) + 1 - \delta]) \left(1 + \frac{\partial K_{t+1}^R}{\partial K_{t+1}}\right)$$

From these two equations we know that around the steady state $G_{K_t}(.) < -G_{K_{t+1}}(.)$ therefore $0 < \frac{dK_{t+1}}{dK_t} < 1$ holds around steady state, ensuring monotonic dynamics for the system

APPENDIX B

Proof that Decentralized Model Has a Steady State

Condition 4: In none of the optimal paths, capital stock goes to zero.

If K_t converges to zero then equation (27) implies an ever increasing consumption pattern, However if $K_t \to 0$ then $F_E(t+1) < F_E(t)$ and $F_{EE}(t)E(t) > F_{EE}(t+1)E(t+1)$ these two conditions with equation 28 implies that K^R is negative. This contradicts with equation 26 since an increasing consumption implies K_t goes to negative infinity in the long run.

Condition 5: In none of the optimal paths renewable capital stock goes to zero

If K^R converges to zero at the optimal path then from equation (28) we need to have $F_{EE}(t)E(t) + F_E(t) > F_{EE}(t+1)E(t+1) + F_E(t+1)$. In order to have we just need to have $F_E(t) > F_E(t+1)K^R < 0$ since we have $F_{EE}(t+1)E(t+1) \to 0$ when $K^R \to 0$. Note that $K^R \to 0$ implies $F_E(t) < F_E(t+1)$ unless we have capital stock decreasing faster than renewable energy capital stock. However if K converges to zero faster than K^R than we will have $F_K(t) \to +\infty$ (or in the case $\gamma = -1$ we will have $F_K(t) \to 1$. This implies a contradiction since $\Delta C_t / C_t$ will be always positive. Again we have contradiction because feasibility constraint is violated since we cannot have both capital stocks approaching to zero and consumption grow forever at the same time.

Condition 6: It is not possible to have one of the capital stock go to infinity and other not.

Assume $K^R \rightarrow \infty$ but not K; then equation (30) implies $K^R > 0$ that means $\pi_t \rightarrow -\infty$ in equation (26') Therefore a violation of feasibility constraint occurs since energy firm is better off when he does not produce energy at all. Now assume $K \rightarrow +\infty$ but not K^R

Now remember the term in the brackets at equation (28) $F_{EE}(t)E(t) + F_E(t)$. This is equal to $\left[K_t^\gamma + (E_t)^\gamma\right]^{\frac{1-\gamma}{\gamma}}(E_t)^{\gamma-1}\frac{\gamma K_t^\gamma + (E_t)^\gamma}{K_t^\gamma + (E_t)^\gamma}$ and if $K \rightarrow +\infty$ but not K^R, then we need K^R to go to zero. However this contradicts with energy firm's problem since $\lim F_E(t) \rightarrow 1$ and the firm can make positive profits by producing a higher amount of energy.

Condition 7: It is not possible to have both K and K^R go to infinity

From earlier analysis we know that on the optimal path we have K_t^N and Z_t converging to zero and their effect on decisions becomes insignificantly small. Then (26') becomes

$$\pi_t = E_t^R\left\{F_E(t) - A - (R+\delta)(E_t^R)^{\frac{1-\alpha}{\alpha}}\right\}$$

R in the above equation is the growth rate of renewable capital stock. Let's define the growth rate of capital stock as M. Then; if M > R we have $\lim F_E(t) \rightarrow 1$ if M < R we have $\lim F_E(t) \rightarrow 0$ and if M = R we have $\lim F_E(t) \rightarrow p$ where $p \in (0,1)$. Note that these values are calculated for $\gamma = -1$, but in any case as long as γ is negative we will have $\lim F_E(t) \rightarrow constant$ and the following argument would still be valid.

Claim: Since $E_t^R \to +\infty$ then the term in the brackets will be negative infinite. This is obviously not optimal for the energy firm. At worst he could do zero profits by choosing $K_t^R = 0$ therefore we cannot have both capital stocks go to infinity

Conclusion: Dependent on the parameters A and γ optimal solution for this economy either leads us to a steady state or a corner solution where the whole economy collapses.

APPENDIX C

Proofs of the Effects of Parameters

Remember that the steady state level of capital stock is given as;

$$K^{\frac{\mu-1}{\mu}} = \frac{H^{\frac{1-\mu}{\mu\gamma}} \frac{\beta^{-1}+\delta-1}{\mu}}{(\beta^{-1}+\delta-1)H^{\frac{\gamma-1}{\gamma}} - A} \quad \text{eqn (17")} \qquad \text{where } H \text{ is equal to } (\beta^{-1}+\delta-1)^{\frac{\gamma}{1-\gamma}} - 1.$$

The Effect of the Parameter γ on Steady State Levels of Capital Stocks

Now let's check the right hand side of above equation. If we take derivatives of nominator and denominator with respect to γ we will have

$$\frac{\partial Nominator}{\partial \gamma} = \frac{\beta^{-1}+\delta-1}{\mu} H^{1/\gamma} \ln H \, (-1) \frac{1}{\gamma^2} (\beta^{-1}+\delta-1)^{\frac{\gamma}{1-\gamma}} \ln(\beta^{-1}+\delta-1)^{\frac{\gamma}{1-\gamma}} \frac{1}{(1-\gamma)^2}$$

$$\frac{\partial Denominator}{\partial \gamma} = \ln(\beta^{-1}+\delta-1)^{\frac{\gamma}{1-\gamma}} (\beta^{-1}+\delta-1)^{\frac{\gamma}{1-\gamma}} \frac{1}{(1-\gamma)^2} \left[(\beta^{-1}+\delta-1)^{\frac{\gamma}{1-\gamma}} - 1\right]^{\frac{\gamma-1}{\gamma}}$$

$$\ln\left[(\beta^{-1}+\delta-1)^{\frac{\gamma}{1-\gamma}} - 1\right] \frac{1}{\gamma^2}$$

If $(\beta^{-1}+\delta-1)^{\frac{\gamma}{1-\gamma}} - 1$ is larger than one than if γ decreases than by checkin the derivatives we take at above, we can see that at the right hand side of the equation (17") nominator decreases and denominator increases. This implies right hand side to be decreasing with γ, hence K increases with the parameter γ. Now let's return to our other variable of interest K^R. If we rewrite above equation by using $K^R = \left(KH^{\frac{1}{\gamma}}\right)^{\frac{1}{\mu}}$ we will

get $K^R = \frac{\left(\frac{\beta^{-1}+\delta-1}{\mu}\right)^{\frac{1}{\mu-1}}}{\left\{(\beta^{-1}+\delta-1)H^{1-\frac{1}{\gamma}}-A\right\}^{\frac{1}{\mu-1}}}$. Note that when γ decreases the denominator decreases (the term in the parenthesis increases but its power is negative) thus we have renewable energy capital stock increasing hence renewable energy production is increasing with γ.

The Effect of the Parameter μ on Steady State Levels of Capital Stocks

We can rewrite the steady state value of K^R as $\left\{\frac{\beta^{-1}+\delta-1}{(\beta^{-1}+\delta-1)H^{1-\frac{1}{\gamma}}-A}\right\}^{\frac{1}{\mu-1}} \mu^{\frac{-1}{\mu-1}}$. In this new equation the second term is increasing with μ since it is between zero and one. First term is increasing if the term in the brackets is smaller than 1, then we have both capital stocks increasing with μ, otherwise its effect on steady state levels of capital stock is ambiguous.

The Effect of the Parameter *A* on Steady State Levels of Capital Stocks

The effect of the cost parameter A is clear. When it increases the right hand side of the capital stock equation, $K^{\frac{\mu-1}{\mu}} = \frac{H^{\frac{1-\mu}{\mu\gamma}}\frac{\beta^{-1}+\delta-1}{\mu}}{(\beta^{-1}+\delta-1)H^{\frac{\gamma-1}{\gamma}}-A}$, decreases. Since the power of K in this equation is smaller than 0, capital stock decreases when unit cost of energy increases. Since parameter A does not affect the choice between capital and energy, energy production declines too.

The Effects of the Parameters δ and β^{-1} on the Steady State Levels of Capital Stocks

The effects of the parameters δ and β^{-1} are the same. If we take derivative of the right hand side of above equation with respect to one of these variables (let it be δ) then we have to calculate $\frac{\partial R.H.S.}{\partial \delta}$.

$\frac{\partial R.H.S.}{\partial \delta} = \frac{\frac{1}{\mu}H^{\frac{1-\mu}{\mu\gamma}}\left\{\left[1+H^{-1}\frac{1-\mu}{1-\gamma}(\beta^{-1}+\delta-1)^{\frac{\gamma}{1-\gamma}}\right]\left[(\beta^{-1}+\delta-1)H^{\frac{\gamma-1}{\gamma}}-A\right]-\left[H-(\beta^{-1}+\delta-1)^{\frac{\gamma}{1-\gamma}}\right]\left[(\beta^{-1}+\delta-1)H^{\frac{-1}{\gamma}}\right]\right\}}{\left((\beta^{-1}+\delta-1)H^{\frac{\gamma-1}{\gamma}}-A\right)^2}$ this equation looks a little hard to evaluate. However note that the third term in the brackets is equal to -1. Since it is assumed that $(\beta^{-1}+\delta-1)H^{\frac{\gamma-1}{\gamma}}-A$ is positive than the derivative of R.H.S. with respect to δ is positive. Since K is decreasing with right hand side then capital stock decreases as we increase the parameters δ and β^{-1}. Note that H is decreasing with these two parameters, since $K^R = \left(KH^{1/\gamma}\right)^{1/\mu}$. Then it is a little ambiguous to interpret from that equation, instead we will directly analyze K^R. When we use $K^R = \left\{\frac{\frac{\beta^{-1}+\delta-1}{\mu}}{(\beta^{-1}+\delta-1)H^{1-\frac{1}{\gamma}}-A}\right\}^{\frac{1}{\mu-1}}$ and do the same calculation again for parameters δ and β^{-1}. We found that the term in the parenthesis is increasing with them hence K^R is decreasing with these parameters.

The Effects of the Parameters on the Steady State Level of Resources Left

The steady state level of resources left is; $X_{SPP}^{SS} = \left\{(\beta^{-1}+\delta-1)H^{\frac{\gamma-1}{\gamma}}-B\right\}^{-1/2}$ the term in the brackets is increasing with γ, δ and $\beta^{-1}$and decreasing with B which implies

steady state level of resource stock left is increasing with B and decreasing with γ, δ and β^{-1} while the productivity of renewable capital stock μ does not affect it.

The Effects of Parameters in the Decentralized Model

In the decentralized model except for the parameters A and B we can only talk about the baseline case, where we have γ =-1 μ=1/2. With these parameters, an increase in γ implies decrease in both capital stocks. Check that the equation that gives the capital stock resource, $\left[(H+\gamma)(\beta^{-1}+\delta-1)^{\frac{1-2\gamma}{1-\gamma}}K^{1-\gamma}-A\right]K^{\frac{\mu-1}{\mu}}=H^{\frac{1-\mu}{\mu\gamma}}\frac{\beta^{-1}+\delta-1}{\mu}$ can be written as; $(H+\gamma)(\beta^{-1}+\delta-1)^{\frac{1-2\gamma}{1-\gamma}}K^{1-\gamma}-A=K^{\frac{1-\mu}{\mu}}H^{\frac{1-\mu}{\mu\gamma}}\frac{\beta^{-1}+\delta-1}{\mu}$. From previous calculations we know that the multiplier of K on the right hand side decreases as γ increases and we can show that if $ln(\beta^{-1}+\delta-1)<\frac{1}{1-\gamma}$ holds (which in fact holds for our baseline parameters) than the multiplier of K on the left hand side is increasing with γ. Since $H^{1/\gamma}$ is increasing with γ these two conditions imply that for equality to hold capital stock should decrease.

If we make a similar analyze for the parameter μ we can see that a change in the parameter would not affect the left hand side of the equation but decreases the multiplier of the right hand side plus it also decrease the power of the capital stock on the right hand side. Therefore for our baseline parameters the equality can only hold if the amount of K decreases. However for parameter values smaller than $1/3$ we need the value of K to be increasing with the parameter μ. Since $K^R=\left(KH^{1/\gamma}\right)^{1/\mu}$ then at the steady state we need K^R to be decreasing too.

APPENDIX D

Decreasing Prices at the Beginning of Centralized Model

First we will examine equation (9). If we assume the effect of resource stock on costs is insignificant then for $y_{t+1} < y_t$ to be optimal we need to have left hand side of equation (9) to be smaller than right hand side of it whenever $y_{t+1} = y_t$. The reason for this is $y_{t+1} < y_t$ implies a decrease in right hand side and an increase in left hand side of (9) compared to $y_{t+1} = y_t$. For this to be optimal choice at the beginning left hand side should be smaller than right hand side therefore creating the opportunity to adopt that kind of policy.

Above conditions imply $\lambda_0 < \beta\lambda_1$ from here equation (2) implies that if we start at relatively low levels of productivity for physical capital stock $F_K(0) < \delta$ then $F_K(1) < \delta$ will become optimal. If all these conditions are met then at next period if $y_1 = y_2$, we will have left hand side still smaller than right hand side since $F_K(1) = F_K(2) < \delta$ still holds. As long as we will have $F_K(t) < \delta$ our above statement will continue to hold for all subsequent periods. and we will have decreasing prices until costs become significant and our optimal choice gives us a productivity ratio $F_K(t) > \delta$. A similar argument can be employed to show that same conditions will not hold if $y_1 > y_0$, with low levels of productivity for capital stock therefore ending the proof.

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